A MORE ACCURATE HALF–DISCRETE HILBERT–TYPE INEQUALITY WITH A GENERAL NON–HOMOGENEOUS KERNEL AND OPERATOR EXPRESSIONS

QILIANG HUANG, AIZHEN WANG AND BICHENG YANG

Abstract. In this paper, by the use of the methods of weight functions and technique of real analysis, a more accurate half-discrete Hilbert-type inequality with a general non-homogeneous kernel and a best possible constant factor is given. The equivalent forms and some reverses are obtained. We also consider the operator expressions with the norm and some particular examples.

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