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A MOTION GENERATION METHOD OF INDUSTRIAL ROBOTS IN CARTESIAN SPACE

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ABSTRACT

Cartesian positions, speeds and accelerations are planned to describe the desired motion of manipulators. We examine the cases of point to point and via points motions, assuming that the reference values of the manipulator are represented by C^2 polynomial trajectories. Usual kinematical constraints are not always sufficient, then we focus on voltage and current DC actuator constraints.

I - INTRODUCTION

This paper is primarily concerned with motion generation when the end effector of the robot must follow a prescribed trajectory as a straight line or a circular arc in cartesian space. Most precedent studies [2], [3], [6], [9] refer to joint motion generation while inverse kinematics are used to transform cartesian points to joint ones. We employ another approach where all parameters are reported in cartesian space.

The minimum time motion generation has been solved in a number of ways, following the usual approach, i.e. taking as feasible limits purely kinematical constraints on velocity and acceleration [2], [3]. Conventional motion generation uses a constant bound on the acceleration [6]. This bound must represent the global least upper bound of all operating accelerations so as to enable the manipulator to move under any operating conditions. It implies that the full capabilities of the manipulator cannot be utilized if the conventional approach is taken. The efficiency of the robotic system can be increased by considering the characteristics of the robot dynamics at the motion generation stage. [5] had applied the classical approach of point to point minimum time control to robot arms, where only a linear approximate model was used. [1] has presented a trajectory generation based on optimal control formulation. Assuming that joint torques are constrained and using the Hamiltonian formulation of the dynamic model, a minimum time cost criterion was considered. [3] have shown that most often the structure of the minimum time control requires that at least one of the actuators is always in saturation whereas the others adjust their torques so that constraints on motion are not violated while enabling the arm to reach its final desired destination.

Although the obtained results are very important theoretically, practically they are not applicable directly to an industrial robot. From an user view point, it would be preferable to have a somewhat suboptimal but simpler solution to implement. For this purpose, we have chosen, a priori, a polynomial trajectory and we find parameters of the trajectory, for a C^2 minimal time motion. Position and orientation are considered. The resulting motion for the end effector position is usually obvious, but the end effector orientation motion depends on the parameters adopted ([11] choose for instance the Euler angles).

In this paper, using the formal calculus software MAPLE, we will show that the simple expressions previously obtained [3], [7], can be numerically extended when we include different actuator constraints.

The remainder of this paper is divided into six sections. While the models and the proposed problem are formulated in the second and third section, the resolution method is stated in the fourth paragraph. Some simulation results are given in the fifth paragraph and some conclusions added in the last section.

II - MODELS

2-1 Manipulator model

The manipulator is assumed to be made of rigid links. Its dynamic model depends on q, $\dot{q}$ and $\ddot{q}$, respectively the joint position, velocity and acceleration :

$$\Gamma = A(q)\ddot{q} + \dot{q}^T B(q)\dot{q} + F(q)\dot{q} + G(q) \qquad (1)$$

The vector Γ is the joint input torque, G is the gravitational force vector, B is the n x n x n Coriolis and Centrifugal force matrix. F is the viscous friction and A is the n x n inertial. Coulomb frictions are included in the gravitational force G and we suppose that the friction derivatives with respect to time are equal to zero.

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2-2 Actuator model

In a permanent magnet DC motor, the magnetic field is developed by permanent magnets. For such a motor, the torque Γ is proportionnal to armature current I. For a non-redundant multi-degrees-of-freedom robot, there are usually as many actuators as the number of degrees-of-freedom. Then actuator dynamics for the whole robot can be characterized in a matrix form as :

$$\Gamma = K\,I \quad \text{and} \quad U = L\,\frac{dI}{dt} + R\,I + K\,\dot{q} \tag{2}$$

L, R and K are square regular diagonal matrices representing the inductance, resistance and torque constant of the robot actuators. U is the motor voltage vector.

2-3 Actuator constraints

DC motors are supplied by Pulse Width Modulation voltage amplifiers. Motors and amplifiers have limitied voltage and current. Thus usual current constraints must be coupled with constraints on voltage and current slew rate, for every joint $1 \le j \le n$:

$$\left|I_j\right| \le I_{max,j} \tag{3}$$

$$\left|U_j\right| \le U_{max,j} \tag{4}$$

$$\left|\frac{dI_j}{dt}\right| \le dI_{max,j} \tag{5}$$

As actuator demagnetization must be avoided, $I_{max,j}$ is the maximum armature pulse current, or amplifier current limit. $U_{max,j}$ is the actuator or amplifier voltage limit and $dI_{max,j}$ is the amplifier current slew rate limit.

If $I_{max,j}$ is greater than the maximum allowable permanent current $Ieff_{max,j}$ in continuous operation, the thermal limit must be taken into account to prevent overheating. Power losses are mainly resistance losses so as iron and friction losses will be neglected. We will only consider the case of a periodic motion with period t_f. This is common in robotic application. The power losses are periodic and filtered through a lowpass thermal model whose time constant is very large versus t_f. Then the motor temperature can be easily obtained with the average power which is proportionnal to the root mean square value of the current. This must be limited by the following relation :

$$\sqrt{\frac{1}{t_f}\int_0^{t_f} I_j^2(t)\,dt} \le Ieff_{max,j} \tag{6}$$

Finally, we also have to fulfill a limitation on joint speeds because of mechanical considerations :

$$\left|\dot{q}_j\right| \le qp_{max,j} \tag{7}$$

Most of the time, people use more restricting relations than (3)-(7) that do not optimize the motion. They prefer to define maximal accelerations and velocities, making approximations for the worst case in (1)-(7), [6].

III - PROBLEM FORMULATION

3.1 Point to Point Motion

The desired trajectory must be chosen smooth enough not to excite the high frequency unmodelled dynamics. It is the reason why we will choose polynomials allowing zero speed and acceleration motion at start and end points.

In order to satisfy the previous assumption we assume that the position and orientation of the robot is represented as a fifth degree polynomial interpolation of time between two points [9] :

$$X(t) = X_i + D\,r(t/t_f) \quad \text{with} \quad D = X_f - X_i \tag{8}$$

$$r(t/t_f) = 10(t/t_f)^3 - 15(t/t_f)^4 + 6(t/t_f)^5 \tag{9}$$

For instance, if $X = [x,y,z,\varphi,\theta,\omega]^T$ with the Euler angles, the position and orientation are given by [10], [11] :

$$P(t) = [x(t), y(t), z(t)]^T \tag{10}$$

$$A(t) = \mathrm{Rot}(Z,\varphi(t))\,\mathrm{Rot}(X,\theta(t))\,\mathrm{Rot}(Z,\omega(t)) \tag{11}$$

Using Inverse Geometrical Model (IGM), with $x = t/t_f$, gives :

$$q(t) = f_{IGM}(X(x)) = \breve{q}(x) \tag{12}$$

With kinematical models of first and second order, we obtain :

$$\dot{q}(t) = \frac{1}{t_f}\breve{q}_x(x) \qquad \ddot{q}(t) = \frac{1}{t_f^2}\breve{q}_{xx}(x) \tag{13}$$

with $\breve{q}_x(x) = J^{-1}(\breve{q}(x))\,\frac{dX}{dx}$ and

$$\breve{q}_{xx}(x) = J^{-1}(\breve{q}(x))\left[\frac{d^2X}{dx^2} - \left\{J_q(\breve{q}(x))\,\breve{q}_x(x)\right\}\breve{q}_x(x)\right]$$

We suppose that the jacobian matrix is regular. The study of singular points is beyond the scope of this paper. We chose cartesian trajectories with no singularity.

We can see that the joint velocities and accelerations can be written as separate function of x and t_f. Then we are able to rewrite the constraints (3)-(7) introducing (13) in the dynamic model of the robot (1) and the actuator model (2) [9] :

$$\pm qp_{max}\, t_f - \breve{q}(x) = 0 \qquad (14)$$

$$\left[\pm I_{max} - K_{em}^{-1}\widetilde{Q}(x)\right] t_f^2 - K_{em}^{-1}\widetilde{F}(x)\, t_f - K_{em}^{-1}\widetilde{A}(x) = 0 \qquad (15)$$

$$\pm dI_{max} t_f^3 - K_{em}^{-1}\hat{Q}(x)\, t_f^2 - K_{em}^{-1}\hat{F}(x)\, t_f - K_{em}^{-1}\hat{A}(x) = 0 \qquad (16)$$

$$\begin{aligned}\left[\pm U_{max} - RK_{em}^{-1}\widetilde{Q}(x)\right] t_f^3 - \hat{C}(x)\, t_f^2 \\ - \hat{B}(x)\, t_f - LK_{em}^{-1}\hat{A}(x) = 0\end{aligned} \qquad (17)$$

where all matrices are given by :

$$\widetilde{A}(x) = A(\breve{q}(x))\, \breve{q}_{xx}(x) + H(\breve{q}(x), \breve{q}_x(x))$$
$$\widetilde{Q}(x) = Q(\breve{q}(x))$$
$$\widetilde{F}(x) = F_V\, \breve{q}_x(x)$$
$$\hat{A}(x) = A(\breve{q}(x))\breve{q}_{xxx}(x) + \frac{dA}{dq}(\breve{q}(x))\breve{q}_{xx}(x)\breve{q}_x(x) + 2H(\breve{q}(x), \breve{q}_x(x))\breve{q}_{xx}(x) + \frac{dH}{dq}(\breve{q}(x), \breve{q}_x(x))[\breve{q}_x(x)]^3$$
$$\hat{F}(x) = F_V\, \breve{q}_{xx}(x)$$
$$\hat{Q}(x) = \frac{dQ}{dq}(\breve{q}(x))\breve{q}_x(x)$$
$$\hat{B}(x) = LK_{em}^{-1}\hat{F}(x) + RK_{em}^{-1}\widetilde{A}(x)$$
$$\hat{C}(x) = K_{em} D\, \breve{q}_x(x) + LK_{em}^{-1}\hat{Q}(x) + RK_{em}^{-1}\widetilde{F}(x)$$

Equations (14), (15), (16) and (17) are related to the constraints (7), (3), (5) and (4) respectively.
In the same way, we have for the constraint (6) :

$$\begin{aligned}Ieff_{max}^2 t_f^4 - t_f^4\int_0^1 \vec{E}(x)dx - t_f^3\int_0^1 \vec{D}(x)dx - \\ t_f^2\int_0^1 \vec{C}(x)dx - t_f\int_0^1 \vec{B}(x)dx - \int_0^1 \vec{A}(x)dx = 0\end{aligned} \qquad (18)$$

with :

$$\vec{A}(x) = \left[\left(K_{em,j}^{-1}\widetilde{A}_j(x)\right)^2\right]_{1\le j\le n}$$
$$\vec{B}(x) = \left[2K_{em,j}^{-1}\widetilde{F}_j(x)K_{em,j}^{-1}\widetilde{A}_j(x)\right]_{1\le j\le n}$$
$$\vec{C}(x) = \left[\left(K_{em,j}^{-1}\widetilde{F}_j(x)\right)^2 + 2K_{em,j}^{-1}\widetilde{Q}_j(x)K_{em,j}^{-1}\widetilde{A}_j(x)\right]_{1\le j\le n}$$
$$\vec{D}(x) = \left[2K_{em,j}^{-1}\widetilde{F}_j(x)K_{em,j}^{-1}\widetilde{Q}_j(x)\right]_{1\le j\le n}$$
$$\vec{E}(x) = \left[\left(K_{em,j}^{-1}\widetilde{Q}_j(x)\right)^2\right]_{1\le j\le n}$$

When a constraint ((3) to (6)) is saturated, the corresponding relation ((14) to (18)) is fulfilled.

3.2 Via Points Motion

Now the manipulator has to pass through m+1 via points, with some imposed straight line trajectories. Velocities and accelerations at these via points are different from zero. The motion is supposed to have a continuous acceleration. Start and end points are chosen to have joint speeds and accelerations equal to zero. For those reasons we represent the motion with a fifth degree polynomial between each crossing point rather than cubic splines that offer less possibilities :

$$X_{j,k}(t) = \sum_{i=0}^{5} a_{i,j,k}\left(\frac{t}{t_{f,k}}\right)^i \qquad \begin{cases}1\le k\le m\\ 1\le j\le n\end{cases} \qquad (19)$$

But, in order to have expressions which are not explicit functions of the final time $t_{f,k}$, we operate the following change of parameters :

$$\dot{X}_k = \hat{\dot{X}}_k / t_{f,k} \qquad \ddot{X}_k = \hat{\ddot{X}}_k / t_{f,k}^2 \qquad \Phi_k = \frac{t_{f,k+1}}{t_{f,k}} \qquad (20)$$

The time $t_{f,k}$ is a function of Φ_k and of time $t_{f,1}$:

$$t_{f,k+1} = \psi_k\, t_{f,1} \qquad \text{with} \qquad \psi_k = \prod_{i=1}^{k}\Phi_i \qquad (21)$$

Additionnal assumptions are added in order to obtain a straight line motion between two adjacent via points. Two adjacent straight lines are connected by classical fifth degree polynomial allowing position, speed and acceleration continuity. Then, for every polynomial k defined on $[0, \psi_{k-1}\, t_{f,1}]$:

$$\widetilde{X}_{k,j}(t) = \sum_{i=0}^{5} \frac{a_{i,k,j}}{\psi_{k-1}^i}\left(\frac{t}{t_{f,1}}\right)^i \qquad \begin{cases}1\le j\le n\\ 1\le k\le m\end{cases} \qquad (22)$$

with :

$$a_{0,k} = X_k \qquad a_{1,k} = \hat{\dot{X}}_k \qquad a_{2,k} = \frac{1}{2}\hat{\ddot{X}}_k \qquad (23)$$
$$a_{3,k} = 10(X_{k+1} - X_k) - \left(6\hat{\dot{X}}_k + 4\frac{\hat{\dot{X}}_{k+1}}{\Phi_k}\right) + \frac{1}{2}\left(\frac{\hat{\ddot{X}}_{k+1}}{\Phi_k^2} - 3\hat{\ddot{X}}_k\right)$$
$$a_{4,k} = -15(X_{k+1} - X_k) + \left(8\hat{\dot{X}}_k + 7\frac{\hat{\dot{X}}_{k+1}}{\Phi_k}\right) - \frac{1}{2}\left(2\frac{\hat{\ddot{X}}_{k+1}}{\Phi_k^2} - 3\hat{\ddot{X}}_k\right)$$
$$a_{5,k} = 6(X_{k+1} - X_k) - \left(3\hat{\dot{X}}_k - 3\frac{\hat{\dot{X}}_{k+1}}{\Phi_k}\right) + \frac{1}{2}\left(\frac{\hat{\ddot{X}}_{k+1}}{\Phi_k^2} - \hat{\ddot{X}}_k\right)$$

Joint speeds and accelerations can be expressed in the same manner of equation (13). Then relations (14) to (18) can be obtained with $t_{f,1}$.

The general problem of minimum time motion may be formulated as follows :

$$\text{Min}\left\{t_f\right\} \text{ subject to path equations and} \tag{24}$$

$$t_f \in \left\{ \begin{array}{l} t_f \in \mathbf{R}^+ / \left|\dot{q}_j\right| \le qp_{max,j}, \left|I_j\right| \le I_{max,j}, \left|U_j\right| \le U_{max,j}, \\ \left|\frac{dI_j}{dt}\right| \le dI_{max,j}, \sqrt{\frac{1}{t_f}\int_0^{t_f} I_j^2(t)dt} \le Ieff_{max,j} \quad 1 \le j \le n \end{array} \right\}$$

Next paragraph introduces the proposed resolution method.

IV - RESOLUTION METHOD

4-1 Proposed minimum time approach

The optimization theory gives the solution of problem (24). It is located in the vertex of the admissible set. The resolution will be organized as follows. First, this problem will be solved for each constraint (14)-(18), then the greatest value of all the proposed times will be taken as the predicted arrival time t_f. Let us assume the robot moves using the maximum motor capabilities.

We present hereafter, the method proposed for the calculus of the time t_f in the case of a point to point motion under technological constraints (3) to (7) or respectively (14) to (18).

First we consider joint speeds limitations (14). The minimal time is obtained when :

$$t_{fj/qp}(x) = \frac{\breve{q}(x)}{\pm qp_{max}} \tag{25}$$

Current bounds (3) lead to the second degree equation in t_f (15). For every joint (j=1,n), the solution $t_{fj/I}(x)$ of (15) can be approximated, neglecting the gravity and viscous friction, by :

$$t_{fj/I}(x) \approx \sqrt{\tilde{A}(x)/(\pm KI_{max})} \tag{26}$$

Then, we can consider that there always exists a real positive root for $t_{fj/I}(x)$.

The candidate times t_f for the constraints on current derivative (16) and voltage (17) are also obtained solving two third degree equations in t_f. A third degree polynomial equation can be solved analytically. It may have 1 or 3 real roots from which we choose the smallest positive value. Such equations can be solved using the formal calculus software MAPLE. The respective solutions are called $t_{fj/dI}(x)$ and $t_{fj/U}(x)$.

The constraint concerning the current root mean square value (18) leads to a fourth degree equation in t_f. The software MAPLE gives all the four solutions. The smallest real positive solution is called $t_{fj/Ieff}$.

4-2 Numerical implementation

All the differents matrices $\tilde{A}$, ..., $\tilde{F}$, $\hat{A}$, ..., $\hat{C}$, and $\vec{A}$, ..., $\vec{E}$ are obtained analytically in a first step for the given robot. Then, in point to point motions, the numerical implementation consists in choosing the start and end points of the path (8). We then calculate, in a second step, for the given trajectory, the solutions of (14), (15), (16) and (17) for every x belonging to [0,1] (i.e. with a sufficient discretisation). Besides, the numerical calculus of the coefficient of (18) allows MAPLE to give all the solutions. The minimal time is the following maximum value :

$$t_f = \max_{1 \le j \le n}\left(\left\{t_{fj/qp}(x), t_{fj/I}(x), t_{fj/dI}(x), t_{fj/U}(x) / x \in [0,1]\right\}, t_{fj/Ieff}\right) \tag{27}$$

In via point problem, we calculate, in the same way, for every polynomial k, the time $t_{f,1}$. The time t_f is then obtained for the maximum value in order to satisfy the contraints (3) to (7) for the whole motion.

The obtained value depends on the variables $\dot{\hat{X}}_k$, $\ddot{\hat{X}}_k$ and Φ_k. Those variables are optimized using a software based on a Sequential Quadratic Programming method. For such a method, we need a initial estimate. This one is obtained considering the point to point motion between two adjacent points.

V - NUMERICAL EXAMPLES

5-1 Robot characteristics

We performed numerical simulations with a two degrees-of-freedom SCARA like robot (simulating our lab robot) that arm lengths are 0.5 m and 0.3 m. The different values of the actuators limitations are :

I_{max} = [11.53,7.29] A U_{max} = [40.0,26.3] V
dI_{max} = [10^4,10^4] A/s $Ieff_{max}$ = [12.0,10.0] A
qp_{max} = [7.0,21.0] rad/s

5-2 Simulation results

In this paper we present an example of a point to point motion for constraints (3)-(7). The trajectory is a straight line (figure 1) between two points represented with a fifth degree polynomial interpolation (9) :

Example 1	cartesian position
start point	[0.6,-0.4]m
end point	[-0.2,0.6]m

table 1

In figure 1, the disc represents the attainable space.

With our new formulation (27), involving actuator constraints, the value of final time is $t_{fa1} = 6.4$ s.

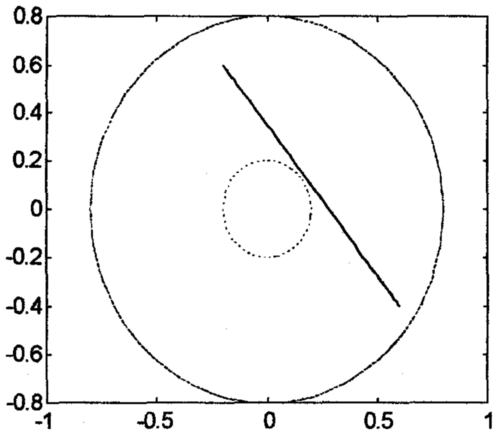

figure 1 : trajectory

Figure 2 presents the current of the first joint, for which the bound is reached.

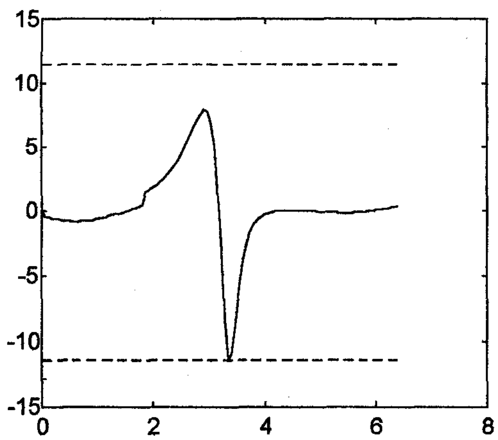

figure 2 : joint 1 current

The two rotational joint robot inverse geometrical model admits two solutions. Then, the same trajectory has been tested with another configuration of the robot arms. The duration of the motion is then $t_{fa3} = 6.6$ s. We give the joint 1 current as a function of time in figure 3.

Usual approaches consider bounds on the cartesian speeds and accelerations [6], or refer to constraints on torques and joint speeds [1]. In the first case, the values of the bounds are not easy to define. In the second case, the values are choosen to realize a compromize beetwen torque and joint speed available. In both cases, those values are lower than the ones obtained considering constraints (3)-(7).

Besides, considering the Coulomb frictions in the dynamical model (1) results in discontinuity on the current when the sign of the joint speed changes.

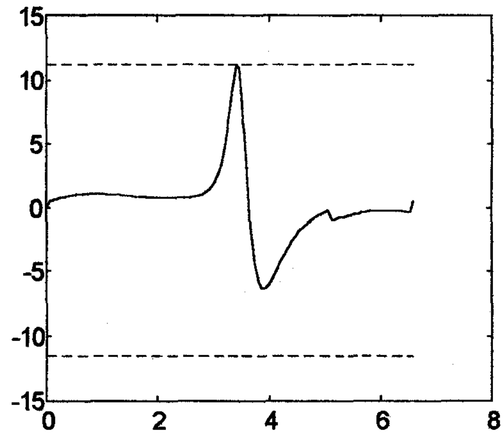

figure 3 : joint 1 current

Our formulation is also applicable to via point trajectory using equation (22). The following points are defined in cartesian space :

Example 2	1	2	3	4	5	6
x (m)	0.6	0.5	0.2	-0.2	-0.4	-0.1
y (m)	0.0	0.3	0.6	0.6	0.4	0.2

table 2

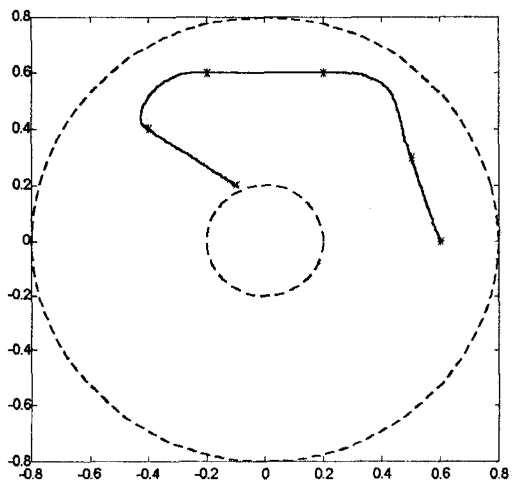

figure 4 : trajectory

The time obtained for a point to point motion between these crossing points is $t_{fa3} = 4.95$ s. For a via point motion with straight lines between points 1 and 2, points 3 and 4 and points 5 and 6, the time is $t_{fa3} = 3.52$ s (figure 4).

For the resulting motion, the current of the first axis reaches its bound (figure 5).

The current of the second axis only adjusts its value in order to allow to follow the prescribed path (figure 6).

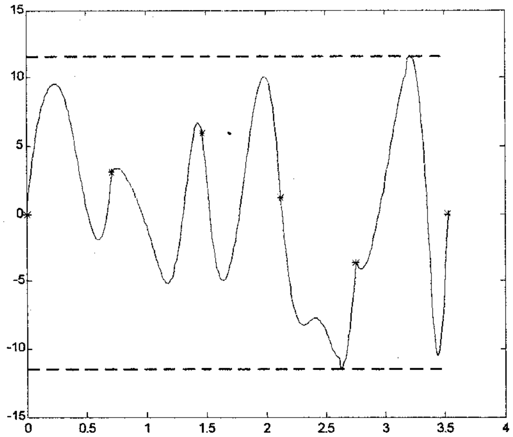

figure 5 : joint 1 current

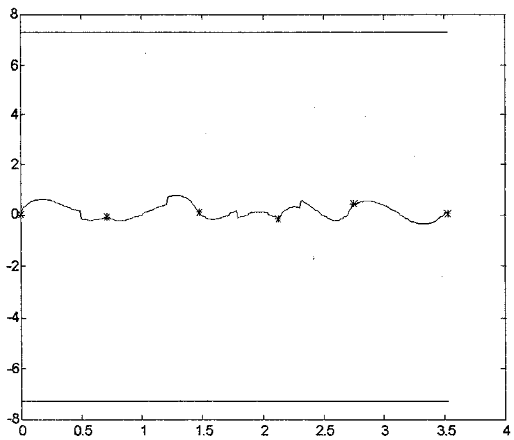

figure 6 : joint 2 current

We show that all constraints are satisfied. Then the motion is admissible. Optimal control approach [1], will certainly lead to shorter times. However, such approaches only consider constraints more restrictive and more unrealistic than (3)-(7). Including those constraints impose a great cost for calculus [8], involving no possibility for on line computation.

Compared with maximal velocity and acceleration problem, even if the computation time for our formulation is longer and depends on the discretisation adopted, it leads to good results. A special fixed motion has often to be repeated thousand of times. In such cases, generation of smooth trajectories which can be performed in minimum time becomes interesting even at the price of longer off line computation times (2 mn). On line computation times, involving few parameters (9) or (21), remain short.

Many concluvive experimentations were also tested on this 2 d.o.f. robot, but we present in this paper only simulation results.

VI - CONCLUSIONS

In specifying a trajectory, the physical limits of the system must be considered. It is common to model these limits as constant maximum values for acceleration and velocity. The trajectory goes from the initial to the final position with initial and final velocities equal to zero, subject to limits on speed and acceleration. These assumptions are often unrealistic. These considerations mean that even for joint level trajectories, any assumptions about fixed acceleration limits must be based on the worst case. This results in motions that are usually slower than necessary or else the actuators may be unable to follow the requested trajectory. A more realistic assumption is that the limits on the amount of voltage and current a motor may generate are given limits.

The proposed motion generation algorithm uses the solution of polynomial equations in t_f to find the predicted arrival time. Besides, the polynomial interpolation with only few parameters, allows to generate easily the path on line.

Although we considered DC motors, other actuators generally present the same constraints since usually both current and voltage of actuators are bounded .

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