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A MODEL OF TRANSIT FREIGHT DISTRIBUTION ON A RAILWAY NETWORK

Summary. The peculiarity of the transit freight transportation by rail in international traffic is the fact that while performing transportation, the railway administrations are in competition among themselves. At the same time, the routes of cargo traffic volumes significantly depend on the conditions of transportation by railways of individual states. The mathematical model for the distribution of transit freight traffic volumes on the railway network, based on the methods of graph theory and game theory, was proposed in this article. The developed model enables the evaluation of the possibilities of attracting transit freight traffic volumes by individual railway administrations by changing the tariff value and transportation conditions.

1. INTRODUCTION

Railway transport is one of the main freight carriers in international traffic in the territory of Eastern Europe. Railways make it possible to carry out transportation of significant cargo volumes from the places of their extraction or production to the places of their consumption or trans-shipment to other modes of transport. Countries with a developed railway network consider transit railway transportation as one of the main directions of transport services export.

Competition for freight traffic volumes between elements of a transport system is a distinctive difference of cargo transportation in international traffic from the domestic communications. A rather good example of such a transportation is the delivery of raw materials by railway from the Russian Federation to the ports of the Black and Baltic Sea. In these directions, transportation with the participation of the transit railways of Ukraine, Lithuania, Latvia, Estonia, and Belarus is possible. Each of them is interested in attracting transit freight traffic volumes. In this respect, the studies carried out to improve the competitiveness and efficiency of transit transportation are relevant for railway transport.

2. LITERATURE REVIEW AND DEFINING THE PROBLEM

Significant numbers of scientific works are devoted to the subject of international railway transportation. Considerable attention has been paid to the setup of transport corridors and reduction of the cargo delivery cost using them. In particular, the paper [1] presents the project OPTIRAIL. It aims to increase the railway transport availability, improve border-crossing interactions, and increase the efficiency of international transportation. The problems of coordinated development of the transport corridor elements are considered in the articles [2, 3]. The works [4, 5] consider international cargo transportation from the perspective of consignors, who search for transportation routes in such a way as to minimize their logistics costs. The railways serving international transportation are in competition

with both road and sea transport routes, and among themselves [6, 7]. Their competitiveness depends on many factors, such as the cost and terms of cargo delivery on the route, carrying capacity, traffic safety, etc. The presence of alternative transportation routes means that the railway infrastructure managers have to determine the cost of their services taking into account the cost of services of other participants in the transportation process. The solution of such problems is arrived at by the methods of game theory.

The development of modern game theory started in 1944, when the book «Theory of Games and Economic Behavior» [8] by John von Neumann and Oskar Morgenstern was published. The use of game theory methods for solving transport problems started to flourish from the end of the 20th century. In the work [9], Colony formulated the route selection problem as a zero-sum game. In this game, one of the players is a driver who chooses whether to use an arterial road, where the traffic volume does not affect driving conditions, or a motorway, where heavier traffic results in a more disturbed drive. The other player is an imaginary entity who chooses the level of service on the road, and tries to disturb the driver's journey as much as possible. Fisk in work [10] investigates a Stackelberg game between the authority that sets traffic signals and all travelers who then find the user-equilibrium solution. This article also presents a formal description of the task of carrier competition for intercity passenger travel.

Nagurny in work [11] developed a multimarket supply chain network design model in an oligopolistic setting. In this model, the firms select not only their optimal product flows but also the capacities associated with the various supply chain activities of production/manufacturing, storage, and distribution/shipment. The relationships between game theory and transportation have been investigated by Hollander and Prashker for a review of games describing transport problems [12].

Currently, the game theory is widely used to simulate the competition of different types of transport. Examples of tasks for modeling the processes in transport systems using the methods of game theory are given in the articles [13-16]. A considerable number of scientific works are devoted to the problem of price competition and planning for the development of competing ports. In particular, the works [17-20] deal with such problems. Analysis of the presented works shows that the methods of game theory make it possible to take into account the features of transport systems functioning in the conditions of competitive struggle. It should also be noted that the railway network functioning during the organization of international transportation has certain features since the competitiveness of individual participants in the transportation process is significantly affected by their geographical position and the network topology. Therefore, the problem under consideration requires further research.

The purpose of this investigation is to determine and evaluate the properties of a mathematical model for the distribution of cargo traffic volumes in the railway transport networks under conditions of competition for freight traffic volumes between their individual elements.

3. METHODOLOGY

A parametric directed graph $G(V, E)$ is used as a model of a railway network. The points of loading V_s and unloading V_d , and the transit points V_t , where the change of conditions for cargo traffic volumes forwarding takes place, correspond to the graph vertices V . The cargo supplies $a_j > 0$ at the loading points ($j = \overline{1; J}$) are put in correspondence with the vertices $v_{sj} \in V_s$. The vertices $v_{dk} \in V_d$ correspond to the maximum cargo volumes that can be unloaded at the unloading points $b_k > 0$ ($k = \overline{1; K}$). In addition, cargo values at the loading P_{sj} and unloading points P_{dk} ($j = \overline{1; J}$, $k = \overline{1; K}$) are put in correspondence with the vertices v_{sj} and v_{dk} .

The transportation costs in the transport network represented by the directed graph $G(V, E)$ are set on each arrow $e_{gq} \in E$. They are denoted by c_{gq} (here g and q are the starting and ending vertices of the arrow, respectively). In addition, the carrying capacity d_{gq} can be put in correspondence with the graph arrows. The arrows' direction determines the direction of transportation between the vertices.

The costs of transportation at certain (controlled) arrows are established by carriers and they can choose them from a discrete set C_{gq} - permissible values, i.e. $c_{gq} \in C_{gq}$ ($e_{gq} \in E$); the costs of transportation at the other (uncontrolled) arcs are fixed. At the same time, the minimum cost of transportation is determined by the cost of the service and its minimum profitability, and the maximum cost of transportation is determined in accordance with the Agreement on the International Railway Transit Tariff [21].

An example of the directed graph describing the transport system is shown in the Fig. 1.

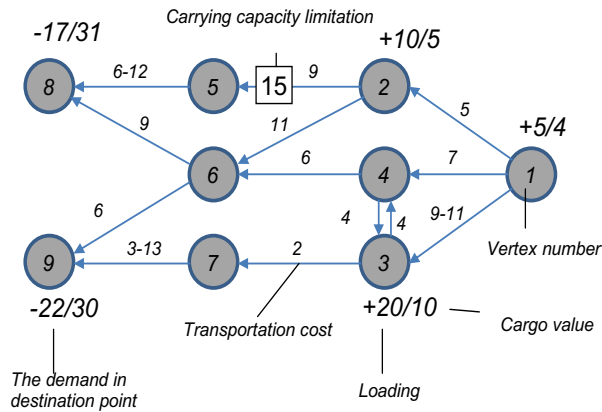


Fig. 1. Directed graph of transport network with an indication of the numerical values of parameters for transportation

Active agents of two types take part in the process of organizing the cargo transportation. They are consignors and freight carriers.

Each consignor chooses the routes from his loading point to the unloading points based on the profit margin. For a unit of cargo, this is determined by the difference between the cargo value at the destination point and the cargo value at the point of departure and delivery costs. In this case, a circuit of arrows connecting some departure vertex v_{sj} and the destination vertex v_{dk} is called the route of transportation. Between the vertices v_{sj} and v_{dk} , there may be several routes of transportation E_{jkm} , differing in the list of arrows included. Transportation routes are characterized by the cost of transportation, which is determined as the total cost of transporting all the arrows included in the route

$$C_{jkm} = \sum_{e_{gq} \in E_{jkm}} c_{gq} \quad (1)$$

As a result of the cargo delivery to the destination points, consignors receive a profit, which is

$$P_{jkm} = P_{dk} - P_{sj} - C_{jkm} \quad (2)$$

when transporting a unit of cargo.

The consignor dispatches cargoes to the destination points in order of decreasing profits. If the profit takes a negative value, the cargo is not transported to this departure point.

The carrier establishes the cost of transportation at the sections ranging from the minimum to the maximum value, seeking to obtain the maximum possible profit, based on the cost accepted by him, as well as the possible costs of transportation from other carriers. It is accepted that the consignors receiving larger profits have an advantage of using scarce resources. With an equal profit margin for different consignors, scarce resources are equally distributed among them. It is necessary to establish the price strategy of carriers.

In the initial graph, the values are put in correspondence with both vertices and arrows. To simplify further calculations, it is necessary to perform the transformation of the graph in such a way that the values were put in correspondence only with its arrows. For this purpose, we should select the vertex with the minimum cargo value at the departure point P_{smin} among all the vertices v_s . For all other vertices of departure, we should add arrows with the cost of transportation $P_{sj} - P_{smin}$. For vertices v_d , it is

necessary to determine the vertex with the maximum cargo value P_{dmax} . For all other vertices of departure, we should add arrows with the cost of transportation $P_{dmax} - P_{dk}$.

The profits of carriers in the transformed graph are defined as

$$P_{jkm} = P_{dmax} - P_{smin} - C_{jkm}, \quad (3)$$

which is equivalent to expression (2).

Transit vertices, incident to only two edges, are excluded from the network. The transportation costs of the joined edges are summed. It should be noted that such a step corresponds to coordinated actions of carriers at some section of the route and somewhat distorts the solution. However, at this stage of research, this fact is neglected. The transformed graph is shown in Fig. 2.

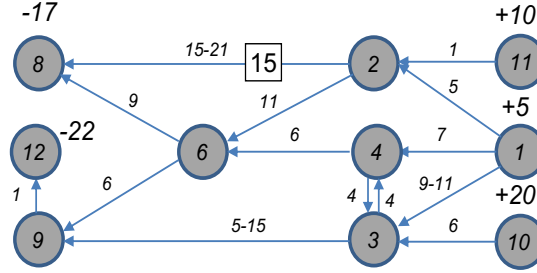


Fig. 2. Directed graph of the transport network after transformations

Let us denote the amount of cargo dispatched from the vertex j to the section of network represented by the arrow e_{gq} (from the vertex j) using x_{gq}^j . The flow values on the edges can only have non-negative values. The traffic volumes on the edges can only have non-negative values $x_{gq}^j \geq 0$.

The distribution of freight traffic volumes in the network has a number of limitations.

The quantity of cargo exported from the vertex j over all arrows e_{jq} incident to it should not exceed the amount of cargo supply at the given vertex

$$a_j - \sum x_{gq}^j \geq 0, \quad j = \overline{1; J}, \quad (4)$$

where $\sum x_{gq}^j$ - is the total volume of cargoes exported from the vertex j .

The amount of cargo discharged at the destination vertex should not exceed its demand or unloading capacity

$$b_k - \sum_{j=1}^J \sum x_{gk}^j + \sum_{j=1}^J \sum x_{kq}^j \geq 0, \quad \sum x_{gk}^j \geq \sum x_{kq}^j, \quad j = \overline{1; J}, \quad k = \overline{1; K}, \quad (5)$$

where $\sum x_{gk}^j, \sum x_{kq}^j$ - the total volume of cargoes dispatched from the vertex j , imported and exported from the vertex k , respectively.

The amount of cargo arriving at the intermediate vertex t from the vertex of departure j along all the incident arrows e_{gt} should be equal to the amount of cargo exported from it. The departure vertices for freight traffic volumes from the other vertices are considered as transit ones

$$\sum x_{gt}^j - \sum x_{tq}^j = 0, \quad j = \overline{1; J}. \quad (6)$$

The amount of cargo transported along the edges with limited carrying capacity should not exceed the value of this capacity

$$\sum_{j=1}^J \sum x_{gq}^j \leq d_{gq}. \quad (7)$$

Consignors choose the routes of transportation proceeding from the task of obtaining maximum profit, which is determined by the expression

$$\sum P_j = \sum_{m=1}^M (P_{dmax} - P_{smin} - C_{jkm}) x_{jkm} \rightarrow \max \quad (8)$$

where $\sum c_{jkm}, x_{jkm}$ - are the total cost and volume of cargo transportation between the vertices v_{sj} and v_{dk} along the route m , respectively.

Since the final values of the demand for cargoes b_k are specified at the unloading points, the consignors cannot determine the routes of their transportation independently from each other. In addition, the limited carrying capacity does not allow using the profitable for transportation sections at the same time to anybody who wants it. In this case, a conflict of interest can take place. It is accepted that passage of the cargo unit, which ensures a greater profit, has an advantage. If there are several consignors with the same profit margin for several destination points or arrows, the unloading and carrying capacity are equally distributed among them. A search for the optimal transportation route for each consignor is carried out using the method of searching the shortest routes in the graph [22]. It was modified to take into account the limitations in the unloading capacity of vertices and the carrying capacity of arrows.

Carriers compete for transportation among themselves to gain the greatest possible profit. Some of them have the opportunity to offer different transportation costs. Let there be n such carriers in the network. Let us change their network designations identical to arrows with two indices, which correspond to the numbers of vertices of the beginning and the end of the arrow, for ordinal designations h_i ($i = 1; n$). Thus, the costs of transportation from the set $C_i \equiv \{c_1, c_2, \dots, c_{u_i}\}$ are put in correspondence with each carrier h_i . After consignors have chosen the transportation routes, the profit of the carriers is evaluated. In particular, the carrier h_i serving the section represented by the arrow e_{gq} looks forward to the profit

$$\Lambda_i^* = c_{gq}^* x_{gq}^*, h_i \sim e_{gq}. \quad (9)$$

All possible cost situations that arise in the network form a set $C = C_1 \times C_2 \times \dots \times C_n$, where C is the Cartesian product of sets C_i consisting of u_i strategies of the carriers. Consignors evaluate each cost situation from the set C and choose the routes of transportation. Therefore, each cost situation gives a certain payoff to the carrier– profit. Therefore, the payoff of the i -th player depends on c . We will determine it using formula (9), i.e. $\Lambda_i(c) = \Lambda_i^*$.

The set of possible payoffs of each player (carrier), depending on the cost situation $c = (c_1, c_2, \dots, c_i, \dots, c_n) \in C$, can be described by the n matrix. Examples of these matrices for players 2 (arrow 5-8) and 3 (arrow 7-7), corresponding to Fig. 1, provided that player 1 (arrow 1-3) has established the cost of transportation equal to 9, are presented in Tables 1 and 2.

Let us consider the case when the conflict of interests n of carriers does not provide for the joint actions of individual groups. Such a conflict can be modeled by a non-cooperative game under the following conditions. The participants cannot conclude mutually binding agreements, their interactions are non-antagonistic, and each player takes his actions independently of the others; the parties to the conflict know the usefulness of each situation that has arisen when choosing actions for themselves and others. Note that the mentioned conflict of carriers one can represent as a distribution of some constant amount between the participants, and the sum of the payoff of all players of the type (9) is not the same in different situations. Thus, to simulate the conflict, one should make a non-cooperative game with a non-zero sum. The analysis of such game models differs from the analysis of antagonistic games.

Let us represent one of the approaches to solving non-cooperative games, based on the principle of equilibrium [23, 13]. Retaining the introduced notations, we represent a non-cooperative game in the form of system

$$G(c) = \{I, \{C_i\}_{i \in I}, \{h_i\}_{i \in I}\} \quad (10)$$

in which $I = 1, 2, \dots, n$; $C = C_1 \times C_2 \times \dots \times C_n$, $C_i \equiv \{c_1, c_2, \dots, c_{u_i}\}$, $h_i(C)$ - are the real functions.

To form the optimal solution of game (6), we introduce the concepts of acceptable and equilibrium situations. Let the set of possible strategies of the players, the game situation,

$c = (c_1, c_2, \dots, c_{i-1}, c_i, c_{i+1}, \dots, c_n) \in C$ be in formula (10), and c_i^j is the random strategy of the player « i ». Further, $c // c_i^j = (c_1, c_2, \dots, c_{i-1}, c_i^j, c_{i+1}, \dots, c_n)$. The situation $c \in C$ is called acceptable for the player « i » if for any strategy c_i^j the inequality

$$h_i(c // c_i^j) \leq h_i(c). \tag{11}$$

Table 1

Payoff matrix of the player 2

		Player 2						
		6	7	8	9	10	11	12
Player 3	3	78	91	92	90	100	55	0
	4	78	91	92	90	100	55	0
	5	78	91	92	90	100	55	0
	6	78	91	92	90	100	55	0
	7	78	91	92	90	90	49,5	0
	8	78	91	92	72	80	44	0
	9	78	91	92	72	80	44	0
	10	78	91	92	45	0	0	0
	11	75	70	80	45	0	0	0
	12	75	70	80	45	0	0	0
	13	75	70	80	45	0	0	0

Table 2

Payoff matrix of the player 3

		Player 2						
		6	7	8	9	10	11	12
Player 3	3	66	66	66	66	66	66	66
	4	88	88	88	88	88	88	88
	5	110	110	110	110	110	110	110
	6	132	132	132	132	132	132	132
	7	154	154	154	147	147	147	147
	8	168	168	168	160	160	160	160
	9	180	180	180	180	180	180	180
	10	200	200	200	170	120	120	120
	11	214,5	187	187	132	77	77	77
	12	234	204	204	144	84	84	84
	13	253,5	221	221	156	91	91	91

is performed. That is, the payoff in an acceptable situation is not less than that in other situations obtained from it by replacing the strategy c_i with any c_i^j . If the inequalities (11) are performed for all $i = 1, \dots, n$; then $c \in C$ is the equilibrium for game (6). Solutions (10) in the form of equilibrium situations in pure strategies $c \in C$ are rare.

To find equilibrium situations, we introduce mixed strategies that establish the probabilities of using the pure strategies « j »: $\sigma_j(c_j)$ by the player « i ». The probability $\sigma(c)$ is called the game situation (6) in the mixed strategies – $G^*(\sigma(c))$:

$$\sigma(c) = \sigma(c_1, c_2, \dots, c_n) = \prod_i \sigma_i(c_i). \tag{12}$$

In this case, the «*i*» player's payoff in $G^*(\sigma(c))$ is understood as the average payoff depending on the probability distributions (12). The situation $\sigma^*(c)$ is called the equilibrium situation of the mixed expansion $G^*(\sigma(c))$ of the game (6) if for any player $i = 1, \dots, n$; and any mixed strategy $\sigma_i(c_j)$ it is performed

$$h_i(\sigma^* // \sigma_i) \leq h_i(\sigma^*). \tag{13}$$

The following statements [6] answer the questions of the existence of and finding a solution for non-cooperative games.

1. In each non-cooperative game, there is at least one equilibrium point in mixed or pure strategies.
2. To make the situation σ^0 the game equilibrium situation (in mixed strategies), fulfillment of the following inequality for any «*i*» and pure strategy c_i is necessary and sufficient.

$$h_i(\sigma^0 // \sigma_i) \leq h_i(\sigma^0). \tag{14}$$

According to the inequality (10), when the pure strategy replaces the mixed strategy in the equilibrium situation σ^0 , the average player's payoff will not be increased. On the contrary, if for some situation σ^0 the average payoff of each player is not less than the average payoff for the situation σ^0 , in which any pure strategy replaced the mixed strategy of each player entering into σ^0 , then σ^0 is an equilibrium situation.

The study of the example under consideration shows that the optimal strategy of player 2 does not depend on the actions of players 1 and 3 and to obtain maximum payoffs, he should always set the cost of transportation equal to 8. Under these conditions, the problem can be reduced to the game of two players and solved by the classical methods [6]. The final distribution of traffic volumes in the network is shown in Fig. 3.

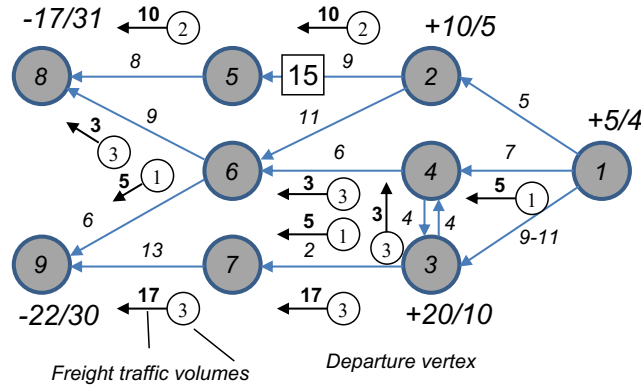


Fig. 3. Traffic volume distribution in the transport network

At present, there are no general mathematical methods for solving non-cooperative games with more than two players (here carriers), having more than two strategies [6]. At the same time, specialized models and algorithms are developed that make it possible to numerically realize such kind of game problems [7]. Let us consider this problem in more detail.

In the games of the form (6) there can be several equilibrium situations. It is these constructive properties of equilibrium situations that were used to develop exhaustive computing algorithms for solving discrete non-cooperative games of n persons [7] of the general type.

We present a generalized scheme for calculating equilibrium situations in the games (6). We denote the unknown probability vector of the mixed strategies of players using the formula

$$\bar{x}_\sigma = (\sigma_1^1, \dots, \sigma_{n_1}^1, \dots, \sigma_1^n, \dots, \sigma_{n_n}^n), \tag{15}$$

where n_j - is the number of pure strategies of the player «*j*».

1. Having the vector (11) taking into account (8), one can calculate the mathematical expectation of the payoffs of each of the players «*i*»

$$M[h_i(\bar{x}_\sigma)], \quad \forall(i \in n) \quad (16)$$

2. Condition parameters of the equilibrium situation in game (6) for formula (15) are calculated according to the following formulas:

$$\Delta_{ij}^k = M[h_i(\bar{x}_\sigma)] - M[h_{ij}^k]. \quad (17)$$

$$\Delta_i = \min_j \min_{k \in nj} \{\Delta_{ij}^k\}. \quad (18)$$

where $M[h_{ij}^k]$ is the mathematical expectation of the player's « i » payoff in case the player « j » uses his pure strategy number « k », and the value (13) is the worst evaluation of the payoff.

3. If the following relation takes place:

$$\exists(i \in n) \Delta_i < 0, \quad (19)$$

then vector (15) is not the equilibrium situation. In case relation (19) is not performed, vector (15) represents the equilibrium situation of game (6).

4. When relation (19) is performed, a certain deviation indicator (15) from the equilibrium situation is formed, using which the optimization task of the form

$$\min \{0, V(\Delta_i)\} \Rightarrow \max, \quad \forall(\Delta_i < 0), \quad (20)$$

is solved; in some way or another the vectors are being formed (15).

In the work [7], we used the random search algorithm to implement formula (20). At the given accuracy of finding solution (15), a searching algorithm over the network is possible.

4. INVESTIGATION RESULTS

The developed model can be used both for research of the cost game of carriers and for the evaluation of various measures on the development of loading and unloading abilities of departure and destination points and carrying capacities of the transport network elements.

As an example, Fig. 4 presents the dependencies of the carriers' profits on the loading volumes at vertex 3.

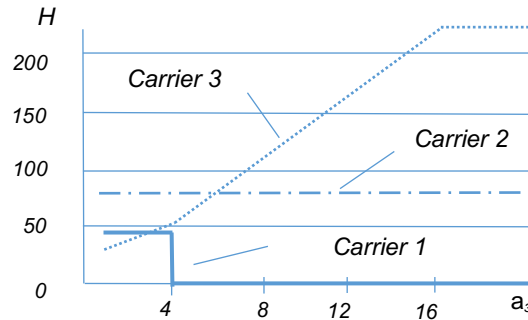


Fig. 4. Dependence of the profits of carriers on the stocks at point 3

Analysis of the obtained dependencies shows that if the loading volume in vertex 3 is less than 5 units, player 3 is interested in choosing a strategy that ensures attraction of cargo traffic volume from vertex 1 to arrows 7-9. In this case, arrows 1-3 of carrier 1 are used to pass cargo traffic volumes. With increasing loading volumes at vertex 3 to 5 or more units, carrier 3 sets the cost of his services based on the objectives of obtaining maximum profit while servicing vertex 3. As a result, the routes using arrows 1-3 (carrier 1) become uncompetitive.

The results of the work can be used to create a system for supporting solutions for tariff evaluation, technical and technological solutions taken in the field of international transit transportations by railway.

5. CONCLUSION

Conditions for carrying out the transit railway transportation in international traffic have significant differences from those in domestic traffic due to the competition for freight traffic volumes between individual elements of the railway network.

When solving the problem of choosing the value of transport tariffs and other transportation conditions, the railway administrations should take into account the factor of interaction with railway administrations located both on the same and on parallel transportation routes. The complexity of the task of choosing a rational tariff value in international railway traffic by a separate administration is related to the fact that tariff changes at one section of the network cause changes in the transportation conditions at its other sections. The originality of this work consists of the development of a mathematical model for solving the problems of distribution of freight traffic volumes on the railway network under conditions of competition for them between the individual network elements. At the same time, the task of choosing the tariff value for individual railway administrations is reduced to solving a non-coalition game with a nonzero amount.

The results of the work can be used to create a system for supporting solutions for tariffs evaluation, technical and technological solutions taken in the field of international transit transportations by railway.

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