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Abstract

A structural model is used to explain activity interactions between heads of households, and, in so doing, to explain household demand for travel. The model attempts to capture links between activity participation and associated derived travel, links between activities performed by male and female heads, links between types of travel, and time-budget feedbacks from travel to activity participation. Data for pairs of opposite gender heads of households are from the 1994 Portland Activity and Travel Survey. The results suggest that a feedback mechanism should be introduced in trip generation models to reflect the effect of activity frequency and duration on the level of associated travel.

1. OBJECTIVES AND SCOPE

Our objective is to explain household activity interactions, and, in so doing, to explain household demand for travel. Activity-based approaches have enhanced our understanding of travel behavior via the development of models of scheduling and activity participation and the examination of the relationships between household members, their activity demands, and the constraints that bind their decision processes. However, there has been very little work, theoretical or empirical, that has dealt with formal relationships between household members.

Townsend (1987) developed a conceptual framework for classification and analysis of travel/activity patterns and used observed task assignments to analyze and classify household patterns using household structure and individual role characteristics. This work was directed toward the development of hierarchical relationships between the travel/activity patterns of the household and its individual members. Townsend first developed a theoretical household time allocation model where individuals participate in activities beyond or below the point of maximum individual satisfaction if household utility maximization is the goal. Substitution, companion, and complementary effects were postulated between individuals. Townsend completed an empirical analysis of household interactions using a combination of trip, tour, and travel/activity pattern statistics. Activities were categorized by purpose (subsistence, maintenance, serve passenger, and leisure) and by performer (single, couple, and multi-person). For couples, several key interactions were identified. With respect to the female's employment status, the partner of working females do not significantly increase their maintenance activities. There was also a shifting of joint maintenance trips to weekends. Townsend also found that working females made fewer maintenance trips than non-working females. He also found that the presence of children reflects more prominently on females. Maintenance trips are greater for mothers and lower for fathers when compared to their childless counterparts. Children tend to increase the amount of work activity for males, and increase the amount of maintenance activity for females. Finally, employed females tend to decrease the amount of leisure activity.

Van Wissen (1989) developed structural models of activity duration for couples using the Dutch Panel data. He proposed to verify the existence of substitution, companion, and complementary relationships as proposed by Townsend (1987) for shopping, recreational, and visiting activities; in general, only the shopping models were very strong. Van Wissen found no evidence of substitution effects, although complementary relations were strong for all activities. He also found that joint activities were important. Overall, the work durations for the male was the primary factor shaping the activity patterns of both partners. The employment status of the female was identified as important in her participation in other activities. The female's employment status did not influence the male's non-work activity hours, however, her actual participation did influence his durations. Van Wissen considered temporal effects using two waves of the panel data set, but he did not investigate activity duration effects on travel time to the activity.

Our modeling approach also allows us to identify potential interrelationships among travel times for different activities and feedbacks from travel times to activity durations. In this way our models can capture “time budget” effects. Zahavi (1979;) and Zahavi and McLynn (1983) demonstrated how travel distances can increase as a result of saving in travel times due to improvements in transportation levels of service. Theoretical bases for these time budget effects are provided by Golob, Beckmann and Zahavi (19981) and Downes and Emmerson (1985). The present model can be viewed as a step “toward taking the total activity pattern and the time budgets per activity group as the basis for explaining individual travel behaviour” (van der Hoorn ,1979).

The initial hypothesis formulated involves the quantification of the relationship between travel and the activity participation from which travel is derived. Several additional research hypotheses are developed to define the role of household interaction in travel behavior. Substitution, companion, and complementary relationships involving both activities and the associated travel of a household are hypothesized to exist both within and between the heads of the household. Finally, as in conventional demand models, a variety of exogenous variables are assumed to affect the relationships between activity participation and travel.

2. MODEL SPECIFICATION

To attack the problem, we limit the analysis to married or unmarried male and female couples who are heads of households. Whether there are other household members is taken into account, but only the activity participation and travel of the two household heads is modeled explicitly. We apply structural equations to simultaneously model the activity participation behavior and travel of these couples and to identify the interactions that define this behavior. Households with two heads of the same gender were excluded. We judged the application to male and female adult heads to be a fair test of the effectiveness of the method.

2.1 The Endogenous Variables

Our endogenous variables are meant to capture the participation of male and female household heads in out-of-home activities and their travel to access these activities. We construct an identical set of variables for both household heads and model their interactions simultaneously.

Highly specific activities are aggregated into three broad activity types:

> Work, which includes activities coded as: work and work-related;
>
> Maintenance (abbreviated "maint."), which includes activities coded as: meals, shopping (general), shopping (major), personal services, medical care, professional services, household or personal business. household maintenance, household obligations, pick up or drop off passenger. school, and religious; and
>
> Discretionary (abbreviated "discr."), which includes activities coded as: visiting, culture, civic, amusements, hobbies, exercise or athletics, rest and relaxation, spectator athletic events, incidental trips, and tag-along trips.

The model specification does not depend on this particular typology of activities. One could use a different allocation of specific activities to three types, or to a different number of types. However, if more activity categories are used, there will be more parameters in the model, and accurate estimation of these parameters will require a larger sample size. Also, the incidence of zero activity duration will increase with the number of activity categories, holding the number of diary days constant at two per person, and this has implication for model estimation, as discussed in the appendix.

For each of the three activity types, total out-of-home duration was computed over each individual's two diary days. The total travel time reported in accessing each activity was also aggregated for each activity type. Some activities, conducted at the same site as a previous activity, might have zero travel associated with them. The twelve endogenous variables in the model are listed in Table 1.

Table 1: The Endogenous Variables

Endogenous Variable	Acronym
1. Total two-day out-of-home activity duration: work & work-related - male	male work activities
2. Total two-day out-of-home activity duration: work & work-related - female	female work activities
3. Total two-day out-of-home activity duration - maintenance - male	male maint. activities
4. Total two-day out-of-home activity duration - maintenance - female	female maint. activities
5. Total two-day out-of-home activity duration - discretionary - male	male discr. activities
6. Total two-day out-of-home activity duration - discretionary - female	female discr. activities
7. Total two-day travel times to out-of-home work and work-related activities - male	male work travel
8. Total two-day travel times to out-of-home work and work-related activities - female	female work travel
9. Total two-day travel times to out-of-home maintenance activities - male	male maint. travel
10. Total two-day travel times to out-of-home maintenance activities - female	female maint. travel
11. Total two-day travel times to out-of-home discretionary activities - male	male discr. travel
12. Total two-day travel times to out-of-home discretionary activities - female	female discr. travel

2.2 The Exogenous Variables

In specifying the model exogenous variables, we restricted ourselves to household and personal characteristics that would in general be available from exogenous sources, such as the US Census. Such characteristics include age of the heads, household membership in terms of the number of children by age category, number of workers, number of vehicles, number of drivers, housing tenure, and income. Detailed personal characteristics, such as employment status, occupation and industry, and personal income were not included as exogenous variables because it is generally not possible to obtain forecasts of such variables for planning purposes.

2.3 The Structural Equation Model Form

The standard structural equations model (without latent variables) is given by:

$$\mathbf{y} = \mathrm{B}\mathbf{y} + \mathrm{G}\mathbf{x} + \mathrm{z} \tag{1}$$

where $\mathbf{y}$ is an (m by 1) column vector of endogenous variables, and $\mathbf{x}$ is an (n by 1) column vector of exogenous variables. In the present application, we have m = 12 endogenous variables and n = 15 exogenous variables.

The structural parameters are the elements of the matrices:

B = (m x m) matrix of causal links between the m=12 endogenous variables,
G = (m x n) matrix of direct causal (regression) effects from the n=15 exogenous variables to the m=12 endogenous variables.

and:

$\mathrm{Y} = E(\mathrm{zz}')$ = variance-covariance matrix of the (m) error terms.

For identification of system (1), B must be chosen such that $(\mathbf{I} - \mathrm{B})$ is non-singular, where $\mathbf{I}$ denotes the identity matrix of dimension m.

It can easily be shown that the total effects of the endogenous variables on each other are given by:

$$\mathbf{T}_{yy} = (\mathbf{I} - \mathrm{B})^{-1} - \mathbf{I} . \tag{2}$$

The total effects of the exogenous variables on the endogenous variables in a structural equations model of this type are given by:

$$\mathbf{T}_{xy} = (\mathbf{I} - \mathrm{B})^{-1} \mathrm{G}, \tag{3}$$

which are the parameters of the reduced-form equations.

2.4 The Postulated Activity - Travel Causal Structure

The postulated structure among the endogenous variables is shown in the flow diagram of Figure 1. In Figure 1 and subsequent flow diagrams. The model endogenous variables are represented by boxes. The boxes in the top row variables are activity durations and those in the bottom row variables are travel times. Variables for the female head are shaded. Each arrow (direct effect) in this diagram represents a postulated free parameter in the B matrix. An arrow from variable k to variable j represents a free parameter corresponding to element $\beta_{j,k}$ in the B matrix. Postulated non-zero direct effects are represented by arrows.)

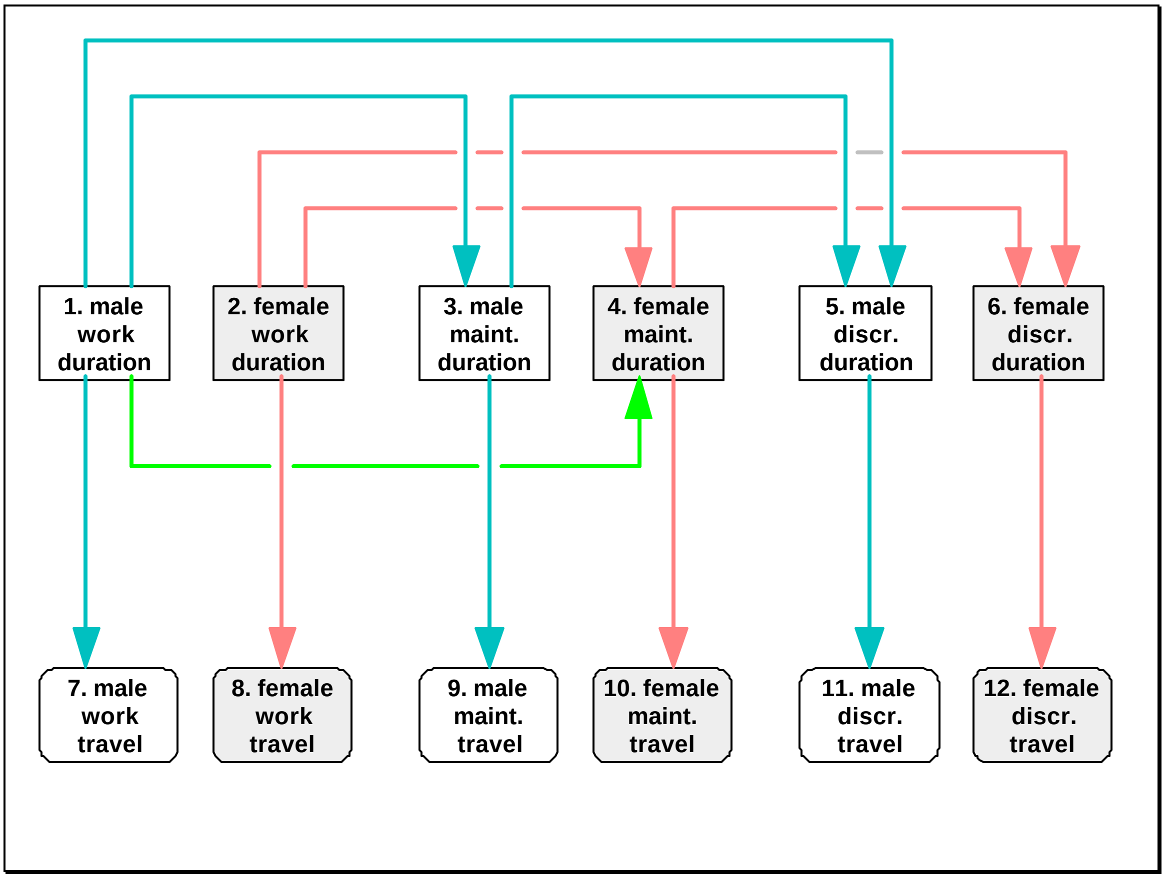
These postulated direct effects can be divided into four types: (1) the travel requirements of out-of-home activities, (2) within-person activity interactions, (3) within-person travel interactions, and (4) cross-person interactions. We next describe specific hypotheses in terms of each of theses four types of effects.

2.4.1 Travel Requirements of Out-of-home Activities

The direct effects from each individual's activity type duration to the travel for that activity type (the vertical arrows in Figure 1 from variable 1 to variable 7, variable 2 to variable 8, etc.) represent travel as a derived demand. The estimated coefficients of these effects can be interpreted in terms of hours of travel time require to access one hour of that activity. We expect these coefficients to be positive and less than one.

If, for any of the three types of activities, men and women display similar travel-activity patterns, the coefficients for male and female household heads should be equal for the same activity type. Test of parameter equality are easily performed in structural equation modeling by estimating a model with the constraint that a pair of coefficients be equal (e.g., $\beta_{7,1} = \beta_{8,2}$) and comparing the goodness-of-fit to an otherwise identical model without the equality constraint. Each of the three pairs of gender differences in travel demand derived from activity demand is tested in our models.

Figure 1: Flow Diagram of Postulated Direct Effects Between Activity Participation and Travel for Paired Male and Female Heads of Households
(Model endogenous variables are represented by boxes. Top row variables are activity durations and bottom row variables are travel times. Variables for the female head are shaded. Postulated non-zero direct effects are represented by arrows.)



2.4.2 Within-person Activity Interactions

The six effects portrayed by six arrows at the top of Figure 1 emanating from variables 1,2,3, and 4 are meant to capture a hierarchy of activities for both men and women. We postulated that this hierarchy is defined by (1) work activities at the top, followed by (2) maintenance activities, and finally (3) discretionary activities. These reflect constraints on available time. A person's engagement in a lower-order activity is controlled in part by his or her level of participation in the higher-order activity or activities. Consequently, there are three hierarchical activity links for both males and females: (1) from work to maintenance, (2) from work to discretionary, and (3) from maintenance to discretionary. We expect all of these coefficients to be negative. No other within-person activity to activity effects are expected to be significant.

2.4.3 Within-person Travel Interactions

Many other within-person effects involving the three travel variables are possible, as long as the we maintain the identity condition that we specify B so that the matrix $(\mathbf{I} - B)$ is non-singular. However, the only three links we postulate involving travel for each person are the three activity to travel links for each activity type that we defined in Section 4.5.1. Hypothesis tests are subsequently used to determine whether or not additional direct effects are required to establish an acceptable overall model.

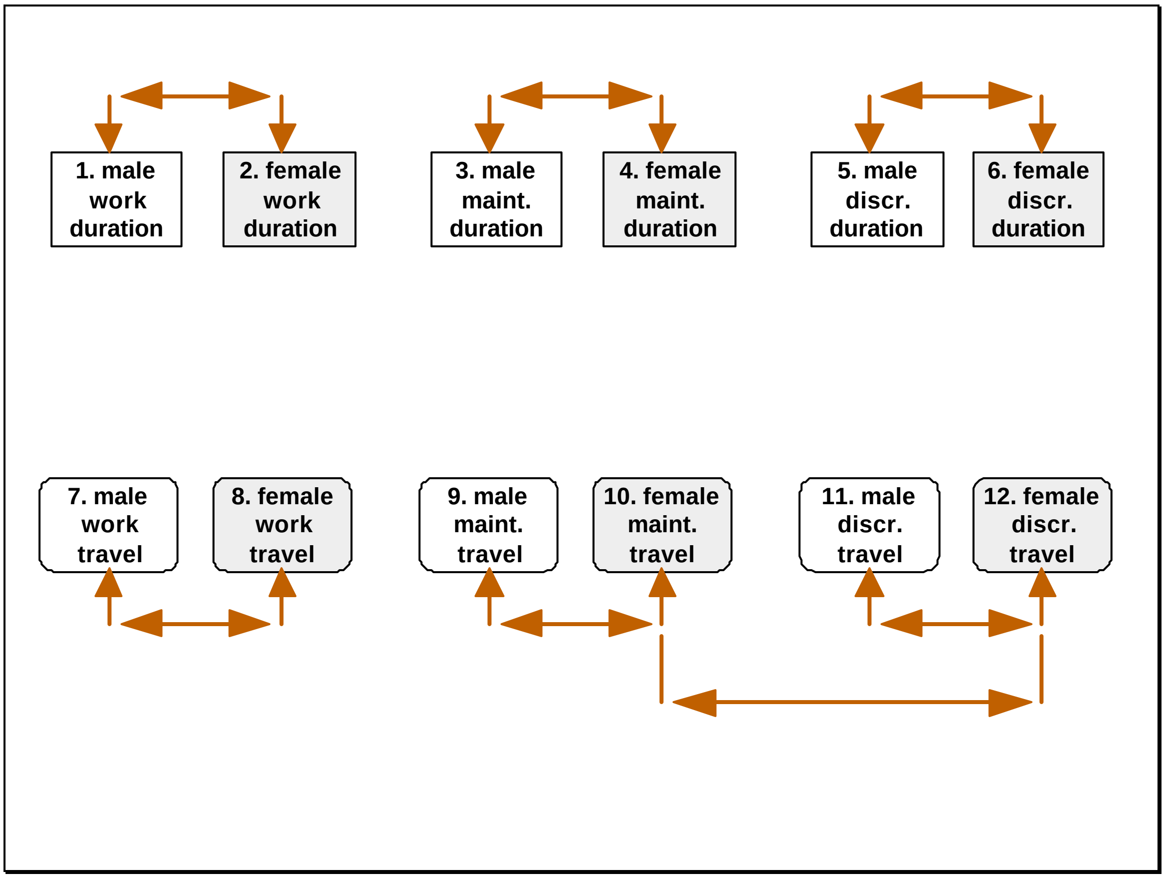
2.4.4 Cross-person Interactions

The model is designed to capture many different kinds of interactions between household heads in terms of their activity participation and travel. As in the case of additional within-person links involving travel (Section 4.5.3), we feel that the best way to find if the data support any of these links is to first establish the simplest theoretically acceptable model, and then to test for significant missing cross-person links. However, we initialize the procedure by postulating one cross-person link: from male work duration to female maintenance duration (the arrow from variable 1 to variable 4 in Figure 1). We expect that male participation in out-of-home work activities will have a positive effect on the out-of-home maintenance activities of female heads, as observed by van Wissen (1989).

2.4.5 Error-Term Covariances

We also expect that certain endogenous variable error terms for male and female would be positively correlated. Such covariance would be induced by joint participation in activities and travel, or by unexplained spatial and lifestyle factors. We postulate non-zero (free) error-term covariance parameters (elements of the Ψ matrix in Equation System 1) for the six pairs of male / female activity and travel variables, as depicted in Figure 2. In addition, exploratory analysis leads us to believe that the unexplained portions of female maintenance and discretionary travel will be correlated, so we specify an additional free error term correlation parameter between these two endogenous variables, as shown in Figure 2. If this specification is correct, we should find that each of these seven error-term covariance parameters is significantly greater than zero.

Figure 2: Flow Diagram Postulated Error-term Covariances for Activity Participation and Travel for Paired Male and Female Heads of Households
(Model endogenous variables are represented by boxes. Top row variables are activity durations and bottom row variables are travel times. Variables for the female head are shaded. Double-headed Arrows Represent Specified Free Error-Term Covariances)



3. PORTLAND DATA

The data to test our model are drawn from the Portland, Oregon *1994 Activity and Travel Survey*. The survey contained coordinated revealed and stated preference components. The revealed preference component utilized in this paper included a two-day (sequential) activity diary recording all activities involving travel and all in-home activities with a duration of at least 30 minutes, for all individuals in the household. A full range of household and person socio-economic data are also included.

The original usable sample contained 2,230 households with 5,120 individuals. To investigate the identified research hypotheses, a sub-sample was drawn of 1,318 married or unmarried adult couples (18 years or older) living in the same residence. After preliminary processing for missing data, a final sample of 1,292 couples was identified. No restriction was placed on other socio-economic characteristics (such as employment status, activity performance, etc.). The activity diaries for the identified couples were processed for both travel days yielding person summaries of total non-home activity duration and total travel time to the non-home activity.

The constructed dataset provides a socio-economic profile of the couple and their household. Summary statistics were appended to capture the effects of other household members (such as number of licensed drivers, distribution of children, and number of employees). The duration and travel time data represents a snapshot of activity participation for the couple only over a sequential two-day period.

A descriptive analysis of the travel and activity participation of these couples is provided in Table 2. A general assessment of the behavior of the couples suggests that the female performs more activities, particularly in-home, and travels more. Furthermore, the average chain complexity for females is greater (defined as average number of sojourns per chain). The total number of activities comprises both out-of-home (sojourns) and in-home activities as well as return home activities, the latter is approximately equal to the number of (home-based) chains. Furthermore, not all reported out-of-home activities required travel, since multiple activities were reported at single destinations.

Through exploratory analyses, we determined that many candidate exogenous variables did not contribute to explanation of the endogenous variables listed in Table 1. In most instances, lack of explanatory power can be attributed to collinearity. For example, after taking into account other household characteristics, we found that the dummy variables designating ages of the household heads for all ranges except the youngest group were ineffective in explaining activity participation and travel. The final set of effective exogenous variables is listed in Table 3.

Table 2: Summary of Activities and Travel in Portland over Two Days for Paired Male and Female Household Heads

	Males (n = 1292)		Females (n = 1292)		Difference in means	
	Mean	Std. Dev.	Mean	Std. Dev.	t-value	*p*
Total Activities (1)	12.80	4.60	14.06	5.05	6.59	0.000
· in-home	4.83	3.99	5.66	4.20	5.12	0.000
· out-of-home	4.98	3.01	5.35	3.33	2.98	0.003
Total Trips	7.36	3.79	7.91	4.42	3.38	0.001
Home-based Chains	2.69	1.37	2.80	1.53	2.03	0.043

(1) Total activities include return home activities (which are approximately equal to the number of chains) as well as in-home and out-of-home activities.

Table 3: The Portland Exogenous Variables

Exogenous Variable	Abbreviation
Number of children under 6 years of age	children 0 - 5
Number of children 6-11 years of age	children 6 - 11
Number of children 12-21 years without driver license	non-drivers 12+
Number of children 12-21 years with driving license	driving children
Number of employed persons in household	number of workers
Number of household vehicles	number of vehicles
Household vehicles per driver	vehicles per driver
Household in current home 1 year or less (dummy)	tenure ≤ 1 yr.
Household is renting (dummy)	household is renting
Male head has a driving license (dummy)	male is driver
Female head has a driving license (dummy)	female is driver
Male head less than 26 years of age (dummy)	male is < 26 yrs.
Household Income less than $20,000 (dummy)	income low
Household income $20,000 to $40,000 (dummy)	income mid-low
Household Income $60,000 or more (dummy)	income high

4. RESULTS

The model was estimated using the two methods -- normal-theory maximum likelihood (ML), and arbitrary distribution function weighted least squares (ADF-WLS) -- described in the Appendix. We compare the results using the ML and ADF-WLS estimation methods. The results using the ML method applied to the variance-covariance matrix are presented first because the coefficients can be more easily interpreted.

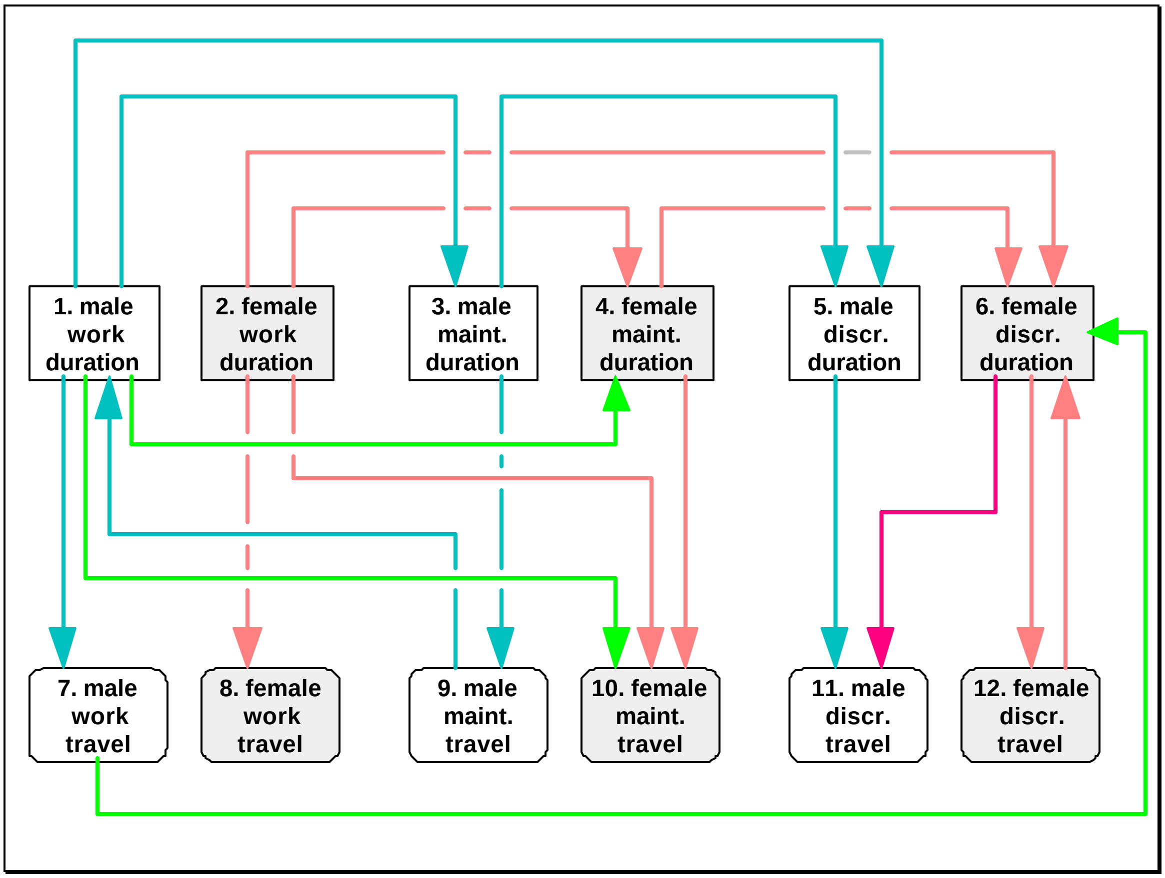
4.1 Endogenous Variable Causal Structure

A model was estimated using the normal-theory ML method applied to the postulated endogenous variable structure (Figure 1) and the best possible exogenous structure. The chi-square value for this model was 228.05 with 194 degrees of freedom (df). This likelihood ratio test statistic is associated with the null hypothesis that the estimated model is consistent with the observed sample variance-covariance matrix. The value of 228.05 with 194 df corresponds to a probability value of $p =$ 0.047, indicating that the model *can* be rejected at the $p = 0.05$ level. Following a series of hierarchical hypothesis tests, we determined that the model could be significantly improved by adding six additional direct effects between endogenous variables. Each of these effects individually improve the model, as measured by differences in chi-square values, and the six effects together reduce the model chi-square value by 79.33 with df = 6, which is a highly significant improvement ($p < 0.0001$).

The final model log-likelihood ratio chi-square of 148.72 with df = 188 corresponds to a probability value of $p = 0.984$. The model definitely cannot be rejected. The adjusted goodness-of-fit Index (AGFI) is a measure of the relative amount of the sample variances and covariances that are predicted by the model, adjusted for the degrees of freedom of the model relative to the number of variables (Bollen, 1989). This index is useful for comparing models fit on the same sample. The AGFI for the model estimated using the ML method is 0.967.

The final model endogenous structure is depicted in Figure 3. This flow diagram has nineteen direct effects, with the six new effects added to the thirteen postulated effects of Figure 1. The estimated coefficients and their *t*-statistics are listed in Table 4. We interpret these results in terms of the four types of effects distinguished in Section 4.5.

Figure 3: Flow Diagram of Final Model Direct Effects Between Activity Participation and Travel for Paired Male and Female Heads of Households
(Model endogenous variables are represented by boxes. Top row variables are activity durations and bottom row variables are travel times. Variables for the female head are shaded. Estimated direct effects are represented by arrows.)



4.1.1 Travel Requirements of Out-of-home Activities

Work activities: Hypothesis test reveal that the direct effects representing travel as a derived demand for work activities are the same for men and women in Portland, the joint coefficient being estimated to be 0.047 (t = 26.9). This translates into 22.6 minutes of travel per eight hours of out-of-home work activity.

Maintenance activities: The direct effects representing travel as a derived demand for maintenance activities are also the same for men and women household heads in Portland, the coefficient being 0.130 (t = 21.1). One hour of out-of-home maintenance activity requires on the average 7.8 minutes of travel time. This effect is also estimated efficiently. As expected, the maintenance activity effect is greater than the work activity effect (by a factor of 2.8).

Table 4: Direct Effects Between the Endogenous Variables - ML Estimation
(*t*-statistics in parentheses)

Effect on:	Causal Variable											
	male work act.	female work act.	male maint. act.	female maint. act.	male discr. act.	female discr. act.	male work travel	female work travel	male maint. travel	female maint. travel	male discr. travel	female discr. travel
male work act.									-1.72 (-4.19)			
female work act.												
male maint. act.	-.078 (-5.4)											
female maint. act.	0.033 (2.23)	-.122 (-8.60)										
male discr. act.	-.148 (-11.3)		-.168 (-4.77)									
female discr. act.		-.148 (-11.3)		-.168 (-4.77)			-.533 (-2.94)					-.808 (-2.35)
male work travel	0.047 (26.9)											
female work travel		0.047 (26.9)										
male maint. Travel			0.130 (21.1)									
female maint. Travel	0.011 (3.97)	-.0154 (-4.93)		0.130 (21.1)								
male discr. Travel					0.092 (16.2)	0.045 (4.94)						
female discr. Travel						0.142 (14.4)						

Discretionary activities: In contrast to work and maintenance activities, in the Portland case study, the equality of male and female direct effects for travel as a derived demand for discretionary activities is rejected. One hour of out-of-home maintenance activity requires on the average 5.5 minutes of travel time for male heads of household, but a similar hour of discretionary activity requires 8.5 minutes of travel for female heads of household. Reasons for this gender difference could be: (1) the average duration of discretionary activities is less for females, requiring more travel per minute of activity; (2) females are less able to link discretionary and other activities; or (3) the destinations of discretionary activities are further from home for female household heads. Further analyses could be conducted to test these hypotheses.

4.1.2 Within-person Activity Interactions

A hierarchy of activities for both men and women in Portland was successfully captured by the negative direct effects between work activity participation and participation in each of the other two types of activities, and between maintenance activity participation and discretionary activity participation. The links to participation in discretionary activities are equal for men and women, being -0.148 (t = -11.3) from work to discretionary and -0.168 (t = -4.8) from maintenance to discretionary. That is, an hour of work activities reduces discretionary activities by about nine minutes, and an hour of maintenance activities reduces discretionary activities by about ten minutes.

However, gender differences were found in terms of the effects of work activities on maintenance activities. Maintenance activities of the female household head are more sensitive to work activities. In conventional gender roles, the responsibility for many household maintenance activities (such as shopping, child care, and cleaning) is assumed primarily by the female. Males, in general, spend a significantly greater amount of time in work activities. When the female increases work participation significantly, a stronger shift away from maintenance activities would be anticipated and maintenance would tend to become more balanced between partners. A second potential effect involves a parallel increase in work hours and a decrease in household maintenance hours for the female due to life cycle changes (such as children leaving home) which would reduce the female's maintenance responsibilities.

4.1.3 Within-person Travel Interactions

The final model has three cross-activity effects involving interactions between travel and activity duration for the same person. One effect is a negative link from maintenance travel to the duration of work activities for males. Male heads with longer maintenance travel exhibit a lower level of work activities, *ceteris paribus*. The coefficient is -1.72, indicating that an increase of ten minutes in maintenance travel time is associated with in a decrease of 17 minutes in the duration of work activities.

A second significant negative direct effect is from female work duration to female maintenance travel. Not only does increased work participation of the female decrease maintenance participation (the effect noted in Section 4.1.2), but, maintenance travel is also reduced over and above the level predicted by the reduction in the participation in maintenance activities. This possibly captures trip chaining and the substitution of more convenient maintenance locations by working women.

The final effect of this type is the negative feedback from discretionary travel to discretionary activity duration for female household heads. This asserts that women who travel further to access discretionary activity sites will exhibit a lower level of activity participation than otherwise expected. The forecasting implication is that increasing accessibility to activity sites will induce latent demand for activity participation for females. The coefficient of slightly less than one is consistent with a total time budget on combined discretionary activity duration and travel time.

4.1.4 Cross-person Interactions

Participation by the male head in out-of-home work activities has a significant positive effect on the female head's participation in maintenance activities, with approximately two minutes of maintenance time being associated with each hour of work. An additional positive link was found between male work activities and female maintenance travel. This means that the total effect on the female's maintenance travel is greater than that captured by the path from work activity through maintenance activity alone. If the male increases his participation in work activities, the model predicts that the female's travel for maintenance activities will increase more than proportionally to the increase in the female's participation in maintenance activities.

We also found a significant negative effect from male travel for work activities to female participation in discretionary activities (the coefficient is -.533; t = -2.9). This predicts that if the male can save work travel time, the female will increase discretionary time by approximately half the amount saved, *ceteris paribus*. Finally, a direct link was needed in the model from female participation in discretionary activities to male travel time to discretionary activities. This may capture trip chaining phenomena.

4.1.5 Error-term Covariances

The postulated error-term covariances (Figure 2) are statistically significant. The estimated error-term covariances (and their corresponding *t*-statistics) are shown in Table 5. We also show standardized versions of the coefficients, estimated using the correlation matrix rather than the variance-covariance matrix. The error terms of male and female maintenance activities and those of male and female discretionary activities are the most strongly correlated, followed by male and female discretionary travel, followed by maintenance travel. The error terms of the work activity and travel variables are the least correlated. These results capture the anticipated effects of joint maintenance and discretionary activity participation and joint travel in pursuit of discretionary activities. Household location and life style factors are also potential sources of these results.

Table 5: Error-term Covariances - ML Estimation

Error-term covariance between	and	Coefficient	Correlation (Std. coeff.)	*t*-statistic
male work activity	female work activity	4.26	0.077	2.58
male maint. Activity	female maint. activity	3.23	0.420	10.4
male discr. Activity	female discr. activity	7.02	0.572	11.0
male work travel	female work travel	0.016	0.052	2.14
male maint. Travel	female maint. travel	0.107	0.213	7.60
male discr. Travel	female maint. travel	0.172	0.367	12.6
female maint. Travel	female discr. travel	0.027	0.059	3.06

4.1.6 Total Effects Between the Endogenous Variables

The total effects between the endogenous variables, computed according to Matrix Equation 2, are listed in Table 6. A comparison between these total effects and the direct effects in Table 4 shows how work activities affect travel for maintenance and discretionary activities through the activity hierarchy paths in combination with the links from each activity to its travel. Specifically, a stronger male participation in out-of-home work activities leads to: (1) a decrease in male maintenance and discretionary activities and travel, (2) an increase in female maintenance activities and travel, and (3) a decrease in female discretionary activities and travel. This is consistent with the findings of van Wissen (1989). On the other hand, a stronger female participation in out-of-home work activities leads to: (1) a decrease in female maintenance and discretionary activities and travel, (2) a decrease in male discretionary travel, but (3) no changes in male maintenance and discretionary activities. These are important gender role differences.

The total effects arising from three of the travel times have potentially important policy implications. First, the model predicts that, if the travel time to work is reduced for male heads of household, perhaps through improvements in traffic flow, the transportation infrastructure, or decreased separation of home and workplace, the female head will increase her duration of discretionary activities and her discretionary travel time. But the male will increase only his discretionary travel time, perhaps by traveling further to more desirable destinations. This supports the concept of the interactions of time budgets within the household setting. Similarly, if the female's discretionary travel time is reduced, perhaps through improved accessibility to activity sites, the female head will increase her duration of discretionary activities and her discretionary travel time, and the male head will increase his discretionary travel time. The discretionary activity time of female heads of household is very sensitive to travel times.

Table 6: Total Effects Between the Endogenous Variables - ML Estimation
(*t*-statistics in parentheses)
(Effects significant at the p = .05 two-tailed level shown in bold)

effect on	Influencing Variable											
	male work act.	female work act.	male maint. act.	female maint. act.	male discr. act.	female discr. Act.	male work travel	female work travel	male maint. travel	female maint. travel	male discr. travel	female discr. travel
male work act.	**0.018 (4.21)**		**-.228 (-4.13)**						**-1.75 (-4.14)**			
female work act.												
male maint. act.	**-.080 (-5.3)**		**0.018 (4.21)**						**0.137 (4.19)**			
female maint. act.	**0.034 (2.23)**	**-.122 (-8.61)**	**-.008 (-1.82)**						-.058 (-1.82)			
male discr. act.	**-.138 (-10.7)**		**-.137 (-3.83)**						**0.237 (3.76)**			
female discr. act.	**-.028 (-3.46)**	**-.115 (-9.91)**	**0.006 (2.58)**	**-.150 (-4.78)**		**-.103 (-2.32)**	**-.533 (-2.94)**		**0.048 (2.59)**			**-.725 (-2.65)**
male work travel	**0.048 (26.8)**		**-.011 (-4.08)**						**-.083 (-4.10)**			
female work travel		**0.047 (26.9)**										
male maint. travel	**-.010 (-5.21)**		**0.133 (20.9)**						**0.018 (4.21)**			
female maint. travel	**0.016 (4.56)**	**-.031 (-8.81)**	**-.004 (-2.91)**	**0.130 (21.1)**					**-.027 (-2.91)**			
male discr. travel	**-.014 (-9.15)**	**-.005 (-4.76)**	**-.012 (-3.63)**	**-.007 (-3.56)**	**0.092 (16.2)**	**0.040 (5.56)**	**-.021 (-2.54)**		**0.024 (3.67)**			**-.032 (-1.96)**
female discr. travel	**-.004 (-3.36)**	**-.016 (-9.32)**	**0.001 (2.45)**	**-.021 (-4.68)**		**0.128 (24.3)**	**-.068 (-2.92)**		**0.007 (2.54)**			**-.103 (-2.32)**

Maintenance travel time by male heads is a different story. The model predicts that males trade off maintenance travel time with participation in out-of-home work activities. Thus a decrease in travel time for maintenance activities will lead through work activities to decreases in maintenance and discretionary activities.

4.2 Exogenous Variable Effects

The estimated structural parameters in the gamma matrix of Equation System (1) give the direct effects of the exogenous variables on the endogenous variables. However, interpretation of influences of the exogenous variables is approached more effectively in terms of the total effects of the exogenous variables on the endogenous variables. These effects, the coefficients of the reduced-form equations, are calculated according to Matrix Equation 3; they are listed in Table 7.

Table 7: Total Effects of the Exogenous Variables - ML Estimation
(*t*-statistics in parentheses)

exog. var.	Endogenous Variable											
	male work act.	female work act.	male maint. act.	female maint. act.	male discr. act.	female discr. Act.	male work travel	female work travel	male maint. travel	female maint. travel	male discr. travel	female discr. travel
children 0 - 5	1.06 (2.16)	-1.37 (-3.05)	-.083 (-2.00)	0.203 (3.26)	-.144 (-2.12)	-.091 (-.5)	0.131 (2.94)	-.065 (-3.03)	-.105 (-2.36)	0.109 (2.61)	-.017 (-1.57)	-.013 (-.5)
children 6 - 11	1.04 (2.10)	-.918 (-2.04)	-.082 (-1.95)	0.147 (2.45)	-.141 (-2.06)	0.077 (1.36)	0.050 (2.10)	-.044 (-2.04)	-.011 (-1.95)	0.045 (2.77)	-.010 (-1.27)	0.011 (1.36)
non-drivers 12+	1.70 (3.00)	-1.07 (-2.11)	-.134 (-2.61)	0.187 (2.65)	-.231 (-2.89)	0.028 (0.4)	0.181 (3.61)	-.051 (-2.11)	-.140 (-2.95)	0.060 (3.16)	-.020 (-2.25)	0.004 (0.4)
driving children		-2.02 (-2.44)		0.247 (2.34)		0.232 (2.37)		-.096 (-2.43)		0.063 (2.35)	0.010 (2.17)	0.033 (2.36)
number of workers	3.59 (10.9)	4.30 (13.7)	-.281 (-4.75)	-.406 (-4.99)	-.486 (-7.65)	-.239 (-1.89)	0.170 (10.1)	0.238 (10.2)	-.037 (-4.67)	-.079 (-3.94)	-.055 (-5.82)	-.034 (-1.87)
number of vehicles						-.031 (-1.90)	0.066 (2.47)	-.072 (-2.57)			-.001 (-1.77)	-.005 (-1.88)
vehicles per driver						0.409 (1.74)		0.166 (3.27)		0.085 (1.73)	0.018 (1.65)	0.058 (1.73)
tenure ≤ 1 yr.		-1.76 (-1.95)		0.214 (1.91)		0.114 (0.94)		-.083 (-1.95)		0.055 (1.91)	0.005 (0.96)	0.138 (2.37)
household is renting						0.708 (2.37)					0.032 (2.12)	0.101 (2.33)
male is driver	1.07 (2.62)		-.084 (-2.63)	0.035 (1.60)	-.145 (-2.51)	-1.84 (-2.43)	0.051 (2.60)		-.621 (-3.37)	-.383 (-2.36)	-.095 (-2.53)	-.262 (-2.43)
female is driver	-.196 (-1.57)		0.875 (1.68)	1.73 (3.16)	-.118 (-1.53)	-.135 (-1.15)	-.261 (-2.44)		0.114 (1.67)	0.504 (3.98)	-.017 (-1.59)	-.019 (-1.14)
male is < 26 yrs.						-.375 (-1.76)					0.671 (3.00)	0.465 (2.32)
income low												
income mid-low						0.075 (1.49)					0.003 (1.33)	-.093 (-1.80)
income high								-.003 (-1.81)				

Some of the strongest total exogenous effects are traceable to the number of children in the household. As expected, the number of children in any of the youngest three age categories is related to the substitution of work and maintenance activities between the male and female heads of the household, confirming the results of Pas (1984) and van Wissen (1989). In contrast, the number of children in the household with driving licenses is not related to the male's activities; but it is negatively related to the female head's participation in work activities and positively related to her participation in both maintenance and discretionary activities. This indicates that household activity interactions change when children become drivers.

Household tenure is related to discretionary activity patterns. Female heads of households that have resided in their current home one year or less exhibit travel more for discretionary activities, *ceteris paribus*. This could reflect either ties to activity sites at a previous residential location or search behavior at the new location.

Female heads of household that are renting have a longer duration of discretionary activities, and both male and female heads of such households spend more time traveling to discretionary activities. This defines a life style. Similar travel behavior for discretionary activities is found for male and female heads in households where the male head is under 26 years of age. However in such young households the female head participates marginally less in discretionary activities. The effects of income are weak.

4.3 Tobit Model Comparison

We next compared the linear model to a model that treats the activity and travel durations as censored (Tobit model). The Tobit model was estimated using the ADF-WLS method. This model is based on a replication of the correlation coefficients of the normally distributed latent endogenous variables. Thus, we must compare the Tobit model results to a standardized linear model estimated using the ML method applied to the observed variable correlation matrix, rather than the observed variable variance-covariance matrix. The Tobit model was specified to have the same parameter structure as the previously described ML linear model.

The Tobit model chi-square value is 286.21 with 188 degrees of freedom, which corresponds to a probability value of p = .00. The chi-square value for the standardized linear model with the same structure is 153.20 with the same number of degrees of freedom (p = .97). However, by optimizing the exogenous effects (the structure of the G matrix) it was possible to establish a Tobit model with a chi-square value of 211.30 with 190 degrees of freedom, corresponding to p = .14. It was not necessary to modify the structure of effects between the endogenous variables. The adjusted goodness-of-fit index (AGFI) for the optimal Tobit is 0.996. The AGFI for the standardized linear model estimated using the ML method is 0.966. Thus, the chi-square value for the Tobit model is lower, but neither model can be rejected, and the Tobit model is equal or better than the linear model on the AGFI measure of fit.

The estimated direct effects between the endogenous variables for the two models are compared in Table 8. There is good correspondence between the coefficients of the effects emanating from the first two endogenous variables: male work activities and female work activities. However, these coefficients in general are estimated with more precision in the Tobit model, where the *t*-statistics are actually asymptotic *z*-statistics. With regard to the remaining direct effects, the ML estimates are substantially higher than the ADF-WLS estimates, particularly for effects involving male discretionary travel and female discretionary travel. The work activity variables are distributed with heavy bimodality, while the discretionary activity variables are heavily skewed. It appears that the skewed variables are more sensitive to differences in estimation treatment. Only one effect, the feedback from discretionary travel to discretionary activities for females, is significant in the linear model but insignificant in the Tobit model..

Table 8: Comparison of Direct Effects Between the Endogenous Variables: Standardized Linear Model (ML estimation) versus Tobit Model (ADF-WLS estimation)

Direct Effect		Std. Linear Model		Tobit Model	
From	To	Coeff.	t-stat.	Coeff.	t-stat.
1.male work act.	3.male maint. act.	-.225	-5.28	-.258	-8.24
1.male work act.	4.female maint. act.	0.091	2.25	0.068	2.24
1.male work act.	5.male discr. act.	-.314	-11.34	-.340	-14.81
1.male work act.	7.male work travel	0.618	26.87	0.806	48.04
1.male work act.	10.female maint. travel	0.123	3.94	0.125	5.29
2.female work act.	4.female maint. act.	-.303	-8.61	-.273	-10.90
2.female work act.	6.female discr. act.	-.314	-11.34	-.340	-14.81
2.female work act.	8.female work travel	0.618	26.87	0.806	48.04
2.female work act.	10.female maint. travel	-.161	-5.10	-.131	-5.91
3.male maint. act.	5.male discr. act.	-.134	-4.83	-.097	-4.63
3.male maint. act.	9.male maint. travel	0.511	21.08	0.598	27.3
4.female maint. act.	6.female discr. act.	-.134	-4.83	-.097	-4.63
4.female maint. act.	10.female maint. travel	0.511	21.08	0.598	27.33
5.male discr. act.	11.male discr. travel	0.490	16.21	0.702	20.96
6.female discr. act.	11.male discr. travel	0.205	4.89	0.094	3.32
6.female discr. act.	12.female discr. travel	0.693	14.33	0.773	28.40
7.male work travel	6.female discr. act.	-.110	-3.05	-.094	-3.19
9.male maint. travel	1.male work act.	-.155	-4.12	-.087	-3.15
12.female discr. travel	6.female discr. act.	-.159	-2.28	-.090	-1.10

We draw three conclusions from this: First, the endogenous structure is relatively independent of the estimation method, while the exogenous structure (which we are unable to show in detail) is dependent on the estimation method. Second, however, the actual coefficient estimates for endogenous variables with heavily skewed distributions are dependent on the estimation method. Third, ML estimation assuming linear endogenous variables overstates the overall model goodness-of-fit chi-square statistic, but potentially also overstates the standard errors of the parameters. Whenever sample size permits, the Tobit model with ADF-WLS estimation should be used in these structural equations applications.

5. Conclusions and Directions for Further Research

We have applied structural equations to simultaneously model the behavior of the male and female heads of household in terms of their activity participation and travel. Several conclusions can be drawn from this exercise.

5.1 Planning Implications

This research has validated hypothesized interactions within households and has identified additional interactions as part of an overall model structure which relates activity participation and travel behavior of household heads. Relationships have been established between the amount of time allocated to work, maintenance, and discretionary activity, and to the travel generated by each activity. The interactions between male and female household heads are modeled endogenously; effects due to the presence of other household members are introduced exogenously. The implication of these results is clear: a feedback mechanism should be introduced in trip generation models to reflect the effect of activity frequency and duration on the level of associated travel. For example, households which have longer commutes should have compensatory reductions in the frequency and duration of participation in and travel to other types of activities. Indeed, Purvis, *et al*., (1995) have recently demonstrated how standard transportation models can be modified to account for such travel time feedback effects. The present research is currently being extended by the authors to explicitly model household vehicle travel distance within the model system.

5.2 Modeling Potential

The general model format can be extended along five directions. First, we can expand or modify the number of types of out-of-home activities. One potential reclassification is to break what we called “maintenance” and “discretionary” activities into more than two subsets, perhaps on the basis of household versus personal activities.

Second, we can expand the model to cover in-home as well as out-of-home activities. This must be done carefully with the Portland Survey dataset, because reported in-home activities are limited to those of one-half hour or more. However, we are motivated by the potential explanation of conditions under which there is substitution between in-home and out-of-home activities, and we place a high priority on extending the model to include in-home activity participation.

Third, travel in the models can be separated by mode. Since it is unlikely that we will find enough public transport travel in a metropolitan area such as Portland, in the present dataset we might be able to distinguish private vehicular travel versus all else (mainly pedestrian) or solo driving versus multiple occupancy.

Fourth, we can increase the number of individuals that we are modeling simultaneously. The obvious choice is to add the activity and travel behavior of children. The possibilities here are highly data dependent. In our sample, approximately half of the 1292 household with male and female heads had at least one other household member, usually a child. Even if we focused only on households with older children, the sample size should be sufficient for the estimation of a linear (ML method of estimation) model with three individuals. Model structures with four or more individuals are problematic, because it would be difficult to retain a sufficient ratio of sample size to the number of estimated free parameters.

Finally, we can add exogenous environmental variables into the models. Such variables could include, but would not be limited to, accessibility indices, level of service indices, and dummy variable representing different residential areas.

5.3 Data Requirements

We also need to consider the issue of weekday versus weekend behavior. There are obviously substantial differences in activity participation behavior across Saturdays, Sundays, and weekdays. Moreover, there might be important differences across weekdays, with particular emphasis on Fridays. In the past, trip diary surveys have often ignored weekends altogether, with their concentration on commuting behavior. In modern travel behavior research we have expanded our focus to weekends as well as weekdays, and now we face trade-offs regarding sampling across days in surveys with multi-day diaries.

The data we used consisted of two-day activity diaries. In this type of modeling there is a clear trade-off between the level of disaggregation of the types of activities and the number of diary days. With more days, we can focus on a finer distinction of activities, because more households will be observed participating in each of the activities over the course of the diary period. A test of the existing model with one versus two diary days is a useful exercise for further research. We might be able to extrapolate results of the one-day versus two-day comparison to assess the advantages of more than two days.

For the Portland Survey data, there are a substantially different number of households beginning their two-day diaries on certain days of the week since the sampling scheme favored weekdays over weekends. Since a “full week sample” was used, Saturdays and particularly Sundays are under-represented in our sample due to the survey favoritism toward weekdays. We should correct for this bias by either re-weighting the sample or creating a smaller sample by randomly selecting an equal number of households from each starting day strata. This is presently relegated to the realm of future research.

In order to study weekday behavior in more detail, we might wish to limit our sample to only those households with diaries on two weekdays. Here we have a problem, even in light of the relatively even distribution over weekday starting days in the Portland Survey. Because Sunday-Monday and Friday-Saturday combinations would be eliminated in a sample limited to two weekdays, the three middle weekdays would carry a weight of approximately twice that of Mondays and Fridays in a "weekday only" two-day sample. If we strive to understand weekend as well as weekday activity and travel behavior, we should over-sample, rather than under-sample, weekend days. This goes against our inherent bias to concentrate our efforts on modeling the most repetitive travel behavior.

5.4 Methodology

Results show that, for accurate assessment of goodness of fit and standard errors, we need to treat activity participation and travel variables in structural equation models as censored (Tobit) variables and use the arbitrary distribution function, weighted least squares (ADF-WLS) estimation method (also known as the asymptotically distribution free, weighted least squares method). However, this method requires a greater sample size, due to its asymptotic properties and the need to use a matrix consisting of computed fourth-order moments. Fortunately, we showed that the main conclusions in our model are consistent between a linear model estimated using the normal-theory maximum likelihood (ML) method and the a Tobit model estimated using ADF-WLS. Our conclusion is that, in situations where low sample size prevents application of the Tobit model, the linear model will provide decent approximation.

Multi-group modeling is one application in which the linear model will likely be the only alternative. In multi-group structural equations modeling, the matrices in Equation System 1 are partitioned along a third segmentation variable. The default form of the model postulates that all structural parameters in the B and G matrices are equal across all segments. The equality restrictions are released where warranted by significant improvements in model goodness-of-fit. Multi-group modeling is a particularly powerful technique for finding statistically significant interactions between individual segmentation groups and structural parameters.

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APPENDIX: ESTIMATION METHODOLOGY

Structural equations systems are generally estimated using methods of moments (also known as variance analysis methods). These methods proceed by defining the sample variance-covariance matrix of the combined set of endogenous and exogenous variables, partitioned with the endogenous variables first:

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{\mathbf{yy}} & \mathbf{S}_{\mathbf{yx}} \\ \mathbf{S}'_{\mathbf{yx}} & \mathbf{S}_{\mathbf{xx}} \end{bmatrix}, \qquad \text{(A1)}$$

where $\mathbf{S}_{yy}$ denotes the variance-covariance matrix of the (n) endogenous variables, $\mathbf{S}_{yx}$ denotes the covariance matrix between the endogenous and exogenous variables, and $\mathbf{S}_{xx}$ denotes the variance-covariance matrix of the (m) exogenous variables. In our model, there are m = 12 endogenous variables and n = 15 exogenous variables, so $\mathbf{S}$ is a (27 by 27) symmetric matrix.

It can be shown using matrix algebra that the corresponding variance-covariance matrix replicated by model system 1 with parameter vector q (denoting all parameters in the B, G, and Y matrices) is:

$$S(q) = \begin{bmatrix} S_{\mathbf{yy}} & S_{\mathbf{yx}} \\ S'_{\mathbf{yx}} & S_{\mathbf{xx}} \end{bmatrix}, \qquad \text{(A2)}$$

where:

$$S_{yy} = (\mathbf{I} - B)^{-1}(G\,\mathbf{S}_{xx}G' + Y)\left((\mathbf{I} - B)^{-1}\right)', \qquad \text{(A3)}$$

$$S_{yx} = (\mathbf{I} - B)^{-1}G\,\mathbf{S}_{xx}, \qquad \text{(A4)}$$

where S_{xx} = S_{xx} is taken as given, defining the variables that are exogenous.

The parameters of the B, G, and Y matrices are estimated by making $S(q)$ be as close as possible to $\mathbf{S}$. There are several estimation methods available, two of which are applied herein: normal-theory maximum likelihood (ML) and arbitrary distribution function weighted least squares (ADF-WLS), also known as asymptotically distribution free weighted least squares.

Normal-theory Maximum Likelihood

The structural equation system can be estimated using several different variance-analysis methods, the most common of which is normal-theory maximum likelihood (ML). This method has the advantage of being computationally swift and the sample size requirements are less than for distribution-free methods. Also unlike distribution-free methods, it can be applied to the variance-covariance matrix in which case the parameters are in terms of the original scales of the variables; we find this useful for interpretation purposes. ML estimates will be consistent, and they have been shown to be fairly robust with respect to common deviations from the assumed multivariate normal distribution (Boomsma, 1983).

The fitting function for structural equations maximum likelihood (ML) estimation is:

$$F_{ML} = \mathit{log}\left|\mathrm{S}(\mathrm{q})\right| - \mathit{log}\left|\mathbf{S}\right| + \mathit{tr}\left[\mathrm{S}(\mathrm{q})^{-1}\mathbf{S}\right] - (m+n) \ , \tag{A5}$$

This fitting function is (-2/n times) the log of the likelihood function that **S** is observed if $\mathrm{S}(\mathrm{q})$ is the true multivariate normal variance-covariance matrix. Minimizing F_{ML} is equivalent to maximizing the likelihood function. Under the assumption of multivariate normality, nF_{ML} is chi-square distributed, providing a test of model rejection and criteria for testing hierarchical models.

ADF - WLS Estimation

The univariate distributions of the endogenous variables are non-normal in that there are substantial numbers of observations for each variable with zero value, which denotes no reported participation in an activity type or associated. For such distributions the ML coefficient estimates will be consistent, but the estimates of parameter standard errors and the overall model chi-square goodness-of-fit will likely be biased (Bentler and Bonett, 1980). Unbiased estimates of standard errors and goodness-of-fit can be generated using the ADF-WLS method (Browne, 1982, 1984).

The ADF-WLS estimation method proceeds in three distinct steps. First, it is assumed each observed endogenous variable is generated by an unobserved normally distributed latent variable. If the latent variable is greater than a censoring level, it is observed; otherwise the censoring level is observed. Each latent variable is assumed to be conditional on the other variables in the system. The problem is to determine the conditional unknown mean and variance of each censored latent variable. A maximum-likelihood solution to the problem was apparently first proposed by Tobin (1958) and was subsequently refined by Amemiya (1973) and Fair (1977) as the Tobit model ("Tobin's probit"). The appropriate maximum likelihood estimation procedure is described in Maddala (1983).

The second step in the estimation method is to obtain estimates of the correlations between the latent censored endogenous variables, and the correlations between each of the latent variables and the continuous exogenous variables in the system. For endogenous variable pairs, the problem is to determine the unknown correlation coefficient between the latent variables that maximizes the likelihood of observing the cross-products where below-censoring level observations are assigned normal scores determined by the Tobit model results of the first step of the estimation method. The solution was apparently developed by Des Raj (1953).

The final step in the ADF-WLS method is to estimate the parameters of the structural equation model by making the model-implied correlation matrix as close as possible to the sample correlation matrix, where the sample matrix is determined in the previous steps. The fitting function is then:

$$F_{WLS} = \left[\mathbf{s} - s(q)\right]' \mathbf{W}^{-1} \left[\mathbf{s} - s(q)\right] \ , \tag{A6}$$

where $\mathbf{s}$ is a vector of censored correlation coefficients for all pairs of endogenous and exogenous variables, $\sigma(\theta)$ is a vector of model-implied correlations for the same variable pairs, and $\mathbf{W}$ is a positive-definite weight matrix. Minimizing F_{WLS} implied that the parameter estimates are those that minimize the weighted sum of squared deviations of $\mathbf{s}$ from $\sigma(\theta)$. This is analogous to weighted least squares regression, but here the observed and predicted values are variances and covariances rather than raw observations. The best choice of the weight matrix is a consistent estimator of the asymptotic covariance matrix of $\mathbf{s}$:

$$W = ACOV\left(s_{ij}, s_{gh}\right) . \tag{A7}$$

Under very general conditions:

$$W = \frac{1}{N}\left(s_{ijgh} - s_{ij}\, s_{gh}\right) \tag{A8}$$

is a consistent estimator, where s_{ijgh} denotes the fourth-order moments of the variables around their means, and s_{ij} and s_{gh} denote covariances. Brown (1982, 1984) demonstrated that F_{WLS} with such a weight matrix will yield consistent estimates which are asymptotically efficient with correct parameter z-statistics and correct chi-square test values. These properties hold for very general conditions, and consequently such estimators are known as arbitrary distribution function, or asymptotically distribution free (ADF) estimators.

ADF-WLS estimators are available in several structural equation model estimation packages. We used the LISREL (Versions 8) and PRELIS (Version 2) programs (Jöreskog and Sörbom,1993a and 1993b).