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A MODIFIED SLM SCHEME WITH LOW COMPLEXITY FOR PAPR REDUCTION OF OFDM SYSTEMS

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Abstract

In this paper, we propose a new peak-to-average power ratio (PAPR) reduction scheme of orthogonal frequency division multiplexing (OFDM) system, called *a modified selected mapping (SLM) scheme*, which considerably reduces the computational complexity with keeping the similar PAPR reduction performance compared with the conventional SLM scheme. The proposed scheme is analytically and numerically evaluated for the OFDM system specified in the IEEE 802.16 standard. For the OFDM system with 2048 subcarriers, the proposed scheme with 4 binary phase sequences can reduce the complex multiplications by 63.5% with the similar PAPR reduction compared with the SLM scheme with 16 binary phase sequences.

I. Introduction

Recently, orthogonal frequency division multiplexing (OFDM) has been regarded and used as one of the core technologies for the communication systems. Especially, OFDM has been adopted for various wireless communication systems such as wireless local area networks (WLANs), wireless metropolitan area networks (WMANs), digital audio broadcasting (DAB), and digital video broadcasting (DVB). OFDM is an attractive technique for achieving high data rate in the wireless communication systems and it is robust to the frequency selective fading channel. However, an OFDM signal can have very high peak-to-average power ratio (PAPR) at the transmitter, which causes the signal distortion such as the in-band distortion and the out-of-band radiation due to the nonlinearity of high power amplifier (HPA), and induces the degradation of bit error rate (BER). Thus, the PAPR reduction is one of the most important research interests for the OFDM systems.

Several schemes have been proposed for reducing the PAPR of OFDM signals, which can be classified in terms of two criteria [1]–[8]. First, the PAPR schemes can be classified according to whether they are multiplicative or additive, where PAPR reduction is carried out in the OFDM modulator. Selected mapping (SLM) and partial transmit sequence (PTS) belong to the multiplicative class because the phase sequences are multiplied to the input symbol sequences or OFDM signal sequences [1]–[3], [5]. On the other hand, tone reservation (TR) and clipping are additive schemes due to the addition of reference signals [4], [6], [7]. Second, the PAPR schemes can be classified according to whether they are deterministic or probabilistic. Deterministic schemes, such as clipping [8], limit the PAPR of the OFDM signals below a given threshold level. Probabilistic schemes, however, statistically improve the characteristics of the PAPR distribution of the OFDM signals without signal distortion [9]. SLM and PTS belong to the probabilistic class because several candidate signals are generated and the one with the minimum PAPR is selected for transmission.

It is well known that SLM is more advantageous than PTS if the amount of side information is limited, but the computational complexity of SLM is larger than that of PTS. In order to improve the PAPR reduction performance of SLM scheme, we have to increase the number of phase sequences. The computational complexity of SLM scheme linearly increases as the number of phase sequences increases, which corresponds to the number of IFFTs required to generate the alternative OFDM signals. Even if the SLM scheme is simple and distortionless, sometimes its computational complexity is burdensome. In this paper, we propose *a modified SLM scheme* which has lower computational complexity with keeping the similar PAPR reduction performance compared with the conventional SLM scheme.

This paper is organized as follows. In Section II, the conventional SLM scheme is explained. In Section III, a new PAPR reduction scheme is proposed and its computational complexity is compared with that of the conventional SLM scheme. In Section IV, simulation results are given to compare the PAPR reduction performances of the proposed scheme and the conventional SLM scheme. Finally, the concluding remarks are

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given in Section V.

II. A Conventional SLM Scheme

Let $\mathbf{A} = [A_0 A_1 \cdots A_{N-1}]$ denote an input symbol sequence in the frequency domain, where A_k represents the complex data of the k-th subcarrier and N the number of subcarriers of OFDM signal. Let T be a period of input symbol and NT a period of OFDM signal. The OFDM signal is generated by summing all the N modulated subcarriers each of which is separated by $1/T$. Then the complex OFDM signal in the time domain is expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi \frac{n}{NT} t}, \ \ 0 \le t < NT \qquad (1)$$

where t is a continuous time index [2]. The OFDM signal sampled at the Nyquist rate can be written as

$$a_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi \frac{n}{N} k}, \ \ k = 0,\ 1, \cdots, N-1,$$

which can also be expressed in the vector form, called an OFDM signal sequence, as $\mathbf{a} = [a_0 a_1 \cdots a_{N-1}]$. In fact, $\mathbf{a}$ corresponds to the inverse fast Fourier transform (IFFT) of $\mathbf{A}$.

The PAPR of OFDM signal sequence $\mathbf{a}$ is defined as the ratio between the maximum instantaneous power and its average power, which can be written as

$$\mathrm{PAPR}(\mathbf{a}) \doteq \frac{\max\limits_{0 \le k \le N-1} |a_k|^2}{\mathrm{E}\left[|a_k|^2\right]}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator.

The conventional SLM scheme is one of the well known PAPR reduction schemes for the OFDM system, which does not cause the in-band distortion and the out-of-band radiation [1]. In this scheme, U alternative input symbol sequences $\mathbf{A}_u, 1 \le u \le U$, are generated by the component-wise vector multiplication of the input symbol sequence $\mathbf{A}$ and U phase sequences $\mathbf{P}_u = [P_{u,0}\ P_{u,1} \cdots P_{u,N-1}], 1 \le u \le U$, that is,

$$\begin{aligned} \mathbf{A}_u &= [A_{u,0}, A_{u,1}, \cdots, A_{u,N-1}] \\ &= \mathbf{A} \otimes \mathbf{P}_u \\ &= [A_0 P_{u,0}, A_1 P_{u,1}, \cdots, A_{N-1} P_{u,N-1})], \ \ 1 \le u \le U \end{aligned} \qquad (2)$$

where $\otimes$ denotes the component-wise multiplication of two vectors. The phase sequence $\mathbf{P}_u$ is generated by using the unit-magnitude complex number, that is, $P_{u,n} = e^{j\phi_{u,n}}$, where $\phi_{u,n} \in [0,\ 2\pi)$. In general, binary or quaternary elements are used for $P_{u,n}$, that is, $\{\pm 1\}$ or $\{\pm 1, \pm j\}$, where $j = \sqrt{-1}$.

IFFT should be performed for each of U input symbol sequences $\{\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_U\}$ to generate U alternative OFDM signal sequences as

$$\mathbf{a}_u = \mathrm{IFFT}(\mathbf{A}_u) = \mathrm{IFFT}(\mathbf{A} \otimes \mathbf{P}_u), \ 1 \le u \le U, \qquad (3)$$

which bear the same input symbol sequence. Then, the OFDM signal sequence $\mathbf{a}_{\tilde{u}}$ with the minimum PAPR among U alternative OFDM signal sequences $\mathbf{a}_u$, $1 \le u \le U$, is selected and transmitted.

Clearly, as U increases, the amount of PAPR reduction for the OFDM signal sequence becomes larger. But, for large U, the computational complexity becomes too high mainly due to the U IFFTs.

III. A Modified SLM Scheme with Low Complexity

In this section, a modified SLM scheme is proposed and shown to have lower computational complexity than the conventional SLM scheme when both schemes have the similar PAPR reduction performance.

A. *A Modified SLM Scheme*

In order to achieve large PAPR reduction in the conventional SLM scheme, we have to generate a sufficiently large number of alternative OFDM signal sequences, that causes high computational complexity because IFFT should be performed to generate each alternative OFDM signal sequence. Therefore, it is desirable if we can reduce the number of IFFTs without compromising the PAPR reduction performance.

Let $\mathbf{a}_i$ and $\mathbf{a}_k$ be the alternative OFDM signal sequences, which are generated by the conventional SLM scheme as in (3). Using the linear property of Fourier transform, the linear combination of these two sequences can be given as

$$\begin{aligned} \mathbf{a}_{i,k} &= c_i \mathbf{a}_i + c_k \mathbf{a}_k \\ &= c_i \mathrm{IFFT}(\mathbf{A} \otimes \mathbf{P}_i) + c_k \mathrm{IFFT}(\mathbf{A} \otimes \mathbf{P}_k) \\ &= \mathrm{IFFT}(\mathbf{A} \otimes (c_i \mathbf{P}_i + c_k \mathbf{P}_k)) \end{aligned} \qquad (4)$$

where c_i and c_k are some complex numbers. If each element of the sequence $c_i \mathbf{P}_i + c_k \mathbf{P}_k$ in (4) has unit magnitude, $c_i \mathbf{P}_i + c_k \mathbf{P}_k$ can also be a phase sequence for the SLM scheme and $\mathbf{a}_{i,k}$ can be considered as the corresponding OFDM signal sequence. Therefore, if we have OFDM signal sequences $\mathbf{a}_i$ and $\mathbf{a}_k$, another alternative OFDM signal sequence $\mathbf{a}_{i,k}$ can be obtained without doing IFFT. Note that the phase sequence $c_i \mathbf{P}_i + c_k \mathbf{P}_k$ is not statistically independent to $\mathbf{P}_i$ and $\mathbf{P}_k$. Now, we will investigate how to make each element of $c_i \mathbf{P}_i + c_k \mathbf{P}_k$ to have unit magnitude under the condition that each element of the phase sequences $\mathbf{P}_i$ and $\mathbf{P}_k$ has unit magnitude. Clearly,

Figure 1: Block diagram of the modified SLM scheme of OFDM systems.

the elements of the sequence $c_i\mathbf{P}_i + c_k\mathbf{P}_k$ have the unit magnitude if the following conditions are satisfied:

i) Each element of $\mathbf{P}_i$ and $\mathbf{P}_k$ takes the value in $\{+1, -1\}$;

ii) $c_i = \pm\frac{1}{\sqrt{2}}$ and $c_k = \pm\frac{1}{\sqrt{2}}j$.

Since two alternative OFDM signal sequences generated from the phase sequences $\pm(c_i\mathbf{P}_i + c_k\mathbf{P}_k)$ have the same PAPR, we only consider the case of $c_i = \frac{1}{\sqrt{2}}$ and $c_k = \pm\frac{1}{\sqrt{2}}j$. Since $|c_i|^2 = |c_k|^2 = \frac{1}{2}$, the average power of $\mathbf{a}_{i,k}$ is equal to one half of the sum of average powers of $\mathbf{a}_i$ and $\mathbf{a}_k$. From U binary phase sequences, we can obtain $2 \cdot \binom{U}{2}$ additional phase sequences with $\binom{U}{2} = U(U-1)/2$ and, thus, total U^2 phase sequences such as

$$\{\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_U, \frac{1}{\sqrt{2}}(\mathbf{P}_1 \pm j\mathbf{P}_2), \frac{1}{\sqrt{2}}(\mathbf{P}_1 \pm j\mathbf{P}_3), \cdots, \frac{1}{\sqrt{2}}(\mathbf{P}_{U-1} \pm j\mathbf{P}_U)\}.$$

A modified SLM scheme can be explained as follows. By combining each pair among U alternative OFDM signal sequences $\mathbf{a}_u$ obtained by using U binary phase sequences as the above, a set $\mathbf{S}$ of U^2 alternative OFDM signal sequences is generated as

$$\begin{aligned}\mathbf{S} &= \{\mathbf{a}_u \,|\, 1 \leq u \leq U^2\} \\ &= \{\mathbf{a}_u \,|\, 1 \leq u \leq U\} \cup \left\{ \frac{1}{\sqrt{2}}(\mathbf{a}_i + j\mathbf{a}_k), \frac{1}{\sqrt{2}}(\mathbf{a}_i - j\mathbf{a}_k) \,|\, 1 \leq i < k \leq U \right\}\end{aligned} \quad (5)$$

where only U IFFTs and additional summations of $U^2 - U$ pairs of OFDM signal sequences are needed. However, the computational complexity for the summations of OFDM signal sequences is negligible compared with that of IFFT. The computational complexity of the proposed scheme is considered in the next subsection.

Next, we have to select and transmit the alternative OFDM signal sequence $\mathbf{a}_{\tilde{u}}$ with the minimum PAPR among the alternative OFDM signal sequences in $\mathbf{S}$, together with the index $\tilde{u}$. When M-ary symbols are used, $\lceil \log_M U^2 \rceil$ symbols should be allocated to transmit the side information corresponding to $\tilde{u}$, which is denoted by $\mathbf{A}_u^{\text{index}}$.

A portion of subcarriers of the OFDM signal is assigned for transmission of the index sequence $\mathbf{A}_u^{\text{index}}$, that is, some part of the input symbol sequence $\mathbf{A}$ should be assigned for $\mathbf{A}_u^{\text{index}}$. Thus, the input symbol sequence $\mathbf{A}$ and the alternative OFDM signal sequence $\mathbf{a}_u$ can be split into the data parts $\mathbf{A}^{\text{data}}$ and $\mathbf{a}_u^{\text{data}}$ and the index parts $\mathbf{A}_u^{\text{index}}$ and $\mathbf{a}_u^{\text{index}}$, respectively. The alternative OFDM signal sequence with the index signal $\mathbf{a}_u^{\text{index}} = \text{IFFT}(\mathbf{A}_u^{\text{index}})$ can be written as, for $1 \leq u \leq U$,

$$\begin{aligned}\mathbf{a}_u &= \text{IFFT}(\mathbf{A}^{\text{data}} \otimes \mathbf{P}_u) + \text{IFFT}(\mathbf{A}_u^{\text{index}}) \\ &= \mathbf{a}_u^{\text{data}} + \mathbf{a}_u^{\text{index}}\end{aligned} \quad (6)$$

and for $U + 1 \leq u \leq U^2$,

$$\begin{aligned}\mathbf{a}_u &= \frac{1}{\sqrt{2}}\text{IFFT}(\mathbf{A}^{\text{data}} \otimes \mathbf{P}_i) + j\frac{b}{\sqrt{2}}\text{IFFT}(\mathbf{A}^{\text{data}} \otimes \mathbf{P}_k) + \text{IFFT}(\mathbf{A}_u^{\text{index}}) \\ &= \frac{1}{\sqrt{2}}\left(\mathbf{a}_i^{\text{data}} + jb\mathbf{a}_k^{\text{data}}\right) + \mathbf{a}_u^{\text{index}}\end{aligned} \quad (7)$$

where $b \in \{+1, -1\}$ and $1 \leq i < k \leq U$.

We may compare the proposed scheme with U binary phase sequences with the conventional SLM scheme with U^2 binary phase sequences. These two schemes show the similar PAPR reduction performance for small U. However, as U increases, the PAPR reduction performance of the proposed scheme becomes worse than that of the conventional SLM scheme with U^2 binary phase sequences, because U^2 phase sequences of the proposed scheme are statistically correlated.

For example, when $U = 3$, let the set of three binary phase sequences be given as $\{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3\}$. In the proposed scheme, the set of nine phase sequences is $\{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \frac{1}{\sqrt{2}}(\mathbf{P}_1 \pm j\mathbf{P}_2), \frac{1}{\sqrt{2}}(\mathbf{P}_1 \pm j\mathbf{P}_3), \frac{1}{\sqrt{2}}(\mathbf{P}_2 \pm j\mathbf{P}_3)\}$. The PAPR reduction performance of the proposed scheme with $U = 3$ is similar to that of the conventional SLM scheme with $U = 9$ as shown in Fig. 2.

B. *Computational Complexity*

In the proposed scheme, the reduction of the computational complexity comes from the generation of the additional $U^2 - U$ alternative OFDM signal sequences from U IFFTed alternative OFDM signal sequences without performing IFFT, whereas the PAPR reduction performance of the proposed scheme with U binary phase sequences is similar to that of the conventional SLM scheme with U^2 binary phase sequences. The complex multiplications and additions are required in IFFT and the complex additions are required for combining the alternative OFDM signal sequences. In this paper, we consider the computational complexity of the PAPR reduction schemes in terms of complex multiplication and complex addition.

When the number of subcarriers is $N = 2^n$ and U the total number of IFFTs, the numbers of complex multiplications and complex additions required for U IFFTs in the conventional SLM scheme are $(N/2)nU$ and NnU, respectively. We also need additional NU^2 complex multiplications for peak power search for U^2 alternative OFDM signal sequences. Thus, the total number of complex multiplications is $(N/2)nU + NU^2$. In the proposed scheme, additional $N(U^2 - U)$ complex additions are needed to generate the additional $U^2 - U$ alternative OFDM signal sequences as in (5). In order to generate U alternative input symbol sequences $\mathbf{A}_u$, we need $N(U-1)$ most significant bit (MSB) inversions, which can be negligible.

The computational complexity reduction ratio (CCRR) [3] of the proposed scheme over the conventional SLM scheme is defined as

$$\text{CCRR}=\left(1 - \frac{\text{complexity of the proposed scheme}}{\text{complexity of the conventional SLM}}\right)\times 100\ (\%). \quad (8)$$

The CCRR of the proposed scheme over the conventional SLM scheme with typical values of U and N is given in Table 1, which tells us that the proposed scheme becomes computationally more efficient as the N or U increases. Note that the complex multiplication is more complicated than the other operations. When $N = 2048$, the proposed scheme with $U = 4$ can reduce the complex multiplications by 63.5% with keeping the similar PAPR reduction performance compared with the conventional SLM scheme with $U = 16$.

(a)

(b)

Figure 2: PAPR reduction performance of the conventional SLM scheme with $U = 4, 8, 9, 16, 25$ and the proposed scheme with $U = 3, 4, 5$ for 16-QAM; (a) $N = 256$, (b) $N = 2048$.

IV. Numerical Analysis

The numerical analysis for the proposed and the conventional SLM schemes is performed for the OFDM system and OFDMA downlink system specified in the *IEEE* 802.16 standard, which use 256 and 2048 subcarriers, respectively, and QPSK, 16-QAM, and 64-QAM modulations. The OFDM system use the 200 subcarriers for data transmission and the remaining 56 subcarriers are set to zero to shape the power spectral density of the transmit signal. The OFDMA system uses 1702 subcarriers for the data transmission and 346 subcarriers as guard carriers. In our numerical analysis, we ignore the guard subcarriers and the number of subcarriers for the input

Table 1: COMPUTATIONAL COMPLEXITY OF THE CONVENTIONAL SLM AND THE MODIFIED SLM SCHEMES WHEN $N = 256,\ 512,\ 1024$, AND 2048

	Conventional SLM, $U = 9$	Modified SLM, $U = 3$	CCRR	Conventional SLM, $U = 16$	Modified SLM, $U = 4$	CCRR
# IFFTs	9	3		16	4	
	$N = 256$					
# Complex multiplications	11,520	5,376	53.3%	20,480	8,192	60.0%
# Complex additions	18,432	6,400	65.3%	32,768	11,264	65.6%
	$N = 512$					
# Complex multiplications	25,344	11,520	54.5%	45,056	17,408	61.4%
# Complex additions	41,472	16,896	59.3%	73,728	24,576	66.7%
	$N = 1024$					
# Complex multiplications	55,296	24,576	55.6%	98,304	36,864	62.5%
# Complex additions	92,160	36,864	60.0%	163,840	53,248	67.5%
	$N = 2048$					
# Complex multiplications	119,808	52,224	56.4%	212,992	77,824	63.5%
# Complex additions	202,752	77,872	61.6%	360,448	81,920	77.3%

symbol sequence $\mathbf{A}$ is 256 or 2048.

The complementary cumulative distribution functions (CCDFs) of PAPR are numerically obtained for the conventional SLM scheme with $U = 4,\ 8,\ 9,\ 16,\ 25$ and the proposed scheme with $U = 3,\ 4,\ 5$, respectively, where the rows of cyclic Hadamard matrix are used as the binary phase sequences. The simulation results are shown in Fig. 2 for 1,000,000 input symbol sequences, where Figs 2 (a) and (b) show the probabilities that the PAPR of OFDM signal sequences exceeds the given PAPR_0 for $N = 256$ and 2048, respectively.

In Fig. 2 (a) with $N = 256$, the proposed scheme with $U = 3$ has almost the same performance compared with the conventional SLM scheme with $U = 8$. The performance of the proposed scheme with $U = 4$ is worse than that of the conventional SLM scheme with $U = 16$ by 0.2 dB. Similar simulation results for 2048 carriers are given in Fig. 2 (b), that is, the proposed scheme with $U = 4$ reduces the complex multiplications by 63.5% and shows the similar PAPR reduction performance compared with the conventional SLM scheme with $U = 16$.

V. Conclusions

We have proposed a modified SLM scheme for the PAPR reduction of OFDM system, which considerably reduces the computational complexity while it maintains the similar PAPR reduction performance compared with the comparable conventional SLM scheme. The performance of the proposed scheme is numerically confirmed for the OFDM system proposed in the IEEE 802.16 standard. Since the computational complexity reduction ratio increases as the numbers of subcarriers and binary phase sequences increase, the proposed scheme becomes more efficient for the high data-rate OFDM systems.

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