One may consider that the non-zero ϕ_1 in the string solution breaks the $U(1)_D$ symmetry, making it a superconducting string [54]. However, this is not the case. In the soliton background, the generator of the $U(1)_D$ symmetry should be defined as $-T^a n^a + 1/2$ with $n^a = (\phi^\dagger \tau^a \phi)/|\phi|^2$. Thus the solution given as Eq. (A6) is always invariant under this symmetry.

Appendix B. Scattering amplitudes of χ and nucleons

We here provide explicit expressions for the scattering amplitudes shown in Fig. 5. In the following, we denote $h_1 \equiv h$, $h_2 \equiv h'$. It is straightforward to calculate diagrams (a)–(d):

$$i\mathcal{M}_a = i\mathcal{M}_b = -\bar{u}u \sum_{j,k=1,2} g_{ffh_j} \frac{1}{m_{h_j}^2} g_{\chi\chi h_j h_k} g_{\chi\chi h_k} \frac{i}{(4\pi)^2} B_0\left(m_\chi^2, m_{h_k}^2, m_\chi^2\right), \tag{B1}$$

$$i\mathcal{M}_c = \bar{u}u \sum_{j=1,2} \frac{g_{ffh_j} g_{VVh_j}}{m_{h_j}^2} \frac{g_D^2}{4} \frac{i}{(4\pi)^2} \left[\left(-m_V^2 + 4m_\chi^2\right) \frac{\partial}{\partial m_V^2} B_0\left(m_\chi^2, m_V^2, m_\chi^2\right) - B_0\left(m_\chi^2, m_V^2, m_\chi^2\right) + B_0\left(0, m_V^2, m_V^2\right) \right], \tag{B2}$$

$$i\mathcal{M}_d = -\bar{u}u \sum_{j,k,l=1,2} \frac{g_{ffh_j} g_{\chi\chi h_j} g_{\chi\chi h_k} g_{h_j h_k h_l}}{m_{h_j}^2} \frac{i}{(4\pi)^2} C_0\left(m_\chi^2, m_\chi^2, 0, m_k^2, m_\chi^2, m_l^2\right), \tag{B3}$$

where u is the spinor wavefunction and

$$\frac{i}{(4\pi)^2} B_0\left(p^2, m_1^2, m_2^2\right) \equiv \int \frac{d^d l}{(2\pi)^d} \frac{1}{\left(l^2 - m_1^2\right)\left(l+p\right)^2 - m_2^2}, \tag{B4}$$

$$\frac{i}{(4\pi)^2} C_0\left(p_1^2, p_2^2, (p_1+p_2)^2, m_1^2, m_2^2, m_3^2\right) \equiv \int \frac{d^d l}{(2\pi)^d} \frac{1}{\left(l^2 - m_1^2\right)\left[(l+p_1)^2 - m_2^2\right]\left[(l+p_1+p_2)^2 - m_3^2\right]}, \tag{B5}$$

$$g_{\chi\chi hh} = 2\lambda_\phi s_\theta^2 + \lambda_{H\Phi} c_\theta^2, \tag{B6}$$

$$g_{\chi\chi hh'} = -2\lambda_\phi s_\theta c_\theta + \lambda_{H\Phi} s_\theta c_\theta, \tag{B7}$$

$$g_{\chi\chi h'h'} = 2\lambda_\phi c_\theta^2 + \lambda_{H\Phi} s_\theta^2, \tag{B8}$$

$$g_{hhh} = 6v\lambda_H c_\theta^3 - 3v_s\lambda_{H\Phi} c_\theta^2 s_\theta + 3v\lambda_{H\Phi} c_\theta s_\theta^2 - 6v_s\lambda_\Phi s_\theta^3, \tag{B9}$$

$$g_{hhh'} = 6v\lambda_H c_\theta^2 s_\theta + 6v_s\lambda_\Phi c_\theta s_\theta^2 + v_s\lambda_{H\Phi}\left(c_\theta^3 - 2c_\theta s_\theta^2\right) + v\lambda_{H\Phi}\left(s_\theta^3 - 2c_\theta^2 s_\theta\right), \tag{B10}$$

$$g_{hh'h'} = 6v\lambda_H c_\theta s_\theta^2 - 6v_s\lambda_\Phi c_\theta^2 s_\theta + v\lambda_{H\Phi}\left(c_\theta^3 - 2c_\theta s_\theta^2\right) + v_s\lambda_{H\Phi}\left(-s_\theta^3 + 2c_\theta^2 s_\theta\right), \tag{B11}$$

$$g_{h'h'h'} = 6v\lambda_H s_\theta^3 + 3v_s\lambda_{H\Phi} c_\theta s_\theta^2 + 3v\lambda_{H\Phi} c_\theta^2 s_\theta + 6v_s\lambda_\Phi c_\theta^3. \tag{B12}$$

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