

# A Modern Course in Aeroelasticity

Fourth Revised and Enlarged Edition

by

**EARL H. DOWELL (Editor)**

*Duke University,  
Durham, NC, U.S.A.*

**ROBERT CLARK**

*Duke University,  
Durham, NC, U.S.A.*

**DAVID COX**

*NASA Langley Research Center,  
Hampton, VA, U.S.A.*

**H.C. CURTISS, JR.**

*Princeton University,  
Princeton, NJ, U.S.A.*

**JOHN W. EDWARDS**

*NASA Langley Research Center,  
Hampton, VA, U.S.A.*

**KENNETH C. HALL**

*Duke University,  
Durham, NC, U.S.A.*

**DAVID A. PETERS**

*Washington University,  
St. Louis, MO, U.S.A.*

**ROBERT SCANLAN**

*Johns Hopkins University,  
Baltimore, MD, U.S.A.*

**EMIL SIMIU**

*National Institute for Standards and  
Technology, Gaithersburg, MD, U.S.A.*

**FERNANDO SISTO**

*Stevens Institute of Technology,  
Hoboken, NJ, U.S.A.*

and

**THOMAS W. STRGANAC**

*Texas A&M University,  
College Station, TX, U.S.A.*



**KLUWER ACADEMIC PUBLISHERS**

DORDRECHT / BOSTON / LONDON

# Contents

Preface	xvii
Preface to the First Edition	xvii
Preface to the Second Edition	xix
Preface to the Third Edition	xx
Preface to the Fourth Edition	xxi
Short Bibliography	xxiii
1. INTRODUCTION (DOWELL)	1
2. STATIC AEROELASTICITY (DOWELL)	5
2.1 Typical Section Model of An Airfoil	5
Typical section model with control surface	10
Typical section model—nonlinear effects	16
2.2 One Dimensional Aeroelastic Model of Airfoils	18
Beam-rod representation of large aspect ratio wing	18
Eigenvalue and eigenfunction approach	22
Galerkin's method	24
2.3 Rolling of a Straight Wing	26
Integral equation of equilibrium	26
Derivation of equation of equilibrium	27
Calculation of $C^{\alpha\alpha}$	28
Sketch of function $S(y_1, \eta)$	28
Aerodynamic forces (including spanwise induction)	30
Aeroelastic equations of equilibrium and lumped element solution method	32
Divergence	33
Reversal and rolling effectiveness	34

	Integral equation eigenvalue problem and the experimental determination of influence functions	37
2.4	Two Dimensional Aeroelastic Model of Lifting Surfaces	41
	Two dimensional structures—integral representation	41
	Two dimensional aerodynamic surfaces—integral representation	42
	Solution by matrix-lumped element approach	43
2.5	Other Physical Phenomena	44
	Fluid flow through a flexible pipe	44
	(Low speed) fluid flow over a flexible wall	47
2.6	Sweptwing Divergence	47
	References for Chapter 2	51
3.	DYNAMIC AEROELASTICITY (DOWELL)	53
3.1	Hamilton's Principle	54
	Single particle	54
	Many particles	56
	Continuous body	56
	Potential energy	56
	Nonpotential forces	59
3.2	Lagrange's Equations	60
	Example—typical section equations of motion	61
3.3	Dynamics of the Typical Section Model of An Airfoil	64
	Sinusoidal motion	64
	Periodic motion	67
	Arbitrary motion	67
	Random motion	73
	Flutter - an introduction to dynamic aeroelastic instability	81
	Quasi-steady, aerodynamic theory	85
3.4	Aerodynamic Forces	87
	Aerodynamic theories available	91
	General approximations	95
	'Strip theory' approximation	95
	'Quasisteady' approximation	95
	Slender body or slender (low aspect ratio) wing approximation	96
3.5	Solutions to the Aeroelastic Equations of Motion	97
	Time domain solutions	98
	Frequency domain solutions	100

3.6	Representative Results and Computational Considerations	103
	Time domain	103
	Frequency domain	103
	Flutter and gust response classification including parameter trends	105
	Flutter	105
	Gust response	121
3.7	Generalized Equations of Motion for Complex Structures	128
	Lagrange's equations and modal methods (Rayleigh-Ritz)	128
	Kinetic energy	129
	Strain (potential elastic) energy	130
	Examples	133
	(a) Torsional vibrations of a rod	133
	(b) Bending-torsional motion of a beam-rod	134
	Natural frequencies and modes-eigenvalues and eigenvectors	135
	Evaluation of generalized aerodynamic forces	136
	Equations of motion and solution methods	137
	Integral equations of equilibrium	139
	Natural frequencies and modes	141
	Proof of orthogonality	143
	Forced motion including aerodynamic forces	144
	Examples	147
	(a) Rigid wing undergoing translation responding to a gust	147
	(b) Wing undergoing translation and spanwise bending	153
	(c) Random gusts-solution in the frequency domain	155
3.8	Other Fluid-Structural Interaction Phenomena	156
	Fluid flow through a flexible pipe: "firehose" flutter	156
	(High speed) fluid flow over a flexible wall - a simple prototype for plate or panel flutter	158
	References for Chapter 3	165
4.	NONSTEADY AERODYNAMICS (DOWELL)	169
4.1	Basic Fluid Dynamic Equations	169
	Conservation of mass	170
	Conservation of momentum	171
	Irrotational flow, Kelvin's theorem and Bernoulli's equation	172
	Derivation of a single equation for velocity potential	174
	Small perturbation theory	175

Reduction to classical acoustics	177
Boundary conditions	178
Symmetry and anti-symmetry	180
4.2 Supersonic Flow	182
Two-dimensional flow	182
Simple harmonic motion of the airfoil	183
Discussion of inversion	185
Discussion of physical significance of the results	187
Gusts	189
Transient motion	190
Lift, due to airfoil motion	191
Lift, due to atmospheric gust	192
Three dimensional flow	195
4.3 Subsonic Flow	201
Derivation of the integral equation by transform methods and solution by collocation	201
An alternative determination of the Kernel Function using Green's Theorem	204
Incompressible, three-dimensional flow	207
Compressible, three-dimensional flow	211
Incompressible, two-dimensional flow	215
Simple harmonic motion of an airfoil	218
Transient motion	224
Evaluation of integrals	229
4.4 Representative Numerical Results	232
4.5 Transonic Flow	238
References for Chapter 4	270
5. STALL FLUTTER (SISTO)	275
5.1 Background	275
5.2 Analytical formulation	276
5.3 Stability and aerodynamic work	278
5.4 Bending stall flutter	279
5.5 Nonlinear mechanics description	281
5.6 Torsional stall flutter	282
5.7 General comments	285
5.8 Reduced order models	288

5.9	Computational stalled flow	289
	References for Chapter 5	294
6.	AEROELASTICITY IN CIVIL ENGINEERING (SCANLAN AND SIMIU)	299
6.1	Vortex-induced Oscillation	301
	Vortex shedding	301
	Modeling of vortex-induced oscillations	305
	Coupled two-degree-of-freedom equations: wake oscillator models	306
	Single-degree-of- freedom model of vortex-induced response	310
6.2	Galloping	314
	Equation of motion of galloping bodies. The Glauert-Den Hartog necessary condition for galloping instability	314
	Description of galloping motion	320
	Chaotic galloping of two elastically coupled square bars	321
	Wake galloping : physical description and analysis	321
6.3	Torsional Divergence	327
6.4	Flutter and Buffeting in the Presence of Aeroelastic Effects	328
	Formulation and analytical solution of the two- dimensional bridge flutter problem in smooth flow	330
	Bridge section response to excitation by turbulent wind in the presence of aeroelastic effects	334
6.5	Suspension-Span Bridges	336
	Wind tunnel testing of suspended-span bridges	336
	Torsional divergence analysis for a full bridge	338
	Locked-in vortex-induced response	340
	Flutter and buffeting of a full-span bridge	350
	Reduction of bridge susceptibility to flutter	360
6.6	Tall Chimneys and Stacks, and Tall Buildings	361
	Tall chimneys and stacks	361
	Tall buildings	365
	References for Chapter 6	367
7.	AEROELASTIC RESPONSE OF ROTORCRAFT (CURTISS AND PETERS)	377
7.1	Blade Dynamics	379
	Articulated, rigid blade motion	379
	Elastic motion of hingeless blades	390

7.2	Stall Flutter	403
7.3	Rotor-Body Coupling	409
7.4	Unsteady Aerodynamics	433
	Dynamic inflow	434
	Frequency domain	440
	Finite-state wake modelling	441
	Summary	444
	References for Chapter 7	444
8.	AEROELASTICITY IN TURBOMACHINES (SISTO)	453
8.1	Aeroelastic Environment in Turbomachines	454
8.2	The Compressor Performance Map	455
8.3	Blade Mode Shapes and Materials of Construction	460
8.4	Nonsteady Potential Flow in Cascades	462
8.5	Compressible Flow	467
8.6	Periodically Stalled Flow in Turbomachines	471
8.7	Stall Flutter in Turbomachines	475
8.8	Choking Flutter	477
8.9	Aeroelastic Eigenvalues	479
8.10	Recent Trends	481
	References for Chapter 8	487
9.	MODELING OF FLUID-STRUCTURE INTERACTION (DOWELL AND HALL)	491
9.1	The Range Of Physical Models	491
	The classical models	491
	The distinction between linear and nonlinear models	494
	Computational fluid dynamics models	495
	The computational challenge of fluid structure interaction modeling	495
9.2	Time-Linearized Models	496
	Classical aerodynamic theory	496
	Classical hydrodynamic stability theory	497
	Parallel shear flow with an inviscid dynamic perturbation	497
	General time-linearized analysis	498
	Some numerical examples	500
9.3	Nonlinear Dynamical Models	500
	Harmonic balance method	503

System identification methods	503
Nonlinear reduced-order models	504
Reduced-order models	504
Constructing reduced order models	505
Linear and nonlinear fluid models	506
Eigenmode computational methodology	507
Proper orthogonal decomposition modes	508
Balanced modes	509
Synergy among the modal methods	509
Input/output models	509
Structural, aerodynamic, and aeroelastic modes	511
Representative results	512
The effects of spatial discretization and a finite computational domain	512
The effects of mach number and steady angle of attack: subsonic and transonic flows	516
The effects of viscosity	521
Nonlinear aeroelastic reduced-order models	522
9.4 Concluding Remarks	524
References for Chapter 9	529
Appendix: Singular-Value Decomposition, Proper Orthogonal Decomposition, & Balanced Modes	538
10. EXPERIMENTAL AEROELASTICITY (DOWELL)	541
10.1 Review of Structural Dynamics Experiments	541
10.2 Wind Tunnel Experiments	543
Sub-critical flutter testing	543
Approaching the flutter boundary	544
Safety devices	544
Research tests vs. clearance tests	544
Scaling laws	544
10.3 Flight Experiments	545
Approaching the flutter boundary	545
When is flight flutter testing required?	545
Excitation	545
Examples of recent flight flutter test programs	546
10.4 The Role of Experimentation and Theory in Design	546
References for Chapter 10	548



11. NONLINEAR AEROELASTICITY (DOWELL, EDWARDS AND STRGANAC)	551
11.1 Introduction	551
11.2 Generic Nonlinear Aeroelastic Behavior	552
11.3 Flight Experience with Nonlinear Aeroelastic Effects	554
Nonlinear aerodynamic effects	556
Freeplay	556
Geometric structural nonlinearities	557
11.4 Physical Sources of Nonlinearities	557
11.5 Efficient Computation of Unsteady Aerodynamic Forces: Linear and Nonlinear	558
11.6 Correlations of Experiment/Theory and Theory/Theory Aerodynamic forces	560 560
11.7 Flutter Boundaries in Transonic Flow	566
11.8 Limit Cycle Oscillations	573
Airfoils with stiffness nonlinearities	573
Nonlinear internal resonance behavior	575
Delta wings with geometrical plate nonlinearities	577
Very high aspect ratio wings with both structural and aerodynamic nonlinearities	578
Nonlinear structural damping	581
Large shock motions and flow separation	581
Abrupt wing stall	594
Uncertainty due to nonlinearity	595
References for Chapter 11	598
12. AEROELASTIC CONTROL (CLARK AND COX)	611
12.1 Introduction	611
12.2 Linear System Theory	612
System interconnections	612
Controllability and observability	615
12.3 Aeroelasticity: Aerodynamic Feedback	617
Development of a typical section model	617
Aerodynamic model, 2D	619
Balanced model reduction	622
Combined aeroelastic model	623
Development of a delta wing model	627
Transducer effects	630

Aerodynamic model, 3D	633
Coupled system	634
12.4 Open-Loop Design Considerations	636
HSVs and the modal model	637
Optimization strategy	638
Optimization results	641
12.5 Control Law Design	642
Control of the typical section model	644
Control of the delta wing model	647
12.6 Parameter Varying Models	647
Linear matrix inequalities	648
LMI controller specifications	649
An LMI design for the typical section	652
12.7 Experimental Results	654
Typical section experiment	655
LPV system identification	656
Closed-loop results	658
Delta wing experiment	664
12.8 Closing Comments	667
References for Chapter 12	669
13. MODERN ANALYSIS FOR COMPLEX AND NONLINEAR UNSTEADY FLOWS IN TURBOMACHINERY (HALL)	675
13.1 Linearized Analysis of Unsteady Flows	676
13.2 Analysis of Unsteady Flows	683
13.3 Harmonic Balance Method	688
13.4 Conclusions	699
References for Chapter 13	701
Appendices	704
Appendix A: A Primer For Structural Response To Random Pressure Fluctuations	705
A.1 Introduction	705
A.2 Excitation-Response Relation For The Structure	705
A.3 Sharp Resonance or Low Damping Approximation	709
Nomenclature	710
References for Appendix A	710

Appendix B: Some Example Problems	711
B.1 For Chapter 2	711
B.2 For Section 3.1	724
B.3 For Section 3.3	730
B.4 For Section 3.6	735
B.5 For Section 4.1	738
Index	743