A multi-criteria analysis of the banking system in the Republic of Croatia

VIŠNJA VOJVODIĆ ROSENZWEIG, PhD*
HRVOJE VOLAREVIĆ, PhD*
MARIO VAROVIĆ, MSc*

Article**
JEL: G21
doi: 10.3326/fintp.37.4.4

* The authors would like to thank two anonymous referees for their useful comments and suggestions.
** Received: June 1, 2012
Accepted: July 23, 2013
The article was submitted for the 2012 annual award of the Prof. Dr. Marijan Hanžeković Prize.

Višnja VOJVODIĆ ROSENZWEIG
Zagreb School of Economics and Management, Jordanovac 110, 10000 Zagreb, Croatia
e-mail: vojvodic.visnja@gmail.com

Hrvoje VOLAREVIĆ
Zagreb School of Economics and Management, Jordanovac 110, 10000 Zagreb, Croatia
e-mail: hrvoje.volarevic@zsem.hr

Mario VAROVIĆ
Zagreb School of Economics and Management, Jordanovac 110, 10000 Zagreb, Croatia
e-mail: mario.varovic@zsem.hr

Abstract

This paper analysis business strategies of banks by solving a goal programming model using a multi-criteria decision making approach. Multi-criteria business performance is represented as the weighted sum of selected indicators, and the weights or importance of the indicators are a solution of the corresponding problem of goal programming. The ten biggest commercial banks (according to size of balance sheet assets) in the Republic of Croatia were chosen. For an analysis of the operations of the ten banks, three groups of indicators were chosen – profitability, security/risk and liquidity – which were calculated from the banks' financial reports for the year 2010.

Keywords: commercial banks, multi-criteria modelling, goal programming, business performance

1 INTRODUCTION

The purpose of the paper is to show the usefulness of multi-criteria decision-making in an analysis of the strategies of economic agents that are all in the same economic activity. The analysis will use a mathematical model of multi-criteria decision making, which will contain a number of the different individual criteria that are usually used in the framework of this branch of the economy. The analysis and ranking of the banking system according to the criteria selected in the model are applied in accordance with the preferences of the decision makers. The mathematical model for multi-criteria decision making presented will contain nine individual criteria classified into three basic groups – profitability, security (or risk) and liquidity – which are the interlinked components of financial management. The paper will formulate the problem of goal programming in which the goal of the bank is determined by the level of a single indicator from a group of cognate indicators, and the closest operational performance to the goal established is sought.

The usual procedure in multi-criteria analysis is to calculate the score of each bank, i.e. the weighted sum of relative indicator values. The score can also be called the multi-criteria business performance of the elements of the research. At various choices of weights, that is, the importance that is assigned to the indicators, the score of the bank changes and accordingly the position of the bank on the ranking list is also changed. An extreme case is when great importance is assigned to one indicator, and little importance to all the other criteria. This is a single-criterion problem, in which it is easy to see which bank is best. As the indicators listed are in conflict (for example, great profitability is achieved with rather great risk and little liquidity), it is clear that some banks will be the best in the appropriate single-criterion problem. Because of this the main problem is to define the weighting of the indicators in such a way as to avoid decision-maker subjectivity.

The selection of the procedure for calculating the distance of two formulated vectors can be carried out using one of the norms, and so the selection of the norm will also have an impact on the value of the weights obtained, and accordingly on the score of the bank. This paper uses the augmented Chebyshev norm, in which great weights are given to indicators that are in conflict. This paper differs from previous papers in the selection of indicators and in the assumed goals of the banks, that is, the set of banks, goals and indicators considered (Garcia, Guijarro and Moya, 2010).

The paper selects for analysis the banking system of the Republic of Croatia, which comprises thirty commercial banks. Although the ten Croatian banks chosen are the biggest in terms of equity and total balance sheet assets, they are not named, for the emphasis of the work is on the promotion of the mathematical model used, without any consideration of the financial position or operations of the individual banks. For this reason too, just one business year (2010) is analysed in the paper, which means that the model is not restricted in its application, rather, a wider use is enabled in future research (with the use of a time series of data).

The remainder of the paper is presented as follows. Chapter two relates to the definition of the objects of commercial banks' operations. Chapter three shows all the nine individual criteria, and the fourth chapter gives the necessary concepts and multi-criteria decision making and presents a model of goal programming. Conclusions are given in the final chapter of the paper.

2 THE BUSINESS OBJECTIVES OF COMMERCIAL BANKS

This chapter discusses the theoretical fundamentals of the main objectives in the operations of commercial banks, which are subsequently empirically analysed in the mathematical model. In a market economy, banks have diverse objectives, some of which are strategic or long-term goals and others are operational or short-term objectives. For the purposes of this paper, three groups of objectives are picked out, that is, three components that are independent and yet simultaneously in conflicting relationships – profitability, security/risk and liquidity.

Profitability can be considered the basic long-term objective of the operations of commercial banks, and indeed, not of the banking sector alone, but in all the business entities in an economy (Nguyen, 2011). But an insistence on the optimum profitability of operations in a commercial bank will ultimately be bound to bring about changes in the remaining objectives of the bank.

On the other hand, liquidity of operations in commercial banks belongs to the group of short-term objectives. Liquidity is the basis for the proper functioning of the deposit mechanism, in that a bank's liabilities to all its depositors can be met promptly (Van Horne and Wachowich, 2002).

Security in banking operations derives from the management of the risks that appear in banking operations, such as currency risk deriving from changes of the exchange rate affecting open foreign currency positions, interest risk from changes in interest rates and credit risk for loans extended to clients (Toby, 2011).

The basic task of a bank's management is to ensure the realisation of its strategic objectives through the definition of operational objectives (Brealey, Myers and Marcus, 2007). While maintaining liquidity on a daily basis and managing operating risks, ultimately a satisfactory profitability has to be provided for the bank. Such activity will bring the management of the bank in some business situations into conflicting situations in which it is in practice impossible to achieve all three objectives at the same time. Accordingly, it is necessary to define operational priorities, or to find an optimum combination of priorities. Here it needs mentioning that the legislative background has a considerable influence on banking operations (in the Croatian case, primarily the Credit Institutions Act and the sub-laws of the Croatian National Bank).

These objectives of banking operations will accordingly be expressed as individual criteria in the mathematical model, enabling the presence of the basic components of financial management in the characteristics of the individual criteria. Nine individual criteria will thus be selected, each of the components of financial management being represented by three criteria. Thus the final order of the components of financial moment will, according to their relative importance, be subject to changes because of the definition of the priorities in the operations of the commercial banks.

3 SELECTION OF CRITERIA FOR THE MATHEMATICAL MODEL

The ranking of commercial banks is a classic multi-criteria decision making problem. Firstly, it is necessary to select the criteria according to which the ranking of the banks in a decreasing order will be made (from best to worst bank performance). Nine individual criteria have been chosen here, i.e. indicators that are provisionally allocated to the three basic groups (profitability, security/risk and liquidity). Most of the indicators selected are much employed in financial analysis and in commercial bank supervision; in this paper they are a specific choice of the authors, in conjunction with certain modifications in their calculation. This does not mean that in the application of the model some other indicators cannot be used, or some other ways of allocating the indicators to the given groups or sets.

1) **Net interest margin** is one of the best known profitability indicators, and is used exclusively in the banking system, for it refers to the interest margin obtained in the operations of the bank as compared to the total assets of the bank used (Berrios, 2013). The value of this indicator is calculated according to the following ratio:

$$X_1 = \text{net interest margin} = \text{net earnings from interest} / \text{total bank assets} \quad (1)$$

Net interest earnings can be seen from the profit and loss accounts for 2010, while the total bank assets can be derived from the final balance sheet for 2010. Interest earnings are the main profit generator for every commercial bank. The values obtained are expressed in percentages, and it is desirable that they should be as great as possible for each bank (max).

2) **Return on average equity** or ROAE is also one of the best known profitability indicators, just like interest margin (Kosak and Čok, 2008). But this is used not only in the banking but also in the real sector (where sometimes as well as average equity end of year equity is also used for the calculation). Unlike interest margin, this indicator shows the realized return on investment in the average equity of the bank. The value of this indicator is calculated as follows:

$$X_2 = \text{return on average equity} = \text{after-tax profit} / \text{average equity of the bank} \quad (2)$$

After-tax profit is the last item in the profit account, while the average equity of the bank is calculated as the arithmetical mean of the balance sheet positions of equity for two successive business years, in this case, for 2009 and 2010. The fiscal policy of the country in which a commercial bank has its principal place of business will have an effect on the amount of this indicator for after-tax profit is a residual magnitude after deduction of profit tax (in the case of Croatia, corporate income is taxed at a rate of 20%). The values obtained are also expressed as percentages, and it is desirable that for each bank they should be as big as possible (max).

3) **The weighted interest income to weight interest expense ratio** is the third in order, and like net interest margin is a specific profitability indicator that is used only in the banking sector (Bulletin about banks, 2011). The value of this indicator is calculated as follows:

$$X_3 = \text{the ratio of weighted interest income to weighted interest expense} = (\text{interest earnings} / \text{average interest assets}) / (\text{interest expenses} / \text{average interest liabilities}) \quad (3)$$

Interest earnings and interest expenses are the starting position in the profit and loss account of every commercial bank, for they define the operating result that derives from the basic activity of banking – receiving deposits and granting loans. Interest assets comprise the sum of all positions of the assets of the balance sheet that represent the basis for the calculation of asset interest in banking earnings. On the other hand, interest liabilities comprise the sum of all positions of the liabilities of the balance sheet, which are the basis for the calculation of liabilities' interest that contribute to the expenses of the bank. Interest earnings are weighted by the average interest assets, and the interest expenses are weighted by the average interest liabilities. The results obtained are expressed in absolute values, and it is desirable that the results of this ratio are as great as possible, since this confirms the profitability of the bank's operations (max).

4) **Ratio of defaulted loans to total loans** is an indicator that is commonly used in the banking sector for an appraisal of the security or risk of bank investments in all kinds of own loans (Kundid, Skrabić and Ercegovac, 2011). The value of this indicator is calculated as follows:

$$X_4 = \text{ratio of defaulted loans to total loans} = (\text{total value adjustments} + \text{reservations}) / (\text{total loans} + \text{contingent liabilities}) \quad (4)$$

The nominator of the indicator contains the sum of value adjustments and reservations, in which value adjustments constitute the sum of all acknowledged losses for dubious or disputed loans for which no returns are expected, that is, collections, while the term reservations refers to the balance sheet position in the liabilities that the bank has acknowledged as costs for future observed and estimated liabilities (an example is reservations for lawsuits against the bank already filed). In the denominator of the indicator there are the total loans that make up the sum of all positions of bank assets that represent bank loans, which are the basis upon which it makes its earnings, while the second part of the indicator refers to contingent liabilities that are as a rule recorded off-balance-sheet, and relate to guarantees made and letters of credit that constitute typical banking business. This indicator calculates the expected costs of the bank as against its total loans, in which the current cost represents an anticipation of operational loan losses. The values obtained are expressed in percentages, and it is desirable that the results of this ratio obtained are as great as possible, which implies that the bank management is aware of possible risks or uncertainty in its operations and the need to anticipate them through an increase in timely value adjustments and reservations (max).

5) **Security of deposits** is an indicator that like the previous indicator comes within an appraisal of security or risk in bank operations, since it evaluates the percentage coverage of deposits received from all the clients of the bank by available average equity of the bank (Cernohorska and Cernohorsky, 2007). Accordingly, it is used only in the banking sector. The value of this indicator is calculated as follows:

$$X_5 = \text{security of deposits} = \text{average bank equity} / \text{deposits received} \quad (5)$$

This equation juxtaposes two positions from the balance sheet liabilities of the bank. Average bank equity, as with ROAE, is calculated as the arithmetical mean of the balance sheet positions of equity in two consecutive business years, in this case, 2009 and 2010. Deposits received constitute all the liabilities from the balance sheet that a bank has to other banks and its other clients (as a rule the sight and time deposits of the retail and corporate sectors made with the bank). This indicator in fact is a ratio of the bank's own means (the bank's equity) and other people's funds (deposits received from clients). Depositor security depends above all on the quality of the bank's loans, but it has to be said that a high degree of capitalisation has a positive effect on bank stability and security of deposits confided to it. The values of this indicator are shown in percentages and it is desirable that they should be as great as possible for in this way the bank can to as great a percentage as possible secure the loans of its clients with its own equity (max).

6) **Leverage** is a well known indicator that can also be included in the category of indicators of the security or riskiness of banking operations. Since this indicator is a ratio of the total equity (original and earned) of the bank and its total assets, it can be concluded that it is desirable that the value of this indicator should be as great as possible (Chortareas, Girardone and Ventouri, 2009). Since the liabilities of a bank consist of equity and liabilities, the rule holds: the greater the proportion of equity, the smaller the proportion of bank liabilities in total liabilities. The liabilities of the bank to its clients should be reduced (the third component of the balance sheet equation that is missing in this relationship), which will guarantee a certain security in its operations. This indicator in this form is often used in the real as well as in the banking sector. The value of the indicator is calculated as follows:

$$X_6 = \text{leverage} = \text{total equity of bank} / \text{total assets of bank} \quad (6)$$

The values in this equation are obtained from the final bank balance sheet for 2010. The results of this indicator are expressed in percentages, and it is desirable that they be as great as possible (max).

7) **Cash ratio** is a classic example of a liquidity indicator that is used not only in the banking but also in the real sector, with certain modifications. This indicator is the ratio of all the currently available cash of the bank to the liabilities of the bank to its clients (Siddiqui, 2008). This is a criterion that as against all other liquidity indicators has by far the smallest result (which is intelligible because of the present paucity of highly liquid resources in operations) but the results can be quite telling with respect to some banks. It answers the question how ready the bank is to meet unexpected and unplanned demands for money from its depositors, which will have a cash outflow as its consequence. The value of the indicator is calculated as follows:

$$X_7 = \text{cash ratio} = (\text{cash on account} + \text{cash on current accounts at the banks} + \text{cash kept with the central bank}) / \text{liabilities to clients} \quad (7)$$

The nominator in this relation is obtained from the short-term position of monetary assets in the bank in the balance sheet assets, while the denominator is obtained from the position of all liabilities to clients (short- and long-term) in the balance sheet liabilities. The results obtained are standardly expressed in absolute values, and as with all previous indicators it is desirable that it should be as great as possible (max).

8) **Loan to deposit ratio** is a specific example of a liquidity indicator that is used only in the banking sector (Kundid, Skrabic and Ercegovac, 2011). This indicator is a ratio of loans made and deposits received by the banks, including all its clients in the calculation (both debtors and creditors). A commercial bank makes loans from the resources of deposits received, i.e. from the basis of resources in the bank's liabilities it makes loans in its assets. This is a criterion that unlike all the other indicators interpreted in the paper should have as small as possible a result, in order to ensure the greatest possibility liquidity in the operations of the bank. Banks that do not go too far in making loans as against availability of deposits can be sure not to have liquidity problems in their operations (if this exceptionally crucial relation is observed only in the context of making sure of operational liquidity). The value of this indicator is calculated according to the following relation:

$$X_8 = \text{loan to deposit ratio} = \text{loans made / deposits received} \quad (8)$$

The nominator in this equation is obtained from the position of loans made to all debtors in the balance sheet assets, while the denominator is obtained from the position of deposits received from all creditors in the balance sheet liabilities. The results are also expressed in absolute values, and unlike those of other indicators, it is desirable that they be as small as possible (min).

9) **Interest bearing assets to interest bearing liabilities ratio** can be observed in this paper in the context of liquidity, but it can also be considered in the context of the loan activities of the bank since a growth in loans will affect the security of banking operations. This indicator is similar to X_3, but it is different in that it directly establishes a ratio of all the assets of the bank that create earnings from interest and all the liabilities of the bank that create interest expenses (Bulletin on Banks, 2011). It is desirable that the result of this indicator be as great as possible, i.e. at least larger than one, for it can thus be assumed that in the given period the bank has handled its assets properly (this assumption is based only on the amount of the principal on which interest is charged, and not on the amount of the rates that are charged). This would mean that the bank has made more loans producing positive interest, which constitute a monetary inflow, than those that create negative bank interest, which constitute a monetary outflow. The value of this indicator is calculated according to the following:

$$X_9 = \text{interest bearing assets to interest bearing liabilities ratio}$$
$$= \text{interest on assets / interest on liabilities} \quad (9)$$

As mentioned earlier, the numerator in this equation is obtained from the total position of the bank's assets, which only create interest earnings, while the denominator is obtained from the total position of the bank's liabilities, which only create interest expenses. The results are also shown in absolute values, and unlike indicator number 8, it is desirable that they should be as large as possible (max).

Pursuant to the previous formulae, the values of all the nine individual benefit criteria (X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8 and X_9) are calculated for the ten selected banks (BANK 1, BANK 2, BANK 3, BANK 4, BANK 5, BANK 6, BANK 7, BANK 8, BANK 9 and BANK 10), and then all the results obtained are shown in the decision making table (table 1), as follows:

Table 1

Values of the nine individual benefit criteria within the three basic sets (profitability, security/risk and liquidity) for the ten banks selected

Bank	I) Profitability			II) Security (risk)			III) Liquidity		
	X_1 (%)	X_2 (%)	X_3	X_4 (%)	X_5 (%)	X_6 (%)	X_7	$1/X_8$	X_9
BANK 1	3.1796	10.5500	2.2679	4.3702	13.2299	11.9127	0.2131	1.1372	0.9831
BANK 2	2.6476	5.5350	2.1353	5.4420	7.3459	8.0478	0.1683	1.2261	0.9305
BANK 3	2.7016	3.5649	1.7827	5.7059	21.6997	16.4943	0.2713	1.0058	1.1275
BANK 4	2.9890	10.0295	2.4288	5.8505	14.7364	12.6907	0.1883	1.2931	0.9680
BANK 5	2.9585	4.8826	2.1145	3.6519	13.8575	11.6291	0.1488	1.1982	0.9727
BANK 6	2.9131	8.5368	2.3136	3.1443	17.9160	15.3611	0.2170	1.0748	0.9997
BANK 7	3.1247	3.5183	2.2133	5.5790	15.3765	13.3672	0.1954	1.3215	0.9478
BANK 8	3.1171	6.7656	2.1882	2.8526	16.8241	14.0295	0.1621	1.1366	1.0444
BANK 9	3.0889	2.0075	2.0401	3.8299	28.9380	21.5735	0.2653	0.9086	1.1302
BANK 10	2.8620	9.0294	2.1152	3.4044	17.8698	14.8134	0.1949	1.0992	1.0228
Average value	2.9582	6.4420	2.1599	4.3831	16.7794	13.9919	0.2025	1.1401	1.0127

Source: Authors' calculation from the banks' financial reports for 2010.

All the results of the individual indicators were positively directed (max – the greater the value the better) except for criterion X_8 which shows the loan to deposit ratio, which is negatively directed (min – the smaller the value the better). To be able to get all the benefit criteria in the decision making matrix, it was necessary to treat the expense criterion X_8 as a benefit criterion by putting into the decision making table or matrix the transformation of the value of X_8, by registering its reciprocal value $1/X_8$.

In this way a decision making matrix for all nine benefit criteria that are not expressed in identical units of measurement (some in percentages and some in absolute values) is created. For this reason the next step is the transformation of the values of positively directed criteria. Here a percentage transformation is used.

This transformation is carried out because it produces proportional changes in the results. The results obtained are shown in the following table (table 2).

TABLE 2

Transformed values of the nine individual benefit criteria in the three basic sets (profitability, security/risk and liquidity) for the ten banks selected

Bank	I) Profitability			II) Security (risk)			III) Liquidity		
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	$1/X_8$	X_9
BANK 1	0.1075	0.1638	0.1050	0.0997	0.0788	0.0851	0.1052	0.0997	0.0971
BANK 2	0.0895	0.0859	0.0989	0.1242	0.0438	0.0575	0.0831	0.1075	0.0919
BANK 3	0.0913	0.0553	0.0825	0.1302	0.1293	0.1179	0.1340	0.0882	0.1113
BANK 4	0.1010	0.1557	0.1124	0.1335	0.0878	0.0907	0.0930	0.1134	0.0956
BANK 5	0.1000	0.0758	0.0979	0.0833	0.0826	0.0831	0.0735	0.1051	0.0961
BANK 6	0.0985	0.1325	0.1071	0.0717	0.1068	0.1098	0.1072	0.0943	0.0987
BANK 7	0.1056	0.0546	0.1025	0.1273	0.0916	0.0955	0.0965	0.1159	0.0936
BANK 8	0.1054	0.1050	0.1013	0.0651	0.1003	0.1003	0.0801	0.0997	0.1031
BANK 9	0.1044	0.0312	0.0945	0.0874	0.1725	0.1542	0.1310	0.0797	0.1116
BANK 10	0.0967	0.1402	0.0979	0.0777	0.1065	0.1059	0.0963	0.0964	0.1010
Total value	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Source: Authors' calculation.

4 MULTI-CRITERIA DECISION MAKING AND GOAL PROGRAMMING MODEL

A multi-criteria decision-making problem (MP) consists of p > 1 objective functions by which one wishes to achieve their maximum in the set of alternatives, feasible solutions or decisions, and it is written in the following way:

$$\max\left\{\left(f_1(a),\ldots,f_p(a)\right):a\in A\right\}$$

The notations represent:

- $f_1,\ldots f_p$ – objective functions
- A – set of alternatives
- a – alternative.

The set of alternatives can be represented in various ways, as a set of solutions to a system of equations and/or inequalities or, as in our case, the alternatives may be explicitly stated. In this paper the alternatives are the banks listed, and the objective functions are the indicators selected. In table 2, which we call the decision making table or matrix, there are ten alternatives in the rows and nine indicators in the columns of the matrix. If we first look at a single criterion problem in which the only criterion is return on average equity, in table 2 we shall look for the biggest number in column X_2 and thus we know that the highest value of this indicator was achieved by Bank 1, and according to this criterion, it is the best. On the other hand, deposit security (column X_5 in table 2) is the largest in Bank 9. At the end, cash ratio (column X_7 in table 2) is the greatest in Bank 3. We have listed three partial, single criterion problems, and we can also list the remaining six. Accordingly, the conclusion is that there is not a single bank that is best in all the nine indicators at the same time. For this purpose the concept of a solution in a multi-criteria problem (MP) is introduced, called an efficient or Pareto-optimal solution (alternative, decision). Alternative $b \in A$ is efficient or Pareto-optimal or non-dominated if there is no alternative $a \in A$ such that $f_i(a) \geq f_i(b)$ for all $i = 1, \ldots, p$ and $f_i(a) > f_i(\mathrm{b})$ for at least one $i = 1, \ldots, p$.

One of the most common approaches to determine one of the efficient solutions is an approach in which a multi-criteria problem is reduced to a single-criterion problem using a function that we call the score of the alternative.

The score of the alternative *a* is the weighted sum of individual objective functions or the indicators:

$$S(a) = \sum_{j=1}^{p} w_j f_j(a)$$

The weights w_j, $j = 1, \ldots, p$ are positive or non-negative numbers and assign importance to individual indicators and most often for calculating reasons it is taken that their sum is equal to one. The alternative that has the greatest score along with positive values of weights is efficient or Pareto-optimal. If some weights have the value of zero, or if the weights are non-negative numbers and only one alternative has the greatest score, then it is efficient. The score is used as multi-criteria operational performance and there can be no alternative in the first place on the ranking list if there is a better. By a choice of differing values of weights, various efficient solutions are obtained, which are called supported efficient solutions. Because of the structure of the problem that we are analysing there are efficient solutions that cannot be obtained with the help of the score, unsupported efficient solutions and in this case some other approaches are used.

In this paper the values of the indicators are aggregated into a score that has accordingly reduced all the relevant information about bank operations to a number and thus by comparison of the score of the banks obtained we can compare and rank them.

The score is called multi-criteria operational/business performance. The score of S_i bank i, $i = 1, \ldots, 10$ depends on the indicators selected and because of the multi-dimensional nature of the data in the decision making table the procedure of reducing the data to relative values is carried out. Through this procedure the problem becomes one-dimensional and the calculation of the score has a point. The score also depends on the weights that are conjoined to each indicator. The weight reflects the importance ascribed to each indicator, and can be any non-negative number at all and for reasons of calculation we say that the sum of weights is equal to one. It is obvious that the score of a given bank will change with the various selections of weights. Thus the selection of weights becomes a problem in which it is necessary to discount decision-maker subjectivity.

The banks themselves are oriented to the capital market and the performances of competitive banks. Each bank has its own business strategy in which it evaluates how it is going to bring to fruition the operating goals it has placed before it for a given period. Pursuant to the results of the criteria achieved (table 2), this model takes as its point of departure the assumption that all banks did not have the same operating goals in 2010.

From this point of view, the problem of goal programming will be formulated. The notations in the model are as follows:

1) i – bank, $i = 1, ..., 10$
2) j – indicator, $j = 1, ..., 9$
3) w_j – weight of indicator j, $j = 1, ..., 9$
4) x_{ij} – relative value of indicator j of bank i, $i = 1, ..., 10$, $j = 1, ..., 9$
5) S_i – score of bank i, $i = 1, ..., 10$
6) g_i – goal of bank i, $i = 1, ..., 10$
7) d_i^- – under-achievement of goal i, $i = 1, ..., 10$
8) d_i^+ – over-achievement of goal i, $i = 1, ..., 10$.

S_i, score of bank i, is defined as follows:

$$S_i = \sum_{j=1}^{9} w_j x_{ij}$$

We give labels to the ten banks as shown in the following table (table 3).

TABLE 3
Numbering the ten selected banks

BANK 1	BANK 2	BANK 3	BANK 4	BANK 5	BANK 6	BANK 7	BANK 8	BANK 9	BANK 10
1	2	3	4	5	6	7	8	9	10

The general problem of goal programming is the well known technique of multi-criteria decision making and it consists of finding a solution that is closest to the goal established (Ignizio and Romero, 2003). In this case that means it is necessary to find such weights of indicators in which deviation of the score, i.e. of the performance from the goal established, is the least. The distance of two vectors is defined in general with the help of some metric.

The first vector is the score vector, the components of which are the score of the corresponding bank. The second vector is the goal vector $g = (g_1, ..., g_{10})$, and its components are the goals of the corresponding bank. Depending on the metric chosen, various solutions and various indicator weights are obtained. Two metrics are highlighted as the two extreme cases of measuring the distance of the vectors according to the interpretation of the weights obtained. The first is defined as the sum of absolute values of deviation obtained from norm 1 and the solution obtained from it gives lower weights to conflicting criteria. The second is defined as the maximum absolute deviation, obtained from the norm ∞ also known as the Chebyshev norm and the solution obtained from it gives greater weights to conflicting criteria. The augmented Chebyshev norm is close to the Chebyshev norms depending on the selection of parameter α and is recommended because it allows an unsupported efficient solution to be obtained while weak efficient solutions can be avoided. The augmented Chebyshev norm is also known as the Dinkelbach-Isermann norm.

We form the problem of goal programming (P_α) with the help of the augmented Chebyshev metric in the following way:

$$\min\left(\max\left\{\left|g_i - S_i\right| : i = 1,\dots,10\right\}\right) + \alpha\sum_{i=1}^{10}(\mathrm{d}_i^- + \mathrm{d}_i^+) \tag{10}$$

$$S_i + d_i^- - d_i^+ = g_i, \qquad i = 1,\dots,10 \tag{11}$$

$$\mathrm{S}_i - \sum_{j=1}^{9} w_j x_{ij} = 0, \qquad i = 1,\dots,10 \tag{12}$$

$$\sum_{j=1}^{9} w_j = 1,\ d_i^-, d_i^+ \geq 0,\ i = 1,\dots,10 \tag{13}$$

$$w_j \geq 0, \qquad j = 1,\dots,9$$

The parameter α is a small positive number. With the non-negativity of the variables in the mathematical model we have the following constraints. The value of the score can deviate from the established goal, which is defined in the set of constraints (11). The set of constraints (12) defines the score. Constraint (13) relates to normed weights. Because of the objective function (10) in the given mathematical model, in the optimal solution at least one of the variables d_i^- or d_i^+ has the value of zero or in other words its value is:

$$d_i^- d_i^+ = 0, \qquad i = 1,\dots,10 \tag{14}$$

This statement (14) can be verified in the book of Sawaragi, Nakayama and Tanino (1985).

Then we introduce the notation:

$$y = \max\left\{\left|g_i - S_i\right| : i = 1,\dots,10\right\} \tag{15}$$

and because of relations (14) and (15) the following holds:

$$y \geq d_i^- + d_i^+, \qquad i = 1,\dots,10 \tag{16}$$

Now we transform the problem (P_α) into the equivalent problem ($P_{y\alpha}$) with the aid of transformation (15) and (16). Problem ($P_{y\alpha}$) is as follows

$$\min y + \alpha\sum_{i=1}^{10}\left(d_i^- + d_i^+\right)$$

$$S_i + d_i^- - d_i^+ = g_i, \qquad i = 1,\dots,10$$

$$\mathrm{S}_i - \sum_{j=1}^{9} w_j x_{ij} = 0, \qquad i = 1,\dots,10$$

$$\sum_{i=1}^{9} w_j = 1$$

$$y \geq d_i^- + d_i^+, \qquad i = 1,\dots,10$$

$$d_i^-,\ d_i^+ \geq 0, \qquad i = 1,\dots,10$$

$$w_j \geq 0, \qquad j = 1,\dots,9$$

$$y \geq 0$$

($P_{y\alpha}$) is a linear programming problem that is easily and rapidly solved with programme support.

5 IMPLEMENTATION AND INTERPRETATION OF THE MODEL

In the model of goal programming ($P_{y\alpha}$) that needs to be solved and its optimal solutions found, all the parameters in table 2 are given, apart from the parameter $g = (\mathrm{g}_1,\dots,\mathrm{g}_{10})$ which represents the goals of the banks that have been set up. We shall solve three problems of goal programming that differ according to the goals chosen.

First of all we will make it the goal of every bank to achieve a certain level of profitability. Since we have three profitability indicators, each bank chooses as its goal the greatest value that is achieved by one of the profitability indicators. Accordingly we have:

$$g_i = \max\left\{x_{ij} : j = 1,2,3\right\}, \qquad i = 1,\dots,10 \tag{17}$$

Problem ($P_{y\alpha}$) is solved for the value of the parameter and the optimal solution is obtained:

$$w_2 = 0.5015, w_5 = 0.4985 \tag{18}$$

As a result we have a score for every bank:

$$S_i = 0.5012x_{i2} + 0.4985x_{i5}, \qquad i = 1,...,10 \tag{19}$$

while the other weights are equal to zero.

Table 4

The scores of banks and their positions on the ranking list in achievement of the goal of profitability

BANK 1	BANK 2	BANK 3	BANK 4	BANK 5	BANK 6	BANK 7	BANK 8	BANK 9	BANK 10
0.1214	0.0649	0.0922	0.1219	0.0792	0.1200	0.073	0.1027	0.1016	0.1234
3	10	7	2	8	4	9	5	6	1

According to values of weights obtained (18) if the goal is obtaining an appropriate value of one of the profitability indicators (17), the indicator of return on average capital (X_2) and the deposits security indicator (X_5) have the greatest weights. The first indicator corresponds to the objective set, which is the maximisation of the profitability of the bank's operations in the sense of the greatest possible return on equity in the operational process. On the other hand, for the achievement of this goal it is essential that the bank should collect as much in deposits as it can from its clients in order to transform them into loans made. Accordingly, a bank that has a greater capital will give its depositors greater security, but this will directly result in a fall in profitability in its business operations. Pursuant to the calculated scores of all banks, the best position on the ranking list was obtained by Bank 10, followed by Bank 4, Bank 1 and then all the remaining banks. Last in the list is Bank 2, which meets this set business goals the least effectively.

The second problem that we solve is the problem of goal programming ($P_{y\alpha}$) in which we observe a model, in which the goal of every bank is to achieve a certain level of liquidity, or:

$$g_i = \max\left\{x_{ij} : j = 7,8,9\right\}, \qquad i = 1,...,10 \tag{20}$$

The problem is solved for the value of the parameter and all the weights of the indicator are obtained:

$$w_4 = 0.2874, w_5 = 0.1838, w_9 = 0.5288 \tag{21}$$

while the other weights are equal to zero.

From this we obtain the score of each bank:

$$S_i = 0.2874x_{i4} + 0.1838x_{i5} + 0.5288x_{i9}, \qquad i = 1,...,10 \qquad (22)$$

TABLE 5

Scores of banks and positions on the ranking list for meeting the liquidity goal

BANK 1	BANK 2	BANK 3	BANK 4	BANK 5	BANK 6	BANK 7	BANK 8	BANK 9	BANK 10
0.0945	0.0923	0.0922	0.12	0.0899	0.0924	0.1029	0.0917	0.1158	0.0953
6	8	1	3	10	7	4	9	2	5

According to the values of weights obtained (21) in the second problem, if the goal attaining an appropriate value for one of the indicators of liquidity (20), two security indicators (X_4 – ratio of NPL to total loans and X_5 – security of deposits) and one liquidity indicator (X_9 – ratio of interest assets and interest liabilities) have a share in the total weight. Indicator X_9, which gives the ratio of interest assets and interest liabilities, corresponds to the given goal. In the case of an increased value of indicator X_9 through an increase in loans (assets interest), there will be a reduction in the value of indicator X_4 and vice versa. On the other hand, by an increase in deposits (interest on liabilities) the indicator X_9 will be reduced, and accordingly there will be a reduction of indicator X_4. Pursuant to the calculated scores of all the banks, in this case the best position on the ranking list was taken by Bank 3, after that by Bank 9 and Bank 4, and then all the remaining banks. On the bottom of the list is Bank 5.

Finally, we shall consider the problem in which some banks have established as their goal the level of profitability, and some have established the level of liquidity as their goal, and so we have:

$$g_i = \max\left\{x_{ij} : j = 1,2,3,7,8,9\right\}, \qquad i = 1,...,10 \qquad (23)$$

The problem is solved for the value of parameter and the weights of the indicator are obtained as follows:

$$w_2 = 0.4093, w_4 = 0.0082, w_5 = 0.0964, w_7 = 0.4861 \qquad (24)$$

while the other weights are equal to zero.

Accordingly we shall obtain a score for each bank:

$$S_i = 0.4093x_{i2} + 0.0082x_{i4} + 0.0964x_{i5} + 0.4861x_{i7}, \quad i = 1,...,10 \qquad (25)$$

TABLE 6
Scores of banks and positions on the ranking list in the case of achieving the goal of profitability or liquidity

BANK 1	BANK 2	BANK 3	BANK 4	BANK 5	BANK 6	BANK 7	BANK 8	BANK 9	BANK 10
0.1266	0.0808	0.1013	0.1185	0.0754	0.1172	0.0787	0.0921	0.0938	0.1151
1	8	5	2	10	3	9	7	6	4

According to the values of weights obtained (24) in the third problem, if the goal of a given bank is achieving a corresponding value of one of the indicators profitability or liquidity (23), one profitability indicator (X_2 – return on average equity) and two security indicators (X_4 – ratio of NPL and total loans and X_5 – deposit security) have a share in the total weight, as does one liquidity indicator (X_7 – cash ratio). In this case the interpretation of the results of the profitability and liquidity indicators can be indirectly connected via the value of the security of deposit indicator (X_5). If the value of equity is increased, then there is a reduction in return on equity (X_2) and at the same time an increase in deposit security. On the other hand, increased deposit security also augments the cash ratio (X_7). The influence of the second security indicator (X_4 – ratio of loan losses to total loans) on the interpretation of the results is practically negligible because of the share in the total weight displayed.

On the basis of the calculated scores of all banks, Bank 1 is in first place, after which comes Bank 4, and after it Bank 6, and then all the remaining banks. Bank 5 brings up the rear, as in the previous problem.

6 CONCLUSION

Multi-criteria analysis of commercial banks can be successfully carried out through the application of goal programming. The first step is to define the criteria pursuant to which the multi-criteria analysis will be carried out, and in accordance with this to seek the best operational performance of the selected banks. The second step is the formulation of the mathematical model of goal programming in which the decision maker is given the opportunities to use various goals.

The analysis was carried out for a single business year, 2010, and it indicates the operational goals of the banks that their managements carried out for this reporting period. From the results obtained it can be seen, considering the different goals established for the banks in each of the three analysed business situations, that we have different banks in the number 1 positions. It can be concluded that there is not a single bank in a dominating position in the banking sector of the Republic of Croatia, because it is in such a position in which by achieving the set objectives it will be necessary to ignore some other objectives. The conclusion is then that the obtained results in the framework of the multi-criteria decision making model can be identified with the definition of Pareto efficiency, because of which the management of a bank has to be ready for conflicting situations in its operations because it will be constrained in its ability to attain all the objectives concurrently. Accordingly, the banks have to define their priorities in their operations, or find an optimum combination for the achievement of their objectives.

Future testing of a model of goal programming established in this way assumes inclusion into the analysis of new indicators or new indicator groups used in given industrial sectors. It is also possible to use some other metrics (norms) and longer data time series.

REFERENCES

1. Berrios, M. R., 2013. The relationship Between Bank Credit Risk and Profitability and Liquidity. *The International Journal of Business and Finance research*, 7(3), pp. 105-118.
2. Brealey, R. A., Myers, S. C. and Marcus, A. J., 2007. *Osnove korporativnih financija.* Zagreb: MATE.
3. Černohorska, L. and Černohorsky, J., 2007. Potential Attitudes Solving the Problems of Banking Stability. *Working Papers in Economics.* Tallinn: School of Economics and Business Administration, pp. 25-40.
4. Chortareas, G. E., Girardone, C. and Ventouri, A., 2009. Efficiency and Productivity of Greek Banks in the EMU Era. *Applied Financial Economics*, 19 (16), pp. 1317-1328. doi: 10.1080/09603100802599506
5. Ehrgott, M., Klamroth, K. and Schwehm, C., 2004. An MCDM approach to portfolio optimization. *European Journal of Operational Research*, 155(3), pp. 752-770. doi: 10.1016/S0377-2217(02)00881-0
6. Garcia, F., Guijarro, F. and Moya I., 2010. Ranking Spanish Savings Banks: A Multicriteria Approach. *Matematical and Computer Modelling*, 52(7-8), pp. 1058-106. doi: 10.1016/j.mcm.2010.02.015
7. CNB, 2011. Banks Bulletin, 11 (23). Available at: http://www.hnb.hr/publikac/bilten-o-bankama/ebilten-o-bankama-23.pdf.
8. Ignizio, J. M. and Romero, C., 2003. Goal Programming, Encyclopedia of Information Systems. *Academic Press*, 2, pp. 489-500.
9. Košak, M. and Čok, M., 2008. Ownership Structure and Profitability of the Banking Sector: The Evidence from the SEE Region. *Zbornik radova Ekonomskog fakulteta u Rijeci*, 26(1), pp. 93-122.
10. Kundid, A., Škrabić, B. and Ercegovac, R., 2011. Determinants of Bank Profitability in Croatia. *Croatian Operational Research Review (CRORR)*, 2, pp. 168-182.
11. Nguyen, J., 2011. Market Concentration and other Determinants of Bank Profitability: Evidence from Panel Data. *International Research Journal of Finance & Economics*, (70), pp. 7-19.
12. Sawaragi, Y., Nakayama, H. and Tanino, T., 1985. *Theory of Multiobjective Optimization.* Orlando, USA: Academic Press, Inc.
13. Siddiqui, A., 2008. Financial Contracts, Risk and Performance of Islamic Banking, *Managerial Finance*, Vol. 34, Issue 10, pp. 680-694. doi: 10.1108/03074350810891001
14. Toby, A., J., 2011. Modelling Financial Fragility and Bank Profitability in an International Context. *International Journal of Business Insights & Transformation*, 4(2), pp. 15-24.

15. Triantaphyyllou, E., 2000. *Multi-Criteria Decision Making Methods: A Comparative Study.* Amsterdam: Kluwer Academic Publishers. doi: 10.1007/978-1-4757-3157-6

16. Van Horne, J. C. and Wachowich, J. M., 2002. *Osnove financijskog menedžmenta.* Zagreb: MATE.