A Modified Discrete Control Law for UPS Applications

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***Abstract* – This paper presents a repetitive controller with parameters tuned *a priori* by a robust controller for uninterruptible power supply applications. The proposed control scheme can minimize periodic distortions resulted from unknown periodic load disturbances. The implemented PWM inverter system with sinusoidal reference employs a low cost microcontroller. Due to the fact that the most of the low cost microcontrollers available in the market presents low processing speed, it is imperious to develop simple algorithms with few operations, so that its implementation is realizable. Stability and special features of the control scheme are discussed. Simulation and experimental results (110 V_{RMS}, 1 kVA) are presented to demonstrate the performance of the proposed control approach under periodic load disturbances.**

I. INTRODUCTION

Uninterruptible power supply (UPS) systems have been used to protect critical loads in case of power failure. A typical UPS system consists of a battery (dc source), a dc-ac inverter, and a LC filter. Closed-loop regulated pulsewidth modulated (PWM) inverters have been widely applied in UPS systems. The output voltage of the high-performance UPS system must be sinusoidal with low total harmonic distortion (THD) under both transient or periodic load disturbances. The performance of the system is measured in terms of transient response due to sudden changes at load, waveform distortion with linear and nonlinear loads, and efficiency.

Repetitive control theory [1] provides an alternative to minimize periodic error occurred in a dynamic system. The repetitive control improves accuracy of steady state response of a control system when reference input signals and disturbances are periodic, consisting of the harmonic components of a common fundamental frequency. Several repetitive control schemes have been developed and applied to various industrial applications [2]-[4]. Haneyoshi, Kawamura and Hoft [5]-[6] had applied the repetitive control technique to eliminate periodic distortions that can appear in a PWM inverter. However, the output voltage error is increased by the repetitive control if the disturbance is non-harmonic. Eliminating this problem, Yeh and Tzou [7] presented a adaptive repetitive control scheme that employs an auxiliary compensator to stabilize the closed-loop system even with variations in the plant. Gründling, Carati and Pinheiro [8] presented a robust model reference adaptive controller including a repetitive control for UPS applications, it can effectively eliminate periodic waveform distortion resulting by unknown periodic disturbances, and it is globally stable in the presence of unmodeled dynamics.

In this paper a repetitive controller based on auxiliary states is proposed, using control parameters tuned *a priori* from a modified least-squares algorithm, which has been presented in [8] for uninterruptible power supply applications. The implementation of this algorithm is done using a low cost microcontroller, yielding an algorithm with few instructions.

In Section II the plant model of the system is described. The OSAP controller with a repetitive controller is described in Section III. Its performance is compared with the repetitive controller based on auxiliary states presented and discussed in Section IV. Section V presents simulation results with nonlinear loads and a discussion of the results is performed. In the Section VI is presented experimental results obtained for linear and nonlinear loads based on a microcontroller-controlled system.

II. DESCRIPTION OF THE PLANT MODEL

Fig. 1 shows the single-phase full-bridge PWM inverter, where the inverter, LC filter, and resistive load R are considered as the plant of a closed-loop system with a sinusoidal reference. The nonlinear (triac plus resistor or rectifier plus RC filter) load causes a periodic disturbance.

The power switches are turned on and off once during each sampling interval *T*, such that $v_{in}(t)$ is a voltage pulse of magnitude V_B, 0 (zero) or $-V_B$ and width ΔT.

The system transfer function of Fig. 1 is given by

$$\frac{y}{u} = G(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}\ , \tag{1}$$

where $\omega_n = 1/\sqrt{LC}$ and $\varsigma = 1/(2RC\omega_n)$, $y = v_c(t)$ is the system output and $u = v_{in}(t)$ is the system input.

From (1) a difference equation can be obtained using an appropriate sampling interval *T*,

$$y(k+1) = a_1 y(k) + a_2 y(k-1) + b_1 u(k) + b_2 u(k-1) + v(k+1) \tag{2}$$

where v is the measurement noise in output of the plant and $v(k) \approx (0, \sigma^2)$.

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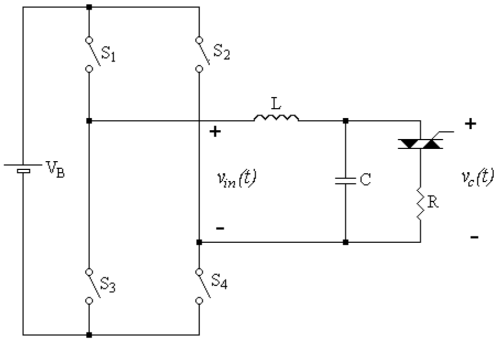


Fig. 1 – PWM inverter system.

Also, (2) can be represented by,

$$y(k+1)=\phi^T(k)\theta+v(k+1)\,, \qquad (3)$$

where $\phi^T(k)=[y(k)\quad y(k-1)\quad u(k)\quad u(k-1)]$ and $\theta^T=[a_1\quad a_2\quad b_1\quad b_2]$.

III. OSAP WITH REPETITIVE CONTROLLER

Consider the closed-loop control system show in Fig. 2.

It is desired to obtain a control law such that the output error is minimized and that satisfies the following quadratic performance index J,

$$J(k+1)=E\left[\left(y(k+1)-r(k+1)\right)^2\right], \qquad (4)$$

where E denotes mathematical expectation.

From (3) and (4) is obtained,

$$J(k+1)=E[(\phi^T(k)\theta-r(k+1))^2]+\sigma^2\,. \qquad (5)$$

assuming that $E[v(k+1)]=0$ and $E[(v(k+1))^2]=\sigma^2$.

Therefore, the minimal value of the performance index is obtained when

$$\phi^T(k)\theta-r(k+1)=0\,. \qquad (6)$$

From (6), the control law that minimizes (4) is given by

$$u_{OSAP}(k)=\frac{r(k+1)-a_1y(k)-a_2y(k-1)-b_2u(k-1)}{b_1}\,, \qquad (7)$$

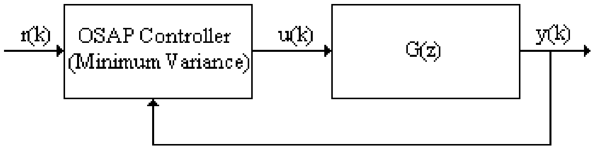


Fig. 2 – Block diagram of control system with OSAP.

which is denominated OSAP (*One Sample Ahead Preview*) controller.

For the case in which the load is linear, the use of an OSAP controller (7) is sufficient to minimize the output error of the plant. This scheme does not minimize the inherent errors of the measurements and errors of the generation of the PWM signal ($v_{in}(t)$) at the input of filter. Otherwise, for a nonlinear load varying periodically in the time, a repetitive controller is added to OSAP controller, as shown in Fig. 3. This procedure minimizes the periodic disturbances due to load cyclic variation. In the same form that in [6], the equation of the repetitive controller (RP controller) is given by

$$u_{RP}(k)=c_1e(k+N-n)+c_2\sum_{i=1}^{\infty}e(k+N-i.n)\,, \qquad (8)$$

where $e(k)=r(k)-y(k)$ is the tracking error, c_1 and c_2 are the gains of the repetitive controller, N is the delay steps and n is the number of samples in a period of output voltage. The control law $u_P(k)$ is given by

$$u_P(k)=u_{OSAP}(k)+u_{RP}(k)\,. \qquad (9)$$

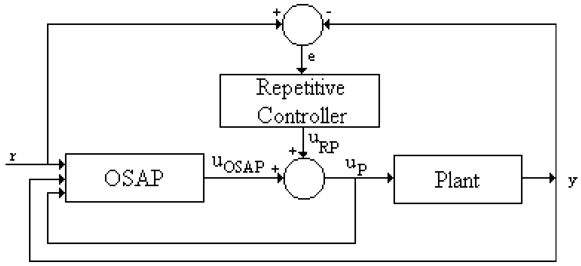


Fig. 3 – Block diagram of the control system with OSAP-RP.

IV. REPETITIVE CONTROLLER BASED ON AUXILIARY STATES

A. Structure of the Controller

Consider a single-input single-output plant as presented in Fig. 4. The input u and the output y are used to generate a m-1 dimensional auxiliary vectors, where m is the order of the system to be controlled. Such that

$$\begin{aligned}\omega_1(k+1)&=F\omega_1(k)+q.y(k)\\ \omega_2(k+1)&=F\omega_2(k)+q.u(k)\end{aligned}\,, \qquad (10)$$

where F is a stable matrix and (F,q) is a controllable pair.

Such as it is defined in [9], (F,q) is the state space realization of $\alpha(z)/\Lambda(z)$, i.e.,

$$(z\mathrm{I}-F)^{-1}q=\frac{\alpha(z)}{\Lambda(z)}\,. \qquad (11)$$

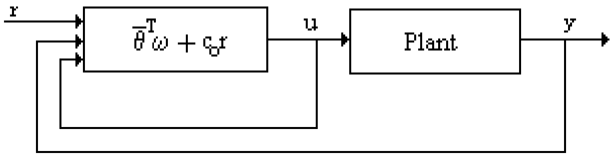


Fig. 4 – Block diagram of control system with the controller based on auxiliary states.

The polynomial Λ(*z*) is chosen as being monic, Hurwitz of degree *m*-1 and its poles must be matched with the dynamics of the model reference as well as with the dynamics of the known part (nominal model) of the plant.

The control law is given by

$$u(k)=\bar{\theta}^T\omega(k)+c_o r(k+1)\,, \tag{12}$$

such as in [8], where $\bar{\theta}^T=[\bar{\theta}_1 \quad \bar{\theta}_2 \quad \bar{\theta}_3]$ is a (2*m*-1) dimensional control parameter vector. It is tuned *a priori* by a robust controller of the type RMRAC, using a modified least-squares adaptation algorithm. And $\omega^T=[\omega_1 \quad \omega_2 \quad y]$ is a vector containing the auxiliary states ω_1 and ω_2 and the output *y*, and c_o is a scalar feedforward parameter.

In Fig. 5, a repetitive controller is added to the controller based on auxiliary states to minimize the periodic disturbance due to periodic load variations. The control law u_P is given by

$$u_P(k)=u(k)+u_{RP}(k)\,. \tag{13}$$

B. Stability Analysis

After z-transforming of (1) and (12), the system transfer function presented in Fig. 4 becomes

$$\left[1-\bar{\theta}_1 G(z)(z-F)^{-1}q-\bar{\theta}_2(z-F)^{-1}q-\bar{\theta}_3 G(z)\right]y=c_o G(z)z.r\,, \tag{14}$$

or

$$y=G^*(z)r\,, \tag{15}$$

where

$$G^*(z)=\frac{c_o G(z)z}{1-\bar{\theta}_1 G(z)(z-F)^{-1}q-\bar{\theta}_2(z-F)^{-1}q-\bar{\theta}_3 G(z)}\,, \tag{16}$$

and *G(z)* is the z-transform of the plant transfer function.

Therefore, a necessary and sufficient condition for that the closed-loop system shown in Fig. 4 be stable, is that the closed-loop characteristic polynomial has all its poles inside the unit circle.

With the inclusion of the repetitive control, the system transfer function which is shown in Fig. 5, after z-transforming of (1) and (13), is given by

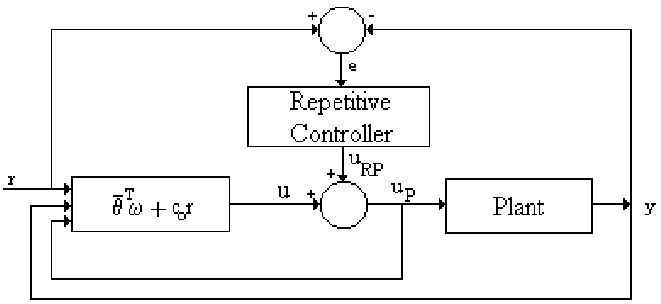


Fig. 5 – Block diagram of the proposed control system.

$$\frac{E(z)}{R(z)}=\frac{(1-G^*(z))(1-z^{-n})}{1-z^{-n}H^*(z)}\,, \tag{17}$$

where the z-transform of the output error *e* is *E(z), R(z)* is the transformed reference input, and

$$H^*(z)=1-\frac{z^{N-1}}{c_o}(c_1+c_2-c_1 z^{-n})G^*(z)\,. \tag{18}$$

Assuming that (16) is stable, then the stability of the system is determined by the repetitive control. From (17), a sufficient condition [6] for the stability is

$$\left|H^*(j\omega)\right|\le 1\,, \tag{19}$$

where *ω* is the angular frequency of the reference input.

V. SIMULATION RESULTS

Table I lists the fundamental parameters of the single-phase PWM inverter system and reference model used in digital computer simulation.

In Fig. 6 is presented the response of the OSAP-RP controller for a rated resistive load with phase commutated at angle 72º. Fig. 7 shows the adaptation of the controller's parameters for a resistive load and a rated resistive load with phase commutated at angle 72º, using a modified least-squares adaptation algorithm. The parameters were adapted through a robust model reference adaptive controller, using the following discrete reference model

$$W_m(z)=\frac{0.1508.z+0.1359}{z^2-1.4477.z+0.7344}\,. \tag{20}$$

TABLE I

PARAMETERS OF PWM INVERTER AND REFERENCE MODEL

Filter inductance	L = 1 mH
Filter capacitance	C = 25 µF
DC input voltage	V_B = 200 V
Reference voltage	V_{REF} = 110 V_{RMS}, f = 60 Hz
Load resistance	R = 12 Ω
Sampling frequency	f_S = 10800 Hz
Sampling time	*T* = 92.6 µs

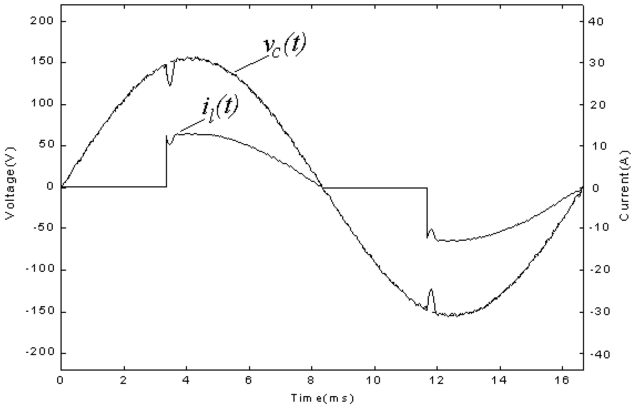
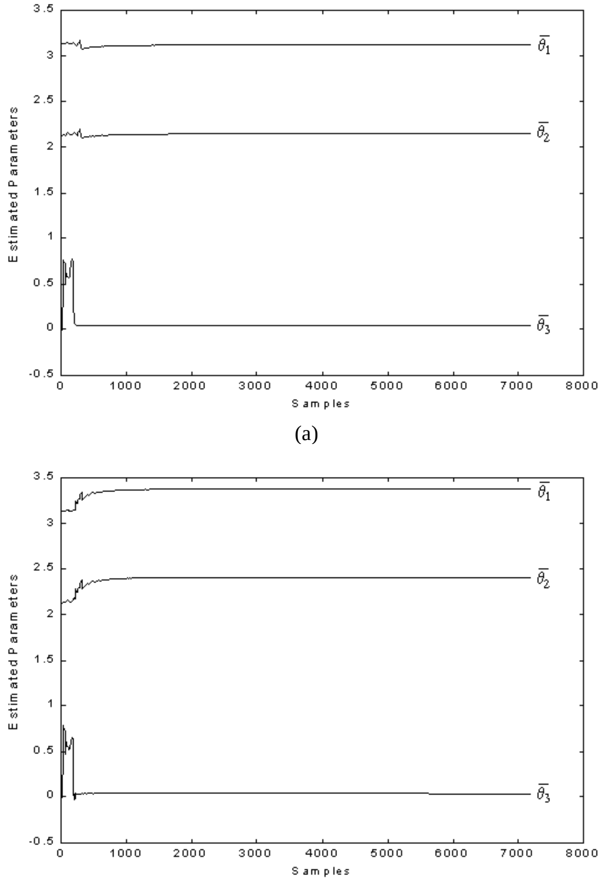


Fig. 6 – Output voltage and load current with OSAP-RP controller.



(a)

(b)

Fig. 7 – Estimated parameters using the RMRAC controller. (a) Resistive load. (b) Resistive load with phase commutated at angle 72º.

Taking into account that the parameters of controller does not change significantly, as it can be seen in Fig. 7, it becomes possible to fix these parameters *a priori*. Therefore, as results, a reduced computation effort is required in the implementation. Fig. 8 shows the response of the repetitive controller based on auxiliary states with parameters tuned *a priori*, for a rated resistive load with phase commutated at angle 72º.

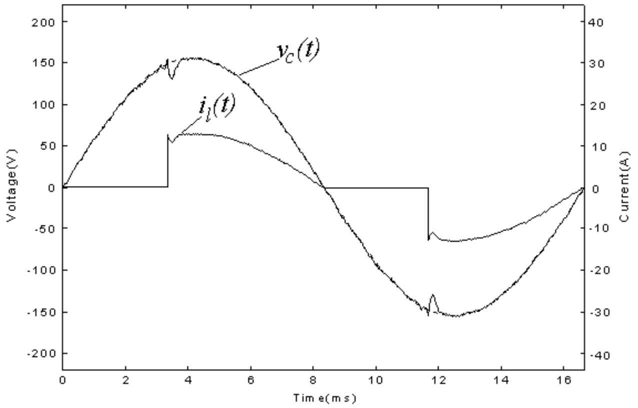


Fig. 8 - Output voltage and load current with repetitive controller based on auxiliary states.

In Fig. 9 and Fig. 10 are presented, respectively, the response of the OSAP-RP and of the repetitive controller based on auxiliary states with a rated resistive load with phase commutated at angle 72º including an unmodeled dynamic (zero @ 50000 rad/s).

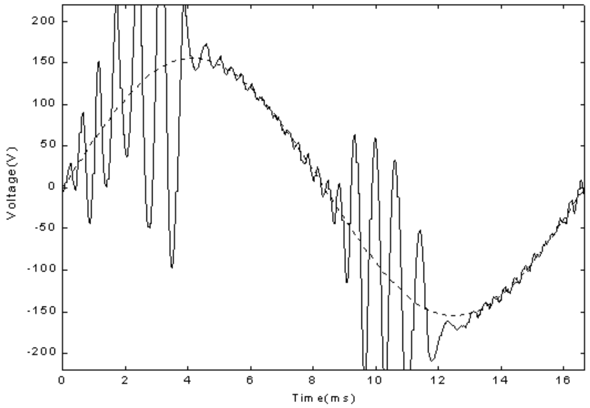


Fig. 9 - Output voltage with the OSAP-RP controller.

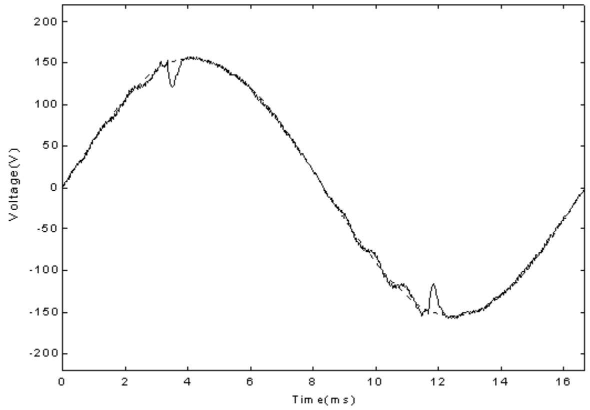


Fig. 10 - Output voltage with repetitive controller based on auxiliary states.

TABLE II

THD OF THE SIMULATION RESULTS FOR A RATED RESISTIVE LOAD WITH PHASE COMMUTATED AT ANGLE 72º.

	Without unmodeled dynamic	With unmodeled dynamic
OSAP-RP	4.62 %	82.19 %
Repetitive controller based on auxiliary states	3.87 %	6.87 %

Table II shows the THD of the simulation results of OSAP-RP controller and repetitive controller based on auxiliary states, for a rated resistive load with phase commutated at angle 72º with and without unmodeled dynamic (zero @ 50000 rad/s).

VI. EXPERIMENTAL RESULTS

A prototype of the PWM inverter has been built in laboratory to verify the performance of the repetitive controller based on auxiliary states with parameters tuned *a priori*. The block diagram of the system is shown in Fig. 11. The component values of the inverter and rectifier-RC load are given in Table III.

The controller has been implemented using an eight bits wide data word microcontroller (PIC17C756 of Microchip Technology Inc.). It has an embedded 10 bits A/D converter and a PWM signal generator. These features reduce significantly the PWM inverter control circuitry without penalizing the cost.

Fig. 12 shows the waveform of the output voltage $v_c(t)$ and load current $i_l(t)$ for nominal resistive load (12 Ω/1 kVA) and Fig. 13 shows the waveforms of reference voltage and output voltage $v_c(t)$ for nominal resistive load, demonstrating the tracking capability of the proposed controller. The THD for nominal resistive load is 1.45%. Fig. 14 shows the output voltage $v_c(t)$ and load current $i_l(t)$ and Fig. 15 shows the output voltage $v_c(t)$ and the input filter voltage $v_{in}(t)$ waveform with a load (D_{1-4}, C_L, R). Fig. 16 shows the waveforms of reference voltage and output voltage for a rectifier-RC load, to demonstrate the tracking capability of this controller for a nonlinear load. The total harmonic distortion for rectifier-RC load is 2.01%.

TABLE III

PARAMETERS OF PWM INVERTER AND LOAD.

Filter inductance	L = 1 mH
Filter capacitance	C = 25 µF
DC input voltage	V_B = 200 V
Reference voltage	V_{REF} = 110 V_{RMS}, f = 60 Hz
Load resistance	R = 25 Ω
Load capacitance	C_L = 330 µF
Sampling frequency	f_S = 10800 Hz
Sampling time	T = 92.6 µs

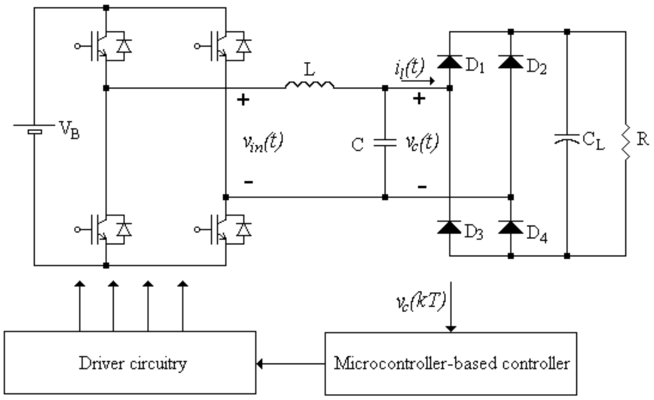


Fig. 11 – Block diagram of experimental setup.

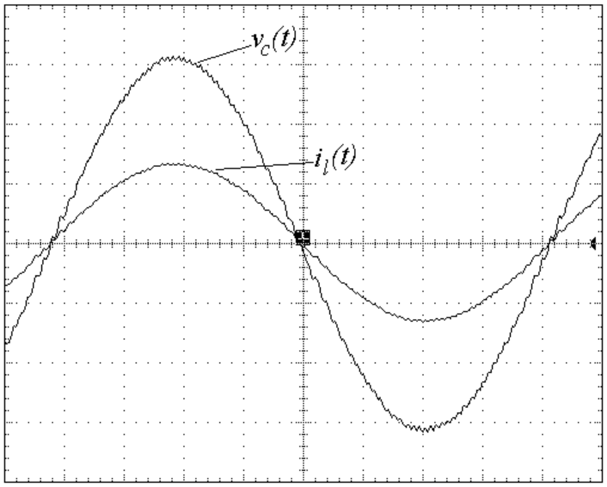


Fig. 12 - Output voltage (50 V/div) and load current (10 A/div) with R = 12 Ω. Time scale 2 ms/div.

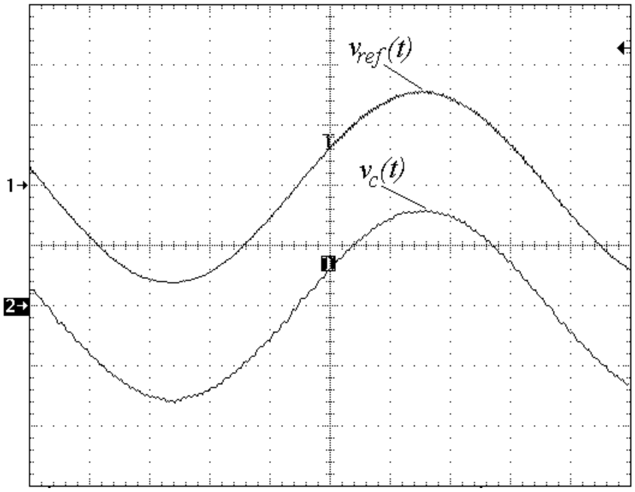


Fig. 13 – Reference voltage (5 V/div) and output voltage (100 V/div) with R = 12 Ω. Time scale 2 ms/div.

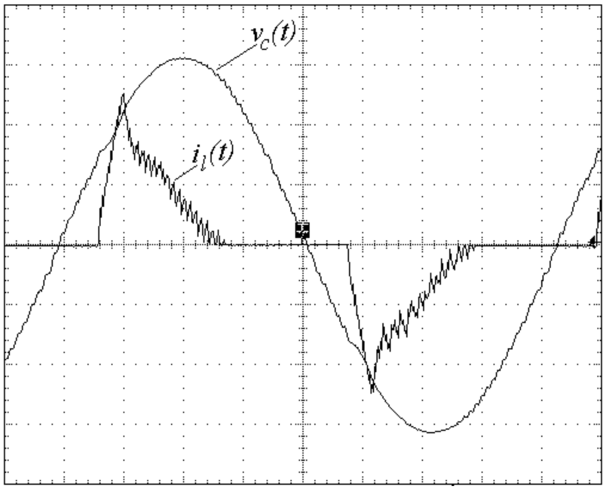


Fig. 14 – Output voltage (50 V/div) and load current (10 A/div) with ($D_{1\text{-}4}$, C_L, R)$_{LOAD}$. Time scale 2 ms/div.

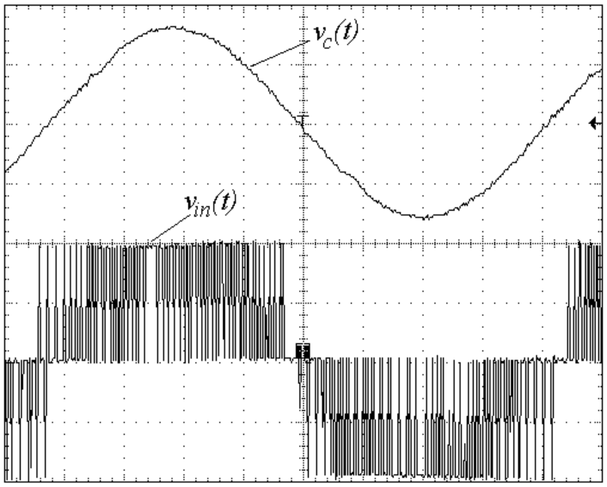


Fig. 15 – Output voltage (100 V/div) and input filter voltage (100 V/div) with ($D_{1\text{-}4}$, C_L, R)$_{LOAD}$. Time scale 2 ms/div.

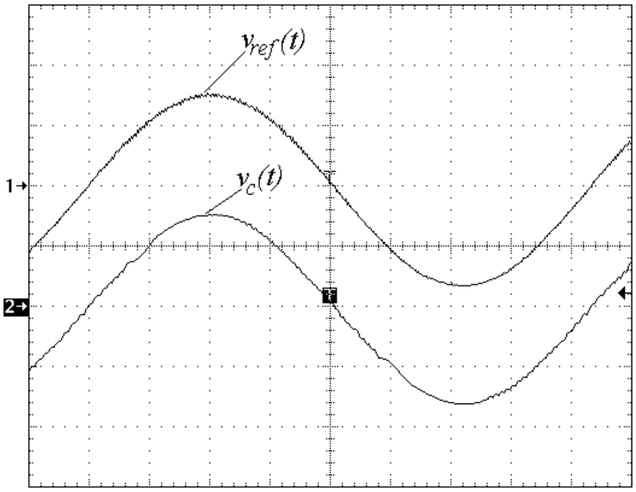


Fig. 16 - Reference voltage (5 V/div) and output voltage (100 V/div) with ($D_{1\text{-}4}$, C_L, R)$_{LOAD}$. Time scale 2 ms/div.

VII. CONCLUSIONS

This paper describes a repetitive controller based on auxiliary states for UPS applications. Simulation and experimental results show that the proposed control scheme can minimize the periodic waveform distortion resulting by unknown periodic disturbances and unmodeled dynamics. The use of the vector of parameters $\bar{\theta}^T$ tuned *a priori* from simulation based on RMRAC reduces the computational effort significantly. Therefore, making it possible to implement it in a low cost microcontroller, at low and medium switching frequencies. Even with the limitations of the processing speed and fixed-point routines, this control algorithm presents good response performance. It has been experimentally demonstrated the good performance of the proposed controller under different load conditions.

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