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A MODEL OF COMPARATIVE STATICS FOR CHANGES IN STOCHASTIC RETURNS WITH DEPENDENT RISKY ASSETS

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ABSTRACT

In this paper we show how the order of Greater Central Riskiness proposed by Gollier (199 can be applied to situations with dependent risky assets. This order was shown to be the lea constrained necessary and sufficient condition to guarantee that all risk-averse agents reduce the demand for a risky position when a marginal increase in risk is imposed in a model with o risky asset. Recently Hadar and Seo (1990) and Meyer and Ormiston (1994) have proposed tw models of comparative static changes with more than one risky asset. They obtained that standar conditions on preferences for comparative statics with one random parameter remain valid. He we introduce a different extension by considering restrictions on changes in risk instead c restrictions on utility functions. We show how the concept of Linear Stochastic Dominanc generates the desired results for mean preserving spreads and first order stochastic dominance Strong increases in risk (Meyer and Ormiston, 1985) are examples of mean preserving spread that satisfy the condition. Other examples are discussed.

Key words : Changes in risk, linear stochastic dominance, risky assets, simple increases in risk

RÉSUMÉ

Dans cet article nous montrons comment le concept d'accroissement de risque central proposé par Gollier (1995) peut être appliqué à des situations avec plusieurs actifs risqués dépendants. Ce concept est connu comme étant la condition nécessaire et suffisante la moins restrictive qui garantit que tous les individus riscophobes vont réduire leur demande pour une position risquée lorsque des accroissements de risque marginaux sont imposés dans un modèle avec une seule variable aléatoire. Récemment, Hadar et Seo (1990) et Meyer et Ormiston (1994) ont proposé deux modèles de statique comparative pour des situations avec plus d'une variable aléatoire. Ils ont vérifié que les conditions classiques sur les préférences des modèles avec une variable aléatoire demeurent valides. Ici nous introduisons une extension différente en considérant des restrictions sur les changements de risque plutôt que sur les fonctions d'utilité. Nous montrons que le concept de dominance stochastique linéaire génère les résultats désirés pour des accroissements de risque à moyenne constante et pour des dominances stochastiques de premier ordre. Les accroissements de risque forts (Meyer et Ormiston, 1985) sont des exemples qui satisfont la condition. D'autres exemples sont discutés.

Mots-clefs : changements de risque, dominance stochastique linéaire, actifs risqués, accroissements de risque simples.

1. INTRODUCTION

In a recent article Gollier (1995) proposed the least constrained necessary and sufficient conditior on a change in distribution for signing the effect of a marginal change in risk on the optimal decision variable for all risk averse agents. The model was applied to the class of linear payoffs where the order of "Greater Central Riskiness" was shown to be necessary and sufficient to guarantee that all risk - averse agents reduce their optimal demand for a risky position. The standard portfolio problem is a good example of such application; the coinsurance problem and the competitive firm with constant marginal cost are other examples of linear payoffs[1].

The object of this paper is to extend the above results to problems with two stochastically dependent risky assets. Although these more general problems correspond to many economic applications such as the composition of optimal portfolios in finance or the determination of optimal capital structures (see Dewatripont and Tirole (1994) and Boyd and Smith (1994)), only few authors have analyzed the effect of changes in risk on the optimal decision variables. Hadar and Seo (1990) did a first contribution to the literature by considering general changes in asset returns with risky returns that are independently distributed both before and after one return is altered. They showed that the conditions on preferences proposed by Rothschild and Stiglitz (1971) remain necessary and sufficient with independence. One condition, to obtain the desired result following a mean preserving spread, states that ZU'(Z) be concave in Z where Z is final wealth and U(Z) is a concave utility function. For a first order stochastic dominance ZU'(Z) has to be non decreasing. More recently, Meyer and Ormiston (1994) extended their results to dependent risky returns and showed that the same conditions on utility remain necessary and sufficient when the independence assumption is replaced by stochastic dependence.

In this paper we propose a different extension of Hadar and Seo (1990) contribution by considering restrictions on the set of changes in risk for all risk averse individuals instead of restrictions on utility functions. We show how the order of Greater Central Riskiness can be extended to models with two dependent risky assets. This extension can be applied to mean preserving spreads (MPS) and first-order stochastic dominances (FSD). Its relationship with second order stochastic dominances (SSD) is also discussed.

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[1] See Dionne, Eeckhoudt and Gollier (1993) for an analysis of increases in risk and linear payoffs.

II. MODEL

Consider a risk averse investor with a twice continuously differentiable, strictly increasing and concave von Neuman - Morgenstern utility function of wealth[2] (W); $u'(W) > 0$, $u''(W) < 0$. He must allocate his unit of capital in two risky assets with returns $\tilde{x}$ and $\tilde{y}$. His terminal wealth is then equal to $\tilde{z} = b\tilde{x} + (1 - b)\tilde{y}$ with b being the proportion of funds invested in asset X.

Stochastic dependence is allowed. Therefore, the initial risky situation is described by a joint cumulative distribution of returns $H_1(x,y)$ and its corresponding joint density $h_1(x,y)$. Returns $\tilde{x}$ and $\tilde{y}$ take their values in the respective intervals $[\underline{x}, \bar{x}]$ and $[\underline{y}, \bar{y}]$ and rectangle $[\underline{x}, \bar{x}] \times [\underline{y}, \bar{y}]$ contains the support of $H_1(x,y)$. The problem of the investor is to select a portfolio that maximizes his expected utility :

$$b\,(u, H_1) \in \text{argmax}\ U\,(b; u, H_1) = \int_{\underline{x}}^{\bar{x}} \int_{\underline{y}}^{\bar{y}} u(bx + (1-b)y)dH_1(x,y). \tag{1}$$

Let hereafter b_1 denote $b\,(u, H_1)$.

The objective of the paper is to determine the effect of an increase in portfolio returns distribution of asset x on the optimal portfolio choice. Suppose that $H_1(x,y)$ is replaced by a new distribution $H_2(x,y)$ and denote b_2 as the optimal solution with distribution $H_2(x,y)$ having its support in the rectangle $[\underline{x},\bar{x}] \times [\underline{y},\bar{y}]$, i.e. $b_2 = b(u, H_2)$. The question is then to find the least restrictive sufficient condition which guarantees that all risk averse investors (with no other restriction than risk aversion) will react to the new risky situation by reducing the demand for the asset x, i.e. $b_2 \leq b_1$. Both FSD and MPS will be analyzed formally.

If b_1 is finite[3], it must satisfy the following first - order condition :

$$\frac{\partial U}{\partial b}(b_1) = \int_{\underline{x}}^{\bar{x}} \int_{\underline{y}}^{\bar{y}} u'(b_1x + (1 - b_1)y)\,(x - y)dH_1(x,y) = 0. \tag{2}$$

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[2] Since no restriction will be imposed on the utility function, risk aversion is sufficient to get all the results. However, Hadar and Seo (1990) and Meyer and Ormiston (1994) had to impose that $U'''(W) \geq 0$ to obtain their results for a MPS.

[3] If b_1 equals + infinity, no restriction is required to guarantee that $b_2 \leq b_1$.

By risk aversion the second order condition is satisfied. Then a necessary and sufficient conditic for $b_2 < b_1$, is that the derivative of the objective function with the new distribution $H_2(x,$ evaluated at $b = b_1$ be negative. In other words, $b_2 < b_1$, if and only if :

$$\frac{\partial U}{\partial b}(b_1) = \int_{\underline{x}}^{\bar{x}} \int_{\underline{y}}^{\bar{y}} u'(b_1 x + (1 - b_1)y)\,(x - y)dH_2(x,y) < 0. \qquad (3$$

Before presenting detailed analysis with two random parameters, we first recall some results fror models in the literature with only one risky asset in order to make connection with our results.

III. CHANGES IN RISK WITH A SAFE ASSET AND ONE RISKY PARAMETER

To obtain direct comparisons, we assume, for the moment, that $\tilde{y}$ takes value r with probabilit one, both before and after any change in risk. In the interpretation of the portfolio problem, r i the risk-free interest rate. Equilibrium on capital markets implies that $E_i(x) \geq r$ so that b_i is positiv for $i = 1,2$.

Let $F_i(x)$ denote the cumulative function of x. We first restate the definition of *location - weightec probability mass* proposed by Landsberger and Meilijson (1990). The location - weightec probability mass of the tail $[\underline{x}, x]$ with respect to the distribution F_i around r is given by :

$$T_i(x,r) = \int_{\underline{x}}^{x} (t - r)\, dF_i(t). \qquad (4)$$

$T_i(x,r)$ has two properties that will be useful :

$$T_i(\underline{x},r) = 0,$$

$$\text{and} \quad \frac{\partial T_i}{\partial x}(x,r) = (x-r)F_i'(x)$$

This derivative is continuous in x and its sign is given by (x-r); it is then independent of the distribution function.

This concept ($T_i(x,r)$) is useful for the following definition :

Definition 1 : Consider two random parameters $\tilde{x}_1$ and $\tilde{x}_2$ with respective distribution functions $F_1(x)$ and $F_2(x)$. $\tilde{x}_1$ dominates $\tilde{x}_2$ in the sense of Linear Stochastic Dominance of factor γ for a given r, i.e. $\tilde{x}_1$ $LSD_{\gamma,r}$ $\tilde{x}_2$, if $\exists\gamma$ such that $T_2(x;r) \leq \gamma\, T_1(x,r)$ $\forall x \in [\underline{x},\bar{x}]$. Moreover $\tilde{x}_1$ Centrally Dominates $\tilde{x}_2$ around r, i.e. $\tilde{x}_1$ CD_r $\tilde{x}_2$, if there exists $\gamma \in R$ such that $\tilde{x}_1$ $LSD_{\gamma,r}$ $\tilde{x}_2$

We know the following result :

Proposition 1 : (Gollier (1995)) : Consider the standard portfolio problem with risk-free rate $r < E(\tilde{x}_1)$. All risk-averse investors reduce their demand for the risky asset when its returns undergo a change in distribution from $\tilde{x}_1$ to $\tilde{x}_2$ if and only if $\tilde{x}_1$ CD_r $\tilde{x}_2$.

The examples of CD_r that restrict $\tilde{x}_1$ and $\tilde{x}_2$ to have the same mean are Strong Increases in Risk (SIR, Meyer and Ormiston (1985)), Relatively Strong Increases in Risk (RSIR, Black and Bulkley (1984)) and Relatively Weak Increases in Risk (RWIR, Dionne, Eeckhoudt and Gollier (1993)). The two last examples imply that $\exists\gamma$ such that $\tilde{x}_1$ $LSD_{\gamma,r}$ $\tilde{x}_2$. γ is not necessarily equal to one while it has to be equal to one in SIR. Another definition implying $\gamma=1$ is a Simple Increase in Risk around r (sIR_r, Dionne and Gollier (1992)). Hammond (1974) considered a similar kind of restrictions on increases in risk. He used a single-crossing condition on cumulatives distributions to asses risk aversion. Finally, note that a Second-Degree Stochastic Dominance (SSD) is neither necessary nor sufficient for CD_r (Corollary 2 in Gollier (1995)).

We may also consider another example of CD_r that does not restrict the two distributions to the same mean : $\tilde{x}_1$ dominates $\tilde{x}_2$ in the sense of Strong Linear Stochastic Dominance of factor γ aroud r, i.e. $\tilde{x}_1$ $SLSD_{\gamma,r}$ $\tilde{x}_2$, if and only if

$$F'_2(x) \geq \gamma F'_1(x) \qquad \text{when } x \leq r$$

$$F'_2(x) \leq \gamma F'_1(x) \qquad \text{when } x \geq r$$

or, if and only if

$$\min_{x\leq r} \frac{F'_2(x)}{F'_1(x)} \geq \max_{x\geq r} \frac{F'_2(x)}{F'_1(x)} .$$

To obtain a SLSD, enough probability mass must be transferred from returns greater than r returns less than r. Figure 1 is an example of SLSD :

(Figure 1 about here)

Note that a SLSD with a constant parameter can represent a First-Order Stochastic Dominanc (FSD). This particularity will be important to derive the comparative statics results. The ne proposition establishes the links between SLSD and LSD.

<u>Proposition 2</u> : If $\exists\gamma$ such that $\tilde{x}_1$ $SLSD_{\gamma,r}$ $\tilde{x}_2$, then $\tilde{x}_1$ $LSD_{\gamma,r}$ $\tilde{x}_2$.

Proof : Immediate by observing that $T_2(x,r) - \gamma T_1(x,r) = \int_{\underline{x}}^{x} (t-r)\left(dF_2(t) - \gamma dF_1(t)\right) < 0$ since $(t-r)\left(dF_2(t) - \gamma dF_1(t)\right) \leq 0, \forall t$, by definition of SLSD.

Since SIR and sIR will be used in the next section, we remind their definitions :

SIR (Meyer and Ormiston, 1987) : let the interval $[x_3, x_4]$ be the support of x with distribution F_1.

We say that F_2 is a strong increase in risk of F_1 if and only if

a) the mean is preserved;
b) $F_2(x)$ is always less or equal to $F_1(x)$ inside the support of F_1.

sIR (Dionne and Gollier, 1992) : F_2 is a simple increase in risk across r of F_1 if and only if

a) the mean is preserved;
b) $F_2(x)$ is larger than $F_1(x)$ whenever x is less than r and $F_2(x)$ is less than $F_1(x)$ whenever x is larger than r :

$$\left(F_2(x) - F_1(x)\right)(x-r) \leq 0 \ \forall x \in [\underline{x}, \overline{x}]$$

We now show how these concepts are useful for the comparative statics analysis of changes in risk with dependent risky assets.

IV. INCREASES IN RISK WITH STOCHASTICALLY DEPENDENT ASSETS

We demonstrate in this section how the $LSD_{\gamma,r}$ definition of change in riskiness can be applied to the case of multiple risky assets. We allow for any kind of stochastic dependence between $\tilde{x}$ and $\tilde{y}$. As discussed in detail by Meyer and Ormiston (1994), the main problem that one faces when considering two stochastically dependent variables is to fix the ceteris paribus assumption. Here we use the same assumption that we proposed in our first contribution to the subject (Dionne and Gollier (1992)) and applied by Dewatripont and Tirole (1994) to the choice of optimal capital structure. The central idea is the following : let us define the joint distribution of $\tilde{x}$ and $\tilde{y}$ as $H_i(x,y) = F_i(x|y)\, g_i(y)$ where $F_i(x|y)$, $i = 1,2$, is the distribution of asset x conditional on y and $g_i(y)$ is the marginal density of y. Because we analyze the effect of a change in risk for asset x, we assume that the marginal distribution for asset y remains unchanged, that is $g_1(y) = g_2(y)$. This is the ceteris paribus assumption that we use. Recently, Gagnon (1994) has shown that this assumption implies that the sign of the correlation between $\tilde{x}$ and $\tilde{y}$ remains constant for any mean preserving spread of $\tilde{x}$ that assumes a fix marginal distribution of $\tilde{y}$. See Meyer and Ormiston (1994) for a more detailed discussion of a similar ceteris paribus condition and for an example. We are now ready to prove the following result.

<u>Proposition 3</u> : Assume b_1 be non-negative. Then a sufficient condition for $b_2 \leq b_1$ for all risk-averse investors is that there exists a scalar γ such that $\tilde{x}_1|_y \; LSD_{\gamma,y} \; \tilde{x}_2|_y$ for all y, i.e. $\exists\gamma\, \forall x \in [\underline{x},\overline{x}]$, $\forall y \in [\underline{y},\overline{y}] : T_2(x,y) \leq \gamma T_1(x,y)$.

Proof : From (3), by using the definition of $H_2(x,y)=F_2(x \mid y)g(y)$, we obtain :

$$\frac{\partial U}{\partial b}(b_1) = \int_{\underline{y}}^{\overline{y}} \{\int_{\underline{x}}^{\overline{x}} u'(b_1x+(1-b_1)y)(x-y)dF_2(x|y)\}g(y)dy.$$

Integrating by parts yields

$$\frac{\partial U}{\partial b}(b_1) = \int_{\underline{y}}^{\overline{y}}\Big\{u'(b_1(\overline{x}-y) + y)T_2(\overline{x},y)$$
$$-b_1 \int_{\underline{x}}^{\overline{x}} u''(b_1(\overline{x}-y)+y)T_2(x,y)dx\Big\}g(y)dy.$$

By assumption, we have $T_2(x,y) \leq \gamma T_1(x,y)$ for all x,y. This condition together with the assumpt that b_1 is non-negative yields :

$$\begin{aligned}\frac{\partial U}{\partial b}(b_1) \leq \int_{\underline{y}}^{\bar{y}} \{ & u'(b_1(\bar{x} - y) + y)\gamma T_1(\bar{x} ; y) \\ & - b_1 \int_{\underline{x}}^{\bar{x}} u''(b_1(x-y)+y)\gamma T_1(x ; y)dx\} dG(y) \\ = \gamma \int_{\underline{y}}^{\bar{y}} \{ & u'(b_1(\bar{x}-y)+y)T_1(\bar{x} ; y) \\ & - b_1 \int_{\underline{x}}^{\bar{x}} u''(b_1(x-y)+y)T_1(x,y)dx\} dG(y) \\ = \gamma & \frac{\partial U}{\partial b}(b_1; u, H_1) = 0.\end{aligned}$$

Thus $b_2 \leq b_1$. This completes the proof.

Notice that Proposition 3 provides the weakest condition on the changes in the condition distribution of $\tilde{x}$ that yields the comparative statics property without putting any constraint on th concave utility function, or on the marginal distribution of $\tilde{y}$. Indeed, suppose that for at least on y, there is no γ such that $\tilde{x}_1|y$ $LSD_{\gamma,y}$ $\tilde{x}_2|y$. Then if we assume that the marginal distribution o $\bar{y}$ be degenerated at y, it is a direct consequence of Proposition I that there exists a concav utility function u such that $b(u, H_2)$ is *larger* that $b(u, H_1)$.

Proposition 3 provides the instrument to extend all analyses performed for the one-safe-one-risky-asset model. For example, remember that Strong increases in risk belongs to the larger set o changes in risk that satisfy $LSD_{1,y}$ for any y. Thus, if the changes in the conditional distribution of $\tilde{x}$ all satisfy the SIR condition for all y, Proposition 3 guarantees that the demand for x will be reduced due to these changes. Similarly, if $\tilde{x}_1|y$ dominates $\tilde{x}_2|y$ in the sense of a Simple increase in risk around y, for any y, then the same result holds. Proposition 3 shows how to go beyond these simple results. Indeed, since simple increases in risk around y also belong to the set of $LSD_{1,y}$, an unambigous comparative statics property is also obtained for "combined incrases in risk" performed on the conditional distributions of $\tilde{x}$. We say that $\tilde{x}$ undergoes a combined increase in risk if for any y,$\tilde{x}_1|y$ dominates $\tilde{x}_2|y$ either in the sense of a simple incrase in the risk around y, or in the sense of a strong increase in risk. What is new here is the possibility to mix sIR for some y and SIR for others.

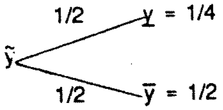
One can also use Proposition 3 for changes in risk that belong to $LSD_{\gamma,y}$, for γ different from unity. For example, if there exists a scalar γ such that all conditional distributions undergo a SLSD of factor γ around the corresponding y, then the comparative statics result holds. Still, we urge the reader to observe that a false extension to the result by Gollier (1995) would be to claim the following result : if for every y, there exists a scalar $\gamma(y)$ such that $\tilde{x}_1|y \;\; LSD_{\gamma(y)y} \;\; \tilde{x}_2|y$, then $b_2 \leq b_1$. Or equivalently, if for every y, $\tilde{x}_1|y \;\; CR_y \;\; \tilde{x}_2|y$, then $b_2 \leq b_1$. In short, the quantifiers "$\exists\gamma$" and "$\forall y$" may not be inverted in Proposition 3. Consequently, we may not infer for this Proposition 3 that if all conditional distributions undergo a relatively strong increase in risk, then $b_2 \leq b_1$. A counter-example is given in the next section. This is due to the fact that different RSIR may belong to $LSD_{\gamma,y}$ sets of different γ.

Another application is a SLSD on x. When a SLSD is a first order stochastic dominance, since this is a particular case of a $LSD_{\gamma,y}$, it is immediate to verify that all risk averters will reduce their position on asset x.

V. A COUNTER-EXAMPLE IN THE CASE OF RSIR

Proposition 3 shows clearly that we cannot apply directly the order of Greater Central Riskiness developed for one risky asset to problems with two stochastically dependent risks. In fact, as discussed above, a false extension to the results in Gollier (1995) would be to claim the following result when $b_1 > 0$: If $\tilde{x}_1|y \; CD_r \; \tilde{x}_2|y$ for all y, then $b_2 \leq b_1$. The following counter example shows that it is necessary to assume that $\tilde{x}_1|y \;\; LSD_{\gamma,y} \;\; \tilde{x}_2|y$ *with the same* γ *for all* y to obtain the desired result.

<u>Counter example</u> : By construction of the example, let



and

$\tilde{x}_1|\bar{y} \equiv \tilde{x}_2|\bar{y} \equiv 13/32$ with probability 1. Obviously $\tilde{x}_1|\bar{y} \quad LSD_{1,\bar{y}} \quad \tilde{x}_2|\bar{y}$.

Now suppose that $\tilde{x}_1|\underline{y}$ is uniformly distributed on [0, 1] and define the following compoun lottery :

We observe that $\tilde{x}_2|\underline{y}$ is a Relatively Strong Increase in Risk (Black and Bulkley (1989)) of $\tilde{x}_1|\underline{y}$ More precisely, $\tilde{x}_1|\underline{y}$ $LSD_{16/15,\underline{y}}$ $\tilde{x}_2|\underline{y}$. However, $\tilde{x}_1|\underline{y}$ does not $LSD_{1,\underline{y}}$ $\tilde{x}_2|\underline{y}$.

We observe that $\tilde{x}_1$ is less risky than $\tilde{x}_2$ but with a more general definition than in Proposition : in the sense that two different values of γ have been used. We now show that $b_2 > b_1$ for a noı decreasing and concave utility function. Let $u(z) = \min(z, 1/2)$. We first show that $b_1 = 1/2$.

$$\frac{\partial U}{\partial b}(1/2) = 1/2\, E\left\{[\tilde{x}_1|_{\underline{y}} - \underline{y}]\, u'\left(\frac{\tilde{x}_1|_{\underline{y}} + \underline{y}}{2}\right)\right\}$$

$$+ 1/2\, E\left\{[\tilde{x}_1|_{\overline{y}} - \overline{y}]\, u'\left(\frac{\tilde{x}_1|_{\overline{y}} + \overline{y}}{2}\right)\right\}$$

$$= 1/2 \int_0^{x \,|\, \frac{x + 1/4}{2} = 1/2} (x - 1/4)dx + 1/2\ (13/32 - 1/2)$$

$$= 1/2\left[\frac{x^2}{2} - \frac{x}{4}\right]_0^{3/4} + 1/2\ (-3/32) = 3/32(1/2 - 1/2) = 0$$

We now verify that $b_2 > 1/2$

$$\frac{\partial U}{\partial b}\left(\frac{1}{2}\right) = \frac{1}{2}\int_0^{\frac{3}{4}} \frac{16}{15}\left(x - \frac{1}{4}\right)dx + \frac{1}{2}\left(-\frac{3}{32}\right)$$
$$= \frac{1}{2}\left(\frac{16}{15}\cdot\frac{3}{32}\right) + \frac{1}{2}\left(-\frac{3}{32}\right) = \frac{1}{320} > 0\ .$$

In fact one obtains that $b_2 = \sqrt{\frac{16}{61}} = 0.5121 > 1/2$.

VI. CONCLUSION

We have extended the order of Greater Central Riskiness to environments with two risky dependent random variables. We have shown that, even when we use a definition of changes in risk that maintains the correlation sign between the two risky assets (ceteris paribus assumption), we must restrict the definition of greater riskiness to obtain intuitive comparative statics results for all risk averse individuals. We obtain that maintaining the same linear stochastic dominance structure between the two random variables ($LSD_{\gamma,y}$) is sufficient to get the desired result for a mean preserving spread.

We must emphasize however that our model generalizes other restrictions on changes in risk with two dependent risky parameters (such as Strong Increase in Risk and Simple Increase in Risk) and allow for any kind of stochastic dependence between the risky parameters. Moreover no other restriction than risk aversion is necessary. This means that our results can be used to study increases in risk in standard portfolio models. Simple increases in risk, for example, can be applied to many distributions of assets returns. Finally, we have shown that a strong linear stochastic dominance (SLSD) can be interpreted as a first order stochastic dominance. However, it cannot be directly associated to a second order stochastic dominance (see Gollier (1995) for more details).

REFERENCES

BLACK, J.M. and G. BULKLEY (1989), A Ratio Criterion for Signing the Effect of an Increase ir Uncertainty, *International Economic Review*, 30, 119-130.

BOYD, J.H. and B.D. SMITH, (1994), The Use of Debt and Equity in Optimal Financial Contracts *Working paper 537*, Federal Reserve Bank of Minneapolis.

DEWATRIPONT, M. and J. TIROLE (1994), A Theory of Debt and Equity : Diversity of Securities and Management-Shareholder Concurrence, *Quarterly Journal of Economic*, 109, 1027-54.

DIONNE, G., L. EECKHOUDT and C. GOLLIER, (1993), Increases in Risk and Linear Payoffs, *International Economic Review*, 34, 309-319.

DIONNE, G and C. GOLLIER, (1992), Comparative Statics Under Multiple Sources of Risk with Applications to Insurance Demand, *The Geneva Papers on Risk and Insurance Theory*, 17, 21-33.

DIONNE, G. and C. GOLLIER, (1992), Simple Increases in Risk and their Comparative Statics for Portfolio Management, *Working Paper 9203*, Economics Department, University of Montreal.

GAGNON, F., (1994), Increases in Risk With Two Dependent Stochastic Parameters : Some New Considerations, Miméo, Université de Montréal.

GOLLIER, C., (1995), The Comparative Statics of Changes in Risk Revisited, *Journal of Economic Theory*, 66, 522-535.

HADAR, J. and T.K. SEO, (1990), The Effects of Shifts in a Return Distribution on Optimal Portfolios, *International Economic Review*, 31, 721-736.

HAMMOND, J.S., (1974), Simplifying Choices Between Uncertain Prospects When Preference is Nonlinear, *Management Science 20*, 1047-1072.

LANDSBERGER, M. and I. MEILIJSON, (1990), A Tale of Two Tails : An Alternative Characterization of Comparative Risk, *Journal of Risk and Uncertainty*, 3, pp. 65-82.

MEYER, J. and M.B. ORMISTON, (1994), The Effect on Optimal Portfolios of Changing the Return to a Risky Asset : the Case of Dependent Risky Returns, *International Economic Review*, 35, 603-612.

MEYER, J. and M.B. ORMISTON, (1985), Strong Increases in Risk and their Comparative Statics, *International Economic Review*, 26, 425-437.

ROTHSCHILD, M. and J. STIGLITZ, (1970), Increasing Risk : I. A Definition, *Journal of Economic Theory*, 2, 225-243.

ROTHSCHILD, M. and J. STIGLITZ, (1971), Increasing Risk : II. Its Economic Consequences, *Journal of Economic Theory*, 3, 66-84.

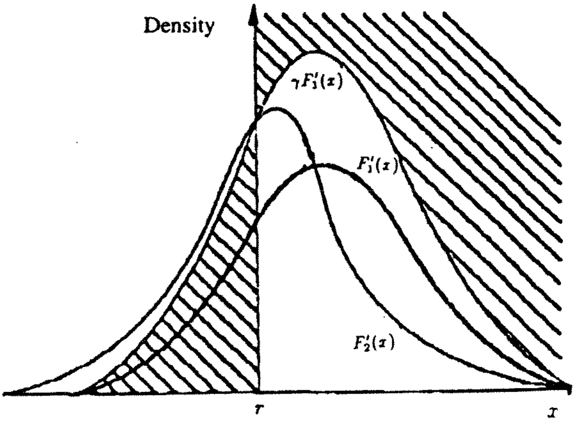


Figure 1. F_1 dominates F_2 in the sence of SLSD.

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