

A modified Li-He's variational principle for plasma

Ji-Huan He

*National Engineering Laboratory for Modern Silk,
College of Textile and Clothing Engineering, Soochow University, Suzhou, China
and School of Mathematics and Information Science,
Henan Polytechnic University, Jiaozuo, China*

A modified
Li-He's
variational
principle

1369

Received 29 June 2019
Revised 29 June 2019
Accepted 30 June 2019

Abstract

Purpose – It is extremely difficult to establish a variational principle for plasma. Kalaawy obtained a variational principle by using the semi-inverse method in 2016, and Li and He suggested a modification in 2017. This paper aims to search for a generalized variational formulation with a free parameter.

Design/methodology/approach – The semi-inverse method is used by suitable construction of a trial functional with some free parameters.

Findings – A modification of Li-He's variational principle with a free parameter is obtained.

Originality/value – This paper suggests a new approach to construction of a trial-functional with some free parameters.

Keywords Variational theory, Semi-inverse method, Burger equation, Plasma, Editorial

Paper type Research paper

1. Introduction

In 2016, Kalaawy obtained the following equation for plasma (El-Kalaawy, 2016):

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left[\frac{2}{3} Au^{3/2} - C \frac{\partial u}{\partial x} \right] = 0 \quad (1)$$

and derived a variational principle by using the semi-inverse method (He, 2004; He, 2017; Wu and He, 2018) through introducing two special functions.

In 2017, Li and He found another variational principle, which reads (Li and He, 2017):

$$J_{Li-He}(u, \Phi) = \iint \left\{ \frac{1}{2} \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial t} + \frac{2}{3} Au^{3/2} \frac{\partial \Phi}{\partial x} - \frac{1}{2} C \left(\frac{\partial \Phi}{\partial x} \right)^2 - \frac{2}{5} Au^{5/2} \right\} dx dt \quad (2)$$

where Φ satisfies the following relations:

$$\frac{\partial \Phi}{\partial x} = u \quad (3)$$

$$\frac{\partial \Phi}{\partial t} = -\frac{2}{3} Au^{3/2} + C \frac{\partial u}{\partial x} \quad (4)$$

2. General variational principle with a free parameter

The semi-inverse method is widely used to search for variational principles directly from governing equations (El-Kalaawy, 2016; El-Kalaawy, 2017; Biswas *et al.*, 2017). By the



semi-inverse method (He, 2004; He, 2017; Wu and He, 2018), we can construct a trial-functional in the form:

$$J(u, \Phi) = \iint \left\{ u \frac{\partial \Phi}{\partial t} + \left[\frac{2}{3} Au^{3/2} - C \frac{\partial u}{\partial x} \right] \frac{\partial \Phi}{\partial x} + F \right\} dxdt \quad (5)$$

where F is a known function of u and its derivative, however we cannot identify F , so equation (5) has to be modified as:

$$J_{New}(u, \Phi) = \iint \left\{ mu \frac{\partial \Phi}{\partial t} + n \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial t} + aAu^{1/2} \left(\frac{\partial \Phi}{\partial x} \right)^2 + bAu^{3/2} \frac{\partial \Phi}{\partial x} - C \frac{\partial u}{\partial x} \frac{\partial \Phi}{\partial x} + F \right\} dxdt \quad (6)$$

where m, n, a and b are constants.

The stationary conditions of equation (6) are as follows:

$$-m \frac{\partial u}{\partial t} - 2n \frac{\partial^2 \Phi}{\partial x \partial t} - \frac{\partial}{\partial x} \left[2aAu^{1/2} \frac{\partial \Phi}{\partial x} + bAu^{3/2} - C \frac{\partial u}{\partial x} \right] = 0 \quad (7)$$

and

$$m \frac{\partial \Phi}{\partial t} + \frac{1}{2} aAu^{-1/2} \left(\frac{\partial \Phi}{\partial x} \right)^2 + \frac{3}{2} bAu^{1/2} \frac{\partial \Phi}{\partial x} + C \frac{\partial}{\partial x} \left(\frac{\partial \Phi}{\partial x} \right) + \frac{\delta F}{\delta u} = 0 \quad (8)$$

In view of equations (3) and (4), we have:

$$-(m + 2n) \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left[(2a + b)Au^{3/2} - C \frac{\partial u}{\partial x} \right] = 0 \quad (9)$$

$$mC \frac{\partial u}{\partial x} + \left(\frac{1}{2}a + \frac{3}{2}b - \frac{2}{3}m \right) Au^{3/2} + C \frac{\partial u}{\partial x} + \frac{\delta F}{\delta u} = 0 \quad (10)$$

Equation (9) should be equivalent to equation (1), this requires:

$$2a + b = 1 \quad (11)$$

$$m + 2n = 1 \quad (12)$$

To identify F in equation (10), we set:

$$m = -1 \quad (13)$$

Equation (10) becomes:

$$\left(\frac{1}{2}a + \frac{3}{2}b + \frac{2}{3} \right) Au^{3/2} + \frac{\delta F}{\delta u} = 0 \quad (14)$$

From equation (14), F can be identified as:

$$F = -\frac{2}{5} \left(\frac{1}{2}a + \frac{3}{2}b + \frac{2}{3} \right) Au^{5/2} = -\frac{2}{5} \left(\frac{13}{6} - \frac{5}{2}a \right) Au^{5/2} \quad (15)$$

We, therefore, obtain the following variational principle:

$$J_{New}(u, \Phi) = \iint \left\{ \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial t} - u \frac{\partial \Phi}{\partial t} + aAu^{1/2} \left(\frac{\partial \Phi}{\partial x} \right)^2 + (1-2a)Au^{3/2} \frac{\partial \Phi}{\partial x} - C \frac{\partial u}{\partial x} \frac{\partial \Phi}{\partial x} - \frac{2}{5} \left(\frac{13}{6} - \frac{5}{2}a \right) Au^{5/2} \right\} dxdt$$

where a is a free parameter.

3. Conclusion

In this paper, we suggest a general construction of a trial functional with some parameters. The semi-inverse method is a powerful tool to establish variational principles from the governing equations. The variational principle is a foundation of the variational iteration method (Anjum and He, 2019; He, 2006), which is now widely applied in fractional calculus (Baleanu *et al.*, 2018; Dogan Durgun and Konuralp, 2018; Inc *et al.*, 2018; Jafari *et al.*, 2018; Wang *et al.*, 2018).

References

- Anjum, N. and He, J.H. (2019), "Laplace transform: Making the variational iteration method easier", *Applied Mathematics Letters*, Vol. 92, pp. 134-138.
- Baleanu, D., Jassim, H.K. and Khan, H. (2018), "A modified fractional variational iteration method for solving nonlinear gas dynamic and coupled KdV equations involving local fractional operator", *Thermal Science*, Vol. 22 No. S1, pp. S165-S175.
- Biswas, A., Zhou, Q., Moshokoa, S.P., Triki, H., Belic, M. and Alqahtani, R.T. (2017), "Resonant 1-soliton solution in anti-cubic nonlinear medium with perturbations", *Optik*, Vol. 145, pp. 14-17.
- Dogan Durgun, D. and Konuralp, A. (2018), "Fractional variational iteration method for time-fractional nonlinear functional partial differential equation having proportional delays", *Thermal Science*, Vol. 22 No. S1, pp. S33-S46.
- El-Kalaawy, O.H. (2016), "Variational principle, conservation laws and exact solutions for dust ion acoustic shock waves modeling modified Burger equation", *Computers and Mathematics with Applications*, Vol. 72, pp. 1013-1041.
- El-Kalaawy, O.H. (2017), "New variational principle-exact solutions and conservation laws for modified ion-acoustic shock waves and double layers with electron degenerate in plasma", *Physics of Plasmas*, Vol. 24 No. 3, p. 32308.
- He, J.H. (2004), "Variational principles for some nonlinear partial differential equations with variable coefficients", *Chaos Solitons and Fractals*, Vol. 19 No. 4, pp. 847-851.
- He, J.H. (2006), "Some asymptotic methods for strongly nonlinear equations", *International Journal of Modern Physics B*, Vol. 20 No. 10, pp. 1141-1199.
- He, J.H. (2017), "Hamilton's principle for dynamical elasticity", *Applied Mathematics Letters*, Vol. 72 No. 2017, pp. 65-69.
- Inc, M., Khan, H., Baleanu, D. and Khan, A. (2018), "Modified variational iteration method for straight fins with temperature dependent thermal conductivity", *Thermal Science*, Vol. 22 No. S1, pp. S229-S236.

- Jafari, H., Jassim, H.K. and Vahidi, J. (2018), "Reduced differential transform and variational iteration methods for 3-D diffusion model in fractal heat transfer within local fractional operators", *Thermal Science*, Vol. 22 No. S1, pp. S301-S307.
- Li, Y. and He, C.H. (2017), "A short remark on Kalaawy's variational principle for plasma", *International Journal of Numerical Methods for Heat and Fluid Flow*, Vol. 27 No. 10, pp. 2203-2206.
- Wang, Y., Zhang, Y.F. and Liu, Z.J. (2018), "An explanation of local fractional variational iteration method and its application to local fractional modified korteweg-de vries equation", *Thermal Science*, Vol. 22 No. 1, pp. 23-27.
- Wu, Y. and He, J.H. (2018), "A remark on Samuelson's variational principle in economics", *Applied Mathematics Letters*, Vol. 84, pp. 143-147.

About the author

Ji-Huan He is an expert on Nonlinear Science and Nanotechnology. He is the owner of some famous analytical methods, such as the semi-inverse method, the variational iteration method, the homotopy perturbation method, the exp-function method and He's frequency formulation. He has published more than 390 articles with an *h*-index of 66. Ji-Huan He can be contacted at: hejihuan@suda.edu.cn