A Modified MIMO Radar Model Based on Robustness and Gain Analysis

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Abstract: Multi-Input Multi-Output (MIMO) radar is a new radar technology in recent years, which partially or completely uses spatial diversity gain of the signal to replace coherent gain in traditional phased-array radar. Using the ideal point source model, we make a detailed analysis of the contributions to radar detection system made by these two kinds of gain. These contributions are divided into two kinds: the contribution to system robustness and the contribution to improving the signal-to-noise ratio. Based on this, it is proposed that the space diversity gain of MIMO radar can make more contribution to the system. The rationality of this proposal is further proved by the modification of the statistical MIMO model. And the theory above is verified by simulation. In addition, this paper illustrates how to analyze other MIMO radar systems from the viewpoints of these two kinds of contributions.

Key-Words: Multi-Input Multi-Output (MIMO) radar, Phased-array radar, Spatial diversity gain, Coherent gain, Detection performance, Swerling model, Stealth target

1 Introduction

MIMO radar is a radar technology of a new generation, which has received widespread attention in the last few years. Its core thought is using spatial diversity gain of the signal partially or completely to replace coherent gain used in traditional phased-array radar. Theory and experiments both indicate that neither the space diversity gain nor the coherent gain is the absolutely optimal choice. Either of them could be superior under different conditions. Among the several most popular radar models, the traditional phased-array radar uses all array elements to get the coherent gain to enhance the radar performance. In generalized MIMO definition, it is also regarded as one violent MIMO pattern. Since the mid-1990s, Bell Laboratories has proposed a new ideal for radar design in which both the receiving and transmitting antenna arrays are placed separately so that all the elements can get the spatial diversity gain [1-5].

This radar is named statistical MIMO [1]. Lincoln Laboratories designs two experimental systems of MIMO digital array radar in 2003, which only use spatial diversity in the transmitting arrays and have the same mode as phased-array radar in the receiving arrays. Therefore the two systems use the transmitting array elements to get space diversity gain, and the receiving array elements to get coherent gain, which can be considered as a compromise between the traditional phased-array and statistical MIMO.

Seeking a new mode like MIMO radar, is to cope with the new challenges for the modern radar [2]: First, the radar should have the ability of long-range detection for weak target, especially for the stealth target. Second, the radar should be able to realize target identification and recognition, as well as assessment of the lethality. Third, the working conditions are very bad for modern radar, especially the active jamming and anti-radiation missile which pose an enormous threat to the survival of the radar, so the radar should have the ability to fight against signal interception. Fourth, the radar should have the ability to search and track the multi-objectives simultaneously, to carry on data fusion, track calculation and threat assessment, etc.

The first point mentioned above is the very reason for using spatial diversity processing in the radar system. On the one hand, the stealth target is characterized by small radar cross-section(RCS) and slow fluctuation between pulses—that is, the RCS fluctuations are coherent during scanning, but not related to scanning gaps, which could be modeled using the Swerling I or III model [6][7]. On the other hand, the target scintillation is so obvious that any small change of its posture and direction will cause serious fluctuation of the radar RCS. Even 1/1000 radian change of location may lead to great influence on received signal power as large as 25dB. Combining the two, the following conclusion could be obtained:

First, the RCS of stealth target is a random variable with small average value and big variance. Second, by time accumulation in one scanning, a number of coherent samples of the RCS would be collected. Third, even at the same time, a number of approximately independent samples of the RCS could be obtained by scanning from different direction. Therefore, detection performance of phased-array radar may be attenuated significantly when all the received signals suffer big fading. While using the MIMO model, as the result of the average processing of independent samples, the system performance could be more stable. As noted, phased-array radar may receive signals that all suffered small decline, thus showing superior performance. Therefore, the original intention of using the space diversity in the radar system is to fight against the target fluctuation to enhance system robustness.

This article makes a thorough analysis of the spatial diversity gain and the coherent gain, the contribution of which are divided into two kinds: one is to fight against the fluctuation of target RCS while the other is to suppress noise interference to increase the signal-to-noise ratio of detection statistics. From the results of the analysis, the contribution of MIMO radar diversity processing to the radar detection system is self-adaptive: when the RCS fluctuates severely, it mainly shows the first kind of contribution and is weak in reducing noise; when the fluctuation of RCS is small, it manifests its second kind of contribution and the ability of reducing noise becomes stronger, but is still weaker than that of coherent processing. Therefore, it is a better choice to use coherent processing to increase the signal-to-noise ratio instead of diversity processing, which can not show evident effects when it is saturated.

But in some cases of target RCS distribution, the contribution to noise reduction can be enhanced by modifying the model of statistical MIMO, and as for the target in the case of small fluctuation, it can be equivalent to that of phased-array radar. This paper will also verify the views by simulation of several radar detection systems. In addition, the view of this paper can be used to analyze other MIMO radar system performance. Taking the MISO system for example, the detail of analysis process is introduced and the simulation result of MISO is also given for confirming this point.

The rest of the paper is organized as follows. Section 2 introduces a MIMO radar ideal point-source model for further analysis. Section 3 derives the contribution of two kinds of gain in MIMO radar for target detection system. In Section 4, with the principle of above sections, the model of statistical MIMO is modified to improve the system performance, and the detection performance of several systems is simulated for comparison. Section V shows how to analyze other MIMO systems with examples by exploiting the method mentioned above, while the last section provides a conclusion.

2 Ideal point-source model of MIMO radar

The MIMO radar is new mode radar which substitutes coherent gain in the tradition phased-array radar with spatial diversity gain partly or completely. When target radar RCS slowly fluctuates between pulses and multiple signals scan from the same angle, only some coherent samples of the RCS will be obtained. While there is certain angular spread between each signal, independent samples of target RCS will be obtained. To meet the condition, the distance of transmitting elements should satisfy the following constraint [1][2][3]

$$d \geq \frac{\lambda R}{D} \tag{1}$$

where D indicates the size of the target, λ indicates wavelength of the transmitting signal, and R indicates the distance from the transmitting antennas to target. The formula is also applicable for situations under which the receiving antenna achieves spatial diversity.

Literature [1] infers this constraint with the target RCS scattering model, and the following accumulation processing makes the model degenerate into point-source model. We no longer derive the constraint of array elements distance in this paper, and model the target RCS directly with point-source model in order to explain the principle of MIMO clearly. At this time, we assume that the size of the target is D, much smaller than distance R from the antennas to target, so the target could be considered as a point.

Suppose in a simple situation, as shown in figure 1, the radar system has M transmitting antennas which are uniformly placed on an arc, the distance to target is R_t, and the distance between two adjacent antennas is d_t; while it has N receiving antennas which are uniformly placed on another arc, and the distance to target is R_r, the distance between two adjacent antennas is d_r. We assume that the distance from the target to radar arrays is far enough, which satisfied

$$R_t \gg (M-1)d_t, R_r \gg (N-1)d_r \quad (2)$$

Note that the assumption is often valid in long-distance target detection. Under such condition, M array elements of the transmitting array could be regarded as uniform linear array approximately, the same as that of the receiving array. Meanwhile, the orientation vectors of transmitter and receiver in coherent processing are simplified, when they both are vectors, the elements of which are equal to 1. That is

$$\begin{aligned} \boldsymbol{a}_r &= (1 \quad 1 \quad \cdots \quad 1)^{\mathrm{T}} \in \mathbb{C}^{M\times 1} \\ \boldsymbol{a}_r &= (1 \quad 1 \quad \cdots \quad 1)^{\mathrm{T}} \in \mathbb{C}^{N\times 1} \end{aligned} \quad (3)$$

The radar beam is exactly pointing to target.

Meanwhile, according to equation(1), the angular spread that the two adjacent antennas need to get independent RCS information of target could be deduced. Taking the line from one edge array element of the transmitting array to the target as the reference, the angle between reference and the line from any other element to target is defined as transmitting-angel. Accordingly, the incident angles of M transmitting antennas are expressed as follows:

$$\theta_1 = 0, \theta_2 = \Delta\theta, \cdots, \theta_M = (M-1)\Delta\theta \quad (4)$$

Similarly the acceptance angle/receiving-angel could be deduced. That is

$$\varphi_1 = 0, \varphi_2 = \Delta\varphi, \cdots, \varphi_N = (N-1)\Delta\varphi \quad (5)$$

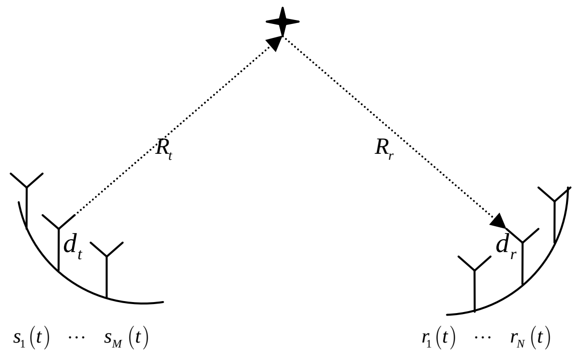


Fig.1 Radar system with M transmitting elements and N receiving elements

According to geometric properties, the following equation is obtained

$$\Delta\theta = \frac{d_t}{R_t}, \quad \Delta\varphi = \frac{d_r}{R_r} \quad (6)$$

As mentioned, when the long-distance assumption is satisfied, the array elements could be regarded as uniform linear array approximately. Therefore, the distance constraint in equation(1) should be satisfied for MIMO setup. By inserting equation(6) into(1), we get

$$\Delta\theta \geq \frac{\lambda}{D}, \quad \Delta\varphi \geq \frac{\lambda}{D} \quad (7)$$

It is proved that the angular spreads through which the transmitting array and the receiving array could achieve spatial diversity are the same. In view of the identical detection goal, it only concerns with the wave length of transmitting signal.

The total signal fade from transmitting array to the receiving array is defined as the signal channel fade coefficient $\alpha(\theta,\phi,R)$, which is related to the channel length R, incident angle θ and receiving angle ϕ. Obviously, the influence of R has nothing to do with the target characteristic, which is a linear function in uniform channel and defined as $\alpha_{\mathrm{R}}(R)$; while signal fade caused by the target RCS is defined as target fading coefficient $\alpha_{\mathrm{T}}(\theta,\phi)$. So

$$\alpha(\theta,\phi,R) = \alpha_{\mathrm{T}}(\theta,\phi)\cdot\alpha_{\mathrm{R}}(R) \quad (8)$$

In this model, the length of the channel $R = R_t + R_r$ is a constant whose influences on all channels are the same. So let

$$\alpha_{\mathrm{R}}(R) = 1 \quad (9)$$

This paper focuses on the situation under which $\alpha_{\mathrm{T}}(\theta,\phi)$ satisfies slow fluctuation condition between pulses. We assume that the expectation and variance are μ and σ^2 respectively, and $\mu \neq 0$. The specific distribution depends on the fluctuation of target, which will be modeled as the Swerling case I, III or V (Case V is non-fluctuation model, and that is the special case when RCS fluctuation of target is very gentle in the following text.). And it also can be illustrated by using the second generation of model, such as lognormal model. The contribution of spatial diversity to target RCS with different distribution is varied, and this paper will analyze the generalized case firstly, and then make a simulation of Swerling case I.

In the above assumptions, the signal of the l th receiving element could be expressed as

$$r_l(t) = \sum_{k=1}^{M} s_k(t-\tau)\alpha_{\mathrm{T}}(\theta_k,\phi_l) + n_l(t) \quad (10)$$

And $s_k(t)$ indicates the k th transmitting signal with period T, which matches the equation $\int_T s_k(t)s_k^*(t) = 1$; $\tau = R/c$ indicates the delay of signal, which is a constant and c is the speed of light; $n_l(t)$ indicates a Gaussian white noise with zero-mean and variance σ_v^2, and it satisfies the equation $E\{n_i(t)n_j(t)\} = 0, \quad i \neq j$.

Rewrite equation(10) in matrix form, that is

$$\boldsymbol{r}(t) = \boldsymbol{Hs}(t) \quad (11)$$

where

$$\boldsymbol{r}(t)=\left[r_1(t) \quad \cdots \quad r_N(t)\right]^{\mathrm{T}} \in \mathbb{C}^{N\times 1}$$
$$\boldsymbol{s}(t)=\left[s_1(t-\tau) \quad \cdots \quad s_M(t-\tau)\right]^{\mathrm{T}} \in \mathbb{C}^{M\times 1}$$
$$\left[\boldsymbol{H}\right]_{lk}=\alpha_{\mathrm{T}}\left(\theta_k,\phi_l\right), \boldsymbol{H}\in \mathbb{C}^{N\times M}.$$

3 Detection performance analysis for MIMO radar system

In this section, the model above will be used to analyze the contributions of the two kinds of gains—the spatial diversity gain and the coherent gain. Thus a basis could be provided for selecting the working mode of radar under various requirements and environments. The generalized MIMO radar concept is adopted here: the traditional phased- array radar is regarded as one kind of MIMO, as well as MISO, SIMO and multi-sub-array MIMO whose elements in the same sub-array are used for coherent gain while sub-arrays for spatial diversity gain, etc.

When both the transmitting and receiving array meet equation(7), all the elements are used to get spatial diversity gain. This radar mode is just the statistical MIMO mentioned above. The corresponding transmitting signals should be orthogonal to each other in order to separate each individual target RCS information from receiving signals. Match the receiving signal with transmitting signals respectively, and then the output should be

$$\begin{aligned} x_{l,k} &= \int r_l(t) s_k^*(t-\tau)\,dt \\ &= \alpha_{\mathrm{T}}\left(\theta_k,\phi_l\right)+\int n_l(t) s_k^*(t-\tau)\,dt \end{aligned} \tag{12}$$

Note that $\int n_l(t) s_k^*(t-\tau)\,dt$ is the linear combination of $n_l(t)$, and it also obeys a Gaussian random variable.

For comparing the gain in single channel case (That helps us to see more clearly how greatly the performance increases to SISO radar.), average the MN outputs in equation(12), which could result in

$$\begin{aligned} c_{\mathrm{SMIMO}} &= \frac{1}{MN}\sum_{l=1}^{N}\sum_{k=1}^{M} x_{l,k} \\ &= \frac{1}{MN}\sum_{l=1}^{N}\sum_{k=1}^{M} \alpha_{\mathrm{T}}\left(\theta_k,\phi_l\right) \\ &+ \frac{1}{MN}\sum_{l=1}^{N}\int n_l(t)\sum_{k=1}^{M} s_k^*(t-\tau)\,dt \end{aligned} \tag{13}$$

The expectation of the above equation is

$$\mathrm{E}\left\{c_{\mathrm{SMIMO}}\right\}=\frac{1}{MN}MN\cdot\mu+0=\mu+0 \tag{14}$$

(Two terminals of the addition sign express the expectation of signal component and noise component separately, and so is the following.)

And its variance is

$$\begin{aligned} \mathrm{var}\left\{c_{\mathrm{SMIMO}}\right\} &= \frac{1}{M^2N^2}\Big(MN\cdot\sigma^2 \\ &+ \mathrm{var}\left\{\sum_{l=1}^{N}\int n_l(t)\sum_{k=1}^{M} s_k^*(t-\tau)\,dt\right\}\Big) \end{aligned} \tag{15}$$

Note that the noises on different receiving elements or at different time are mutually independent, so it can be obtained that

$$\begin{aligned} &\mathrm{var}\left\{\sum_{l=1}^{N}\int n_l(t)\sum_{k=1}^{M} s_k^*(t-\tau)\,dt\right\} \\ &= \sum_{l=1}^{N}\int \mathrm{var}\left\{n_l(t)\sum_{k=1}^{M} s_k^*(t-\tau)\right\}dt \\ &= N\int\left\{\sigma_v^2\cdot\left[\sum_{k=1}^{M} s_k^*(t-\tau)\right]\left[\sum_{k=1}^{M} s_k(t-\tau)\right]\right\}dt \\ &= NM\cdot\sigma_v^2 \end{aligned} \tag{16}$$

The last equal sign uses the orthogonal characteristic of signals. Therefore, equation(15) may change into

$$\begin{aligned} \mathrm{var}\left\{c_{\mathrm{SMIMO}}\right\} &= \frac{1}{M^2N^2}\left(MN\cdot\sigma^2+NM\cdot\sigma_v^2\right) \\ &= \frac{\sigma^2}{MN}+\frac{\sigma_v^2}{MN} \end{aligned} \tag{17}$$

Regardless of the changes of noises provisionally, we examine the target information of c_{SMIMO} alone, whose expectation is μ, and its variance is σ^2/MN. And the expectation and variance of the target fading coefficient $\alpha_{\mathrm{T}}(\theta,\phi)$ is μ and σ^2 respectively. Therefore, after average processing those MN independent components derived from spatial diversity generate a new random variable which has the same expectation and $1/MN$ times variance in comparison with $\alpha_{\mathrm{T}}(\theta,\phi)$. c_{SMIMO} contains all target information in the echo signal, and it can be taken for detection statistics of system.

In the case of target RCS fluctuating, the task of the radar is to examine the random signal in the random noise. Spatial diversity processing reduces the signal variance, which therefore enhances the robustness of the detector. It is the first contribution of spatial diversity gain. It is easy to notice that if the number of diversity is large enough, the variance of statistical will approach zero, and now the information radar obtains from target is no longer random. In other words, the detection work is transformed into examining definite signal in the random noise.

In addition, the changes of signal-to-noise ratio are also under inspection. Take the signal-to-noise ratio of single channel as a standard, whose transmitting and receiving end both have only one element. The SNR of filter output after matching is

$$\mathrm{SNR} = \frac{\sigma^2 + \mu^2}{\sigma_v^2} \tag{18}$$

The SNR of MIMO system output is

$$\mathrm{SNR}_{\mathrm{SMIMO}} = \frac{\sigma^2 / (MN) + \mu^2}{\sigma_v^2 / (MN)} = \frac{\sigma^2 + MN\mu^2}{\sigma_v^2} \tag{19}$$

And the relative SNR gain resulted from spatial diversity is

$$\eta_{\mathrm{SMIMO}} = \frac{\mathrm{SNR}_{\mathrm{SMIMO}}}{\mathrm{SNR}} = \frac{\sigma^2 + MN\mu^2}{\sigma^2 + \mu^2} \tag{20}$$

when $\sigma^2 \gg \mu^2$, $\eta_{\mathrm{SMIMO}} \approx 1$; $\mu^2 \gg \sigma^2$, $\eta_{\mathrm{SMIMO}} \approx MN$.

Based on the analysis, we can conclude that the second kind of contribution of spatial diversity is to enhance the SNR. Associating with SNR gain in limiting condition, then we come to the following conclusion:

(1) When the target fading coefficient fluctuates severely, the first contribution of spatial diversity comes into play, which will reduce the fluctuation to enhance robustness of the system;

(2) As we can see from the equation above and Figure 2, the smaller target fading coefficient is, the larger SNR gain is. So when the target fading coefficient fluctuates slightly, the second contribution of spatial diversity manifests, which will improve the signal-to-noise ratio of the detection statistics. Note that the influence of diversity processing on the variance of detection statistics does not change, but due to the high robustness of detection system, it plays a much less influential role in enhancing system performance.

Next, the influence of coherent gain on radar performance will be discussed here. In the phased-array mode, all the transmitting elements transmit the same signal $s(t)$ from almost the same angle, while the receiving elements are very close to each other. Therefore, the MN fading coefficients $\alpha_{\mathrm{T}}(\theta,\phi)$ are approximately equal to each other, expressed as α_{T} for short. Then the l th receiving signal only needs one match, and the output is

$$\begin{aligned} x_l &= \int r_l(t) s^*(t-\tau) dt \\ &= M \cdot \alpha_{\mathrm{T}} + \int n_l(t) s^*(t-\tau) dt \end{aligned} \tag{21}$$

Actually, every output contains M input signal components. For comparing the gain in single channel case, averaging MN signal components, that is

$$\begin{aligned} c_{\mathrm{PHASE}} &= \frac{1}{NM} \sum_{l=1}^{N} x_l \\ &= \alpha_{\mathrm{T}} + \frac{1}{NM} \sum_{l=1}^{N} \int n_l(t) s^*(t-\tau) dt \end{aligned} \tag{22}$$

Its expectation is

$$\mathrm{E}\{c_{\mathrm{PHASE}}\} = \mu + 0 \tag{23}$$

And the variance is

$$\begin{aligned} \mathrm{var}\{c_{\mathrm{PHASE}}\} &= \sigma^2 + \frac{N \cdot \int \sigma_v^2 s^*(t-\tau) s(t-\tau) dt}{M^2 N^2} \\ &= \sigma^2 + \frac{\sigma_v^2}{M^2 N} \end{aligned} \tag{24}$$

Similar to the MIMO, c_{PHASE} contains all receiving information in the echo signal, and it can be taken for detection statistics. The expectation and variance of target information are μ and σ^2 respectively. Consequently, through coherent processing, the influence of target fluctuation on detection system does not change. So for coherent gain, the first kind of contribution is zero.

Considering the changes of signal-to-noise ratio, the average SNR of output is expressed as

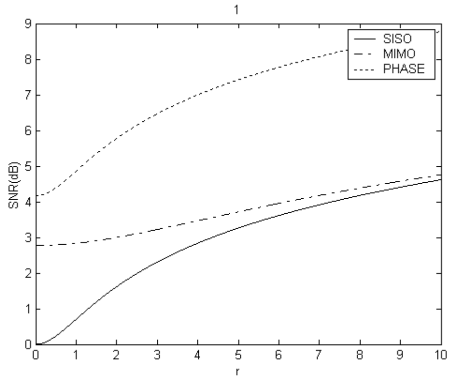
$$\mathrm{SNR}_{\mathrm{PHASE}} = \frac{\sigma^2 + \mu^2}{\sigma_v^2 / (M^2 N)} = \frac{M^2 N \sigma^2 + M^2 N \mu^2}{\sigma_v^2} \tag{25}$$

And the relative SNR gain resulted from coherent processing is

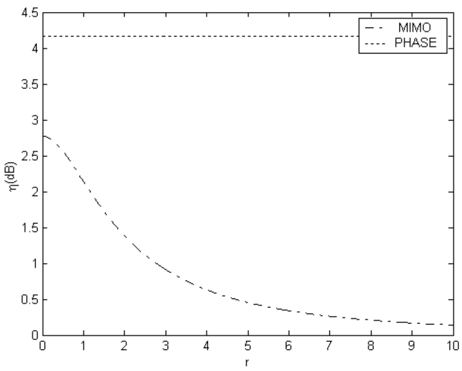
$$\eta_{\mathrm{PHASE}} = \frac{\mathrm{SNR}_{\mathrm{PHASE}}}{\mathrm{SNR}} = M^2 N \tag{26}$$

Therefore, the conclusion for coherent gain is that coherent gain shows the second kind of contribution only, and the effect does not change with the target fluctuation. It always increases the SNR of statistics by $M^2 N$ times.

Since coherent processing has no contribution to the robustness of system, the spatial diversity processing has overwhelming superiority in this aspect. Next we make a comparison between their second kind of contribution. Figure 2 shows the SNR of a radar detection system which has 4 transmitting elements and 4 receiving elements, while a SISO(single channel case) SNR curve is also drawn for reference. The x-axis is $r \triangleq \sigma / \mu$, which indicates the target RCS fluctuation level: the larger r is, the more severe the corresponding fluctuation is. While $r = 0$, the target remains unchangeable. The figure (a) is the logarithm SNR and the figure (b) is the SNR gain curve for MIMO and phased-array relative to SISO. From above we can conclude that the SNR gain of phased-array is stable while the SNR gain of MIMO decreases with the RCS fluctuation level increasing. Regarding the target which fluctuates very severely, the SNR gain relative to SISO almost approaches zero. Even in the case of stable target, the performance of MIMO radar is still worse than phased-array in improving SNR. In the next section we will analyze this and find method for improvement.



(a)



(b)

Fig. 2 SNR curve of statistics for MIMO and phase-array radar

(a) The original SNR
(b) The gain of SNR relative to SISO

4 Modified statistics MIMO radar and simulation

The contribution of diversity gain and coherent gain has been discussed in the last section. From above, we can draw a conclusion that when target fading coefficient is so small that it can be regarded as quantum, diversity processing can improve the SNR by MN times and the coherent gain by M^2N times. So the diversity processing is a suboptimal choice in comparison.

So, is this an inevitable weak point caused by spatial diversity processing? We make an analysis on the mechanism of the phenomenon in this section. And for statistics MIMO radar, we propose one modified method to increase the contribution of diversity processing to increasing SNR.

First, comparing equation(12) with equation(21), it is easy to find that the phased-array radar only matches each receiving signal once and the statistics MIMO radar matches M times when processing information of target from receiving elements. The difference of the matching process results in the difference in noise-component of the statistics c_{PHASE} and c_{SMIMO}, which are equal to $\left[\sum_{l=1}^{N}\int n_l(t)s^*(t-\tau)dt\right]/NM$ and $\left[\sum_{l=1}^{N}\int n_l(t)\sum_{k=1}^{M}s_k^*(t-\tau)dt\right]/NM$. And it can be obtained from equation(24) and equation(17) that the power of the former is only $1/M$ times of the latter. That is the reason why coherent SNR is almost M larger than diversity SNR in the limiting case.

And then, by analyzing the characteristics of the transmitting signal of statistics MIMO, we can find that M orthogonal signal transmitted by array elements is the direct reason for matching M times. Compared with the coherent signal, the unique function of orthogonal signal is to tag the target information obtained by the different transmitting elements. Therefore, M independent components such as $s_k(t-\tau)\alpha_{\text{T}}(\theta_k,\phi_1), k=1,\cdots M$ can be separated from one receiving signal $r_1(t)$. And for all the receiving signals $r_l(t), l=1\cdots N$, we can get total MN independent components to structure the statistics of detection system. And whether it is necessary to separate the components depends on construction of the statistics.

Again, we make an analysis of the construction of detection statistics. One way is to use the average of the sample observations, which means taking the expectation of random variable as features for detection. Another way is to employ the mean squares value of the sample observations as detection feature, which actually is the second order moment of the random variable. Moreover, the higher order moment also could be used for detection feature. The method mentioned in this paper is the first way, which is related to the RCS distribution of the target. The second or higher moment should be chosen when the expectation of target fading coefficient is equal to zero; while the expectation is a simple and feasible alternative in other cases. Actually even if the lower order moment could be used to detect, it is still better to select the higher one which may improve system performance. However, it is hard to realize because of its complexity, which is beyond the scope of the paper.

At last, we come to the conclusion that the average of the sample observation could be chosen as statistics when the RCS of target obeys a distribution with non-zero expectation, such as Sweiling case I and III. The combining form of the components in the receiving signal of equation(10) is in accordance with the average form, so there is no need to separate every component from receiving signal by using orthogonal transmitting signal. Using the same transmitting signal, the noise components could be further reduced when the transmitting signals match receiving signals. When the second or higher moment is chosen for detection feature, the transmitting components should be separated from each receiving signal and the orthogonal signal is irreplaceable.

After modification in statistical MIMO model, the detection statistics is changed into

$$\begin{aligned}\tilde{c}_{\text{SMIMO}} &= \frac{1}{MN}\sum_{l=1}^{N}\tilde{x}_l \\ &= \frac{1}{MN}\sum_{l=1}^{N}\sum_{k=1}^{M}\alpha_{\text{T}}\left(\theta_k,\phi_l\right) \\ &\quad +\frac{1}{MN}\sum_{l=1}^{N}\int n_l\left(t\right)s^*\left(t-\tau\right)dt\end{aligned} \tag{27}$$

In the equation, $s(t)$ is the transmitting signal, $\tilde{x}_l$ is the output of the l th receiving signal after matching. It can be expressed as

$$\begin{aligned}\tilde{x}_l &= \int r_l\left(t\right)s^*\left(t-\tau\right)dt \\ &= \sum_{k=1}^{M}\alpha_{\text{T}}\left(\theta_k,\phi_l\right)+\int n_l\left(t\right)s^*\left(t-\tau\right)dt\end{aligned} \tag{28}$$

Then

$$\begin{aligned}\text{E}\left\{\tilde{c}_{\text{SMIMO}}\right\} &= \mu+0 \\ \text{var}\left\{\tilde{c}_{\text{SMIMO}}\right\} &= \frac{\sigma^2}{MN}+\frac{\sigma_v^2}{M^2N} \\ \tilde{\text{SNR}}_{\text{SMIMO}} &= \frac{M\sigma^2+M^2N\mu^2}{\sigma_v^2} \\ \tilde{\eta}_{\text{SMIMO}} &= \frac{\tilde{\text{SNR}}_{\text{SMIMO}}}{\text{SNR}} = \frac{M\sigma^2+M^2N\mu^2}{\sigma^2+\mu^2}\end{aligned} \tag{29}$$

When $\sigma^2 \gg \mu^2$, $\tilde{\eta}_{\text{SMIMO}} \approx M$; when $\mu^2 \gg \sigma^2$, $\eta_{\text{SMIMO}} \approx M^2N$.

Therefore, when using coherent signals instead of orthogonal signals, the robustness of radar system does not change, while the SNR is increased by M times. Its adaptivity enhances: when the fluctuation of the target is severe, it reduces the influence of fluctuation mainly and improves the SNR to a certain degree; when the fluctuation is gentle, its contribution to the SNR is almost the same as that of phased-array.

Figure 3 presents the SNR gain curves of four radar systems that all have 2 transmitting elements and 4 receiving elements. The MIMO* expresses the modified statistical MIMO radar, whose SNR gain is higher than the original one and is equal to the phase-array in the limiting case, will never reduce to 0dB. The SNR gain of MISO radar is not lower than the original statistical MIMO from beginning to end, and even surpasses the modified one in many cases. This is because the elements used to get coherent gain in MISO are more than that of MIMO. In other words, MISO system benefits more from the elements in increasing SNR. Also, it should be noticed that it plays a smaller role in improving robustness of system, and only reduces variance by M times. The data of MISO will be discussed in the following section.

Corresponding with figure 3, table 1 shows the performance parameters of the four radar systems. By combining the two, a comprehensive assessment could be obtained.

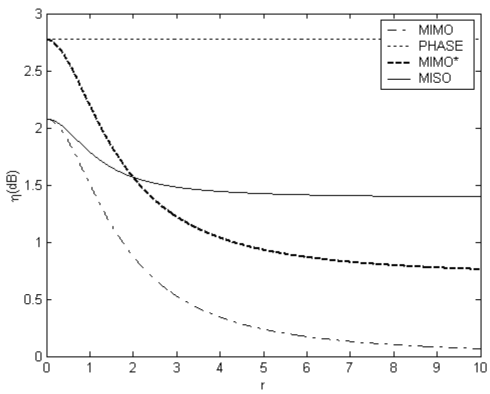


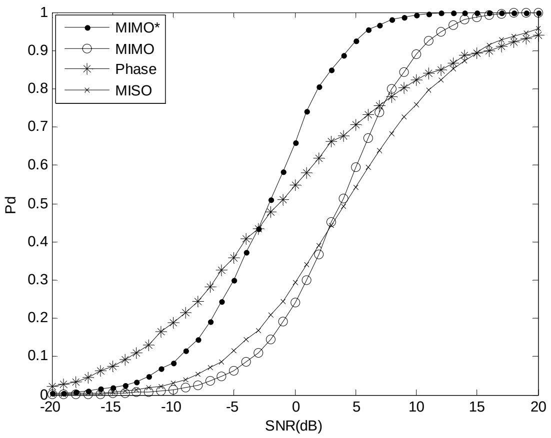
Fig.3 The SNR gain curve for four radar systems

	variance of the target Component	SNR gain	
		$\sigma^2 \gg \mu^2$	$\mu^2 \gg \sigma^2$
phase array	σ^2	M^2N	M^2N
Statistical MIMO	σ^2/MN	1	MN
modified MIMO	σ^2/MN	M	M^2N
MISO	σ^2/M	N	MN

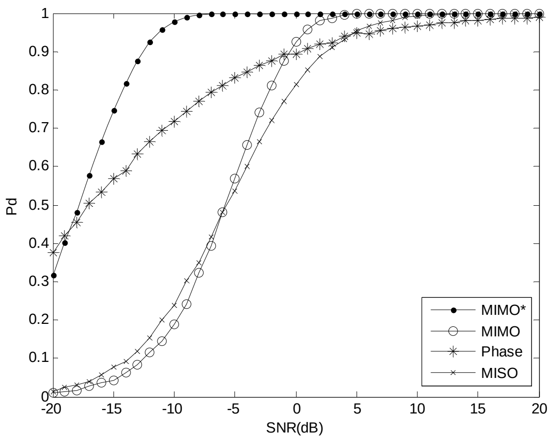
Table 1 Performance parameters of four radar systems

Finally, we make a simulation for the detection performance of the four radar systems. In the experiment, it is assumed that the target RCS obeys the distribution of Sweiling case I, and is under the zero-mean Gaussian white noise environment. We uses the constant false-alarm (CFAR) detector to detect one target, and get the detection probability from data statistics by repeated Monte Carlo experiments.

The false-alarm probability in Figure 4 is $P_f = 10^{-5}$. The figure (a) shows the detection probability of the radars with 2 transmitting elements and 4 receiving elements. As we can see clearly, phase-array radar gets the best detection performance when the SNR is in the lowest level, and the other three which use the spatial diversity technology surpass it gradually.



(a)

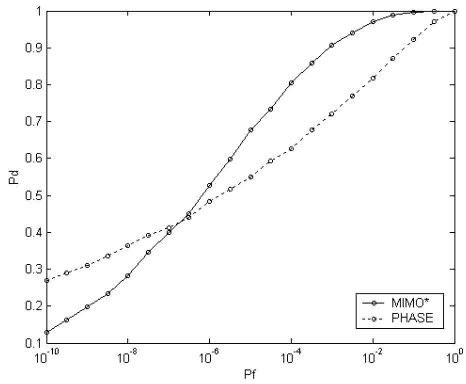


(b)

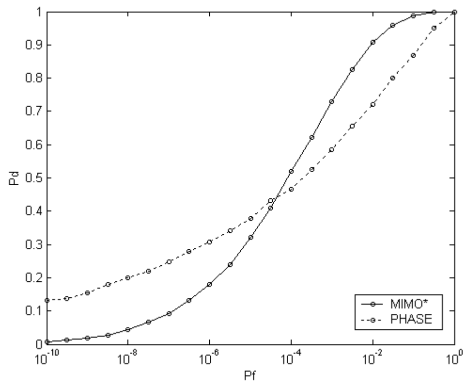
Fig.4 Curves of detection probability with SNR
(a) 2 transmitting elements and 4 receiving elements, $P_f = 10^{-5}$;
(b) 4 transmitting elements and 6 receiving elements, $P_f = 10^{-5}$

The phenomenon can be well-explained with the views about gains and contributions mentioned above: the phased-array makes the largest contribution to improving the SNR of statistics, and it gets the best performance in extremely low SNR. When the SNR increases to a certain level, target fluctuation brings more negative effect on the system than noise does, both of which are the two main factors leading to the detection performance degradation. As for MIMO, MISO and modified MIMO radars, all of which use the spatial diversity technology to resist target fluctuation, they have obvious advantage over the phase-array in detection performance. Among them, MISO system gets the smallest diversity gain, so its speed of surpassing phase-array radar is the lowest; the modified statistical MIMO provides the best performance, for it reduces more noise than the original one.

The number of transmitting and receiving elements increases to 4 and 6 respectively in figure 4(b). Compared with figure 4(a), detection performance improves significantly due to the increasing of elements, and the other three MIMO radars could surpass phase-array in the lower SNR. In fact, this is because added array elements improves the SNR of statistics so that the threshold of original SNR reduces, in which the target fluctuation is the main negative factor affecting the performance rather than noise.



(a)



(b)

Fig.5 Curves of detection probability with false-alarm probability
(a) 2 transmitting elements and 4 receiving elements, SNR = 0dB;
(b) 4 transmitting elements and 6 receiving elements, SNR = −20dB

Figure 5 is the curve of the detection probability change with false alarm probability for the modified statistical MIMO and phase-array. As we can see, with the number of radar array elements increasing, the weaker target could be detected.

5 Quick analysis on other MIMO systems

The conclusions about diversity gain and coherent gain derive from two extreme MIMO model, statistical MIMO and phased-array. In fact, all sorts of generalized MIMO models use the two kinds of gain which possesses different weight to enhance the system performance. They could always be considered as combination of statistical MIMO and phased-array. Therefore, the method used in this paper may be applied to analyzing other MIMO systems. Here we will take the MISO system for example to introduce the procedures of analysis, and verify the conclusion once more.

In the MISO system, M transmitting elements adopt spatial diversity processing while N receiving elements use coherent processing. Its transmitting signals were orthogonal. Similarly select the sample average for statistics of detection

$$c_{\text{MISO}} = \frac{1}{MN}\sum_{l=1}^{N}\sum_{k=1}^{M} x_{l,k} \tag{30}$$

Instead of analyzing its concrete structure, here we directly use the view about gains and theirs contributions to analyze the properties of c_{MISO}.

First, taking the diversity gain into consideration, it is regarded as a MIMO system which has M transmitting elements and only one receiving element. Then it could be obtained that the expectation of c_{MISO} is μ, the variance of the signal part is σ^2 / M, and the SNR gain relative to SISO is $\eta_{\text{MISO1}} = \frac{\sigma^2 + M\mu^2}{\sigma^2 + \mu^2}$.

Then as for the coherent gain, it was regarded as a phased-array system with 1 transmitting elements and N receiving elements, so the SNR gain was $\eta_{\text{MISO2}} = N \cdot 1^2$;

Finally, synthesizing the two, the total SNR gain is

$$\begin{aligned}\eta_{\text{MISO}} &= \eta_{\text{MISO1}} \cdot \eta_{\text{MISO2}} \\ &= \frac{N\sigma^2 + NM\mu^2}{\sigma^2 + \mu^2}\end{aligned} \tag{31}$$

And the contribution to the system robustness is to reduce the variance of target component to $1/M$ times.

This result is in accordance with the simulation and also same as the result from concrete structure analysis of c_{MISO}. The procedure of structure analysis is similar with the section III, and we no longer repeat here.

6 Conclusion

This paper has mainly analyzed the two kinds of contributions made by spatial diversity gain and the coherent gain in the MIMO radar. Through analysis it is shown that diversity processing focuses more on enhancing robustness of the system, but is weaker in increasing the SNR of statistics compared with coherent processing. The paper further makes an analysis on the principle of this, and proposes a modified method to increase the contribution of diversity processing to SNR. The result of simulation indicates that the modified MIMO could enhance the performance of the detection system. Based on this, to cope with the various target RCS distributions and more complex environment, it is valuable to continue studying how to use the two kinds of gain reasonably to maximize the performance of radar detection system.

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