A model for warehouse layout

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Abstract
This paper describes an approach to determine a layout for the order picking area in warehouses, such that the average travel distance for the order pickers is minimized. We give analytical formulas by which the average length of an order picking route can be calculated for two different routing policies. The optimal layout can be determined by using such formula as an objective function in a non-linear programming model. The optimal number of aisles in an order picking area appears to depend strongly on the required storage space and the pick list size.

1 Introduction

Order picking is the process by which products are retrieved from storage to satisfy customer demand. In its simplest form, an order arrives at the warehouse and an order picker is sent into the picking area with the customer's list to retrieve the requested items from storage. Much research has been done to find methods to retrieve products from storage as efficiently as possible. The high efforts in this area are partly caused by the fact that order picking is an extremely expensive activity. Typically, the order picking contributes most to the operational costs of a warehouse (Tompkins *et al.*, 2003). For the time required to pick an order, we can roughly distinguish three components: traveling between items, picking of items and remaining activities. Picking the items consists of a series of actions ranging from positioning the vehicle to putting the picked items on a product carrier. The remaining activities include picking up an empty pick carrier, the acquisition of information, and dropping off the full pick carrier at some point after picking is complete. Most efforts to improve the operational efficiency of order picking can be categorized into three groups of operating policies, namely routing, batching, and storage assignment. Each of these approaches generally focuses on reducing travel times since these are easiest to influence. The time required for picking and remaining activities is influenced by aspects such as the chosen rack type and training of personnel.

Routing concerns the traveling of the order picker from location to location to retrieve products. Usually, a large part of an order picker's time is spent on traveling. Therefore this is an important aspect to consider. Research has focused on developing and comparing various routing methods. For example, Petersen (1997) gives a number of routing methods for warehouses with a ladder-structure, i.e. a warehouse with a number of parallel aisles where order pickers can change aisles in the front and rear cross aisle of the warehouse. Cross aisles are aisles perpendicular to the pick aisles and can be used to change from one aisle to the next. Routing methods vary from simple heuristics to optimization schemes using dynamic programming. Two frequently used heuristics will be used in this paper and are explained below. A dynamic

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programming approach for routing in a warehouses with an added cross aisle in the middle is given in Roodbergen and De Koster (2001).

With *batching* several orders or partial orders are combined to create one or more picking routes. One form is to aggregate all available orders and to pick each product type individually. Another form is to combine several complete orders into one picking route. Numerous intermediate forms exist. Much research is performed in this area, especially on methods that combine complete orders into a single route, see e.g. De Koster *et al.* (1999b) or Ruben and Jacobs (1999). If an order picker retrieves several orders at the same time, there are two possibilities: either the products are sorted by order while picking (*sort-while-pick*) or the order integrity is restored after picking is completed (*pick-and-sort*). For sort-while-pick a special vehicle may be required to separate the various orders. For pick-and-sort a manual or automated sorting system may be needed. A situation where multiple pickers work on the same order can be found in areas with *zoning*, i.e. where each order picker only picks those products from an order that are located in his assigned part of the warehouse.

Research concerning *storage assignment* focuses mainly on rules to assign products to locations. Existing rules range from random, where new storage locations are assigned to products randomly, to full-turnover storage, where products with the highest pick frequency are assigned to the easiest accessible locations. Intermediate forms also exist, such as ABC-storage, where the A, B, and C categories are determined based on pick frequencies but storage within the categories is random (see e.g. Petersen and Schmenner, 1999).

A common objective for order-picking systems is to maximize the service level subject to resource constraints such as labor, machines, and capital. The service level is composed of a variety of factors such as response time, order integrity, and accuracy. A crucial link between order picking and service level is that the faster an order can be retrieved, the sooner it is available for shipping to the customer. If an order misses its shipping due time, it may have to wait until the next shipping period. Also, short order retrieval times imply high flexibility in handling late changes in orders. Minimizing the order retrieval time (or picking time) is, therefore, a need for any order-picking system. Possible improvements by changing the operating policies are restricted by the physical layout of the area. That is, given a certain layout of the picking area, a good mix of operating policies can be chosen. In this paper, we will take the reverse approach. We propose a method that can find a layout that optimizes the order picking efficiency, given certain operating policies. Such a method allows designers to take operational efficiency factors into account while designing the warehouse. Specifically, we formulate a non-linear programming model to optimize the layout with respect to average travel distances.

In Section 2 we describe a model to find the best layout in a picking area consisting of one block. In Section 3 estimates are developed for the average travel distance of two common routing policies. In Section 4 the location of the depot is discussed. In Section 5 the results from the travel distance estimates are compared with simulation and with existing estimates from the literature. Section 6 describes some layout experiments. Layout differences resulting from the optimization with different routing policies are analyzed in Section 7. Conclusions are given in Section 8.

2 A model for layout optimization

We consider a manual order picking operation, where order pickers walk or drive through a picking area to retrieve products from storage. Picked items are placed on a vehicle, which the order picker takes with him on his route. With some minor changes, we can also optimize other picking environments with this model, see Section 3.4. The picking area is rectangular with no unused space and consists of a number of parallel aisles. At the front and rear of the picking area there is a cross aisle to enable aisle changes. An example of a layout of such a picking area is given in Fig. 1. Solid black squares in the figure indicate sections in the rack where items have to be picked.

Order pickers are assumed to be able to traverse an aisle in either direction and to change direction within an aisle. Items are stored on both sides of the aisles. Item locations are determined randomly according to a uniform distribution. We consider this storage assignment policy because it can be considered as a base-line against which layouts with other storage assignment policies can be compared. Furthermore, random storage is frequently used in practice. This occurs, for example, in situations where the product assortment changes too fast to produce reliable statistics about demand frequency (see e.g. De Koster *et al.*, 1999a). Previous work on travel distance estimation for random storage is described in Hall (1993). Estimations in an activity based storage environment are described in for example Caron *et al.* (1998) and Chew and Tang (1999). Each order consists of a number of items, which can and will be picked in a single route (such a "pick order" can be the result of a batching procedure, though). Picked orders have to be deposited at the depot, where the picker also receives the instructions for the next route. The depot is located in the front cross aisle.

We use two routing policies called the S-shape (or traversal) policy and the largest gap policy. These policies are widely used in practice (see Hall, 1993). With the *S-shape policy*, any aisle containing at least one item is traversed through the entire length. Aisles where nothing has to be picked are not entered. After picking the last item, the order picker returns to the front end of the aisle. With the *largest gap policy*, the picker enters the first aisle and traverses this aisle to the back of the warehouse. Each subsequent aisle is entered up to the 'largest gap' and left from the same side as it was entered. A gap represents the distance between any two adjacent items, or between a cross aisle and the nearest item. The last aisle is traversed entirely and the picker returns to the depot along the front entering again each aisle up to the largest gap. Thus, the largest gap is the part of the aisle that is not traversed. Fig. 1 contains routes found by applying the S-shape and largest gap policies to an example situation.

XXXXXXXXXXXXXX
Insert figure 1
XXXXXXXXXXXXXX

To find the layout that results in minimal average travel distances, we first have to find an analytical expression that expresses the average travel distance as a function of a number of layout arguments. We distinguish the following variables that influence average travel distance:

n the number of aisles (integer);

y the length of each of the aisles (real);

d the depot location, $1 \leq d \leq n$ (real).

The depot can be located anywhere in the front cross aisle between the left-most aisle (aisle 1) and the right-most aisle (aisle n). The position is indicated with a number. For example, $d = 1$ indicates that the depot is located at the head of aisle 1; $d = 3.5$ indicates that the depot is located between aisles 3 and 4.

Furthermore, we define the following parameters:

m the number of picks per route (integer);

w_a center-to-center distance between two adjacent aisles (i.e. width of an aisle including the storage racks);

w_c width of a cross aisle;

S total aisle length, measured along the pick face.

Once we have an expression for the average travel distance, then we can try to minimize this expression. If $T_m^X(n, y, d)$ gives the average travel distance of routing policy X in a picking area with n aisles of length y and the depot located at d, given that m products have to be picked per route, then our problem can be formulated as:

$$\begin{aligned}
&\min T_m^X(n, y, d) \\
&n \cdot y = S \\
&n \geq 1 \quad \text{(integer)} \\
&y \geq 1.0 \\
&1 \leq d \leq n.
\end{aligned}$$

Thus, we try to find values for n, y and d such that $T_m^X(n, y, d)$ is minimized, under the conditions that total aisle length equals S, the number of aisles is 1 or more, the depot is located in the front cross aisle between aisle 1 and n, and the minimum length of an aisle is 1.0 meter (which is the minimum that would be physically possible to build). The model can be used for any routing method, provided that an expression $T_m^X(n, y, d)$ is available to calculate average travel distances. In the next section we will derive such expressions for two commonly used routing methods.

3 Average travel distance estimation

In this section, we will give explicit formulas to calculate the average travel distance in a picking area consisting of one block. Previous work on travel distance estimation in similar situations includes Hall (1993) and Kunder and Gudehus (1975). The estimates presented in these papers are fairly simple formulations and may therefore provide an easy-to-use method to obtain a rough estimate for the expected travel distance. We give more precise estimates, which will allow us to accurately determine layouts with the model of the previous section and to prove an optimal location for the depot. A comparison between our estimates, the estimates of Hall (1993), the estimate of Kunder and Gudehus (1975), and simulation is given in Section 5. The travel distance consists of two components: (1) distance traveled within the aisles and (2) distance traveled in the cross aisles. We will derive estimates for both components separately.

3.1 S-shape travel within the aisles

Under the assumption that products are distributed uniformly over the aisles and locations, we can easily derive that the number of aisles containing at least one pick location, has an expected value of:

$$E[A] = n \cdot \left(1 - \left(\frac{n-1}{n}\right)^m\right), \tag{1}$$

which is n times the probability that an aisle contains at least one pick. This is similar to the formulations in Caron *et al.* (1998), Chew and Tang (1999), and Hall (1993). Actually this is a good approximation of the expected number of aisles (see Kunder and Gudehus, 1975).

The expected value of the distance traveled inside the aisles for S-shape, D_y^S, can then be stated as:

$$E[D_y^S] = y' \cdot E[A] + C \tag{2}$$

where $y' = y + w_c$. That is, y' is the length of an aisle plus two times the distance to go from the end of an aisle to the center of the cross aisle (two times $\frac{1}{2}w_c$). This distance is added since we assume that the order pickers walk through the middle of the cross aisles. C is a correction term that accounts for extra travel in the last aisle that is visited. This extra travel distance occurs if the number of aisles that has to be visited is an odd number. In this case the last aisle is both entered and exited from the front (see Fig. 1 for an example). In Hall (1993) and Kunder and Gudehus (1975) it is assumed that if the order picker has to turn in the last aisle, then the distance traveled in this aisle is $2 \cdot y'$. That is, the last aisle first has to be traversed entirely to the back of the warehouse before the order picker can return to the front. In manual picking operations, however, the order picker will generally return to the front directly after picking the last item, instead of first going to the back of the warehouse. Caron *et al.* (1998) use a slightly different routing policy. They assume that the order picker makes the turn in the aisle where the turn will be the shortest, instead of always turning in the last aisle. Furthermore, they assume that the warehouse always has an even number of aisles. In practice, it will be difficult for the order picker to know in which aisle to make the turn and the warehouse may have an odd number of aisles.

We will estimate the correction term C for the most common situation where the turn (if any) is made in the last aisle with an item. Furthermore, the turn occurs directly after the last item has been picked. Now, suppose that a turn has to be made and the number of picks in this last aisle is b, then the distance traveled for a turn in this aisle would equal:

$$2 \cdot \frac{b}{b+1} \cdot y + w_c$$

which is based on the well-known property that the maximum of b continuous uniformly distributed [0,1] variables equals $b/(b+1)$. Since we already accounted for a distance of y' in the estimate $y' \cdot E[A]$, we find that the additional travel for turns is given by:

$$2 \cdot \frac{b}{b+1} \cdot y + w_c - y' = 2 \cdot \frac{b}{b+1} \cdot y - y.$$

Let us now determine the probability that such a turn occurs. First, we need to determine the probability that all picks fall into exactly g aisles out of the n aisles, where g is an odd number. This probability is given by:

$$\binom{n}{g} \left(\frac{g}{n}\right)^m \cdot X \tag{3}$$

where X is 1 minus the probability that all m picks fall into less than g aisles, conditional on the fact that all m items fall into at most g specific aisles. X equals:

$$X = 1 - \sum_{i=1}^{g-1} (-1)^{i+1} \binom{g}{g-i} \left(\frac{g-i}{g}\right)^m.$$

This result relies on the inclusion-exclusion rule. We start with a probability of 1 that all items are in g aisles. We then subtract the probability that the items are in $g-1$ or less aisles, which equals $\binom{g}{g-1}\left(\frac{g-1}{g}\right)^m$. However, we have now subtracted the probability that all items are in $g-2$ or less aisles $\binom{g}{g-1}\binom{g-1}{g-2}$ times, but we should have only subtracted it $\binom{g}{g-2}$ times. Therefore we have to add $\binom{g}{g-1}\binom{g-1}{g-2}-\binom{g}{g-2}=\binom{g}{g-2}$ multiplied by $\left(\frac{g-2}{g}\right)^m$. However, now we have added the probability that all items are in $g-3$ aisles $\binom{g}{g-3}$ too often. And so on.

The correction term for the extra travel distance in the last aisle can be formulated as:

$$C=\sum_{g\in G}\left[\begin{pmatrix} n \\ g \end{pmatrix}\left(\frac{g}{n}\right)^m\cdot X\cdot\left(2\cdot y\cdot\frac{\frac{m}{g}}{\frac{m}{g}+1}-y\right)\right]$$

where

$$G=\{g \quad | \quad 1\le g\le n,\, g\le m \text{ and } g \text{ is odd}\}\,.$$

3.2 Largest gap travel within the aisles

In this section we will give an average travel distance estimate for another commonly used routing policy: largest gap. Based on results from Hall (1993) an estimate for the travel distance within the aisles with the largest gap policy can be obtained as:

$$n\cdot y'\cdot\sum_{i=0}^{m}\left[\binom{m}{i}\left(\frac{1}{n}\right)^i\left(\frac{n-1}{n}\right)^{m-i}D_i\right] \tag{4}$$

where D_i is a factor that denotes the expected travel distance on a line of length 1 if i items are randomly distributed over this line. The values D_i were obtained by simulation and tabulated in Hall (1993) for $i=1,...,10$.

Formula 4 does not take into account that the first and last aisle are always entirely traversed with largest gap routing. Furthermore, if all items are in one aisle, then the route should just consist of entering and leaving that aisle from the front side. We will give a new estimate for largest gap for which we distinguish between (1) the case where all items are in a single aisle and (2) the case where items are distributed over two or more aisles. For the second case we add the additional time required to entirely traverse the first aisle and the last aisle with picks.

The probability that all items are in one aisle equals $n\cdot\left(\frac{1}{n}\right)^m=\left(\frac{1}{n}\right)^{m-1}$ and the average distance traveled in that situation is simply $2y\cdot\frac{m}{m+1}+w_c$. What remains to be estimated is the distance traveled given that at least 2 aisles are to be visited. This event occurs with a probability of $1-\left(\frac{1}{n}\right)^{m-1}$. Given that at least two aisles contain picks, we can estimate the number of aisles to visit by:

$$E[A\mid A\ge 2]=\frac{E[A]-\left(\frac{1}{n}\right)^{m-1}}{1-\left(\frac{1}{n}\right)^{m-1}}$$

where $E[A]$ is given by equation 1.

To estimate the distance to be traveled in an aisle, we first obtain the probability:

$$P(i \text{ items in an aisle}\mid i\ge 1)=\frac{1}{1-\left(\frac{n-1}{n}\right)^m}\binom{m}{i}\left(\frac{1}{n}\right)^i\left(\frac{n-1}{n}\right)^{m-i}.$$

Similar to Hall (1993) we can then multiply these probabilities by the related travel distance to obtain an estimate for travel in the aisles if at least two aisles have to be visited:

$$\frac{1}{1-\left(\frac{n-1}{n}\right)^m} \cdot (E[A \mid A \geq 2]) \cdot \sum_{i=1}^{m} \binom{m}{i} \left(\frac{1}{n}\right)^i \left(\frac{n-1}{n}\right)^{m-i} (yD_i + w_c E_i) \tag{5}$$

where D_i is identical to Hall (1993) and $E_i \quad (1 \leq E_i \leq 2)$ is a simulated estimate for the number of times an aisles needs to be entered, given a certain number of picks in that aisle (the idea to use the estimate E_i is inspired by Đukić and Oluić, 2001). The values for $i = 1, ...50$ are given in Table 1.

Finally, we note that two of these aisles actually should be traversed entirely. We, therefore, subtract two aisles from equation 5 and add the distance of 2 full aisles, which is equal to $2(y+w_c)$, to the total estimate. This gives us as a total estimate for travel within the aisles with largest gap routing:

$$\begin{aligned} E[D_y^{LG}] \;&=\; \left(\frac{1}{n}\right)^{m-1} \left(2y \cdot \frac{m}{m+1} + w_c\right) + \left(1 - \left(\frac{1}{n}\right)^{m-1}\right) \cdot 2(y + w_c) + \\ &\quad \frac{1-\left(\frac{1}{n}\right)^{m-1}}{1-\left(\frac{n-1}{n}\right)^m} \cdot (E[A \mid A \geq 2] - 2) \cdot \sum_{i=1}^{m} \binom{m}{i} \left(\frac{1}{n}\right)^i \left(\frac{n-1}{n}\right)^{m-i} (yD_i + w_c E_i)\,. \end{aligned}$$

3.3 Travel within the cross aisles

The estimate for travel in the cross aisles is identical for S-shape and largest gap and consists of three components (1) travel from the depot to the left-most aisle with picks, (2) travel from aisle to aisle while picking items, and (3) travel from the right-most aisle with picks to the depot. See also Fig. 2 for a graphical illustration of the components.

XXXXXXXXXXXXXX

Insert figure 2

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Both Hall (1993) and Kunder and Gudehus (1975) assume that the depot is located in the middle of the front cross aisle. Kunder and Gudehus (1975) determined the probability that all items are distributed over a certain number, say f, aisles and estimated the distance traveled in the cross aisles by $2 \cdot w_a \cdot \left(\frac{f-1}{f+1} \cdot n - 1\right)$. That is, the distance in the cross aisles is estimated using a property of f points distributed over a line according to a continuous uniform distribution. Hall (1993) uses the same property in a different way, estimating the distance in the cross aisles by $2 \cdot w_a \cdot (n-1) \cdot \frac{m-1}{m+1}$. Both approximations have the same two problems. First of all, a discrete process is approximated using expected values for the continuous case. Secondly, it is assumed that for each route the depot is located between the left-most and right-most aisle with picks. Situations in which all aisles with picks are on one side of the depot are neglected. As an example of the possible impact of this deviation, consider a situation with $d = 1$ and $m = 1$. In this example, the actual expected travel distance in the cross aisles equals $w_a \cdot (n-1)$, which is the distance from the depot to the middle of the warehouse and back. Both the formulation of Hall (1993) and Kunder and Gudehus (1975) will return an estimate of zero. This difference will decrease if the number of picks m increases. Section 5 contains a comparison of the various distance estimates.

Firstly, we estimate the average distance from the depot to the right-most aisle containing items. The probability that i is the right-most aisle to be visited is $\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m$ which is the probability that all picks fall in aisles $1, ..., i$ minus the probability that all picks fall in aisles $1, ..., i-1$. The distance to be traveled is then the distance from depot location to the right-most aisle. If aisle i would be the right-most aisle with items, then the distance to travel would be $w_a \cdot |i - d|$. Taking the sum over all values of i and multiplying by their probability of occurrence gives the expected distance to be traveled from the depot to the right-most aisle:

$$w_a \cdot \sum_{i=1}^{n} \left(|i-d| \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right). \tag{6}$$

Similarly, we can find the distance from the depot to the left-most aisle to be:

$$w_a \cdot \sum_{i=1}^{n} \left(|i-(n-d+1)| \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right). \tag{7}$$

Next we determine the estimate for the distance traveled in the cross aisles while picking items. We have n aisles numbered $1, ..., n$ from left to right. The estimated distance between the left-most aisle containing picks and the right-most aisle containing picks is given by:

$$w_a \cdot \sum_{\ell<r}^{n} (r-\ell) \cdot \left[\left(\frac{r-\ell+1}{n}\right)^m - 2\left(\frac{r-\ell}{n}\right)^m + \left(\frac{r-\ell-1}{n}\right)^m \right] = w_a \cdot \left((n-1) - 2 \cdot \sum_{i=1}^{n-1} \left(\frac{i}{n}\right)^m \right)$$

where the term between square brackets gives the probability that all m items fall in aisles $\ell, ..., r$, at least one pick falls in ℓ and at least one pick falls in r.

The expected value of the distance traveled in the cross aisles, D_x, can now be obtained by adding all three components for cross aisle travel:

$$\begin{aligned} E[D_x] &= w_a \cdot \left(n - 1 - 2 \cdot \sum_{i=1}^{n-1} \left(\frac{i}{n}\right)^m \right) \\ &\quad + w_a \cdot \sum_{i=1}^{n} \left((|i-d| + |i-n+d-1|) \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right). \end{aligned}$$

3.4 Estimate for total average travel distance

Adding the two components (distance traveled in aisles and in cross aisles) gives the total expected travel distance in the picking area:

$$T_m^X(n, y, d) = E[D_y^X] + E[D_x].$$

This formulation has been developed for a manual picking operation. Therefore, we were able to use the average travel distance as a performance criterion. If we analyzed, for example, an automated storage / retrieval system, then a better performance criterion would be average travel time, because travel speed in aisles and cross aisles is unequal in such an environment. If we define t_y as the travel speed in the aisles and t_x as the travel speed in the cross aisles then the estimate for average travel time would be $t_y \cdot E[D_y] + t_x \cdot E[D_x]$. Furthermore, additional time may be required for each change of aisles to position the vehicle correctly, especially in an environment with a relatively large vehicle in narrow aisles. Since this extra time has to be added each time the vehicle enters an aisle, we can just add $t_c \cdot E[A]$ to the travel time estimate of S-shape, where t_c is the time needed to enter an aisle. For largest gap we can easily build an appropriate expression using values of E_i from Table 1.

In practical situations it will often be the case that the pick list size is variable. The formulas we developed are valid only for a fixed pick list size. However, they can easily be adapted for variable pick list sizes. For example, assume that we know for every pick list size m that it will occur with probability p_m, then the estimate for average travel distance is given by $\sum_{m=1}^{\infty} p_m \cdot T_m^X(n, y, d)$.

Clearly, due to the nature of the approach we used (e.g. we assume that expected travel distance in aisles and expected travel distance in cross aisles are independent), these formulations give approximations of actual average travel distances. We will test the performances of the formulas in Section 5.

4 Optimization of the depot location

Bassan *et al.* (1980) show that under the condition of random storage, the depot should be located in the middle of the front cross aisle to minimize average travel distance. Their proof was given in the context of a single command environment (i.e. each pick list contains only one item). We will show that it holds for any pick list size. Intuitively, a depot in the middle of the front cross aisle seems to be the best with respect to average travel time. In this way, the probability seems to be highest that the depot is located between the left-most and right-most aisle of a picking route, thus preventing extra travel time to go forth and back to this region from a depot that is out of the middle. However, in the literature, as well as in practice, depot locations vary. A depot located in the middle is used in Goetschalckx and Ratliff (1988), Hall (1993), Kunder and Gudehus (1975) and Petersen (1999). A depot located in a corner is used in Chew and Tang (1999), De Koster *et al.* (1999b), Gibson and Sharp (1992), and Rosenwein (1996). Both middle and corner options are considered in Jarvis and McDowell (1991), Petersen (1997) and Petersen and Schmenner (1999).

Theorem

The depot location that minimizes average travel distance is the exact middle of the front cross aisle.

Proof

The depot location only influences average travel distance through the term (see equations 6 and 7):

$$w_a \cdot \sum_{i=1}^{n} \left((|i-d| + |i-n+d-1|) \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right).$$

We distinguish 4 cases.

Case 1: if $i \geq d$ and $i+d \geq n+1$ then $|i-d|+|i-n+d-1| = i-d+i-n+d-1 = 2i-n-1$. Since $2i-n-1$ is independent of d there is no influence of the depot location on average travel distance.

Case 2: if $i \leq d$ and $i+d \geq n+1$ then $|i-d|+|i-n+d-1| = d-i+i-n+d-1 = 2d-n-1$. Travel distance is minimized by choosing d as small as possible under the condition that $i \leq d$ and $i+d \geq n+1$. Substituting i by d gives $2d \geq n+1$ or $d \geq \frac{n+1}{2}$. This implies that travel distance is minimized if $d = \frac{n+1}{2}$, i.e. the depot is located in the middle of the front cross aisle.

Case 3: if $i \geq d$ and $i+d \leq n+1$ then $|i-d|+|i-n+d-1| = i-d-i+n-d+1 = -2d+n+1$. Travel distance is minimized by choosing d as large as possible under the condition that $i \geq d$

and $i + d \leq n + 1$. Substituting i by d gives $2d \leq n + 1$ or $d \leq \frac{n+1}{2}$. This implies that travel distance is minimized if $d = \frac{n+1}{2}$, i.e. the depot is located in the middle of the front cross aisle.
Case 4: if $i \leq d$ and $i+d \leq n+1$ then $|i - d|+|i - (n - d + 1)| = d-i-i+n-d+1 = -2i+n+1$. This is independent of the depot location.
Thus, average travel distance is minimized by taking $d = \frac{n+1}{2}$. ■

We have proven that the depot is to be located in the middle of the front cross aisle under the conditions that random storage is used and that the depot location is restricted to the front cross aisle. The proof holds for both S-shape routing and largest gap routing, since both have the same amount of travel in the cross aisles. The above proof holds also for some other routing heuristics (for a description of other routing heuristics see e.g. Petersen, 1997) because they have exactly the same amount of cross aisle travel per route. An evaluation of the percentage difference in average travel distance between a picking area with a depot located at the left and a picking area with a depot located in middle is included in the next section.

5 Model accuracy experiments

We compare the values calculated with the formulas from Section 3 with the results from simulation. Furthermore, we will include the estimates of Hall (1993) and Kunder and Gudehus (1975) into our comparisons. For this comparison, we consider a manual picking operation in a shelf area, where several order pickers may be assigned to the same zone. The center-to-center distance between two neighboring aisles is 2.5 meters. Order pickers are assumed to travel through the exact middle of the aisles and cross aisles. Cross aisles are 2.5 meters wide. For this type of picking areas we assume the following measures to be representative. Aisle length varies between 10 and 30 meters. Each order picker works in a zone consisting of 7 to 15 aisles. We use the extremes of these values for our comparison, which gives four different layouts. For each layout we consider three different values for the number of picks per route, namely 1, 10 and 30. Furthermore, for each situation we consider two depot locations: at the left (at the head of aisle 1) and in the middle of the front cross aisle. For the simulation we generated 2000 orders for each situation. This number of replications is sufficient to obtain a 95% confidence interval with a half-width of less than 2% of the sample mean. Pick locations are distributed over the aisles and locations according to a uniform distribution. No picks occur in the cross aisles. Tables 2 and 3 give the results from the simulation, our formulas, and the existing formula(s) for respectively S-shape routing and largest gap routing. The percentage difference in Tables 2 and 3 between simulated and calculated values has been calculated before rounding of the route lengths.

From the results, we can see that the formulas of Hall (1993) and Kunder and Gudehus (1975) become less accurate if the number of picks decreases; deviations of more than 50% occur. The results from our formulas follow the behavior of the simulation closely. For each situation, the results of our formulas are close to the simulation results. Therefore, average route length can be determined by straightforward calculations with our formulas instead of developing a simulation model for the order picking area. The quality of the estimates is especially important since we wish to use it to determine a layout. Errors in the distance estimates may result in an erroneous ranking of the alternatives and, therefore, lead to the choice of a layout that is not as efficient as possible.

XXXXXXXXXXXXXX
Insert tables 2 and 3
XXXXXXXXXXXXXX

6 Some layout considerations for S-shape

We can use the model of Section 2 to determine the best layout for various values of the total aisle length. Using formula 2, we determine average route length for 1, 2, 3, ... aisles while decreasing aisle length such that total aisle length is kept constant. The depot can be located in the middle of the front cross aisle, because we already showed in the previous section that the middle is the best location. Aisle length is determined by dividing total aisle length by the number of aisles and adding twice the distance needed to go from the end of the aisle to the middle of the cross aisle ($y' = S/n + w_c$). By taking the number of aisles that give the minimum average route length, we obtain the best layout with respect to travel distance. Essentially, this means we suggest solving the layout model by complete enumeration. Even though enumeration is generally not the obvious approach, there is no strong need to search for a more efficient solution method here. The solution space is very small, because the minimum aisle length is 1 meter, and because the number of aisles is an integer value. Thus, for example, for a warehouse layout problem with $S = 150$, there are only 150 possible solutions, ranging from a layout with 1 aisle of 150 meters to a layout with 150 aisles of 1 meter each. Calculation time for these 150 possible layouts is less than 1 second on a 1.5 GHz personal computer.

XXXXXXXXXXXXXX
Insert figure 3
XXXXXXXXXXXXXX

Fig. 3 depicts average route length for S-shape routing as a function of the number of aisles for $S = 300$ and $m = 3, 10, 30$. The minimum route length for $m = 3, 10, 30$ can be found at respectively 17, 21 and 2 aisles. First of all, the erratic behavior of the upper curve ($m = 30$) is striking. This may be explained as follows. If the picking area has an odd number of aisles and if all aisles have to be visited, then the order picker has to make a turn in the last aisle. Thus, part of the last aisle is traveled twice. If the number of picks is high then the probability that all aisles have to be visited is high and the distance traveled twice in the last aisle is large. This effect diminishes if the number of aisles increases, because the probability decreases that all aisles must be visited. Considering the formulas for S-shape from section 3, it can be derived that *if all aisles are visited*, then the travel distance equals:

$$2w_a(n-1) + nw_c + S \text{ if the number of aisles is even} \tag{8}$$

$$2w_a(n-1) + nw_c + S + 2y \cdot \left(\tfrac{m}{n}/(\tfrac{m}{n}+1)\right) - y \text{ if the number of aisles is odd.} \tag{9}$$

To investigate the difference in travel distance between layouts that differ by one aisle only, we replace n by 2ν in equation 8 and by $2\nu + 1$ in equation 9. Then, we subtract equation 8 from equation 9, which gives us a difference of

$$2w_a + w_c + 2y \cdot \left(\tfrac{m}{2\nu+1}/(\tfrac{m}{2\nu+1}+1)\right) - y.$$

Since $\frac{m}{n}/(\frac{m}{n}+1) \geq \frac{1}{2}$ for situations with $m \geq n$ (which must be true, because otherwise not all aisles would be visited), it follows that in this situation the travel distance in a layout with an even number of aisles is lower than the travel distance in a layout with one aisle more (and therefore an odd number of aisles). Note that the travel distance in a layout with 2ν aisles can be either smaller or larger than the travel distance with $2\nu - 1$ aisles. Furthermore, this disadvantage may be outweighed by the gains from having one more aisle, which increases the probability of not having to visit one (or more) of the aisles. This effect can be observed in Fig. 3, where the curves become more smooth if n increases.

From Fig. 3 we can also see that two aisles was the best option for the situation with a pick list size of $n = 30$. It would be interesting to know whether a higher pick density in general tends to favor layouts with just 2 aisles. We evaluate the behavior of $T_m^S(n, y, d)$ if m approaches infinity.

It is fairly straightforward to show that

$$\begin{aligned}
\lim_{m\to\infty} E[A] &= n \\
\lim_{m\to\infty} C &= \begin{cases} y \text{ if } n \text{ is odd} \\ 0 \text{ if } n \text{ is even} \end{cases} \\
\lim_{m\to\infty} E[D_x] &= w_a(n-1).
\end{aligned}$$

Because the probability that all aisles are visited approaches 1 if $m \to \infty$ it holds that there is a shorter layout with an even number of aisles for each layout with an odd number of aisles. We, therefore, can restrict our search for a minimum to the situations for which n is even. Thus,

$$\begin{aligned}
\lim_{m\to\infty} T_m^S(n, y, d) &= \lim_{m\to\infty} E[D_y] + \lim_{m\to\infty} E[D_x] \\
&= \lim_{m\to\infty} ((y + w_c) \cdot E[A]) + \lim_{m\to\infty} C + \lim_{m\to\infty} E[D_x] \\
&= (y + w_c)n + w_a(n-1) \text{ if } n \text{ is even.}
\end{aligned}$$

The minimum is obtained at $n = 2$ (there are no smaller even values). From a practical point of view, it may be noted that a single picking area consisting of two aisles may not always be convenient. Firstly, the shape may be difficult to fit into the overall warehouse design. Secondly, there is a risk of congestion, since all pickers would then be working in the same two aisles. This could result in a zone-based approach for order picking, where each order picker is responsible for picking in two aisles. Obviously, this may require system changes, such as the inclusion of a sorting area or the introduction of a method where picked items are passed from one picker to the next ("pick-and-pass"). These changes would have to weighted against the benefits of travel time reductions.

Finally, it is interesting to note from a practical point of view that curves appear to be quite flat around the optimum (at least if the optimum is not equal to 2 aisles). That is, around the optimum there are a large number of other layouts that have average travel distances which are only a few percent higher. A designer may be able to use this flexibility to meet other requirements. Such requirements may include prevention of congestion, flexibility for redesign or stability with respect to changes in the pick list size. Instead of finding "the best" layout, a designer may use the model to determine the k-best layouts and then choose from these layouts based on other considerations than travel time alone.

7 Influence of the routing policy on the layout

We have formulated a model to determine optimal layouts for a given routing policy. Furthermore, we have derived two expressions for average travel distance, which can be used as objective functions in the model. One objective function consists of a travel distance estimate for S-shape routing and the other for largest gap routing. Given a certain choice for a specific routing method, it is now fairly straightforward to determine the best layout based on expected pick list size and the required storage space. In design practice, however, the layout is often chosen before the routing policy. It would, therefore, be interesting to know what the actual influence of the routing policy is on the layout. Or, formulated differently, what is the loss in efficiency if the layout is optimized with another routing policy than the policy that is actually going to be used? We investigate this issue as follows. We take the same manual picking operation in a shelf area (i.e. $w_a = 2.5$ and $w_c = 2.5$) and determine the optimal layout for both routing policies for a range of sizes ($S = 50, 60, 70, ..., 500$) and pick list sizes ($m = 2, 3, 4, ..., 30$). We then calculate the expected travel distance when S-shape is used in the layout that has been optimized for S-shape and we calculate the expected travel distance when using S-shape in the layout optimized for largest gap. The percentage difference between these two travel distances is given in Table 4. A table for largest gap routing would be very similar.

XXXXXXXXXXXXXX
Insert table 4
XXXXXXXXXXXXXX

As can be seen from Table 4, there is only a small difference in travel distances for most situations. The upper-right corner of the table contains only zeros because the optimal layout in these situations consists of exactly 2 aisles regardless of the routing policy. The lower-left corner contains situations where the optimal layout consists of strictly more than 2 aisles for both routing methods (though the exact number may differ). The grey area consists of situations where the optimal layout for S-shape consists of exactly 2 aisles and the optimal layout for largest gap consists of strictly more than 2 aisles. As can be seen from the table, the only significant differences occur in the last situation.

From the above experiment it can be seen that in many practical situations (namely those where the majority of pick list sizes fall outside the grey area) there is no significant efficiency loss if another routing method is used for the optimization than for the actual operation. On the other hand, large deviations (in our experiment up to 18%) may occur depending on the required storage space and the pick list sizes. If the routing policy is undecided when the layout has to be chosen, it may therefore be advisable to perform a sensitivity analysis on the various routing policies.

Another interesting point to consider is whether conditions can be identified when S-shape or largest gap is to be preferred as a routing policy. From Hall (1993), it is known that S-shape is preferred over largest gap if the number of picks per aisle exceeds 4. This rule, however, assumes that the layout is already determined in advance. Using our model, we can compare routing policies together with their optimal layout. To illustrate the possibilities, we analyze an initial design for a shelf area ($w_a = 2.5$; $w_c = 2.5$) with 18 picks per route and 4 aisles of 75 meters each. Travel distance is then 324.4 meters for S-shape and 340.3 meters for largest gap. The optimal layout, however, consists of 23 aisles for S-shape and 20 aisles for largest gap. The resulting travel distances are then 300.3 meters for S-shape and 246.6 meters for largest gap. Largest gap thus outperforms S-shape here. We have completed this experiment for $S = 50, 60, 70, ..., 500$ and $m = 2, 3, 4, ..., 30$. For all these 1334 situations the largest gap policy gave travel distances that are equal to or shorter than the distances for the S-shape policy. The best layout - when optimized for largest gap - is for all situations such that the number of aisles equals 2 (i.e. largest gap and S-shape routing are the same) or the number of aisles is such that there are less than 4 items per aisle. We repeated this experiment for pallet racks ($w_a = w_c = 4$) with the same results. It seems, therefore, that if layout still needs to be determined, then largest gap routing is generally to be preferred over S-shape routing.

8 Concluding remarks

In this paper we have evaluated the relation between the layout of the order picking area and the average length of a picking route for areas consisting of one block. Analytical formulas are presented by which the average route length can be calculated for two different routing policies. The outcomes of the formulas depend on the number of aisles, the aisle length, the depot location, the number of picks per route and some physical parameters of the racks. The results from the formulas are compared with simulation results. It appeared that the difference between simulated and calculated values was minor in all of our experiments. Therefore, we consider the analytical formulas to be accurate enough to be used instead of simulation.

The analytical formulas can be used as objective function in the presented non-linear programming model to find an optimal layout for the order picking area in warehouses. We proved that the best depot location is in the middle of the front cross aisle. The proof is valid for several routing heuristics under the condition that random storage is used. Given that S-shape will be used for routing, we derived some general characteristics of "good" layouts. For high picking densities, S-shape routing is best employed in a layout with an even number of aisles instead of an odd number of aisles. From the viewpoint of strict travel distance minimization, a very high pick density is best dealt with in a picking area where each picking zone consists of exactly two aisles. For practical implementations, other considerations than travel distance can be taken into account easily too, since there are generally several layouts that have an average travel distance that is close to the optimum.

In a comparison of more than 2500 different layout problems, we found that largest gap routing performed better than or equal to S-shape routing in all situations if the optimal layout was used for each routing method. The optimal layout is sensitive to the routing policy used in the optimization. We found efficiency losses of up to 18% when optimizing the layout for another routing policy than was used for the actual operation. It must also be noted that there are large regions where the resulting travel distance is highly insensitive to the routing method used in the optimization. Given these uncertain effects, it is advisable to perform the optimization for more than one routing method before deciding on the layout.

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9 Captions

Fig. 1. Example of an order picking area, including example routes as found by the S-shape heuristic (upper image) and the largest gap heuristic (lower image).

Fig. 2. Illustration of the three components for travel in the cross aisles.

Fig. 3. Average route length in meters as a function of the number of aisles for three different pick list sizes (indicated next to the curves). Total aisle length (S) equals 300 meters. Asterisks indicate the minimum for $m = 3$ and $m = 10$.

Table 1. Values of D_i and E_i for $i = 1, \ldots, 50$ (based on 1,000,000 replications).

Table 2. Average route length in meters for picking an order with the S-shape routing policy. For each of the formulas the percentage difference with the distance obtained from the simulation is also given.

Table 3. Average route length in meters for picking an order with the largest gap routing policy. For each of the formulas the percentage difference with the distance obtained from the simulation is also given.

Table 4. Percentage difference in travel distance when using the S-shape routing policy in a layout optimized for S-shape and in a layout optimized for largest gap.

10 Biographical sketches

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i	D_i	E_i	i	D_i	E_i
1	0.500	1.000	26	1.712	1.926
2	0.778	1.333	27	1.719	1.929
3	0.958	1.500	28	1.727	1.931
4	1.087	1.600	29	1.734	1.933
5	1.183	1.667	30	1.740	1.936
6	1.259	1.715	31	1.746	1.937
7	1.321	1.750	32	1.752	1.939
8	1.371	1.778	33	1.758	1.941
9	1.414	1.800	34	1.763	1.943
10	1.451	1.818	35	1.768	1.944
11	1.483	1.833	36	1.773	1.946
12	1.511	1.846	37	1.777	1.947
13	1.535	1.857	38	1.782	1.949
14	1.558	1.867	39	1.786	1.950
15	1.577	1.875	40	1.790	1.951
16	1.595	1.883	41	1.794	1.952
17	1.612	1.889	42	1.798	1.954
18	1.627	1.895	43	1.801	1.955
19	1.640	1.900	44	1.805	1.956
20	1.653	1.905	45	1.808	1.956
21	1.665	1.909	46	1.811	1.957
22	1.675	1.913	47	1.814	1.958
23	1.685	1.917	48	1.817	1.959
24	1.695	1.920	49	1.820	1.960
25	1.703	1.923	50	1.823	1.961

TABLE 1. Values of D_i and E_i for i = 1,…, 50 (based on 1,000,000 replications).

S-shape										
aisle length	number of aisles	number of items	depot location	simulation	Kunder & Gudehus		Hall		new formula	
					distance	difference	distance	difference	distance	difference
10	7	1	left	27.0	15.0	-44.3%	18.8	-30.4%	27.5	2.0%
10	7	1	middle	20.9	15.0	-28.3%	18.8	-10.3%	21.1	0.8%
10	7	10	left	98.5	86.1	-12.6%	103.7	5.3%	99.0	0.5%
10	7	10	middle	97.4	86.1	-11.6%	103.7	6.5%	97.7	0.4%
10	7	30	left	121.8	100.8	-17.2%	125.6	3.2%	122.4	0.5%
10	7	30	middle	121.7	100.8	-17.2%	125.6	3.2%	122.3	0.5%
10	15	1	left	46.3	15.0	-67.6%	18.8	-59.5%	47.5	2.5%
10	15	1	middle	30.9	15.0	-51.5%	18.8	-39.4%	31.2	0.7%
10	15	10	left	159.1	127.9	-19.6%	161.1	1.2%	159.6	0.3%
10	15	10	middle	154.4	127.9	-17.2%	161.1	4.3%	155.0	0.4%
10	15	30	left	235.2	211.8	-9.9%	240.2	2.2%	235.1	0.0%
10	15	30	middle	234.4	211.8	-9.7%	240.2	2.5%	234.4	0.0%
30	7	1	left	46.8	55.0	17.5%	48.8	4.1%	47.5	1.5%
30	7	1	middle	40.8	55.0	34.9%	48.8	19.5%	41.1	0.7%
30	7	10	left	210.5	205.9	-2.2%	223.7	6.3%	212.0	0.7%
30	7	10	middle	209.7	205.9	-1.8%	223.7	6.6%	210.7	0.5%
30	7	30	left	270.8	258.1	-4.7%	274.3	1.3%	272.6	0.7%
30	7	30	middle	270.6	258.1	-4.6%	274.3	1.3%	272.6	0.7%
30	15	1	left	66.2	55.0	-16.9%	48.8	-26.4%	67.5	2.0%
30	15	1	middle	50.8	55.0	8.2%	48.8	-4.1%	51.2	0.7%
30	15	10	left	309.2	268.2	-13.3%	320.6	3.7%	310.6	0.5%
30	15	10	middle	304.4	268.2	-11.9%	320.6	5.3%	306.0	0.5%
30	15	30	left	501.0	483.9	-3.4%	512.4	2.3%	501.2	0.0%
30	15	30	middle	500.2	483.9	-3.3%	512.4	2.4%	500.4	0.0%

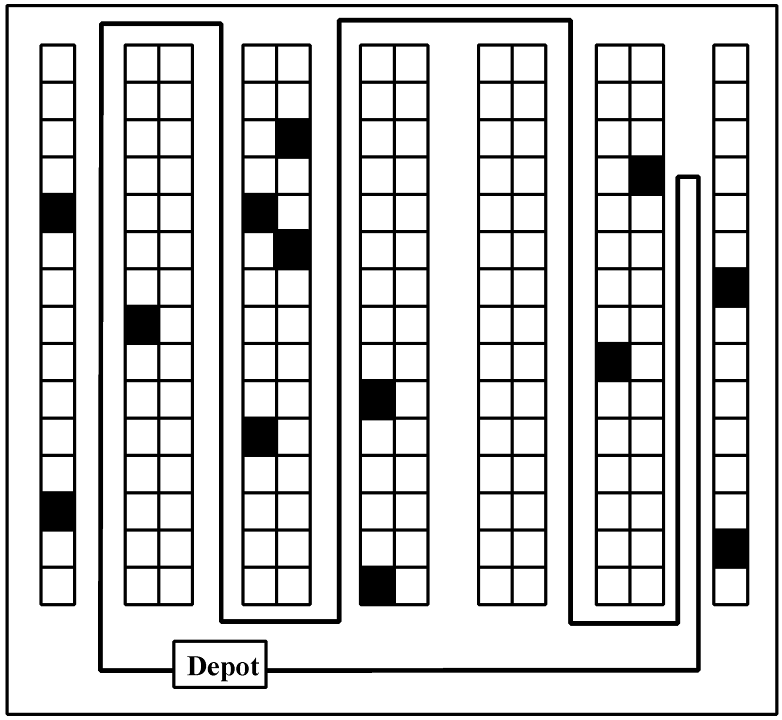
TABLE 2. Average route length in meters for picking an order with the S-shape routing policy. For each of the formulas the percentage difference with the distance obtained from the simulation is also given.

Largest Gap								
aisle length	number of aisles	number of items	depot location	simulation	Hall		new formula	
					distance	difference	distance	difference
10	7	1	left	27.0	6.2	-76.8%	27.5	2.0%
10	7	1	middle	20.9	6.2	-70.1%	21.1	0.8%
10	7	10	left	88.3	76.6	-13.3%	88.8	0.6%
10	7	10	middle	87.0	76.6	-12.0%	87.6	0.7%
10	7	30	left	125.8	125.4	-0.3%	127.0	0.9%
10	7	30	middle	125.8	125.4	-0.3%	127.0	1.0%
10	15	1	left	46.3	6.2	-86.5%	47.5	2.5%
10	15	1	middle	30.9	6.2	-79.8%	31.2	0.7%
10	15	10	left	137.1	116.3	-15.2%	137.7	0.4%
10	15	10	middle	132.5	116.3	-12.2%	133.1	0.5%
10	15	30	left	217.3	198.8	-8.5%	217.9	0.3%
10	15	30	middle	216.6	198.8	-8.2%	217.2	0.3%
30	7	1	left	46.8	16.2	-65.3%	47.5	1.5%
30	7	1	middle	40.8	16.2	-60.2%	41.1	0.7%
30	7	10	left	176.7	153.2	-13.3%	177.6	0.5%
30	7	10	middle	175.4	153.2	-12.6%	176.4	0.6%
30	7	30	left	271.6	273.8	0.8%	272.6	0.4%
30	7	30	middle	271.5	273.8	0.8%	272.5	0.4%
30	15	1	left	66.2	16.2	-75.5%	67.5	2.0%
30	15	1	middle	50.8	16.2	-68.0%	51.2	0.7%
30	15	10	left	240.9	204.2	-15.2%	242.1	0.5%
30	15	10	middle	236.3	204.2	-13.6%	237.5	0.5%
30	15	30	left	432.9	404.6	-6.5%	432.3	-0.1%
30	15	30	middle	432.2	404.6	-6.4%	431.6	-0.1%

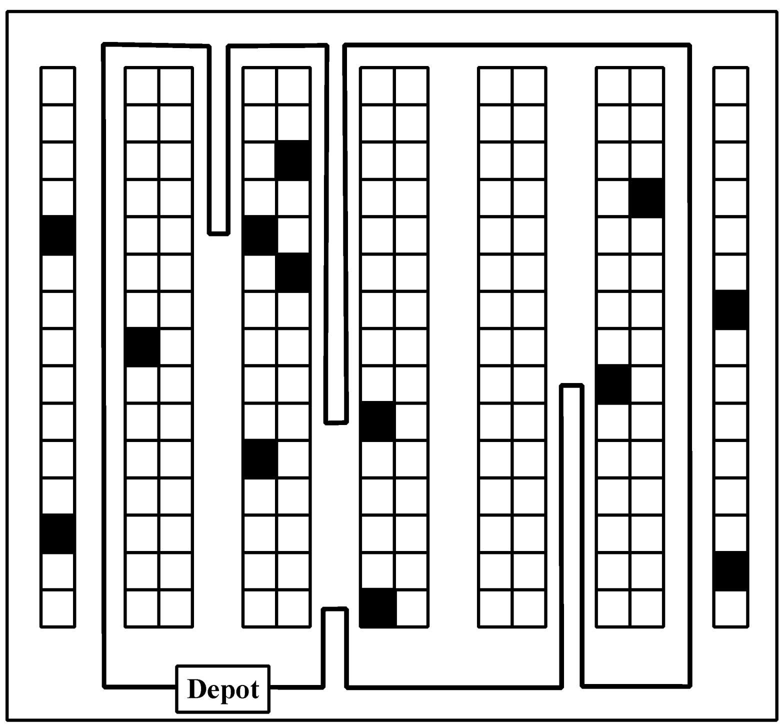
TABLE 3. Average route length in meters for picking an order with the largest gap routing policy. For each of the formulas the percentage difference with the distance obtained from the simulation is also given.

S \ m	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
50	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	3	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	5	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	0	0	0	1	0	1	8	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
90	0	0	1	0	0	0	4	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	6	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	3	8	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	4	9	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	1	1	0	0	0	0	2	6	10	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
140	0	0	0	0	0	0	0	0	0	3	8	11	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0	0	0	0	1	1	1	1	0	1	5	9	12	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
160	0	0	1	1	0	0	0	0	0	0	3	6	10	13	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
170	0	0	0	0	1	1	1	1	0	0	0	4	7	11	14	16	0	0	0	0	0	0	0	0	0	0	0	0	0
180	0	0	1	1	0	0	0	0	0	0	0	2	5	8	11	14	17	0	0	0	0	0	0	0	0	0	0	0	0
190	0	0	0	0	1	1	1	1	0	0	0	0	3	6	9	12	15	0	0	0	0	0	0	0	0	0	0	0	0
200	0	0	0	1	1	1	0	0	0	0	0	0	1	4	7	10	13	15	0	0	0	0	0	0	0	0	0	0	0
210	0	0	1	1	0	1	1	1	0	0	0	0	0	3	6	8	11	13	16	0	0	0	0	0	0	0	0	0	0
220	0	0	0	0	1	1	0	0	1	1	0	0	0	1	4	7	9	12	14	16	0	0	0	0	0	0	0	0	0
230	0	0	1	1	1	1	1	1	1	0	0	0	0	0	2	5	7	10	12	14	16	0	0	0	0	0	0	0	0
240	0	0	1	0	1	1	1	0	1	1	1	1	0	0	0	3	6	8	10	13	15	17	0	0	0	0	0	0	0
250	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	2	4	7	9	11	13	15	17	0	0	0	0	0	0
260	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	3	5	7	10	12	14	15	17	0	0	0	0	0
270	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	4	6	8	10	12	14	16	18	0	0	0	0
280	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	4	7	9	11	13	14	16	18	0	0	0
290	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	3	5	7	9	11	13	15	16	0	0	0
300	0	0	1	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	2	4	6	8	10	12	13	15	17	0	0
310	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	3	5	7	9	10	12	14	15	17	0
320	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	3	5	7	9	11	13	14	16	17
330	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	4	6	8	10	11	13	14	16
340	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	3	5	7	8	10	12	13	15
350	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	4	6	7	9	11	12	14
360	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	3	5	6	8	9	11	12
370	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	2	3	5	7	8	10	11
380	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	2	4	6	7	9	10
390	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	3	5	6	8	9
400	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2	4	5	7	8
410	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	3	4	6	7
420	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	2	3	5	6
430	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	2	4	5
440	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	3	4
450	0	0	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	2	4
460	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	3
470	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	2
480	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1
490	0	0	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
500	0	0	1	1	1	1	1	1	2	1	2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0

TABLE 4. Percentage difference in travel distance when using the S-shape routing policy in a layout optimized for S-shape and in a layout optimized for largest gap.



(a) Example route for S-shape



(b) Example route for largest gap

Figure 1: Example of an order picking area, including example route as found by (a) the S-shape heuristic and (b) the largest gap heuristic..

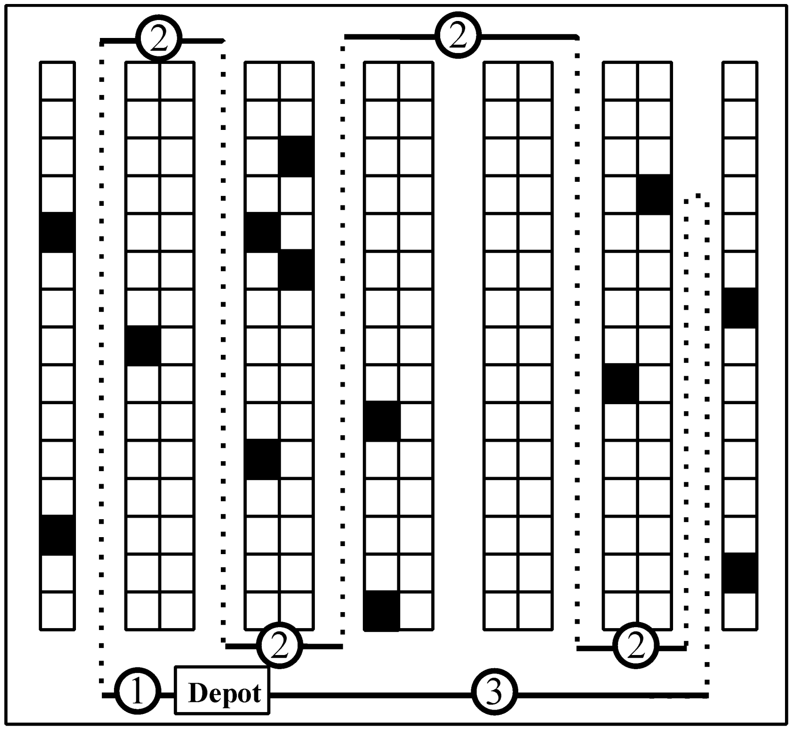


Figure 2: Illustration of the three components for travel in the cross aisles.

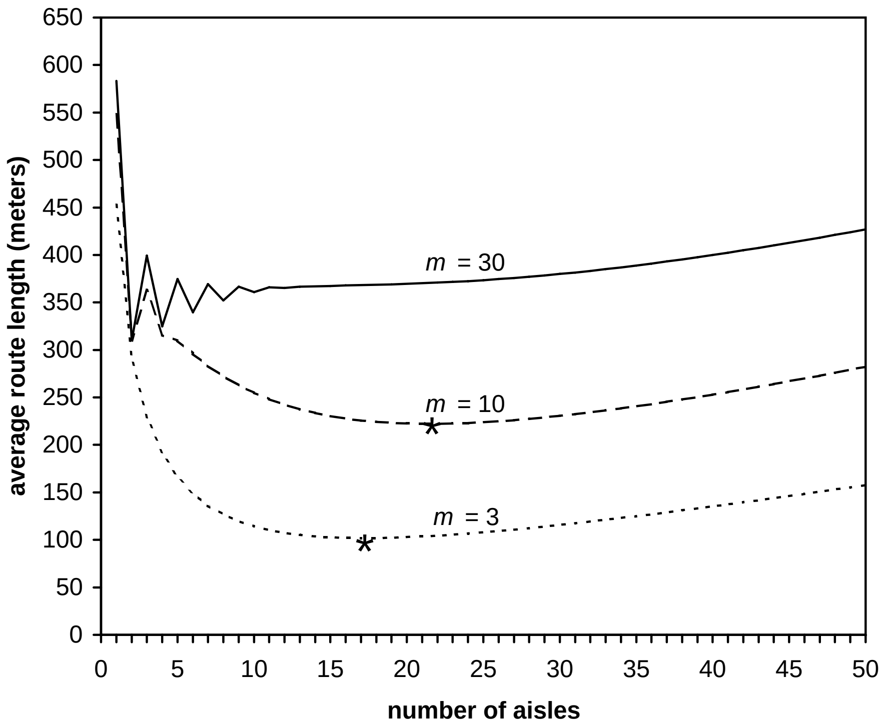


FIGURE 3. Average route length in meters as a function of the number of aisles for three different pick list sizes (indicated next to the curves). Total aisle length (S) equals 300 meters. Asterisks indicate the minimum for m=3 and m=10 .