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A monopoly pricing model for diffusion maximization based on heterogeneous nodes and negative network externalities (Case study: A novel product)

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A B S T R A C T

Social networks can provide sellers across the world with invaluable information about the structure of possible influences among different members of a network, whether positive or negative, and can be used to maximize diffusion in the network. Here, a novel mathematical monopoly product pricing model is introduced for maximization of market share in noncompetitive environment. In the proposed model, a customer's decision to buy a product is not only based on the price, quality and need time for the product but also on the positive and negative influences of his/her neighbors. Therefore, customers are considered heterogeneous and a referral bonus is granted to every customer whose neighbors also buy the product. Here, the degree of influence is directly related to the intensity of the customers' relationships. Finally, using the proposed model for a real case study, the optimal policy for product sales that is the ratio of product sale price in comparison with its cost and also the optimal amounts of referral bonus per customer is achieved.

1. Introduction

Numerous studies concerning monopoly pricing in social networks have been published as of year 2003 which reviewed models for estimation of the market acceptance for new products. This market acceptance is measured in terms of probable adopters and also for the product types where the buying cycles are shorter, in terms of initial adopters and repeat buyers. Obviously, knowing the repeat purchase rate for repeatedly purchased items is necessary (Rosenzweig et al., 2016).

The social influence mechanism may describe the S-shaped model of product acceptance by customers over time. The generalized model proposed by Mansfield (1961), Fourt and Woodlock models (1960) and the Bass diffusion model (Bass, 1969) are the most well-known models for the past 50 years. Diffusion models typically define two sources of influence for buying a product, the speed and model

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of diffusion. These two sources of influence are expressed in two different forms where customer decision 1) is independent of the neighbors' decisions and 2) has certain dependency on the neighbors' decisions. The external and internal influence coefficients are used to compute the two sources of impact, respectively (Huang et al., 2016; Midgley & Dowling, 1978). Positive influence manifests itself in the form of persuasion for buying the product in interactive social media and verbal communication environments, thus, presenting itself as an important source of information. According to research conducted by the TRAP company, an unsatisfied customer shares its negative experience with on average of 22 individuals whereas a satisfied customer shares its positive experience with only 8. (Tatikonda, 2013). Therefore, the influence of positive information as opposed to negative information is an important factor (Mayshak et al., 2016; Pal, 2015; Relling et al., 2016; Sadatrasool et al., 2016; Sandhya & Garg, 2016; Sadi-Nezhad, 2017). Surprisingly, so far the diffusion models have been often independent cascade or linear threshold models in which a decision for buying a product is either based on a simply assumed constant probability influence from neighbors or a threshold value of acceptance.

A number of researches concerning monopoly pricing in social networks focus on competitive environment and fall outside the scope of the present work in which non-competitive environment is targeted. Findings show approaches for studying the problem of diffusion in social networks with monopoly pricing under non-competitive environment may be divided into the following areas:

- Algorithmic model development
- Analytical model development
- Optimization model development

It should be mentioned in all of aforesaid areas, researchers tried to maximize revenue/profit or diffusin.

1.1. Algorithmic model development

Revenue/profit maximization

Marketing strategies aiming to maximize the revenue based on positive externalities, which may require enforcement of different pricing schemes for various customers were studied by Hartline et al. (2008). Therefore, they used a primary influence strategy where products are offered to a population for free or at very low prices in order to exploit the remaining customers; named IE[1]-strategy in short. In their algorithmic model, the optimum marketing strategy is described in polynomial time when customer relations are perfectly symmetric. Arthur et al. (2009) considered a generalization of the linear threshold and independent cascade models and employed viral marketing strategies in social networks to develop IE O(1)-approximation algorithm based on the construction of a max-leaf spanning tree that seeks to maximize the revenue via the sales of a single product. A strategy was also proposed for which the revenue is guaranteed and is a fixed coefficient of the optimum strategy for a wide spectrum of models. Akhlaghpour et al. (2010) investigated positive externalities in network pricing models and proposed two iterative pricing models for optimization of expected revenue in the presence of myopic consumers. Ajorlou et al. (2015) modified the IE-strategy slightly to make the price valid only at the exploitation step and employed this strategy to demonstrate a non-negative and non-uniform expected revenue for the concave graph model. They proposed the 1/2-approximation model for achieving an optimal design for the two stage fixed price marketing strategy.

In addition, Haghpanah et al. (2013) studied positive externalities in the design of auctions. They proved that the optimal auction problem is APX-hard and proposed a type of fixed approximation algorithm for a special case of externalities in step functions such that total revenue is maximized. Alon et al. (2013) proposed a novel model for nodes pricing in a social network based primarily on inequity

[1] Influence and exploit

aversion such that differences in the cost allocated to two neighbors are constrained via some endogenous parameters so that price differences related to closed agents are not significant. Here, the challenge is to find a feasible price vector such that node constraints are satisfied and at the same time, total revenue is maximized. Fotakis and Siminelakis (2014) examined the problem of revenue maximization by focusing on IE-strategy. They obtained algorithms with polynomial time that estimate the approximate revenue from the best influence and exploit strategy by an approximate coefficient of 0.9.

Moreover, Cao et al. (2016) investigated pricing of indivisible products with the aim of maximizing vendor revenue via development of an iterative pricing model in which customers are offered a sequence of prices over time. They assumed perfect knowledge of the network to propose a 2-approximation algorithm for weighted networks with intrinsic non-uniform values. Amanatidis et al. (2016) used the model proposed by Alon et al. (2013) to maximize revenue in social networks by proposing an approximate algorithm for a natural class of instances that is referred to single-value revenue functions class. Their results demonstrated improvements especially in cases where the numbers of distinct prices are small. Ehsani et al. (2012) studied optimum pricing strategies for profit maximization in the presence of both internal and external network influences. In their work, network influences were modelled using a weighted graph and given the production costs, they assumed that customers enter the market online and only then should vendors offer their prices. So, the FPTAS algorithm was proposed that estimates the general optimum price with high probability. A polynomial time algorithm was also proposed to comply with the assumption that the monopolist should offer a customized price to every customer.

Diffusion maximization

Optimum pricing strategies for budget distribution under the assumption of limited budget were studied by Zhang et al. (2015). They approached the problem by targeting primary customers and devising incentive policies in order to maximize the number of individuals who would make a decision to purchase a product. The greedy discrete pricing strategy was suggested for this purpose.

1.2. Analytical model development

For the first time in the context of social networks, Shi (2003) studied pricing methodologies by which every individual is entitled for paying a different price based on the intensity of his/her social relations with other closely related individuals. Accordingly, he proposed discriminative pricing strategies, which could be designed based on the type of social relations in order to maximize the profit owned by the provider of communications services. Hartline et al. (2008), Mirrokni et al. (2012), Fotakis and Siminelakis (2014) and also Arthur et al. (2009) analyzed the effect of IE-strategy on revenue gain in addition to proposing algorithms for maximizing the revenue. Hande et al. (2010) investigated pricing for internet connection services in the exclusive domain of the ISP (sales of bandwidth to customers) and considered utility of bandwidth consumers which is prone to change in time under the influence of typically unexpected demand. Given the legal requirements for non-discriminant pricing of products, they determined the extent of ISP revenue reduction when economic approaches are employed and used non-linear pricing as a clearly non-discriminative approach to analyze the resulting relative improvement in revenue reduction.

In addition to presenting a revenue optimization algorithm, Akhlaghpour et al. (2010) compared the complexity of the pricing problem in states of Basic(k) and Rapid(K). Nejad and Estelami (2012) analyzed the effect of the time length within which competitors may enter the market and also the dependence of profitability on customer price sensitivity. In their work, the set of preliminary pricing strategies were identified using the agent-based simulation framework such that profitability is maximized under various market conditions. In addition, Nejad (2013) studied how the primary price of a product may determine its maximum market share and analyzed the effect of customer price sensitivity on widespread diffusion in the network of customers using relevance between revenue net present value and price. In spite of the emphasis given to market price elasticity in other studies, extensive simulation experiments of such works have indicated the importance of inhomogeneity as a factor influencing customer price sensitivity. Campbell (2012) inspected the effect of network centrality measures, mean communication between nodes and sparseness distribution of communication on optimum pricing. According to his findings, vendors should target the group of customers with the highest communication level (degree) only when the customers' expected price is higher than the sale price, and vice versa.

Therewith, Ajorlou et al. (2015) studied the optimum dynamic pricing for a monopoly vendor in social networks and demonstrated variability in the validity of results in the presence of strategic customers, various internal and external network influences, and customer indifference to price. They also examined how the amount and duration of discounts would affect final profitability. Crapis et al. (2015) investigated social learning mechanism and its respective impact on vendor pricing decisions where customers followed intuitive decision making for purchasing a product. Given the study circumstances, he showed that customers would eventually learn about the product quality and established a technique by which the learning trajectory could be estimated using the mean-field approximation method in high demand situations. This method emphasizes on the dynamic nature of the learning process, its price dependency and market heterogeneity based on different customer preferences.

The optimum preliminary pricing problem for financial services in an attempt to address the research gap concerning the incentives of various dimensions in the problem of sales in social networks was proposed by Nejad and Kabadayi (2016). They conducted agent-based simulation experiments to show that the amount of discount and the time span within which a new customer receives the discount are important factors determining the current net present value of the firm profit. Zubcsek et al. (2016) examined the similarity and difference between the expected price of a specific product for individuals with high social interaction and its consequent effect on the optimum price and promotion levels of the product. For this purpose, an analytical model was presented for informative promotion and pricing in social networks and the terms "homophily" and "heterophily" were introduced[2] (Gatignon et al., 2016). Shin (2015) presented a dynamic pricing and product diffusion model in networks in which the product is a subscription social network good. Here, customers undertake the best decision for subscription on the one hand, and the other hand, providers select a set of subscription rights such that given the discount offered the total profit for every period is maximized. In this work, the effect of variations in discount coefficient and social network density on subscription rights, subscription rate and loss in stable conditions were studied.

1.3. Optimization Model development

In all researches concerning monopoly pricing in social networks under non-competitiveness just Hande et al. (2010) presented an analytical model for ISP[3] and in addition, studied the optimum price combination based on bandwidth use and uniform rate in order to maximize revenue via a mathematical model given the restrictions on the available capacity for demand satisfaction.

In this work, we address a problem that is significantly different from its previous counterparts by constructing in terms of the underlying assumptions. Here, a novel mathematical monopoly pricing model is proposed for a given novel product in a social network in which customers are heterogeneous and the environment is non-competitive. In this model, verbal communication or word of mouth (WOM) is not the sole social influence process although it forms the more typical form of

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[2] In the social network literature, homophily (heterophily) and assortativity (disassortativity) are applied to indicate positive (negative) correlation between the features of neighbours.

[3] Internet service provider

communication. It is believed that simple visual observation of a product being used by new customers coupled with simple decision making methods based on cost, quality and need can be sufficient to convince the target group of individuals about the benefits of the product. In other words, a new decision is made when a tradeoff between sale cost and configuration (or quality), need time and sufficient influence from negative or positive opinions of friends and media exposure is accepted. The total seller profitability also depends on the referral bonus granted to the influential customers in the network that succeed to convince their friends to purchase a specific product. The referral bonus for each potential customer is proportional to the number of individuals who also purchased the product through their influences in the network. Determination of the referral bonus is the objective of the proposed pricing model and aims to maximize diffusion in the network. Therefore, this paper contributes to the following items simultaneously:

a) Diffusion maximization in social networks is described via a mathematical model of monopoly pricing in a noncompetitive environment. Previous research suffices to creating algorithms or analytical models only.
b) A definite referral bonus is granted to the influential individuals through which a number of other individuals also purchase a given product. The referral bonus is proportional to the number of influenced individuals and suggests that given the positive or negative influence of the various individuals in the network, they may be entitled to paying different prices for the same product. Previous studies generally consider single-tiered or two-tiered pricing.
c) Network nodes are considered heterogeneous; in other words, the personal and social features and the acceptance thresholds are different for every node. As a result, the multi-criteria decision making approach is used in diffusion modelling. The criteria of interest are the influence of friends, neighbours, media, need time for a novel product given its configuration and quality and also, the financial ability of the customer to pay for the product. The majority of previous studies treated all nodes homogeneously and the diffusion model was generally independent cascade or linear threshold model.
d) Each node experiences positive or negative impact from its local networks. In other words, every customer is directly influenced by its neighbours in the social network structure. In this work, the negative externalities scenario is considered. Compared to the positive externality, negative externality creates numerous interesting situations, however, it is not considered in the studies as much.
e) The extent of influence and impact depends on the strength of the link between two nodes. Therefore, the contact edges between the nodes are considered directed and weighted. Previous studies mostly examined non-directed and non-weighted graph networks.

The distinctions between the present and previous works is clearly evident in Table 1. This section outlined “influence and diffusion in social networks” and reviews the literature of “monopoly pricing in social networks in non-competitive environment” to highlight the difference between the present work and previous research efforts. The remainder of this paper is organised as follows. In section 2, we propose the novel mathematical model. In section 3, the applicabality of the proposed pricing model are demonstrated using a real world dataset and the results are analysed. Finally, Section 4 concludes the study and addresses potential future research directions.

Table 1
Features of studies concerning monopoly pricing models of social networks in noncompetitive environment

Paper	objective					Pricing model			Diffusion approach				parameter							
	Optimization model		Algorithmic model										edge				node		externality	
	Diffusion maximization	Revenue/Profit Maximization	Diffusion maximization	Revenue/Profit Maximization	Analytical model	One rate	Two rates	Several rates	Linear Threshold model	Independent Cascade model	wave model	Multi Criteria Decision making	undirected	directed	weightless	weighted	homogenous	heterogeneous	positive	negative
(Campbell, 2012)					√	√					√			√	√			√	√	√
(Akhlaghpour et al., 2010)				√							√		√			√	√		√	√
(Nejad & Estelami, 2012)					√	√				√			√		√			√	√	
(Mirrokni et al., 2012)				√		√				√			√		√		√		√	
(Ajorlou et al., 2015)					√					√			√		√		√		√	
(Alon et al., 2013)			√			√				√			√	√	√		√		√	
(Hande et al., 2010)					√	√					√			√	√		√		√	√
(Shi, 2003)					√				√				√		√		√		√	
(Arthur et al., 2009)				√	√			√	√				√	√		√	√		√	
(Zhang et al., 2015)					√	√				√			√		√			√	√	
(Hartline et al., 2008)				√				√	√				√		√			√	√	√
(Ehsani et al., 2012)					√	√			√					√	√			√	√	
(Relling et al., 2016)				√				√	√				√		√		√		√	
(Haghpanah et al., 2013)				√		√		√	√				√	√		√	√		√	
(Amanatidis et al., 2016)					√	√				√			√		√		√			√
(Pal, 2015)				√	√		√				√		√			√	√		√	
(Mayshak et al., 2016)				√	√					√			√		√		√		√	
(Cao et al., 2016)		√			√	√			√				√		√		√		√	
(Tatikonda, 2013)				√	√		√		√	√			√		√		√		√	
(Huang et al., 2016)				√	√		√			√			√	√		√	√		√	
(Fotakis & Siminelakis, 2014)					√			√		√			√		√			√	√	
This paper	√							√				√	√	√		√		√	√	√

2. The Proposed Model

2.1. Definitions and Preliminaries

A social network is defined as a directed graph $G = (V, E)$, with $|V| = n$ and with $|E| = l$. Suppose $V = \{\dots, i, \dots\}$ represents the set of all consumers (agents/nodes), i.e. Each potential customer with its distinct approach is considered a node in the influence social network and the number of nodes in V is n. Also, assume $E = \{\dots, (i,j), \dots\}$ represents the set of all relationships between the consumers (edges), i.e. each directed link from customer i to customer j is considered as (i,j). In other words, agent i plays the role of a parent for agent j and agent j is a child of agent i. In the proposed model, a relationship between nodes i and j is only considered when the value of the binary parameter R_{ij} is equal to 1. As mentioned hereinbefore effectiveness is directly related to the intensity of the relationship linking two nodes in the social network. This is characterized in the proposed model by assigning a weight in the range [0,1] to each directed link to show the strength of the X edge influence.

Given the differences in the personalities of the potential customers, these individuals may selectively spread their opinions of a purchased product to other individuals with whom they interact and are represented as connected nodes in the social network, whether positive or negative, or may even decide not to voice any opinion at all. Therefore, given ρ_i be the probability that customer comments on the product and given K_i be the comments, which may be positive, negative or neutral, the model is created[4] (Gafni & Golan, 2016). On the other hand, the purchasing probability K_i of ith customer is also incorporated into the modelling process given the differences in the product prices and qualities. The binary parameter Z_i incorporates the need for purchasing a product and ε_i defines the acceptance threshold of the potential customer i. Then, if the potential customer i becomes an actual one, the binary variable Y_i will be equal to 1. The cost price for the product is given by C and the proposed sale price by S. It is noteworthy that the product is not a luxurious product.

2.2. Mathematical model

The monopoly pricing model for diffusion maximization in a noncompetitive environment in a social network with heterogeneous nodes can be described as follows: Assumed the influence social network $G = (V, E)$, For every node, the overall influence from all parent nodes is computed: $\sum_{i\in V} W_{ji}$. This reflects the fact that every customer may be connected to and thus influenced by more than one individual. Our objective is to maximize the number of individuals who eventually decide to purchase a product via the price referral bonus mechanism, i.e., $\max \sum_{i\in V} Y_i$. This marketing strategy functions by returning to the customer a bonus that is a fraction of the sale price of the product, α_i, that is proportional to the number of individuals connected to the customer who also purchased the product. It is obvious that customers would seek to maximize their bonus and therefore, would attempt to convince greater numbers of friends and neighbors in the network to purchase the product. Maximizing the overall bonuses indeed maximizes the number of actual customers in the network, thus, fulfilling our objective. However, the following constraints should be met:

$$\max \sum_{i\in V} \alpha_i$$

subject to:

$$\varepsilon_i \geq 1 - P_i, \qquad \forall i \in V \tag{1}$$

$$Y_i \geq Z_i, \qquad \forall i \in V \tag{2}$$

$$Y_i\Big(\sum_{j\in V} W_{ji} K_j \rho_j - \varepsilon_i\Big) \geq 0, \qquad \forall i \in V \tag{3}$$

$$Y_i\Big(\sum_{j\in V} W_{ji} K_j \rho_j - \varepsilon_i\Big) \geq \sum_{j\in V} W_{ji} K_j \rho_j - \varepsilon_i, \qquad \forall i \in V \tag{4}$$

$$S\Big(\sum_{i\in V} Y_i\Big) \geq C\Big(\sum_{i\in V} Y_i\Big) + S\Big(\sum_{i\in V} \alpha_i Y_i\Big) \tag{5}$$

[4] Passive nodes often do not comment on the product in contrast to active nodes that mostly share their personal experiences with other network members and actively post (write) consumer reviews in the social network.

$$\alpha_i Y_i \leq \left(\sum_{j \in \frac{V}{i}} Y_j R_{ji} \Big/ \frac{1}{n} \sum_{i \in V} \sum_{j \in \frac{V}{i}} R_{ji} \right), \qquad \forall i^+ \in V \tag{6}$$

$$0 \leq \alpha_i, \varepsilon_i \leq 1, \qquad \forall i \in V \tag{7}$$

$$Y_i \in \{0,1\}, \qquad \forall i \in V \tag{8}$$

Constraint 1 ensures that the minimum acceptance threshold is always greater than the probability by which the product is not purchased. In other words, the acceptance threshold is lower for an individual when the probability of purchasing a product by the individual is higher based on its sale price and quality. Constraint 2 accounts for the importance of need time for purchasing a product as a factor that has greater contribution to the final decision as compared to sale price, quality and opinions of others. So, a potential customer will only become an actual one if and only if he needs the product immediately. Constraint 3 suggests that the product is not purchased by the agent i when Y_i is equal to 0. This is when the overall influences from the parent nodes are smaller than the minimum acceptance threshold value. This constraint was added to account for the possibility of positive, negative or even neutral comments from the parent nodes with respect to spreading their purchasing experience with others. Constraint 4 ensures that provided the overall influence from the parent nodes is greater than the minimum acceptance threshold, Y_i will be equal to 1 and the agent i will purchase the product. Therefore, if a person under influence of his friends decides to purchase the good, constraint 4 assigns 1 to Y_i and if he not influenced by them, constraint 3 assigns zero to Y_i. However, amount of 1 under the first condition and amount of 0 under the second condition are not guaranteed by constraint 3 and constraint 4 respectively.

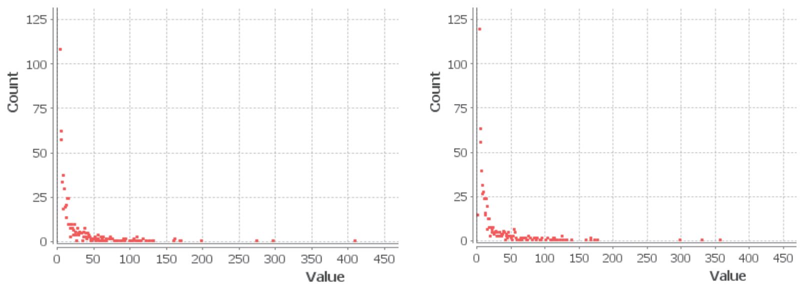
In addition, constraint 5 demonstrates the minimum financial loss threshold for the seller. Recall the referral bonus given to the customer by the seller in proportion to the positive activity of the node in the network with respect to attracting more friends for purchasing the product. Some nodes carry so much value that the bonus may be equal to the sale price of the product; therefore, the seller determines a minimum value such that its revenue always exceeds its costs. This constraint suggests that given the implementation of the referral bonus strategy, it is required to compute the coefficient by which the sale price of the product is greater than the cost price of the product in order to ensure continuous financial benefit of the seller. Constraint 6 determines the maximum fraction of the sale price of the product, α_i, that can be granted to the customer as referral bonus. This value is proportional to the number of friends who are convinced to purchase the product based on positive influence from the agent i divided by the total number of directed connections in the network. As indicated, this referral bonus is intended as appreciation of those who increase the final number of customers for a given product via positive influence in the network. So, no bonus is given to individuals with either negative or neutral influence.

3. A Case study

In order to evaluate the proposed model performance and validation, the empirical study is conducted.

3.1. Data collection and description

A real dataset is used in this section. To create the dataset, an electronic membership form was designed and randomly emailed to a number of individuals in order to create a network of potential customers. Each individual was asked to forward the e-membership form to their friends via email, text messages, social networks, etc. The individual was then entered into a lottery where chances of winning a prize would be higher for agents through which a greater number of individuals also entered the network and purchased the product. To become a member of the network, each individual had to answer a number of questions in the membership form and was then provided with a unique membership code. These individuals were asked to save the code and submit it with the membership form to their friends. In the membership form of the new individual, this code will be used to indicate the intensity of the relationship between the sender and receivers of the form.[5] The sender of the form is the individual who had joined the network earlier and whose promotion efforts for purchasing the product will convince the receiver to also purchase the product. In other words, the new individual will become a member of the network based on the influence of the sender. A directed edge can be drawn from the sender to the receiver. It should be noted that membership in the network does not imply purchasing the product. So, in the first step, only a network of real data is constructed as given in the Gephi 0.9.1 software and shown in Fig. 1. Also, the in-degree and out-degree distributions of the network are shown in Fig. 2.



(a) (b)

Fig. 1. a) in-degree distribution, b)out-degree distribution

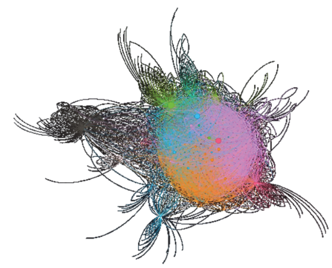


Fig. 2. Schematic of the generated network

Following the membership in the network, members were offered a price for the product and some purchased it. These individuals were notified of the referral bonus that would be granted to them proportional to the number of additional individuals who would also purchase the product based on their influence. The impact of every network member who distributes the form to other can be assessed using the unique sender code that is entered by other individuals in the network at purchase moment. Table 2 and Table 3 show the results of answering the membership form questions after a month starting from distribution of the forms and the questionnaire following product purchase, respectively. Studies show that a negative opinion has 10-12 times more destructive effect on the sales as compared to the constructive effect of positive opinions (Tatikonda, 2013). In other words, every negative comment can be compensated for by 10-12 positive comments. Therefore, our computations also suppose 11 times greater impact of the negative comments. When a price was offered for a medium quality product, 355 network members considered it reasonable, 381 considered it high and 319 considered it low. So, according to Table 2, customers would purchase the product with different probabilities.

Table 2
Probability of purchasing the product based on price and quality

	Price		
quality	low	Medium	high
low	27%	20%	4%
Medium	69%	56%	29%
high	94%	72%	52%

Table 3
Profile of the collected social network database

Number of nodes	1055
Number of directed edges (relations)	15087
Number of individuals who needed the product	103
Number of passive nodes (no comment)	472
Number of active nodes	583
Probability of satisfaction for the purchased product	91.1%
Probability of positive externalities	25.1%
Probability of negative externalities	28.8%

[5] Intensity of relations is the edge weight. It is a number in the range [1,10] that the new individual scores to indicate their relation with the sender of the form. This quantity is normalized for further computations

Table 4 and Table 5 exemplify the node and edge data, respectively for a selected sample of the dataset. Additionally, if recommendation by friends is the sole reason for purchasing the product, at least two closely related friends should have recommended the product for it to have been purchased by the individual. This means that the maximum normalized weight for each edge is 0.5.

Table 3
Sample of node data

Node code	Need (Z_i=0/1)	Comment(K_i= -1/0/+1)	Experience Share (0/1)	probability of purchasing the product regarding price=P and quality=Q, low/medium/high=L/M/H									Suggested Price (low / medium / high)
				P&Q=L	P&Q=M	P&Q=H	P=L &Q=M	P=L &Q=H	P=M &Q=L	P=M &Q=H	P=H &Q=L	P=H &Q=M	
1	0	1	0	0%	50%	50%	80%	100%	20%	80%	0%	30%	low
2	0	0	1	20%	60%	60%	60%	100%	20%	100%	10%	60%	medium
3	0	0	0	10%	45%	80%	55%	80%	30%	60%	5%	45%	low
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.

Table 4
Sample of edge data

Edge code	Child node code	Parent node code	relation Intensity(1/2/…/10)	Normalized weight (Wij=0,…,0.5)
1	2	1	2	0.1
2	2	3	5	0.25
3	2	4	7	0.35
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

3.2. Solving problem and Results analysis

The proposed model can be solved in multiple iterations. The model is solved starting from the nodes that purchased the product in the first place, that is, members of the network who needed the product. Then, primary buyers may influence other network members via edge connections in either a positive or negative way so that these potential secondary buyers decide whether or not they would also buy the product, respectively. How much influence is exerted on the child nodes is a direct function of the personality of the parent node, being either active or passive. In the succeeding iterations, the number of members who are convinced to buy the product increases. This is given in Table 6 and Fig. 3 for different iterations. The solution process for the model terminates when no new network member purchases the product. As evident, no improvement is observed in the number of buyers after iteration 6. Given the number of nodes and edges in the network, the average degree of the network is equal to 14.3. That is, each member of the network will get a full refund on the purchased item if they convince at least 15 friends to also purchase the product. Then, a fraction of the sale price, α_i, is refunded to the customer in the form of credit as appreciation for promoting the product depending on how well they performed. Table 7 shows a number of examples for such fraction. Note that neighbors of a passive agent may still purchase the product as a result of promotion by a second common friend. Therefore, α_i is calculated only for agents with positive externalities in the network and not the others.

Table 5
Total numbers of Y_i in each iteration

Iteration Number	1	2	3	4	5	6	7	8
Total numbers of Y_i	105	130	149	163	196	172	172	172

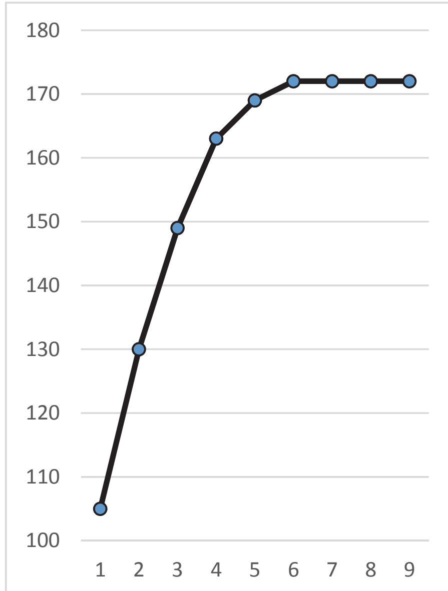


Fig. 3. Total numbers of Y_i in each iteration

Table 6
A sample of resulting X for each node

Parent node code	2	4	16	21	23	27	28	30	39	40	…	Total
Total numbers of buyer child node code	14	36	2	1	4	2	5	1	3	8	…	172
Referral bonus coefficient (α)	0.97	1	0.14	0.07	0.28	0.14	0.35	0.07	0.21	0.56	…	153.3

As mentioned already in the earlier sections, the seller uses constraint 5 to determine its minimum financial loss threshold. In order to yield a feasible referral bonus strategy, it is required that the product is assigned a sale price that is multiple times greater than the cost price of the product solely for the purposes of maximizing diffusion in the network and not maximizing the profit. Therefore, constraint 5 can simply re-written as:

$$({}^{S-C}/_{S})(\sum_{i\in V} Y_i) \geq \sum_{i\in V} \alpha_i Y_i \tag{9}$$

As evident, calculation of $\sum_{i\in V} Y_i$, i.e. the total number of codes for the child nodes that purchased the product and also calculation of the maximum value of $\sum_{i\in V} \alpha_i$, i.e. the total referral bonus coefficient can easily yield the minimum ratio of $({}^{S-C}/_{S})$ that is equal to $(1 - {}^{C}/_{S})$. Given the resulting $\sum_{i\in V} Y_i$ and $\sum_{i\in V} \alpha_i$ values, we have:

$$\left(1 - {}^{C}/_{S}\right) * 172 \geq 153.3 \quad \rightarrow \quad \left(1 - {}^{C}/_{S}\right) \geq 0.89 \quad \rightarrow \quad 0.11 \geq {}^{C}/_{S} \quad \rightarrow \quad S \geq 9.09 * C$$

So, it is concluded based on real network data that the sale price should be approximately 9 times greater than the cost price of the product. It is the case when the referral bonus strategy is implemented for diffusion maximization and not profit maximization, so that no financial loss is incurred.

4. Conclusion and future directions

A multitude of studies have been conducted to investigate diffusion in social networks via monopoly pricing in non-competitive environment. In this work, a novel mathematical model was proposed for monopoly pricing of products in the network in a noncompetitive environment with the aim of maximizing market share and diffusion in the network. In the proposed model, the social network is considered directed and customers exhibit heterogeneous decision making behaviors for purchasing a specific product due to the presence of different decision making criteria such as product sale price, quality and need time to consumption, in addition to positive and negative neighbor externalities. Moreover, a node may be passive and provide no comment to its neighbors on its satisfaction or dissatisfaction of the product or even active. Impact/influence is a function of the intensity of relations between the various customers. Therefore, mathematical modelling is carried out based on positive and negative externalities, passiveness and different relation intensities (i.e. edge weights). Moreover, a fraction of the sale price is refunded to the customer as credit, that is a sign of appreciation for their efforts in promoting the product when they also convince their neighbors to purchase the product. Applicability of the proposed pricing model was revealed using a case study of real network data. Results show that implementation of the referral bonus strategy with the aim of maximizing diffusion and not profitability requires the sale price by the seller to be approximately 9 times greater than the cost price of the product, so that no financial loss is incurred by the seller. These results were obtained given the constraints of the problem. Different results may yield if the maximum value of constraint 6 is varied.

Future work may constitute mathematical modelling of monopoly pricing in weighted social networks in a competitive environment between the seller and the buyer. Here, multiple sellers would compete for selling multiple products to multiple heterogeneous customers. Alternatively, devising a new diffusion model for maximization of diffusion or seller profitability in the network such that the independent cascade and linear threshold models are improved is also favorable.

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