Entropy **2007**, *9*, 108-112

entropy
ISSN 1099-4300

www.mdpi.org/entropy/

Full Paper

A Model of Time

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Received: 19 July 2007 / Accepted: 5 September 2007 / Published: 7 September 2007

Abstract: Based on a mathematical model of quantum measurement we derive some properties of intuitive time.

Keywords: Quantum Entropy, Energy, Measurement, Time.

MSC 2000 Codes: 94A17, 81P15

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1. Introduction

In [7, 8] we present a model of quantum measurement which bases on the notions of decoherence and collapse. Given an apparatus or register $\mathcal{A}$, a system $\mathcal{S}$ and the environment $\mathcal{E}$ at temperature T, the measurement process $\mathcal{M}$ can be thought of as being represented by the following sequence:

$$\mathcal{M} := \mathcal{A} \otimes \mathcal{S} \to^{d} \mathrm{tr}_{\mathcal{E}}(\mathcal{A} \otimes \mathcal{S}) \to^{p} \mathcal{A}_{\iota} \otimes \mathcal{S}_{\iota},\ \iota \in I. \tag{1}$$

The first step, d, is the decoherence of the entangled $\mathcal{AS}$ system and the creation of classical alternatives, mathematically expressed by a density matrix $\rho_{\mathcal{AS}}$. The second step, p, is the projection or collapse to one of the eigenstates and a definite outcome. There is a fluctuation of entropy $S_{\rho_{\mathcal{AS}}}$ connected to $\mathcal{M}$. The fluctuation of entropy creates a pulse of energy whose average satisfies $\overline{E} \geqslant kTS_{\rho_{\mathcal{AS}}}$ where k denotes the Boltzmann constant. The term ”measurement” is somewhat misleading in this context and we want to replace it by ”realization” because what happenes in $\mathcal{M}$ is actually the emergence of a real fact out of a background of possibilities. The dissipation of energy gives proof of the realization to $\mathcal{E}$, the ”world outside”. The realization is real insofar as it is possible for the environment to notice it via the energy pulse. In this sense the saying becomes true that ”the tree only falls if it is being heard”. At this point we calculate that, due to the time-energy inequality [1], it takes $\Delta t \sim \frac{\hbar}{ckTS_{\rho_{\mathcal{AS}}}}$ to notice the realization with a generic a priori precision in $\mathcal{E}$. We interpret this time as the time it takes the realization to "happen".

In this short paper we want to deepen the explanation of the model and discuss some consequences for the ontology of time and, in contrast to some other authors [11] , defend some properties of intuitive time.

2. Interpretation

We know that decoherence cannot be governed by a Hamiltonian flow in finite time and that the paradigm assumes that the collapse is instantaneous. This state of explanation is in point of fact not satisfactory. The ontological basis of our argument will be the standpoint that there is physical reality associated with the quantum state of a single system as opposed to the assumption that a quantum state has physical meaning only for an ensemble of identical systems [12]. We call this in the sequel the single-trial interpretation. Further, and quite consequent, we assume that sequences like $\mathcal{M}$ exist without preparation or test by a conscious observer.

2.1. Erasure

There is a careful analysis of the sequence $\mathcal{M}$ and its implications in [9] . Starting point is the puzzling fact that step d creates entropy which is again lost in step p. Especially the fact that this must be valid for the single trial and not just for an ensemble needs attention. The simple reason is that, if there is no entropy production in a single trial, there will be none in the ensemble either, because n times zero remains zero. To save the second law, the loss of entropy in step p must be compensated. In [9] this happenes by erasing the former (eigen)state of the register $\mathcal{A}$ through thermal randomization by the environment $\mathcal{E}$. There follows $\overline{E} \geq kTS_{\rho_{\mathcal{AS}}}$.

Careful analysis in [9] shows that the calculation of $\overline{E}$ is not a result of Landau tracing over unobserved parts. It is calculated as a convex combination of erasure energies of separate relative realities, namely the eigenstates of the register $\mathcal{A}$. The expectation value then reflects that the measurement is repeatedly done and is, in a way, an average over time. If we stick to the single trial interpretation and do not demand that $\mathcal{M}$ needs a repeatable set up, however, it is difficult to see how $\overline{E}$ can be interpreted as the average energy originating from the erasure of register (eigen)states stemming from former trials. Yet, as mentioned above, the entropy of the single trial in $\rho_{\mathcal{AS}}$ is real and its loss in p has to be compensated. Indeed, with some shift in interpretation, the argument can be saved.

In step d of $\mathcal{M}$ the entageled state of system and register $\mathcal{AS}$ is decohered by thermal contact with the environment. In equilibrium the result is a mixture with statistical probabilities for its eigenstates $q_j = \frac{\exp\left(-\frac{E_j}{kT}\right)}{Z}$, where Z denotes the partition function. We denote by p_j the outcome probabilities of the single trial. Since the entropy of a pure state is zero we calculate for the energy dissipation $\overline{E}_{\mathcal{AS}} = -kT\sum_j q_j \ln q_j$. The dissipation from the environment $\overline{E}_{\mathcal{E}}$ is just minus the difference of the inner energy of $\mathcal{AS}$ after and before decoherence, which is $\overline{E}_{\mathcal{E}} = -\sum_j (q_j - p_j)\, E_j$. Hence

$$\overline{E} = -kT \cdot \sum_j q_j \ln q_j + \sum_j (p_j - q_j) E_j = -kT \sum_j p_j \ln q_j \geq -kT \sum_j p_j \ln p_j = kTS_{\rho_{\mathcal{AS}}}.$$

One can say that the entropy loss in step d is compensated by the erasure of the original pure state and the corresponding irreversible loss of other possible outcomes of the single trial.

2.2. Time

Step d of $\mathcal{M}$ creates classical alternatives and increases entropy until somebody ”looks”. This ”somebody” is in our model the environment. There is only one way for the environment to ”look”: the environment has to evolve from an initial state representing ”nothing has happened” to an orthogonal and hence distinguishable state representing ”something definite has happened” by the dynamics of the energy dissipated by erasure.

So far we have not explicitely made use of a time parameter. The word ”sequence” for $\mathcal{M}$ is, of course, implying time and reflects the fact that our language is deeply saturated with intuitive time. We have to stress that the division of $\mathcal{M}$ into two steps is just a help to explain the physical consequences but does not suggest that there are actually two sequential steps happening.

We assume that there is a time parameter t introduced in the environment $\mathcal{E}$. For the environment the duration of $\mathcal{M}$ is equivalent to the time it takes to distinguish the dissipated energy. All we know is the minimal average energy $\overline{E} = kTS_{\rho_{\mathcal{AS}}}$ but no Hamiltonian H. Hence there applies a time-energy inequality $\Delta E \Delta t \geqslant \hbar$ since the dynamics by H has to be estimated [1] . There is an even more direct link between time and $\overline{E}$ which does not need a detour via a standard deviation ΔE. The initial state and the final state of the environment after registering the pulse represent the alternatives ”nothing has happened” and ”something has happened”. The two states have consequently to be distinguishable. Results by Margolus and Levitin [13] show that a quantum system with average energy $\overline{E}$ takes at least $\Delta t = \frac{h}{4\overline{E}}$ to evolve into an orthogonal and hence distinguishable state. We conclude that in terms of our model the sequence $\mathcal{M}$ takes $\Delta t \sim \frac{h}{4kTS}$ to happen.

Our results have some consequences for the phenomenology of the time parameter t. First of all time becomes a true dimension since to a sequence $\mathcal{M}$ there belongs a time interval $\Delta t_{\mathcal{M}} > 0$ which is naturally interpreted as its duration. Since the time parameter is introduced in $\mathcal{E}$, we do not have the conceptual difficulties with ”instantaneous collapse” or ”approximate decoherence”. The question how long does $\mathcal{M}$ take in internal system-register time makes no sense in our model. In addition, since some of the possible realizations are together with the initial state irreversibly lost, there is a direction given to time. During $\Delta t_{\mathcal{M}}$ a system changes from potential to real. Reality is thus divided into a realized and an unrealized part or the corresponding past and future. Note that the past, defined as the set of realized events, has no ”depth” yet since temporal order has not been defined. Note also, that $\Delta t_{\mathcal{M}}$ is configuration dependent since mutual entropy differs for different $\mathcal{A}$ and $\widetilde{\mathcal{A}}$, $S_{\rho_{\mathcal{AS}}} \neq S_{\rho_{\widetilde{\mathcal{A}}\mathcal{S}}}$. Here enters an aspect of relativity.

If we model reality by carving out an isolated part of it with known energy functional H, there are no entropy fluctuations. $\mathcal{M}$ has no place in such a model. Since H is known to the observer, energy can be measured in arbitrary small time intervals and initial states can be reconstructed. It is no surprise that phenomenological time cannot be explained and that time turns out to be continuous and reversible, actually inexisting $[11]$.

2.3. Space-Time

In special relativity space and time are linked through the assumption that the speed of light in vacuum, c, is a constant and the maximal speed at which any signal can pass through space. Light signals are used to gauge clocks and the framework of relativity is thus derived. In the EPR (Einstein-Podolsky-Rosen) context we learn, however, that a cause can have an immediate effect over an arbitrary distance. This is a puzzling element of non-locality. The paradigm resolves the problem by saying that it still takes a signal to send the news of the effect to make it ”known” to the cause and that special relativity is compatible. The resolution is deeply rooted in classical ideas, especially the idea of local realism. In quantum mechanics values of physical systems are elements of the spectra σ of observables and result from realizations. Our model shows that in the same way any realization, in particular an EPR event, creates an interval of time which we can interpret as the time it takes the EPR event to happen. The event is not ”instantaneous” it just creates time for the environment ”experiencing” it. With the elements in place so far, we can build a brige between quantum mechanics and space-time of special relativity.

In $[8]$ we derived classical velocity from our model. It is simply the quotient between some distance (function of eigenvalues of the space observable) and the time it takes to register a particle at that distance. By the same right we could derive other generalized velocities by simply forming quotients between distance functions of eigenvalues of observables and corresponding times of realization. This way we would derive models of spin - time or polarisation - time. Note, however, that it follows from our estimates in $[8]$ that it is the fact that the space dimension is three, $n = 3$, which leads to the possibility of constant velocity and ultimately to an absolute bound on velocity which singles out space-time.

From the insights we have had so far we realize, that a model which divides and explains past and future relying on a particular derived velocity seems rather arbitrary, though understandable from a phenomenological point of view. We also understand that without the roots of quantum mechanics it is not possible to explain the flow of time.

Our model does not naturally produce an explanation of temporal order which is, of course, another fundamental property of intuitive time. To introduce a temporal order, which means an asymmetric relation between realizations, we need another input. If A is prior to B, then A and/or B somehow have to know that. Cause and effect could serve the purpose. We do not pursue this question further, however, but it is very likely that any relation can at most produce a partial order like in special relativity.

3. Conclusion

We have constructed a mathematical model which serves to defend a large part of intuitive time. It ties elementary, configuration dependent time intervals to realizations. The direction of time can be explained and a past with corresponding future be defined. The basis of our ideas roots in different philosophical areas like idealism (reality of single quantum states) and empirism (measurement by environment). Classical velocity as a non primitive quantity can be derived and a bridge to special relativity be built. We also get a first understanding why classical physics, including the relativity theories, cannot explain the flow of time. Basically it is the fundamental randomness of quantum mechanics which makes intuitive time and its flow possible.

4. References and Notes

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