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A modified Nuclear Model for Binding Energy of Nuclei

K. K. Sirma[1], **L. S. Chelimo**[2] and **K. M. Khanna**[2,*]

[1]Department of Renewable Energy and Technology, Turkana University College,
P.O. Box 69-30500, Lodwar, Kenya

[2]Department of Physics, University of Eldoret, P.O. Box 1125-30100, Eldoret, Kenya

*E-mail address: khannak700@gmail.com

ABSTRACT

A new nuclear model to calculate the binding energy of nuclei is proposed. The nucleus is assumed to be composed of two regions; the inner core region and surface region. The inner core is assumed to be composed of Z proton-neutron pairs (Z = N) and the surface region is composed of the unpaired neutrons for a nucleus in which N>Z. The interaction between the core and neutrons in the surface region is assumed to be such that it leads to an average potential V_o in which each neutron in the surface region can move. Knowing the experimental values for the binding energy of nuclei, this average interaction potential V_o has been calculated for light, medium and heavy nuclei. It is found that V_o varies for isotopes and isotones. For isotopes the value of V_o decreases as the neutron number (N) in the surface region of the nucleus increases. The decrease in V_o is quite large when the neutron number increases by unity in light nuclei compared to heavy nuclei. For isotones, the value of V_o increases with an increase in proton number (Z). This is evident for both light nuclei and heavy nuclei.

***Keywords*:** Potential, Isotopes, Isotones, light Nuclei, Heavy Nuclei, Surface region, Core

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1. INTRODUCTION

From time to time a number of nuclear models have been proposed to understand the properties of nuclei [1-10]. None of the models can explain all the properties of a given nucleus. Due to saturation of density of the nucleus (no other system has a density more than the nuclear density except neutron stars), the nuclear interaction energy is also assumed to be the largest. Assuming that nuclides are made of incompressible matter, and that nuclear force is the same for each nucleon and that the nuclear force saturates [2], Weizsäcker and Bohr [1, 2] proposed what is called a liquid drop model (LDM) that led to the famous Semi-empirical mass formula (SEMF) for the mass of the nucleus. LDM is a model in nuclear physics which treats the nucleus as a liquid drop of incompressible nuclear fluid. In view of similarities between the properties of the liquid drop and atomic nucleus, such as the latent heat of vaporization of fluid which is comparable to the constant binding energy per nucleon and the surface tension effects of nucleus, the quantitative aspect of the model delivers a formula that approximates the mass and binding energy of nuclei [11].

The binding energies predicted by the LDM underestimate the actual binding energies of "magic nuclei". These magic numbers can be explained in terms of the Shell Model of the nucleus, which considers each nucleon to be moving in some potential and classifies the energy levels in terms of quantum numbers in the same way as the wave functions of individual electrons are classified in Atomic Physics. The shell model [4, 12] could explain more successfully a larger number of properties of the nuclei. A phenomenological understanding of nuclear binding energies as function of A, Z and N was presented by SEMF which is often referred to as Bethe-Weizsäcker (BW) formula [3, 13-15]. It has always been at the heart of our understanding of several properties of the atomic nuclei [16]. The nuclear mass depends on the values of A and Z [3], Coulomb force, the Pauli exclusion principle, the nucleon asymmetry, the pairing force etc and the final formula for the mass and binding energy of the nucleus depends on N, Z, N+Z, Z-N or N-Z, [17].

The famous SEMF for nuclear binding energy is given as [17, 18].

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c Z(Z-1)\mathrm{A}^{-1/3} - a_a (N-Z)^2 A^{-1} - 1 \pm \delta + \eta. \qquad (1)$$

The BW formula describes, with good accuracy, the various properties of nuclides such as fission, fusion, and Alpha–decay barrier potential energies with remarkable accuracy [19-21]. However, various suggestions have been made to add additional terms to BW formula, to further improve the results. The terms that are added are the Wigner term, pairing term, Coulomb term and exchange term, surface symmetry term etc. [22]. Then there are quite a few other nuclear models [17] that have been proposed to explain the properties of nuclei. The most successful of them, being the nuclear shell model [4]. Binding energy B and binding fraction f have been studied by many authors [5, 10]. It has been found both experimentally and theoretically [5] that the values of B and f vary drastically as we go from light nuclei to heavy nuclei. For $A \leq 50$, there are some nuclei in which the number of protons (Z) is equal to the number of neutrons (N).

As the mass number, A=N+Z, of a nuclide increases, the neutron number, N, increases faster than the proton number, Z [23]. Nuclei for which N=Z are found to be strongly bound and have large binding energy, B, and also large binding fraction $f = \frac{B}{A}$.

For nuclei with large A and N>Z, the binding energy B and binding fractions f get reduced due to Coulomb repulsion between the protons. Nuclei with equal and even numbers of neutrons and protons (even-even expressed as e-e) are found to be strongly bound. Then there are nuclei with equal and odd numbers of neutrons and protons (odd-odd expressed as o-o) and they are slightly less bound. There are also nuclei with even number of neutrons and odd number of protons (e-o), and odd number of neutrons and even number of protons (o-e) again with slightly less binding. It is found that no two nuclei have the same binding fraction f, and the value of f rises abruptly for nuclei with A<20, rising to a maximum value of around 8.4MeV and then starts decreasing as A increases, to some value 7.4MeV for A>150. The value of B and f depends on how we model the nucleus and what kind of interactions between the neutrons and protons are considered.

2. THEORETICAL DERIVATIONS

Recently [24] designer nuclei have been prepared in the lab in a manner that N>>Z. One such nucleus is ${}^{11}_{3}Li$ [25] whose radius is as big as that of Pb nucleus. Keeping this kind of nuclear structure in mind, a nuclear model is proposed in which it is assumed that the core of the nucleus is composed of equal number of neutrons and protons that form neutron-proton pairs, and the unpaired neutrons reside in the surface region of the nucleus which is also called neutron skin. Thus for a nucleus of mass number A=Z+N, the core of the nucleus will be composed of Z protons-neutron pairs and the unpaired neutrons equals to N-Z, will constitute the surface region of the nucleus. The interaction energy will be the pairing energy, $\in_\rho$ between a pair of nucleons. The core will contain 2Z nucleons, and thus the number of pairs will be,

$$^{2Z}C_2 = \frac{2Z!}{(2Z-2)!} = 2Z(2Z-1) \tag{2}$$

If $\in_\rho$ is the pairing energy associated with each pair, then the energy, E_ρ, of the core of the nucleus due to pairing interaction will be,

$$E_p = \in_p 2z(2z-1) \tag{3}$$

As in the liquid drop model, the core will be assumed to be an incompressible system and thus the volume energy will be assumed to be E_V

$$E_v = 2a_v Z \tag{4}$$

Similarly the Coulomb energy due to the presence of Z protons in the core of the nucleus will be (for large nuclei) [15]

$$E_C = -0.72\frac{Z^2}{(2Z)^{1/3}} \tag{5}$$

Since in this model, the surface region will be treated separately, there will be no such thing as the surface energy. Also N=Z in the core, thus there will be no asymmetry energy of the core. Now we have to decide on the type of interaction that the core of the nucleus may have with the unpaired neutrons in the surface region. To keep matters simple, it will be assumed that the interaction between the core and the neutrons in the surface region leads to an average potential in which each neutron in the surface region can move. This will be close to what the Hartree-Fock theory demands. Thus if the average potential per neutron is denoted by V_0, then total potential energy will be - V_0 (N-Z); denoting this by E_H, we write,

$$E_H = -V_0\,(N-Z) = -V_0\,(A-2Z) \tag{6}$$

Combining Eqs (3, 4, 5 and 6), we get the total binding energy, B_{tot} (A,Z), as

$$B_{tot}(A,Z) = 2a_v Z - \in_\rho\ 2Z(2Z-1) - 0.72\frac{Z^2}{2Z^{1/3}} - V_0(A-2Z) \tag{7}$$

Equation (7) gives the new formula for binding energy according to our model of the nucleus. In Eq (7) all the parameters are known, except the value of V_0, and this will be an adjustable parameter since this will depend on the number of nucleons in the core, and the number of neutrons in the surface region. Exact calculation of B_{tot} and its comparison with the experimental values of B will exactly determine how V_o may vary as A changes. It is important to understand that the value of V_0 may change as the neutron number (N) changes in isotopes of a given nucleus with Z fixed. Calculations will indicate how V_0 changes as the number of neutrons in the surface region changes. The value of V_o is calculated by equating $B_{tot}=B$ (the experimental value of binding energy). Variation of V_o with variation of the number of neutrons is calculated using the formula in Eq (7) where B_{tot} will be the experimental value for different nuclei. Eq (7), is re-arranged to get the value of V_o as,

$$V_0 = \frac{-[B_{tot}(A,Z)+2a_v Z- \in_\rho 2Z(2Z-1)- 0.72\frac{Z^2}{2Z^{1/3}}]}{(A-2Z)} \tag{8}$$

Using the following parameters

$a_v = 15.8$ MeV [26, 27]

$\in_\rho = 2\text{MeV}$

$B_{tot}(A,Z)$. The experimental binding energy values obtained from [5, 28]

Eq (8), changes to

$$V_0 = \frac{-B_{tot}+31.6Z -4Z(2Z-1)-0.72[\frac{Z^2}{2Z^{1/3}}]}{(A-2Z)} \quad \text{MeV} \tag{9}$$

Using the respective Binding energy [5, 28] and mass number we calculate the values of V_0 for some light and heavy nuclei.

3. RESULTS

Table 1. Values of V_0 for light, Medium and heavy Nuclei's.

B (Exp)	Nucleus	A	Z	A-2Z	V_0	N	V_0/N	N-Z
0	H	1	1	-1	12.5714644	0	0	-1
2.225	H	2	1	0	0	1	0	0
8.482	H	3	1	1	21.0534644	2	10.526732	1
7.718	He	3	2	-1	49.5322863	1	49.5322863	-1
28.296	He	4	2	0	0	2	0	0
31.994	Li	6	3	0	0	3	0	0
39.244	Li	7	3	1	126.810081	4	31.70252	1
58.165	Be	9	4	1	207.925	5	41.585	1
64.751	B	10	5	0	0	5	0	0
76.205	B	11	5	1	304.55986	6	50.759977	1
92.162	C	12	6	0	0	6	0	0
97.108	C	13	6	1	420.429603	7	60.061372	1
B (Exp)	**Nucleus**	**A**	**Z**	**A-2Z**	**V_0**	**N**	**V_0/N**	**N-Z**
104.659	N	14	7	0	0	7	0	0
115.492	N	15	7	1	550.13014	8	68.766268	1
127.619	O	16	8	0	0	8	0	0
131.763	O	17	8	1	694.04986	9	77.116651	1
139.807	O	18	8	2	351.04693	10	35.104693	2
147.801	F	19	9	1	854.054245	10	85.405424	1
160.645	Ne	20	10	0	0	10	0	0
167.406	Ne	21	10	1	1033.93103	11	93.99373	1
177.77	Ne	22	10	2	522.147513	12	43.512293	2
186.564	Na	23	11	1	1229.65564	12	102.4713	1
198.257	Mg	24	12	0	0	12	0	0

205.588	Mg	25	12	1	1441.53185	13	110.88707	1
216.681	Mg	26	12	2	726.312424	14	51.879459	2
224.952	Al	27	13	1	1670.02547	14	119.28753	1
236.537	Si	28	14	0	0	14	0	0
245.011	Si	29	14	1	1915.4842	15	127.69895	1
255.62	Si	30	14	2	963.046599	16	60.190412	2
262.917	P	31	15	1	2175.05343	16	135.94084	1
271.781	S	32	16	0	0	16	0	0
280.422	S	33	16	1	2450.47916	17	144.14583	1
291.839	S	34	16	2	1230.94808	18	68.386004	2
308.714	S	36	16	4	619.69279	20	30.98464	4
298.21	Cl	35	17	1	2742.43992	18	152.35777	1
317.101	Cl	37	17	3	920.443639	20	46.022182	3
306.717	Ar	36	18	0	0	18	0	0
327.343	Ar	38	18	2	1530.99632	20	76.549816	2
343.811	Ar	40	18	4	769.615162	22	34.982507	4
333.724	K	39	19	1	3375.03569	20	168.75178	1
341.524	K	40	19	2	1691.41784	21	80.543707	2
351.619	K	41	19	3	1130.9769	22	51.408041	3
342.052	Ca	40	20	0	0	20	0	0
361.896	Ca	42	20	2	1863.05386	22	84.684266	2
369.829	Ca	43	20	3	1244.68024	23	54.116532	3
380.96	Ca	44	20	4	936.292928	24	39.012205	4
398.769	Ca	46	20	6	627.163452	26	24.121671	6
387.848	Sc	45	21	3	1363.73122	24	56.822134	3
398.193	Ti	46	22	2	2228.4514	24	92.852142	2
407.073	Ti	47	22	3	1488.59427	25	59.543771	3

418.7	Ti	48	22	4	1119.35245	26	43.052017	4
426.842	Ti	49	22	5	897.110361	27	33.22631	5
437.781	Ti	50	22	6	749.415134	28	26.764826	6
B (Exp)	**Nucleus**	**A**	**Z**	**A-2Z**	**V_0**	**N**	**V_0/N**	**N-Z**
434.794	V	50	23	4	1216.27364	27	45.047172	4
445.845	V	51	23	5	975.229112	28	34.829611	5
435.049	Cr	50	24	2	2626.5818	26	101.02238	2
456.349	Cr	52	24	4	1318.6159	28	47.093425	4
464.289	Cr	53	24	5	1056.48072	29	36.43037	5
474.008	Cr	54	24	6	882.020434	30	29.400681	6
482.075	Mn	55	25	5	1140.84476	30	38.028159	5
471.763	Fe	54	26	2	3057.08157	28	109.18148	2
492.258	Fe	56	26	4	1533.66454	30	51.122151	4
499.905	Fe	57	26	5	1228.46103	31	39.627775	5
509.949	Fe	58	26	6	1025.39152	32	32.043485	6
517.313	Co	59	27	5	1319.23577	32	41.226118	5
506.459	Ni	58	28	2	3519.0011	30	117.30004	2
526.846	Ni	60	28	4	1764.5973	32	55.143666	4
534.666	Ni	61	28	5	1413.24184	33	42.82551	5
545.262	Ni	62	28	6	1179.46753	34	34.690222	6
561.758	Ni	64	28	8	886.662651	36	24.629518	8
551.385	Cu	63	29	5	1510.36294	34	44.422439	5
569.212	Cu	65	29	7	1081.37738	36	30.038261	7
559.098	Zn	64	30	4	2011.15521	34	59.151624	4
578.136	Zn	66	30	6	1343.94314	36	37.331754	6
585.189	Zn	67	30	7	1152.95883	37	31.16105	7
595.387	Zn	68	30	8	1010.11373	38	26.58194	8

611.087	Zn	70	30	10	809.660984	40	20.241525	10
601.996	Ga	69	31	7	1226.97376	38	32.288783	7
618.951	Ga	71	31	9	956.196817	40	23.90492	9
610.521	Ge	70	32	6	1519.14017	38	39.977373	6
628.686	Ge	72	32	8	1141.62575	40	28.540644	8
635.469	Ge	73	32	9	1015.53211	41	24.769076	9
645.665	Ge	74	32	10	914.9985	42	21.785679	10
661.598	Ge	76	32	12	763.8265	44	17.359693	12
652.564	As	75	33	9	1076.73152	42	25.636465	9
642.891	Se	74	34	6	1705.13471	40	42.628368	6
662.073	Se	76	34	8	1281.24878	42	30.505923	8
669.492	Se	77	34	9	1139.71214	43	26.504934	9
679.99	Se	78	34	10	1026.79073	44	23.336153	10
696.866	Se	80	34	12	857.065273	46	18.631854	12
712.843	Se	82	34	14	735.768591	48	15.328512	14
686.321	Br	79	35	9	1204.48132	44	27.374576	9
704.37	Br	81	35	11	987.125537	46	21.459251	11
675.578	Kr	78	36	6	1901.97944	42	45.285225	6
B (Exp)	**Nucleus**	**A**	**Z**	**A-2Z**	**V_0**	**N**	**V_0/N**	**N-Z**
695.434	Kr	80	36	8	1428.96658	44	32.476513	8
714.274	Kr	82	36	10	1145.05727	46	24.892549	10
721.737	Kr	83	36	11	1041.6396	47	22.162545	11
732.258	Kr	84	36	12	955.713054	48	19.910689	12
749.235	Kr	86	36	14	820.395261	50	16.407905	14
739.283	Rb	85	37	11	1097.64197	48	22.867541	11
757.856	Rb	87	37	13	930.20267	50	18.604053	13
728.906	Sr	84	38	8	1584.79441	46	34.452052	8

748.928	Sr	86	38	10	1269.83773	48	26.454953	10
757.356	Sr	87	38	11	1155.16412	49	23.574778	11
768.469	Sr	88	38	12	1059.82653	50	21.196531	12
775.538	Y	89	39	11	1214.16789	50	24.283358	11
783.893	Zr	90	40	10	1401.12485	50	28.022497	10
791.087	Zr	91	40	11	1274.40387	51	24.988311	11
799.722	Zr	92	40	12	1168.92313	52	22.479291	12
814.677	Zr	94	40	14	1003.00232	54	18.574117	14
828.996	Zr	96	40	16	878.52197	56	15.687892	16
805.765	Nb	93	41	11	1336.03208	52	25.692925	11
796.508	Mo	92	42	8	1920.81406	50	38.416281	8
814.256	Mo	94	42	10	1538.42604	52	29.585116	10
821.625	Mo	95	42	11	1399.23904	53	26.400737	11
830.779	Mo	96	42	12	1283.39862	54	23.766641	12
837.6	Mo	97	42	13	1185.20034	55	21.549097	13
846.243	Mo	98	42	14	1101.16053	56	19.663581	14
860.458	Mo	100	42	16	964.403903	58	16.627653	16
826.496	Ru	96	44	8	2100.48501	52	40.393943	8
844.79	Ru	98	44	10	1682.21741	54	31.152174	10
852.255	Ru	99	44	11	1529.96719	55	27.817585	11
861.928	Ru	100	44	12	1403.27601	56	25.0585	12
868.73	Ru	101	44	13	1295.85493	57	22.734297	13
874.844	Ru	102	44	14	1203.73058	58	20.753975	14
893.083	Ru	104	44	16	1054.40419	60	17.573403	16
884.163	Rh	103	45	13	1353.03903	58	23.328259	13
875.212	Pd	102	46	10	1832.46952	56	32.72267	10
892.82	Pd	104	46	12	1528.52527	58	26.353884	12

899.914	Pd	105	46	13	1411.4921	59	23.923595	13
909.474	Pd	106	46	14	1311.35409	60	21.855901	14
925.239	Pd	108	46	16	1148.42014	62	18.522905	16
940.207	Pd	110	46	18	1021.64946	64	15.963273	18
915.263	Ag	107	47	13	1471.15864	60	24.519311	13
931.727	Ag	109	47	15	1276.10175	62	20.582286	15
B (Exp)	**Nucleus**	**A**	**Z**	**A-2Z**	**V_0**	**N**	**V_0/N**	**N-Z**
905.14	Cd	106	48	10	1989.14313	58	34.295571	10
923.402	Cd	108	48	12	1659.14111	60	27.652352	12
940.646	Cd	110	48	14	1423.35266	62	22.957301	14
947.622	Cd	111	48	15	1328.92755	63	21.094088	15
957.016	Cd	112	48	16	1246.45671	64	19.475886	16
963.556	Cd	113	48	17	1173.52043	65	18.05416	17
972.599	Cd	114	48	18	1108.82724	66	16.800413	18
987.44	Cd	116	48	20	998.686565	68	14.686567	20
963.094	In	113	49	15	1382.80347	64	21.606304	15
979.404	In	115	49	17	1221.08012	66	18.501214	17
953.532	Sn	112	50	12	1795.11085	62	28.953401	12
971.574	Sn	114	50	14	1539.95516	64	24.061799	14
979.121	Sn	115	50	15	1437.79462	65	22.119917	15
988.684	Sn	116	50	16	1348.53014	66	20.432275	16
995.627	Sn	117	50	17	1269.61325	67	18.949451	17
1004.955	Sn	118	50	18	1199.5974	68	17.641138	18
1011.438	Sn	119	50	19	1136.80191	69	16.47539	19
1020.546	Sn	120	50	20	1080.41721	70	15.434532	20
1035.53	Sn	122	50	22	982.878557	72	13.651091	22
1049.963	Sn	124	50	24	901.573385	74	12.183424	24

1026.325	Sb	121	51	19	1181.00715	70	16.871531	19
1042.097	Sb	123	51	21	1069.28133	72	14.85113	21
1017.282	Te	120	52	16	1454.45479	68	21.389041	16
1034.333	Te	122	52	18	1293.79598	70	18.4828	18
1041.263	Te	123	52	19	1226.06619	71	17.268538	19
1050.686	Te	124	52	20	1165.23403	72	16.183806	20
1057.256	Te	125	52	21	1110.05956	73	15.206295	21
1066.369	Te	126	52	22	1060.01653	74	14.324548	22
1081.439	Te	128	52	24	972.309736	76	12.793549	24
1095.941	Te	130	52	26	898.074448	78	11.513775	26
1072.577	I	127	53	21	1151.6155	74	15.562372	21
1046.257	Xe	124	54	16	1564.44554	70	22.349222	16
1063.909	Xe	126	54	18	1391.59892	72	19.327763	18
1080.743	Xe	128	54	20	1253.28073	74	16.936226	20
1087.651	Xe	129	54	21	1193.92965	75	15.919062	21
1096.907	Xe	130	54	22	1140.08084	76	15.001064	22
1103.512	Xe	131	54	23	1090.79929	77	14.166224	23
1112.448	Xe	132	54	24	1045.72165	78	13.406688	24
1127.435	Xe	134	54	26	965.857945	80	12.073224	26
1141.878	Xe	136	54	28	897.383913	82	10.943706	28
1118.528	Cs	133	55	23	1130.13437	78	14.488902	23
B (Exp)	**Nucleus**	**A**	**Z**	**A-2Z**	**V_0**	**N**	**V_0/N**	**N-Z**
1092.722	Ba	130	56	18	1492.95236	74	20.175032	18
1110.038	Ba	132	56	20	1344.52292	76	17.691091	20
1126.696	Ba	134	56	22	1223.05075	78	15.680138	22
1133.668	Ba	135	56	23	1170.17776	79	14.812377	23
1142.775	Ba	136	56	24	1121.79981	80	14.022498	24

1149.681	Ba	137	56	25	1077.20406	81	13.298816	25
1158.293	Ba	138	56	26	1036.10436	82	12.635419	26
1155.774	La	138	57	24	1160.7591	81	14.330359	24
1164.551	La	139	57	25	1114.67982	82	13.593656	25
1138.792	Ce	136	58	20	1438.97126	78	18.44835	20
1156.035	Ce	138	58	22	1308.93947	80	16.361743	22
1172.692	Ce	140	58	24	1200.55522	82	14.640917	24
1185.29	Ce	142	58	26	1108.68936	84	13.198683	26
1177.919	Pr	141	59	23	1294.47414	82	15.78627	23
1185.142	Nd	142	60	22	1397.75656	82	17.045812	22
1191.266	Nd	143	60	23	1337.25079	83	16.111455	23
1199.083	Nd	144	60	24	1281.85772	84	15.260211	24
1204.838	Nd	145	60	25	1230.81361	85	14.48016	25
1212.403	Nd	146	60	26	1183.76559	86	13.764716	26
1225.028	Nd	148	60	28	1099.66179	88	12.496157	28
1237.448	Nd	150	60	30	1026.76501	90	11.4085	30
1195.737	Sm	144	62	20	1637.53785	82	19.969974	20
1217.251	Sm	147	62	23	1424.88135	85	16.76331	23
1225.392	Sm	148	62	24	1365.8505	86	15.881983	24
1231.263	Sm	149	62	25	1311.45132	87	15.074153	25
1239.25	Sm	150	62	26	1261.31808	88	14.33316	26
1253.104	Sm	152	62	28	1171.71871	90	13.019097	28
1266.94	Sm	154	62	30	1094.06533	92	11.892014	30
1244.141	Eu	151	63	25	1352.72644	88	15.371891	25
1258.998	Eu	153	63	27	1253.07474	90	13.923053	27
1251.485	Gd	152	64	24	1452.52769	88	16.505996	24
1266.627	Gd	154	64	26	1341.37717	90	14.904191	26

1273.062	Gd	155	64	27	1291.93487	91	14.197086	27
1281.598	Gd	156	64	28	1246.0992	92	13.544556	28
1287.958	Gd	157	64	29	1203.34957	93	12.939243	29
1295.896	Gd	158	64	30	1163.50252	94	12.377686	30
1309.29	Gd	160	64	32	1091.20217	96	11.366689	32
1302.027	Tb	159	65	29	1240.08706	94	13.192416	29
1278.021	Dy	156	66	24	1541.91646	90	17.132405	24
1294.046	Dy	158	66	26	1423.92385	92	15.477433	26
1309.455	Dy	160	66	28	1322.76532	94	14.071972	28
B (Exp)	**Nucleus**	**A**	**Z**	**A-2Z**	**V_0**	**N**	**V_0/N**	**N-Z**
1315.909	Dy	161	66	29	1277.37528	95	13.446056	29
1324.106	Dy	162	66	30	1235.06933	96	12.865306	30
1330.4	Dy	163	66	31	1195.431	97	12.324028	31
1338	Dy	164	66	32	1158.313	98	11.819518	32
1344.3	Ho	165	67	31	1230.834	98	12.559534	31
1320.7	Er	162	68	26	1508.927	94	16.052412	26
1336.4	Er	164	68	28	1401.709	96	14.601132	28
1351.6	Er	166	68	30	1308.766	98	13.354751	30
1358	Er	167	68	31	1266.755	99	12.795505	31
1365.8	Er	168	68	32	1227.412	100	12.274118	32
1379	Er	170	68	34	1155.601	102	11.329422	34
1371.4	Tm	169	69	31	1303.184	100	13.031837	31
1362.8	Yb	168	70	28	1482.937	98	15.13201	28
1378.1	Yb	170	70	30	1384.586	100	13.845857	30
1384.7	Yb	171	70	31	1340.135	101	13.268664	31
1392.8	Yb	172	70	32	1298.506	102	12.730455	32
1399.1	Yb	173	70	33	1259.351	103	12.226706	33

1406.6	Yb	174	70	34	1222.531	104	11.755101	34
1419.3	Yb	176	70	36	1154.965	106	10.895892	36
1412.1	Lu	175	71	33	1294.539	104	12.447495	33
1418.4	Lu	176	71	34	1256.65	105	11.968093	34
1403.9	Hf	174	72	30	1462.534	102	14.338572	30
1418.8	Hf	176	72	32	1371.591	104	13.188373	32
1425.2	Hf	177	72	33	1330.221	105	12.66877	33
1432.8	Hf	178	72	34	1291.321	106	12.182274	34
1438.9	Hf	179	72	35	1254.6	107	11.725237	35
1446.3	Hf	180	72	36	1219.956	108	11.295885	36
1444.7	Ta	180	73	34	1326.392	107	12.396187	34
1452.2	Ta	181	73	35	1288.712	108	11.932514	35
1444.6	W	180	74	32	1446.686	106	13.647985	32
1459.3	W	182	74	34	1362.021	108	12.611305	34
1465.5	W	183	74	35	1323.283	109	12.14021	35
1472.9	W	184	74	36	1286.731	110	11.697554	36
1485.9	W	186	74	38	1219.349	112	10.887044	38
1478.3	Re	185	75	35	1358.302	110	12.348203	35
1491.9	Re	187	75	37	1285.246	112	11.475414	37
1469.9	Os	184	76	32	1523.787	108	14.109136	32
1484.8	Os	186	76	34	1434.59	110	13.041727	34
1491.1	Os	187	76	35	1393.781	111	12.556589	35
1499.1	Os	188	76	36	1355.287	112	12.100779	36
1505	Os	189	76	37	1318.818	113	11.670954	37
B (Exp)	**Nucleus**	**A**	**Z**	**A-2Z**	**V_0**	**N**	**V_0/N**	**N-Z**
1512.8	Os	190	76	38	1284.317	114	11.26594	38
1526.1	Os	192	76	40	1220.434	116	10.520985	40

1518.1	Ir	191	77	37	1352.824	114	11.866881	37
1532.1	Ir	193	77	39	1283.807	116	11.067302	39
1509.9	Pt	190	78	34	1509.046	112	13.473629	34
1524.96	Pt	192	78	36	1425.63	114	12.50553	36
1539.58	Pt	194	78	38	1350.982	116	11.646394	38
1545.68	Pt	195	78	39	1316.498	117	11.252117	39
1553.6	Pt	196	78	40	1283.783	118	10.87952	40
1567.01	Pt	198	78	42	1222.97	120	10.191416	42
1559.39	Au	197	79	39	1349.605	118	11.437327	39
1551.22	Hg	196	80	36	1497.778	116	12.911882	36
1566.49	Hg	198	80	38	1419.35	118	12.028388	38
1573.15	Hg	199	80	39	1383.127	119	11.622916	39
1581.18	Hg	200	80	40	1348.75	120	11.23958	40
1587.41	Hg	201	80	41	1316.005	121	10.876076	41
1595.17	Hg	202	80	42	1284.856	122	10.531609	42
1608.65	Hg	204	80	44	1226.76	124	9.8932282	44
1600.87	Tl	203	81	41	1348.279	122	11.051465	41
1615.07	Tl	205	81	43	1285.898	124	10.370148	43
1607.51	Pb	204	82	40	1415.299	122	11.600813	40
1622.33	Pb	206	82	42	1348.257	124	10.873039	42
1629.06	Pb	207	82	43	1317.059	125	10.53647	43
1636.43	Pb	208	82	44	1287.293	126	10.216611	44
1640.23	Bi	209	83	43	1348.529	126	10.70261	43
1766.69	Th	232	90	52	1306.915	142	9.2036283	52
1778.57	U	234	92	50	1418.6	142	9.9901422	50
1783.86	U	235	92	51	1390.888	143	9.726492	51
1801.69	U	238	92	54	1313.947	146	8.9996363	54

4. DISCUSSIONS

4. 1. Heavy nuclei

Table 2. Isotopes of Copernicium with Z = 112.

Nuclei	A	N	Z	A-2Z	V_O (MeV)
Cn	276	164	112	52	1904.444
Cn	277	165	112	53	1868.328
Cn	278	166	112	54	1833.576
Cn	279	167	112	55	1800.063
Cn	280	168	112	56	1767.766
Cn	281	169	112	57	1736.578

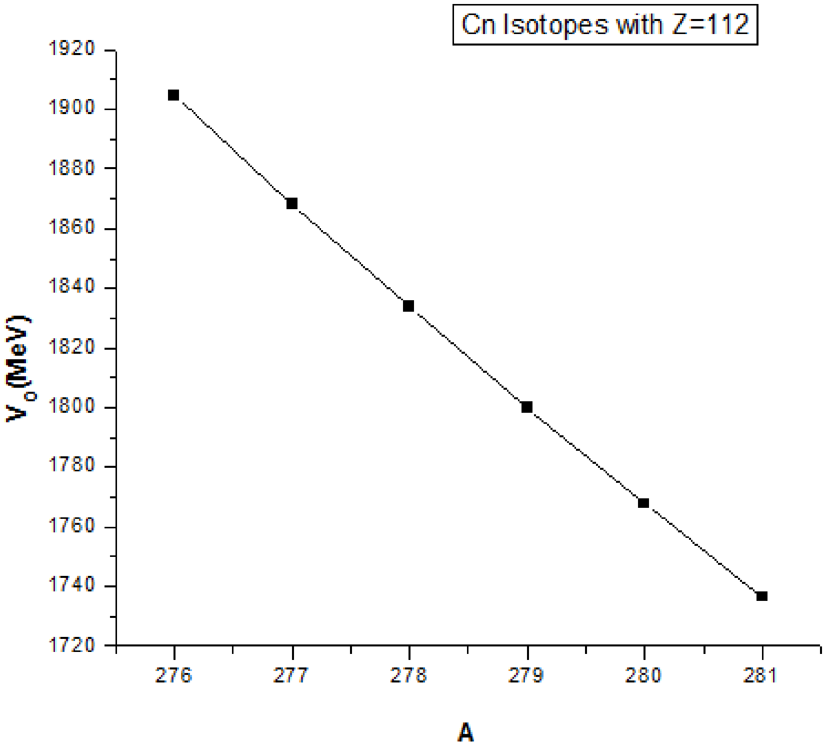

Figure 1. Graph showing variation of V_0 (MeV) against A for Copernicium Isotopes with Proton number Z = 112 using Table 2.

As the neutron number changes by unity in the surface region, V_0 changes by about 33 MeV and it is a linear variation, or more or less a constant. Here V_0 decreases as N increases keeping Z constant. Binding between the core and surface neutrons decreases as N increases. The ratio of N:Z increases with increase in A.

Table 3. Isotones with Neutron number N = 173.

Nuclei	A	N	Z	A-2Z	Vo (MeV)
Cn	285	173	112	61	1622.082
Ed	286	173	113	60	1679.2
Fl	287	173	114	59	1738.538
Ef	288	173	115	58	1800.175
Lv	289	173	116	57	1864.271

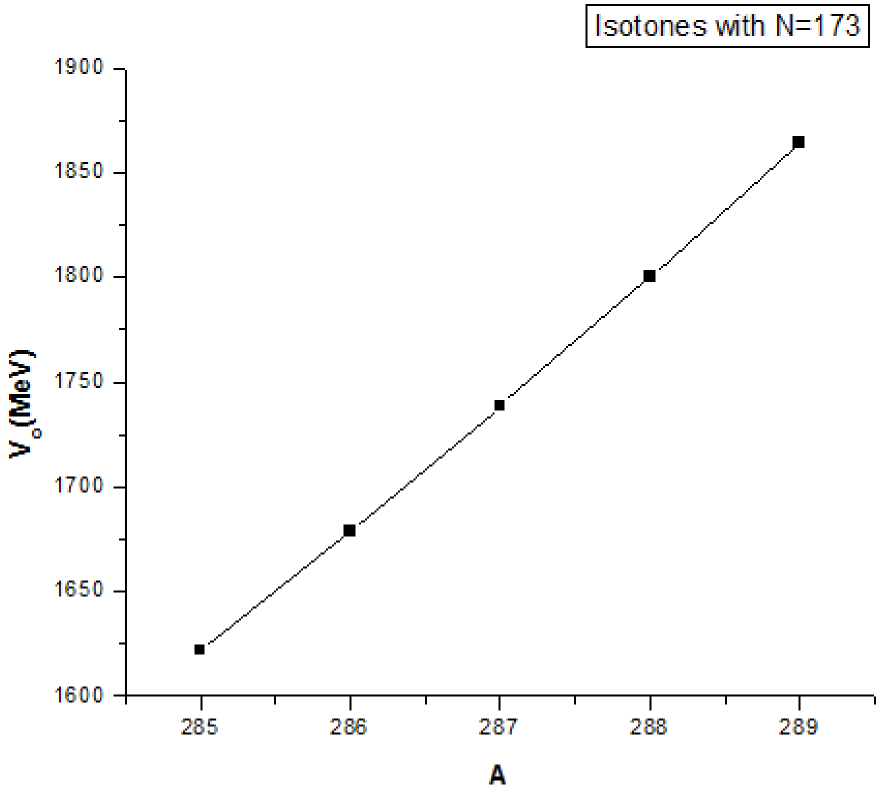

Figure 2. Graph showing variation of V_0 (MeV) against A for Isotones with Neutron number N = 173 using Table 3.

Here V_0 increases as Z increases keeping N constant, interaction (Binding) energy variation V_0 hovers around 59MeV as a proton is added. Binding between the core and surface neutrons increases as Z increases. Table 2 shows that increase in neutron reduces V_0, and table 3 shows that increase in proton increases V_0. This can lead to stability of transuranic elements in the "Island of stability".

4. 2. Light nuclei

Table 4. Isotopes of Sulphur with Proton number Z = 16.

Nuclei	A	N	Z	A-2Z	V_0(MeV)
S	33	17	16	1	2450.4791
S	34	18	16	2	1230.0281
S	36	20	16	4	619.6927

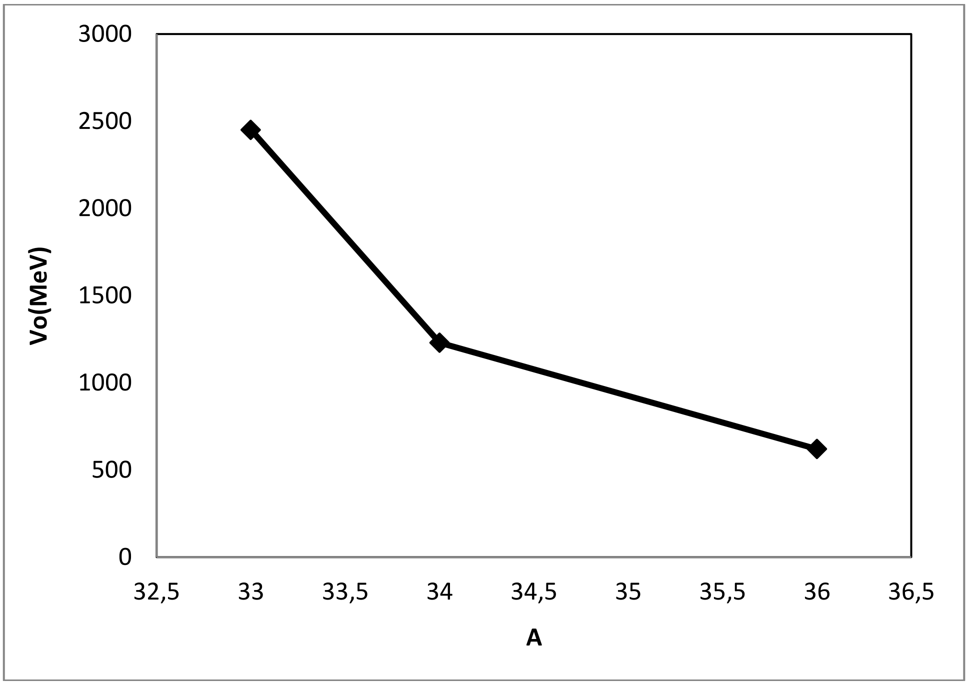

Figure 3. Graph showing variation of V_0 (MeV) against A for isotopes of Sulphur using Table 4.

The interaction potential V_0 decreases with an increase in A for Sulfur isotopes. For Z=N region the interaction potential energy E_H goes to zero because there are no neutrons in the surface region. The ratio N/Z increases with an increase in A. For low mass nuclei, the value of V_0 changes abnormally (drastically) or by a very large amount when compared to the change in V_0 for transuranic elements or super-heavy-nuclei (SHN) or (SHE), but the trend in the variation is the same. V_0 decreases as N increases keeping Z constant (for isotopes).

Table 5. Isotones with Neutron number N = 20.

Nuclei	A	N	Z	A-2Z	V_0(MeV)
S	36	20	16	4	619.69
Cl	37	20	17	3	920.44
Ar	38	20	18	2	1530.99
K	39	20	19	1	1691.41

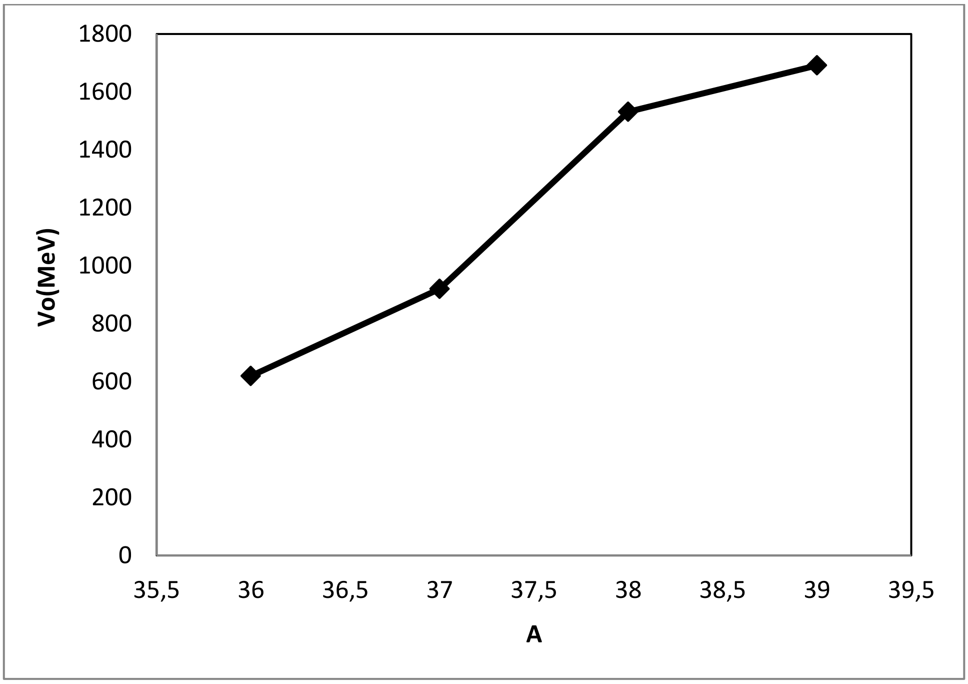

Figure 4. Graph showing variation of V_0 (MeV) against A for Isotopes with Neutron number N = 20 using Table 5.

For isotones with N = 20, the value of V_o increases with an increase in A. The ratio N/Z decreases with an increase in Z. In these light nuclei also, V_0 increases as Z increases, but the increase in the value of V_0 is very large.

5. CONCLUSIONS

For the isotopes, interaction potential energy V_0 generally decreases with an increase in A. This is evident for both light and heavy nuclei. When the ratio N:Z increases as N increases, V_o decreases. Binding between the core and the surface decreases as N increases. The variation in V_o is different for different isotopes as the neutron number changes in the surface region of the nuclei by unity. For heavy nuclei (A>110) the maximum change in V_o is of the order of 100MeV when the neutron number in the surface region changes by unity. For instance, the value of V_o for ${}^{234}_{92}\mathrm{U}$ is 1418.6MeV, but for ${}^{235}_{92}\mathrm{U}$, V_o =1390.888MeV. And for light nuclei (A<40), the change in V_o for a unit change in neutron number is considerably higher. For instance, the value of V_o for ${}^{17}_{8}\mathrm{O}$ is 694.0498MeV, but for ${}^{18}_{8}\mathrm{O}$, V_o = 351.0469MeV. Similarly the value V_o for ${}^{21}_{10}\mathrm{Ne}$ is 1033.9310MeV and that for ${}^{22}_{10}\mathrm{Ne}$ it is 522.1475MeV. From Table 4, it is clear that as the neutron number N increases by unity, V_o decreases. Here the decrease is quite large compared to the decrease in heavy nuclei as contained in Table 2. Since the addition of a neutron carries attractive energy into the surface region, the value of V_o decreases to account for the appropriate binding energy of the nucleus.

For isotones the interaction potential energy generally increases with increase in Z. This is evident for both light and heavy nuclei. Thus as N:Z decrease with increase in Z, V_o increases. From Table 3 and Table 5, it is evident that as Z increases by unity, V_o increases. This means that the interaction energy between the core of the nucleus and surface region increases. This is due to the fact that as Z increases, repulsion interaction energy increases leading to increase in the value of V_o.

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