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A Modification on Applied Element Method for Linear Analysis of Structures in the Range of Small and Large Deformations Based on Energy Concept

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ABSTRACT: In this paper, the formulation of a modified applied element method for linear analysis of structures in the range of small and large deformations is expressed. To calculate deformations in the structure, the minimum total potential energy principle is used. This method estimates the linear behavior of the structure in the range of small and large deformations, with a very good accuracy and low analytical time. The results show that analysis of a console beam by proposed method, even with minimum numbers of elements, in range of small deformations, has a computation error of less than 2%. Meanwhile, solving the same problem by Applied Element Method (AEM), has more than 31% error. Also, the buckling and post-buckling behavior of the structure, within the range of large deformations, is well-suited. So, with minimum number of elements, and very high accuracy, the buckling behavior of the fixed-base column was simulated. Also, the computational time of the proposed method is less than 40 percent of the computational time in the application of the applied elements method with 10 series of connection springs.

Keywords: Applied Element Method, Linear Analysis, Minimum Total Potential Energy Principle, Small And Large Deformations.

INTRODUCTION

Many studies have been conducted on analytical and efficient methods for the accurate simulation of the progressive collapse mechanism. Among these studies, some researches in this field can be cited (Tavakoli and Kiakojouri, 2013, 2014; Tavakoli and Akbarpoor, 2014). Also in the last two decades, methods based on discrete elements have grown dramatically. Various branches of Distinct Element Method (DEM) family developed. DEM was proposed by Cundall in 1971 (Zhang et al., 2019). Generalized Discrete Element Method (Williams et al., 1985), Discontinuous Deformation Analysis (DDA) (Shi, 1992) and the Finite-Discrete Element Method (Munjiza et al., 1995), concurrently developed by several groups. Distinct Element method was

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originally developed by Cundall in 1971 to analysis rock mechanics problems (Zhang et al., 2019). The theoretical basis of this method was established by Sir Isaac Newton in 1697 (Bićanić, 2017). Williams et al. (1990) showed that discrete element method could be viewed as a generalized finite element method and described its application to geomechanics problems. Today distinct element method is widely accepted as an effective method of addressing engineering problems in granular and discontinuous materials, especially in granular flows, powder mechanics, and rock mechanics (Govender et al., 2016). Recently, the method was expanded into the Extended Discrete Element Method (EDEM) taking thermodynamics and coupling to Computational Fluid Dynamics (CFD) and Finite Element Method (FEM) into account (Alizadeh et al., 2019).

First, rigid body and springs method was introduced (Kawai, 1980). This method simulates cracking in a simpler way than the finite element method. But the growth path of the cracks in this method essentially depends on the shape, size and arrangement of the elements. (Ueda and Kambayaashi, 1993; Soltani and Moshirabadi, 2019). Then Meguro and Hakuno (1989) developed modified discrete element method. It was good to follow the extreme non-linear behavior of the structure. But in some cases, the accuracy was not enough and compared with the finite element method and rigid body and springs method, it required a relatively longer CPU time. Meguro and Tagel-Din (1997) presented the Applied Element Method. This method, with simple formulation, can follow the behavior of the structure with high accuracy and acceptable computational time from the initial stages of loading to the destruction stages. But it is less accurate than, FEM, in the range of small deformations (Gohel et al., 2013). Therefore, in order to overcome this problem, more number of elements with smaller size should be used, which will increase the computational time. In addition, due to the nature of the rectangularity of the elements, the path of crack growth, is depended on the shape, size, and arrangement of the elements (Worakanchana and Meguro, 2008). Applied Element Method, is a new method for structural analysis, that combines traits of both the Finite Element Method and Discrete Element Method (DEM). FEM can be accurate until element separation, while DEM can be used when elements are separated (Shakeri and Bargi, 2015) .

Today, a wide range of research is conducted on the progressive collapse analysis by AEM. This includes simulation of an industrial building demolition (Simon and Dragomir, 2013), theoretical research on progressive collapse of Reinforce Concrete (RC) frame buildings (Lupoae and Constantin, 2013), and even progressive collapse analysis of existing building (Prionas, 2016).

In applied element method, the structure is considered as a set of discrete elements. Unlike the FEM, instead of the principle of minimum total potential energy, the concept of equilibrium and hardness matrix are used to solve the problem. Therefore, it is less accurate in the range of small deformations (Liu and Piemoz, 2016). To overcome this problem, the number of elements and springs between the elements should be increased, which will increase the computational time of this method. However, due to the assumption that structural materials are discrete in this method, the simulation of structural behavior in the range of large deformations and cracking is much simpler than the FEM, which makes it possible to model the destructive and post-destructive behavior of the structure with less computational time (Borja and Thomas, 2015).

The proposed modified applied element method, applies two effective modifications to the applied element method. First, due to the use of the principles of minimum total potential energy instead of equilibrium concept, has a better performance than the applied element method. Also the concept of side elements was developed to more accurately model the structure, especially when it is going to model the structure with few elements. Using this method, the outer half of the elements adjacent to the structural boundary are included in the modelling, whereas in the conventional modelling method, this part of the elements is not included in the modelling. Therefore models that are performed by this method, especially when the size of the elements are large, will be far more accurate than the conventional models. As a result, this method yields more accurate answers with fewer elements and lower computational time. Also, due to the use of discrete elements, structural behavior can be easily tracked in the stages of large deformation, cracking and failure. Even in the large deformation range, it is more accurate and faster than the applied element method due to the use of the principles of minimum potential total energy and fewer springs.

In this paper, first, the formulation of the modified applied element method is expressed for linear materials and small deformations. Then, the corrections needed to simulate large deformations are expressed. To investigate the results within the range of small deformations, a sensitivity analysis will also be presented on the effect of the number of elements, and the results will be compared with the applied element method. In the following, three examples, including a simple support beam, a two-bar truss, and a fixed-based column, are considered in the range of large deformations analysis. The verification of the proposed method is done by comparing the results obtained with the theoretical values.

FORMULATION OF MODIFIED APPLIED ELEMENTS METHOD

Analysis of Small Deformations

This article is concerning to implement Applied Element Method for analysis of structures in the range of either small or large deformations. To this end, the domain of the structure is divided into some rigid parts (elements), which are connected with the use of some linear springs at their boundaries covering axial, shear and bending behavior of the structure. The total energy (the sum of internal energy and the work done by the external forces) is considered as the target functional to be minimized. As a result the displacements/rotations of different nodes are the primary unknowns in their analysis. Thus, a system of algebraic equations should be primarily solved to obtain the displacements/rotations of different nodes. The large deformations have been carried out by dividing the nonlinear problem into some consequently linear deformations as usual. Figure 1 shows the elements arrengment of the structure and the connection of the elements by the springs.

The spring between the two elements represents the forces and deformations between the two adjacent elements. Its axial, shear and bending stiffness, obtained from Eq. (1). The effect zone of the spring between the two elements is also shown in Figure 1.

$$K_n = \frac{E \times b \times t}{a} \qquad K_s = \frac{G \times b \times t}{a} \qquad K_\theta = \frac{E \times t \times b^3}{12 \times a} \tag{1}$$

where K_n, K_s and K_θ: are the axial, shear and bending stiffness of the springs, respectively. t: is the thickness of the element, b: represents element's width, a: represents element's length, E and G: represent the modulus of elasticiy and the shear modulus of the material, respectively. These relationships indicate that spring stiffness represents the properties of materials.

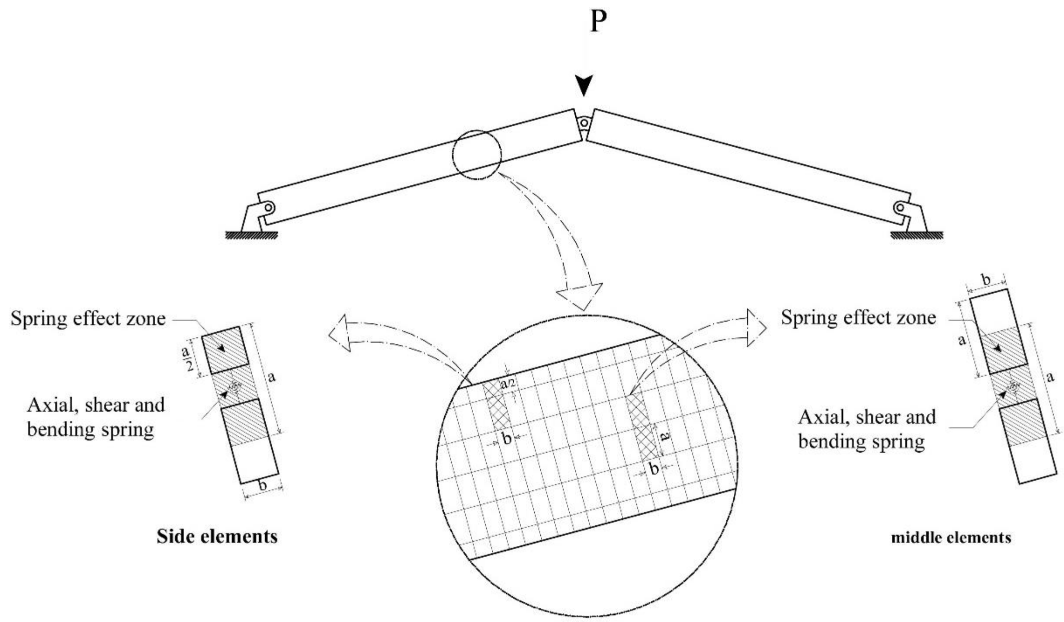


Fig. 1. Modeling of structure in modified applied element method

For better modeling of the structure, the concept of side elements was developed in this method. This concept has been developed due to the effect zone of the springs on the side elements of the structure. As shown in Figure 1, in the middle elements only half of the element is modeled by a spring coneccted to the adjacent element. Therefore, if the side elements are not used, the other half of the element will not be modeled by springs and therefore the correct model of the structure will not be presented. Therefore, the concept of side elements is developed. The degrees of freedom, and the corresponding positive directions, are given in Figure 1.

According Figure 2 δ_i' and ε_i' represent the variations of the axial and shear displacments of i[th] element under angle α, and δ_i and ε_i, are the variations of the horizontal and vertical displacments of i[th] element, and θ_i is the rotation of i[th] element. *a*: is also the size of the element. For calculating δ_i' and ε_i', Eqs. (2a) and (2b) are used in terms of δ_i and ε_i.

$$\delta_i' = \delta_i \times \cos(\alpha) + \varepsilon_i \times \sin(\alpha) \tag{2a}$$

$$\varepsilon_i' = \varepsilon_i \times \cos(\alpha) - \delta_i \times \sin(\alpha) \tag{2b}$$

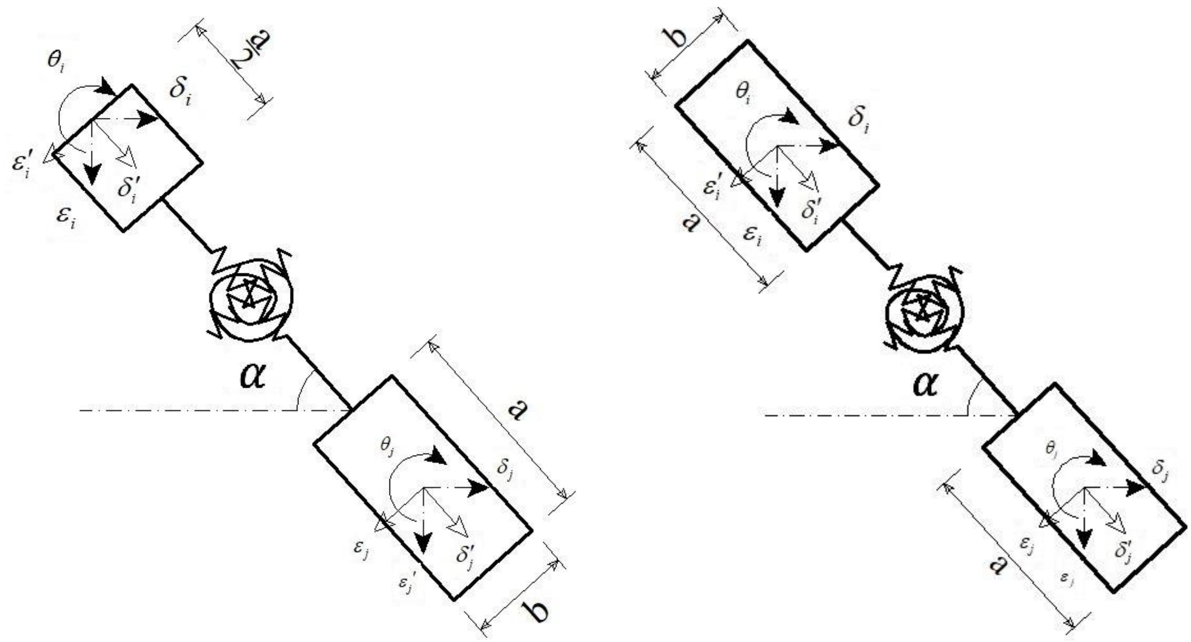
Given the fact that each element has three degrees of freedom on the plane, which includes the horizontal, vertical, and bending degree of freedom, the deformation of the springs between the two elements can be expressed in terms of these degrees of freedom of two adjacent elements. According to Figure 1, and given the positive direction of degrees of freedom agreement, axial, shear and bending deformation of springs can be calculated in the range of small deformations as follows. Axial deformation, is related to the connecting springs of elements i and j (Eq. (3)).

$$\Delta = \delta_j' - \delta_i' \tag{3}$$

Shear deformation, is related to the connecting springs of elements i and j (Eq. (4)).

$$\gamma = \varepsilon_j' - \varepsilon_i' - (\theta_j + \theta_i) \times a/2 \tag{4}$$

Bending deformation, is related to the connecting springs of elements i and j (Eq. (5)).



(a) (b)

Fig. 2. Elements shape, spring position and degrees of freedom: a) side elements; b) middle elements

$$\Phi = \theta_j - \theta_i \quad (5)$$

By placing the Eq. (2) into Eqs. (3) and (4) the deformation of the spring between the two adjacent elements will be obtained as a function of the displacement of these two elements. Therefore, by knowing the values of the displacements of the adjacent elements, the deformation and, consequently, the spring forces between the two elements can be calculated.

Regarding the principle of minimum total potential energy, the actual deformation of the structure is the same deformation that minimize the potential energy of the entire structure. Eq. (6) represents the total potential energy of the structure:

$$\Pi = U + V \quad (6)$$

where Π: is the total potential energy of the structure, U: is the internal potential energy and V: is the external potential energy of the structure.

Due to the use of discrete elementalization in present method, which considers the structure as a set of rigid elements and springs, it is clear that internal potential energy can be derived from the sum of stored energy in all existing springs of the structure, according to Eq. (7).

$$U = \frac{1}{2}\left\{\begin{array}{l}(K_{n_1} \times \Delta_1{}^2 + K_{n_2} \times \Delta_2{}^2 + \cdots + K_{n_i} \times \Delta_i{}^2 + \cdots + K_{n_{nf}} \times \Delta_{n_f}{}^2) + \\ (K_{s_1} \times \gamma_1{}^2 + K_{s_2} \times \gamma_2{}^2 + \cdots + K_{s_i} \times \gamma_i{}^2 + \cdots + K_{s_{nf}} \times \gamma_{n_f}{}^2) + \\ (K_{\theta_1} \times \Phi_1{}^2 + K_{\theta_2} \times \Phi_2{}^2 + \cdots + K_{\theta_i} \times \Phi_i{}^2 + \cdots + K_{\theta_{nf}} \times \Phi_{n_f}{}^2)\end{array}\right\} \quad (7)$$

where K_{n_i}, K_{s_i} and K_{θ_i}: represent axial, shear and bending stiffness of the ith spring, respectively, and n_f : is the total number of structural springs. Also Δ_i, γ_i and Φ_i: are axial, shear and bending deformation of ith spring, that are a function of the displacements of elements, and U: is also a function of the displacements of all elements of the structure, The number of these variables in a two-dimensional model is three times of the number of elements (equal to the total number of degrees of freedom in the structure).

Similarly, Eq. (8) can be used to calculate the external potential energy V:

$$V = -\left\{ \begin{array}{l} (P_1 \times \delta_1 + P_2 \times \delta_2 + \cdots + P_i \times \delta_i + \cdots + P_{n_e} \times \delta_{n_e}) + \\ (F_1 \times \varepsilon_1 + F_2 \times \varepsilon_2 + \cdots + F_i \times \varepsilon_i + \cdots + F_{n_e} \times \varepsilon_{n_e}) + \\ (M_1 \times \theta_1 + M_2 \times \theta_2 + \cdots + M_i \times \theta_i + \cdots + M_{n_e} \times \theta_{n_e}) \end{array} \right\} \quad (8)$$

where n_e: is the total number of elements of the structure, P_i and δ_i: represent horizontal force and displacement of i[th] element, F_m and ε_m: are vertical force and displacement of i[th] element and M_n and θ_n: represent the moment and bending displacement of i[th] element.

External potential energy is composed of the potential energy generated by external forces and support reactions. Also, with respect to Eq. (8), it can be seen that external potential energy is a function of the displacement of the elements under external loads or support reactions, and the number of these variables in general can be the total number of degrees of freedom in structure.

According to the formulation of V and U, it can be concluded that Π is a function of the displacements of the structural elements (whole structure elements degrees of freedom). So it has $3 \times n_e$ variables (where n_e is the total number of structural elements).

Given the principle of minimum total potential energy, to find the deformations in the structure, it is sufficient to derive Π relative to the displacements of the elements and equal to zero. Thus, the displacement values of the elements corresponding to the minimum Π, which are the unknowns of the problem, will be obtained.

$$\frac{\partial \Pi}{\partial \delta_i} = 0 \to \to \quad (9)$$

$$\frac{\partial \Pi}{\partial \varepsilon_i} = 0 \to \to \quad (10)$$

$$\frac{\partial \Pi}{\partial \theta_i} = 0 \to \to \quad (11)$$

$for\ i = 1\ \ to\ \ n_e$ independent linear equation.s

First, according to Eq. (9), the derivative of Π will be set to zero with respect to the horizontal displacement of the elements (δ_i). This gives us n_e linear equations. Then the derivative of Π, with respect to the vertical displacement of the elements, (ε_i) will be set to zero, which according to Eq. (10), n_e more linear equations will be obtained. Finally, the derivative Π will be set to zero with respect to the bending displacement of the elements (θ_i). According to Eq. (11), there are also n_e linear equations in this way. Therefore a linear relation system including $3 \times n_e$ relation will be obtained. By solving this equations, the displacements of the elements, which are unknown will be obtained.

Analysis of Larg Deformations

To analyze the structure under large deformations, the problem can be divided into steps involving small deformations, in each of these steps, the theory of small deformations is acceptable. Therefore, in each step, the method described in the previous section can be used to solve the problem. But in formulating the Π relation at each step, considering the change in the position of the elements and springs, the following points should be noted:

1. In order to calculate the internal energy of the structure (U) in each step, the axial, shear and bending deformations of the springs must be calculated according to the positioning of the elements due to the

previous step.

$$U = \frac{1}{2}\left\{(\sum_{1}^{n_f} K_{n_i} \times \Delta_{i_s}{}^2 + \sum_{1}^{n_f} K_{s_i} \times \gamma_{i_s}{}^2 + \sum_{1}^{n_f} K_{\theta_i} \times \Phi_{i_s}{}^2\right\} \quad (12)$$

In fact, due to the change in the position of the elements in each step compared to the previous one, at the beginning of each step, we will be faced with a new geometry and thus a new problem. Therefore, Δ_{i_s}, γ_{i_s} and Φ_{i_s} in Eq. (12) represent axial, shear and bending deformations of the ith spring in the sth step, which can be calculated from Eqs. (2-5) according to the state of the elements in the previous step.

2. To calculate the external potential energy of the structure V at each step, and considering the change in the direction of the springs at the end of each step, the equilibrium of the elements will not be satisfied with the initial assumption at the beginning of the same step. Therefore, for each element, the non-balancing forces in the preceding step should be considered as an external force at next stage in this way, the cumulative error due to the change in the geometry will be prevented.

$$V = -\left\{\sum_{1}^{n_e} P_{i_s} \times \delta_{i_s} + \sum_{1}^{n_e} F_{i_s} \times \varepsilon_{i_s} + \sum_{1}^{n_e} M_{i_s} \times \theta_{i_s}\right\} \quad (13)$$

where P_{i_s} and δ_{i_s}: represent horizontal force and displacement of ith element in sth step, F_{m_s} and ε_{m_s}: are vertical force and displacement, of ith element in sth step and M_{n_s} and θ_{n_s}: represent the moment and bending displacement of ith element in sth step, respectively.

PROCEDURES FOR THE APPLICATION OF THE MODIFIED APPLIED ELEMENT METHOD

Given that each element has 3 degrees of freedom in two-dimensional mode, the problem containing the n_e element will have a total of $3 \times n_e$ unknowns (elements displacements). Of course, depending on boundary conditions, some of these displacements may be equal to zero or a known value, whereby, in the bounded degree of freedom, support reactions will sometimes occur whose values are unknown. Therefore, the total number of unknowns remain equal $3 \times n_e$, and the number of equations is exactly the same. Since for each degree of freedom, the Π relation is differentiated, and therefore a linear equations system including $3 \times n_e$ equations that can be solved by conventional solving methods. A description of the program steps is given below. The flowchart of the program developed in Matlab is also shown in Figure 3.

First, in the preprocessing stage, the basic information includes the values of elastic module and shear module, the dimensions of the element, the number and position of the elements with constraints and type of constraints, the number and position of loaded elements and the amount of load corresponding to each degree of freedom will be received. Then the program forms Π relation by considering the element numbering matrix, taking into account the number of elements and springs between them, as well as considering the external forces, and the possible support reactions. Then, by computing the derivative of Π relative to each of the elements' displacements, a linear equations system will be formed. By solving this system of equations, the displacements corresponding to each degree of freedom will be calculated. Therefore, the forces and displacements of the springs between the elements are also obtained.

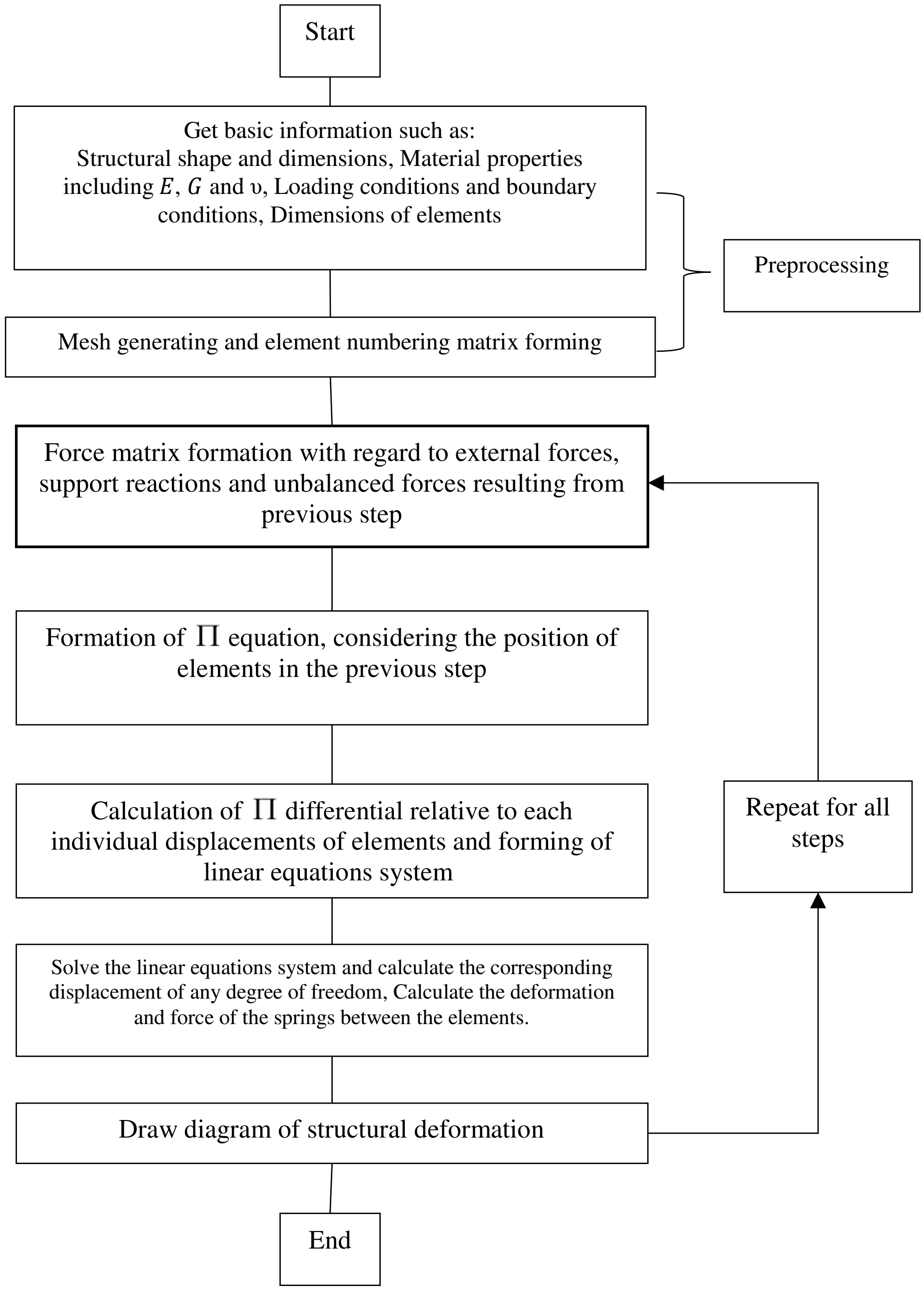


Fig. 3. Stages of linear analysis of structure using the modified applied element method

INVESTIGATION OF MODIFIED APPLIED ELEMENT METHOD FOR STRUCTURAL ANALYSIS OF SMALL DEFORMATIONS

The Effect of Element Size on the Accuracy of the Responses and Comparison of the Modified Applied Method with AEM

A sensitivity test was carried out on the effect of the size of the elements on the computational accuracy and the analysis time of the proposed method. Thus, a console beam with an elastic module E = 2.1×10^8 KN/m^2 and a Poisson coefficient υ = 0.3 and the dimensions as shown in Figures 4 and 5 with thickness t = 0.25 m under the lateral load P = 10 KN were considered. The beam was analyzed for the models 1 to 6, respectively, according to Figures 4 and 5. Deformation of beam were compared with theoretical values, and finally, the error percentage and computational time of each model were compared with other models. In the analysis of these models with applied element method models with 10 and 20 springs for connecting adjacent elements were used.

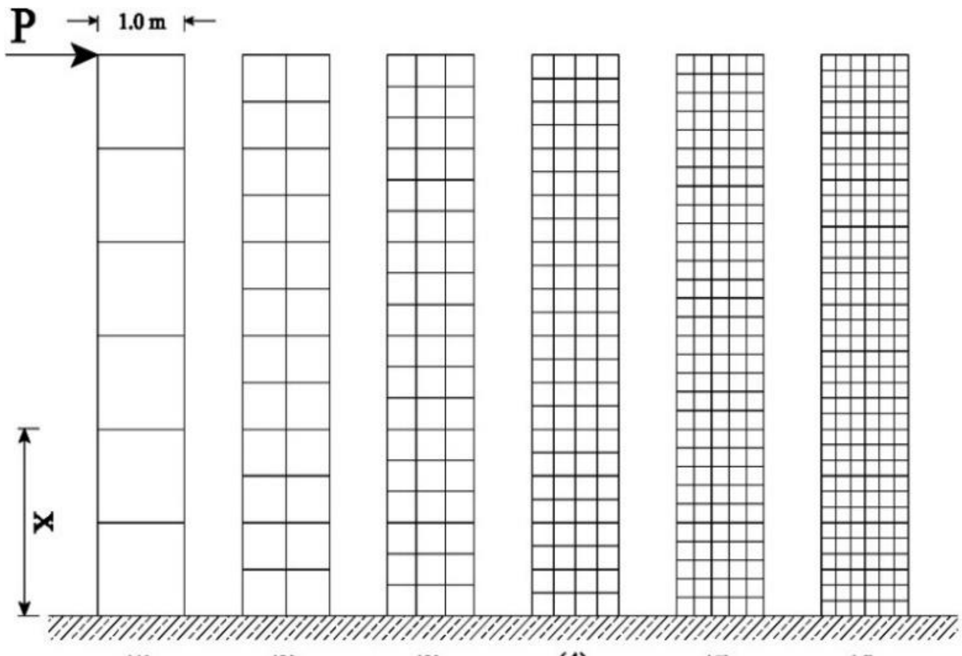


Fig. 4. Size and arrangement of elements for the console beam model under lateral load used in the applied element method

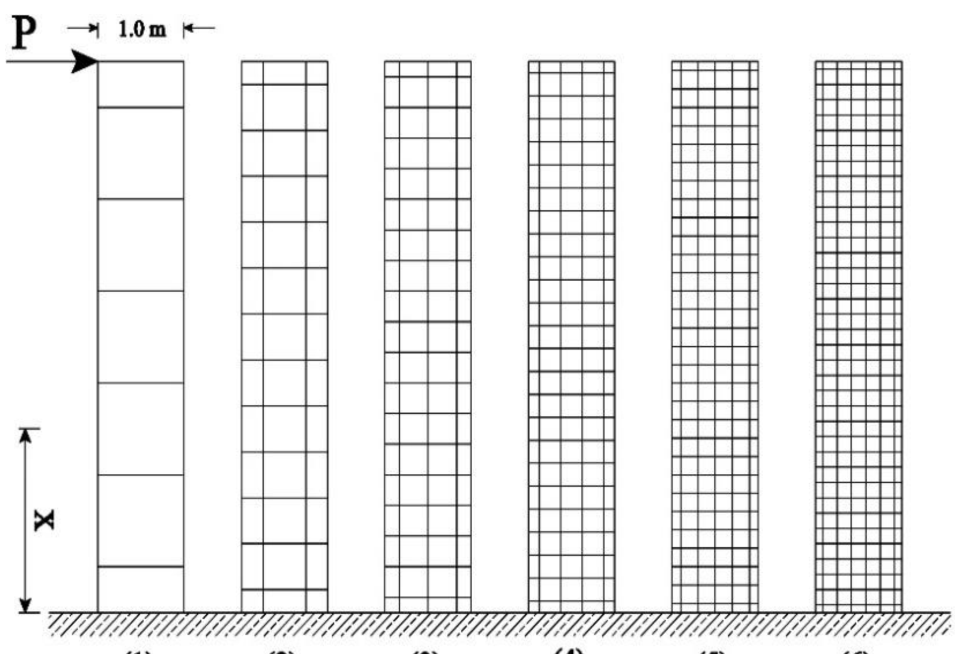


Fig. 5. Size and arrangement of the elements for the console beam model under lateral load in the modified applied element method

The console beam theoretically deformation, can be calculated according to Eq. (14).

$$\delta_x = \frac{PX^2}{6EI}(3L - X) \qquad (14)$$

in which δ_x: represents beam deformation at X meter distance from the support, P: represent lateral force applied to tip of the console, L: is length of the beam, and I: indicates the moment of inertia of the beam section.

Figure 6 shows deformation curve of the beam obtained by AEM and modified applied element method for the model (1) of the console beam. According to the figure, there is a very high correlation between the calculated values of the modified applied element method and theoretical values. It should be noted that model (1) uses 1 m elements and the entire beam is modeled with only seven elements. Also, according to the results of the analysis of the beam in model (1) by applied element method, the difference between computational values and theoretical values can be seen clearly.

According to Figure 7, in the applied element method, the processing time of the program is also reduced by reducing the number of springs connecting elements, so that the processing time in the model with 10 springs for connecting elements, is approximately half of the corresponding time in the model with 20 spring for connecting elements (Tagel-Din, 1998). The same trend was observed in the modified applied element method, so that the processing time in this method was approximately 40% of the processing time associated with the model with 10 connecting springs in the applied element method. This result can be justified due to the fact that, the number of connection springs in the proposed method is reduced to one spring and also due to the use of side elements in this method, which leads to an increase in the number of elements compared to the applied element method.

The results of Figure 7 show that the modified applied element method, using the concept of side elements for better modeling of the structure and using the concept of energy to solve the problem, even with a row of elements in the base of the beam, had the ratio of error less than 2%. While the error of similar model in the applied element method was more than 31%. This error for the proposed method decreases with increasing number of elements, so that the error percentage for 5 or more rows of elements will be less than 0.2%. Therefore, the proposed method, even with the larger dimensions of elements and much lower computational time will produce more accurate answers than the applied element method.

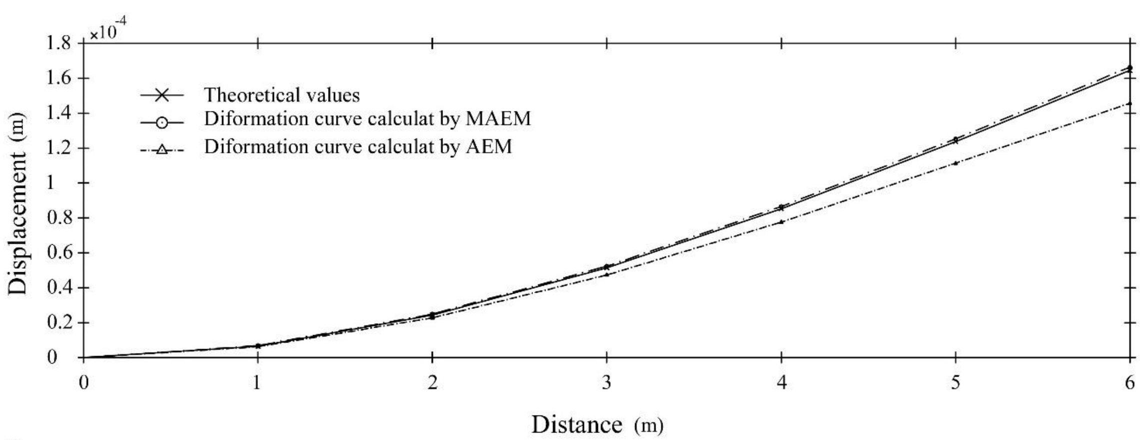


Fig. 6. Deformation of the beam - model (1) - analyzed by applied element method, modified applied element method, and comparison with theory

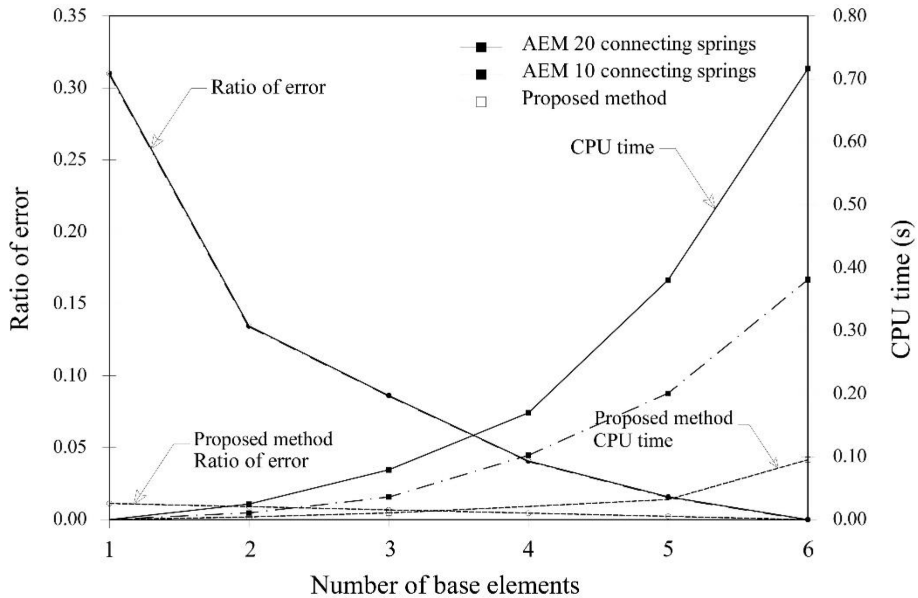


Fig. 7. The effect of dimensions of element on computational time and the computation error ratio

Evaluation of Energy Absorption in the Frame Using Modified Applied Element Method and Comparison with AEM

To investigate the efficiency of the proposed method for solving various problems, a one story one span frame, with simple supports was considered. The elastic modulus E = 2.1×10^8 KN/m^2, Poisson coefficient υ = 0.3 and element thickness t = 0.25 m where considered. The dimensions of the frame are as shown in Figure 8 and frame is under the tensile load of P in the middle of the span.

The behavior of the frame was evaluated using the applied element method and modified applied element method. Deformation curve in BC part of the frame was compared with the theoretical curve. Also, energy absorption of the frame was calculated with respect to the area values below the force-deformation curve in each of the applied element method and modified applied element method and compared with the corresponding theoretical values.

Deformation curvature of BC, calculated by applied element method and modified applied element method, and comparison with the theoretical results, is presented in Figure 9. It can be seen that the modified applied element method has a very high correlation with theoretical values. While the applied element method has a high computational error. So that to calculate the maximum deformation in BC using the modified applied element method, an error of 0.15% will occur, while the corresponding error in the applied element method is estimated to be more than 31%. According to the results obtained from the frame analysis, very good agreement with results of console beam is visible.

Table 1. Comparison of the frame ductility for applied element method and modified applied element method

Analysis method	Displacement ($\times 10^{-5}\ m$)	Displacement error (%)	Area below the force-deformation diagram (m^2)	Area error (%)
Theoretical method	1.6333	-	7.1458×10^{-5}	-
Modified applied element method	1.6362	+0.17	7.3732×10^{-5}	+3.1
Applied element method	1.1219	-31.31	4.1982×10^{-5}	-41.2

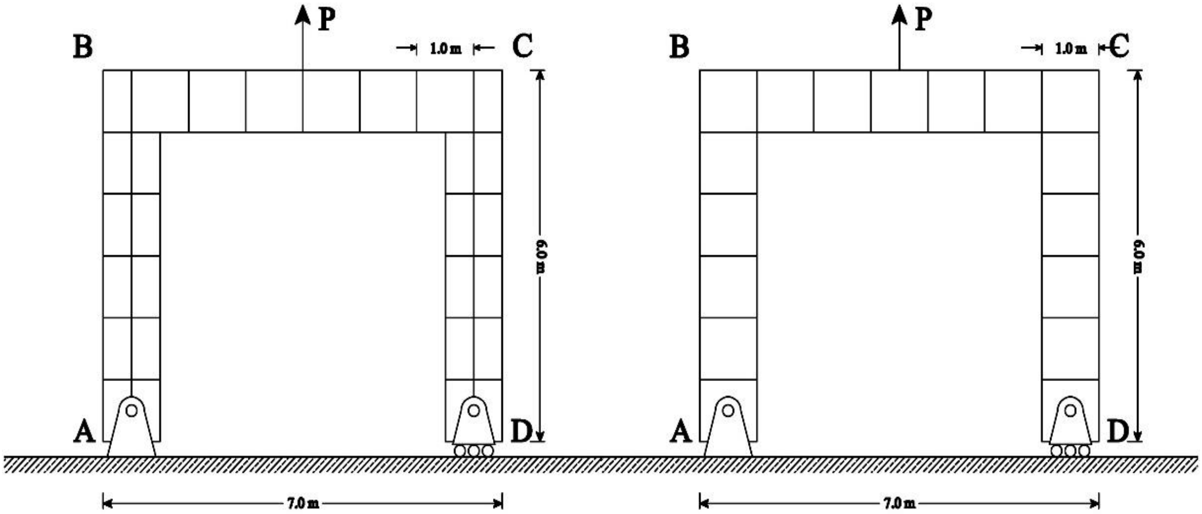


Fig. 8. Size and arrangement of elements for one span frame modeling used in: a) modified applied element method; b) applied element method

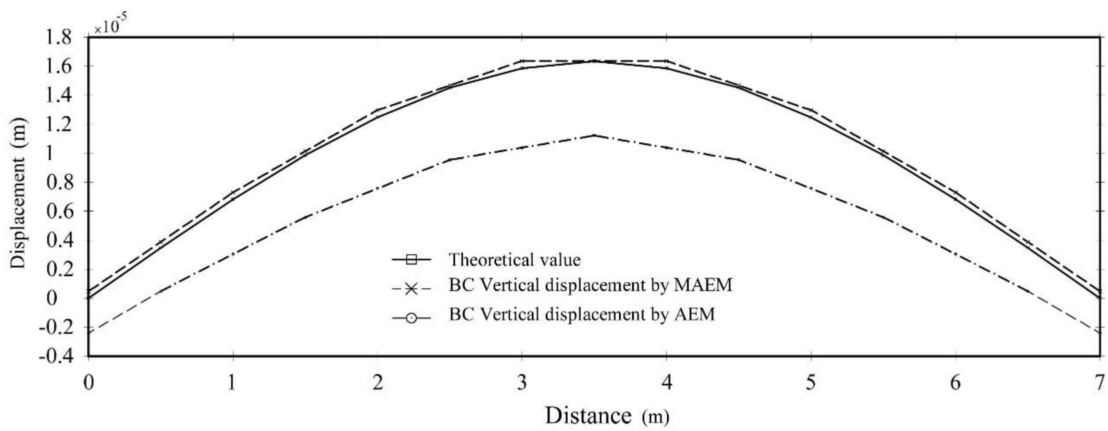


Fig. 9. Deformation of the BC part of the frame with the application of applied element method, modified applied element method and comparison with theory

According to the concept of ductility and its relation to the area below the force-deformation diagram, and Table 1, the results are similar to those of the previous section, so that the error of estimation of ductility in the modified applied element method is 3.1% and the corresponding value in the applied element method is more than 41%.

INVESTIGATION OF MODIFIED APPLIED ELEMENT METHOD FOR STRUCTURAL ANALYSIS OF LARGE DEFORMATION

Simulation of a Simple Support Beam under Large Deformations

A simple support beam with 12 m span and a square cross-section with 1 m length, and an elastic modules $E = 210$ Mpa was considered. As shown in Figure 10, when load increase and taking into account the effect of the large deformations, beam shape changes in a way that the rolled support is displaced. While, without considering the effect of the large displacements, rolled support displacement would not be modeled.

The changes of the applied force against the horizontal displacement of the rolled support, *DX* and vertical displacement in the middle of the span, *DY* is shown in Figure 11. The results of applied element method were obtained with 300 elements for modeling, while the results obtained from modified applied element method were obtained only using 12 elements. Nevertheless, results matched very well and computational time decreased. As can be seen, by load increasing the stiffness of the system increases. This is due to a change in the form of the beam from flat to arc that will increase the stiffness of the beam.

Local Buckling of Symmetrical Two-Bar Truss

Dimensions, shape and the way of truss loading are shown in Figure 12. The elastic module of truss members *E* = 210 Mpa is considered. Bar cross-section are square with dimensions of 0.1 m. Half of the structure was modeled due to the symmetry. Displacement of the middle of the span was increased gradually. The relationship between the applied force (*P*) and the internal force of the member (*S*) against the change in the displacement ratio (*d/H*) was investigated. The comparison of the results with theoretical values (Szabo et al., 1986) show complete accommodation. A model with only a single row of elements, including 11 elements with dimension of 0.1 m was used.

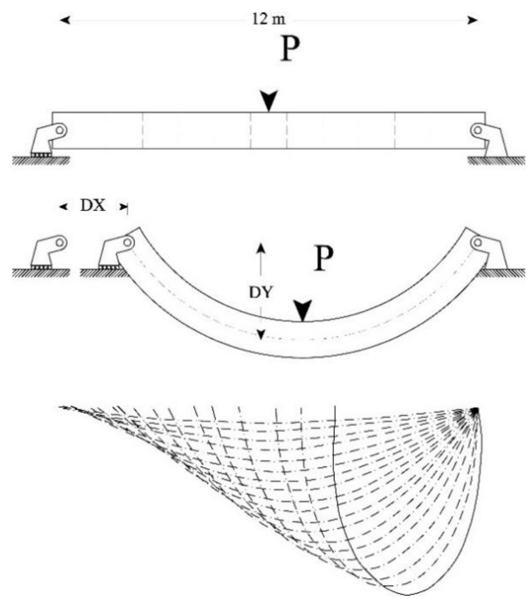


Fig. 10. Simple support beam deformation curve

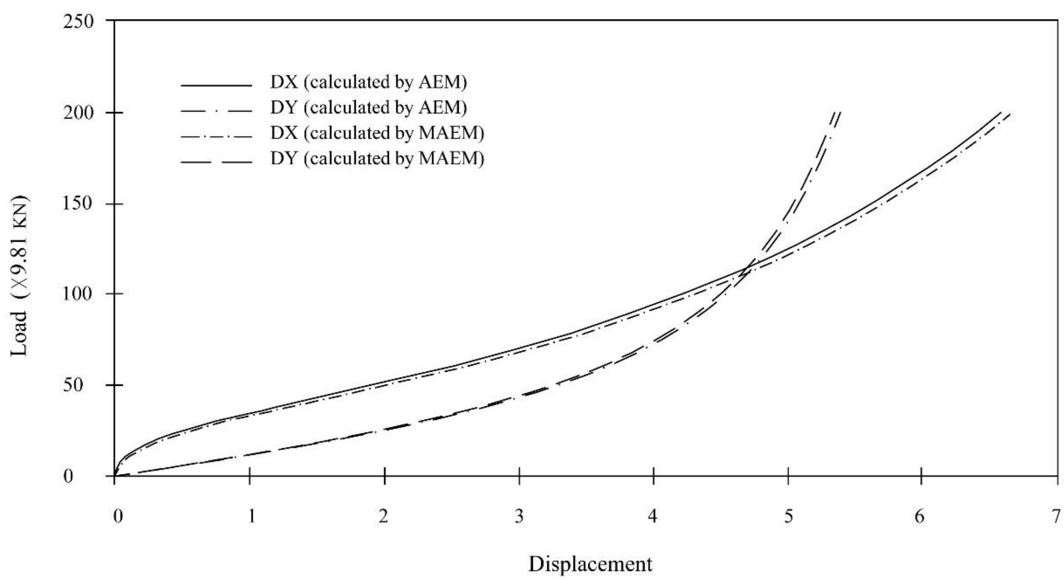


Fig. 11. Beam load- displacement curve with a simple support for large deformations

The steps for changing the truss were as follows:

1. The bar under pressure suffers a decrease in length. This process continues until the bar is in a horizontal position. At this stage, bar resist maximum stress, and applied load *P* would be zero.

2. As the displacement increases, internal force of the bar will be released and direction of the load *P* changes, when the displacement reached 2×(*d/H*), bar reaches its initial length, and therefore the internal force of the bar and the applied load will be zero.

3. As the displacement increases, the bar undergoes an increase in length and thus tensile strength. The direction of the external load also varies.

Buckling Behavior Simulation for Fixed-Base Column

A fixed base column under pressure was checked as shown in Figure 13. The section of the column was square and 1 m in size, the height of the column was 12 m and its elastic module was *E* = 840 Mpa. Column modeled by a row of elements that consist of only 13 squared elements. Horizontal displacement of the column tip was increased with a constant ratio. To change the symmetry, an extremely small moment was inserted into the column tip at the first step. The horizontal and vertical displacement of the tip of the column was calculated under load *P* and was compared with theoretical values of Timoshenko and Gere (1961). Like the applied element method, good accommodation was observed. With the difference that the model made in the modified applied element method simulated the buckling behavior of the column only with 13 elements.

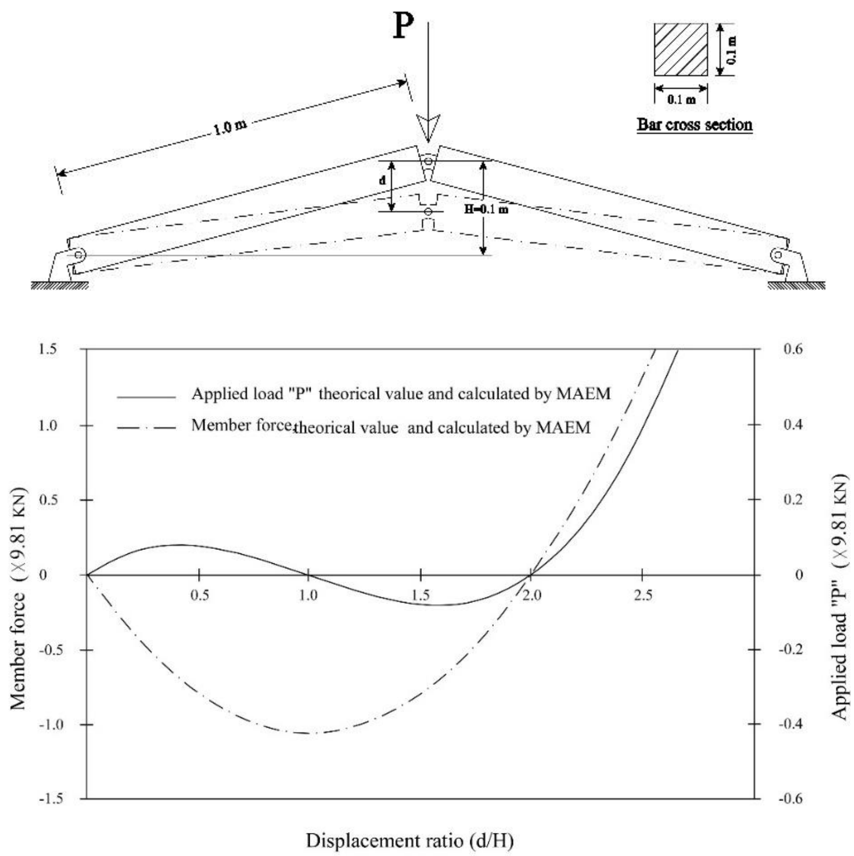


Fig. 12. The curve of change in the applied load and the internal force of the truss relative to the change in the middle of the span

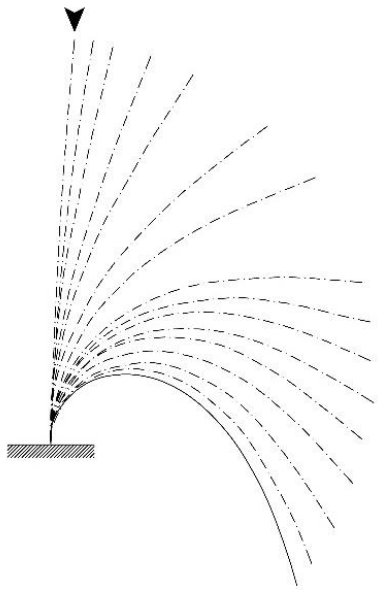


Fig. 13. Deformation curve of fixed base column under axial load

With respect to the load-displacement Curve in Figure 14, it can be concluded that:

1. The buckling load is increased while the load is constant, and this phenomenon is in good accommodation with theoretical results.

2. After reaching the buckling load, it is observed that with a slight increase in load, a great increase in displacement occurs, which is why the load control method cannot be used for analyzing this problem.

3. By increasing the loading and column deformation, the shape of the column and, as a result, the nature of the column stiffness changes, and the stiffness of the structure increases.

CONCLUSIONS

In this study, an effective modification was performed on the applied element method. In this method, instead of using several springs for connecting two elements, one spring with axial, shear and bending stiffness were used. This method estimates the deformations in the structure based on the minimum total potential energy principle. Also more accurate model of the structure will be provided by developing the concept of side elements. The comparison of proposed method and applied element method showed that:

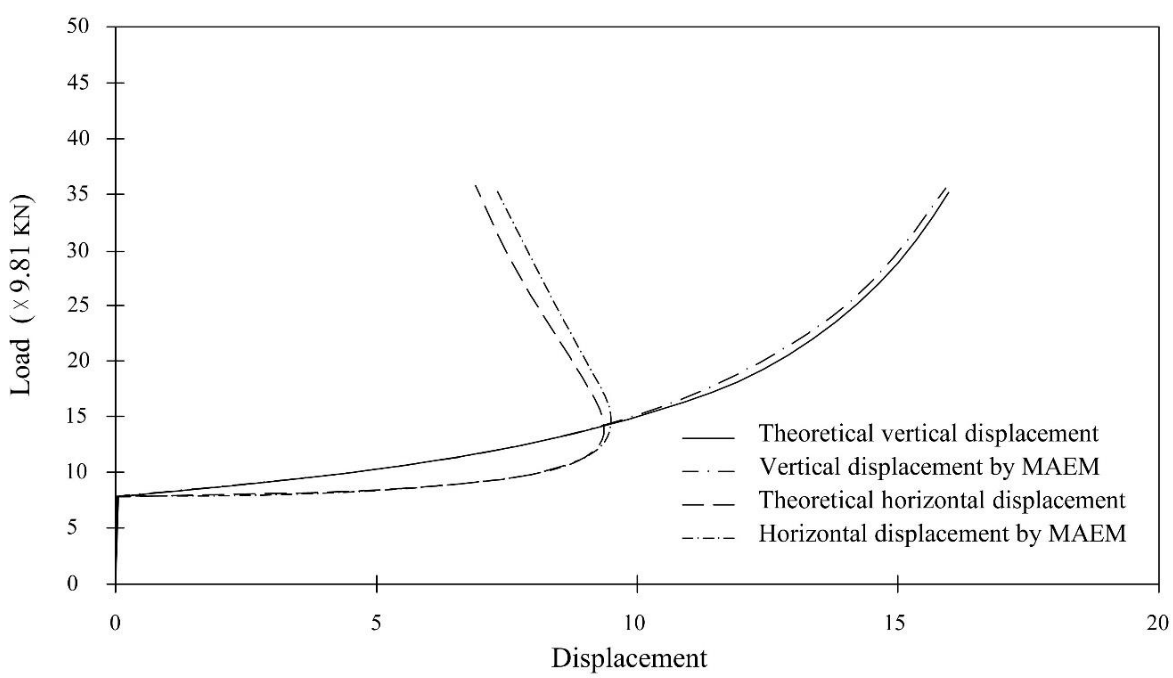


Fig. 14. Load-displacement Curve of fixed base column under axial load

- The computational accuracy in proposed method is obviously higher than applied element method, especially in modeling with larger elements. For example, the computational error for a console beam with a single row of elements in the proposed method was less than 2%, while the corresponding error in the applied element method of the same element size was more than 31%.
- The computational time of modified applied element method was approximately 40% of similar analysis by applied element method with 10 joint springs for connecting adjacent elements. This result can be justified by the fact that the number of joint springs is reduced to one-tenth that decrease the computational time seriously, otherwise by using the concept of side elements that caused a little increases in the number of elements and computational time.
- The behavior of the structures is calculated precisely even with the minimum number of elements.
- This method simulates large deformations like the applied element method. With the difference that it is highly accurate with a very small number of elements and a short computational time.

Therefore, in comparison to applied element method, the advantage of the proposed method is that, by fewer elements and lower computational time, more precise analysis can be performed. Considering the appropriate results obtained for linear analysis by the proposed method, nonlinear materials behavior for large deformations is being studied by modified applied element.

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