A multi-period fuzzy portfolio optimization model with investors' loss aversion

Xingyu Yang
Guangdong University of Technology
Jingui Chen
Guangdong University of Technology
Weilong Liu
Guangdong University of Technology
Xuejin Zhao (zhaoxj2002@163.com)
Guangdong University of Technology https://orcid.org/0009-0002-0165-368X

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A multi-period fuzzy portfolio optimization model with investors' loss aversion

Xingyu Yang[1] · **Jingui Chen**[1] · **Weilong Liu**[1] · **Xuejin Zhao**[2]

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Abstract This paper considers the problem of how to construct the optimal multi-period portfolio for investor with loss aversion in fuzzy environment. Firstly, we regard the return rates of the risky assets as fuzzy numbers, and use the value function in prospect theory to transform the return rate of a portfolio into perceived value, which reflects investors' loss aversion. Moreover, due to the fact that investors' perception level toward risk may vary with the loss aversion degree, we propose a new risk measure based on the perceived value. Then, we formulate the objectives of maximizing the cumulative expected perceived value and minimizing the cumulative perceived risk, and propose a multi-period portfolio selection model with diversification constraint. Furthermore, to solve the proposed model, we design a multiple particle swarm optimization algorithm with respect to its specific situation. Finally, using the data from real financial market, we construct a real case to illustrate the effectiveness of the model and algorithm. The results show that loss aversion has an important effect in investors' investment decisions, and the proposed model could provide more reasonable strategies for investors with different loss aversion degrees.

✉ Xuejin Zhao
zhaoxj2002@163.com

Xingyu Yang
yangxy@gdut.edu.cn

Jingui Chen
gdutchenjg@126.com

Weilong Liu
liuwlweller@outlook.com

[1] School of Management, Guangdong University of Technology, Guangzhou, Guangdong 510520, China
[2] School of Economics, Guangdong University of Technology, Guangzhou, Guangdong 510520, China

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1 Introduction

How to allocate the wealth in multiple assets properly is one of the important problems in financial sector. The mean-variance (MV) portfolio selection model proposed by Markowitz (1952) laid the foundation for the modern portfolio theory, where the return of each risky asset is assumed as random variable. Inspired by this pioneering model, many scholars have studied the portfolio problems by using the expected value and variance of portfolio to measure the investment return and risk, respectively, e.g., Sharpe (1964), Merton (1972), Perold (1984), Best and Hlouskova (2000). Considering the deficiency of variance which views the volatility of high return as risk, Markowitz (1959) proposed the semi-variance, which measures the risk by only taking the volatility of return below the expected value into account. In addition, lower partial moment is also used for measuring the risk, many scholars regarded that this risk measure can characterize investors' perceived levels of risk more properly, such as Harlow and Rao (1989), Chow and Denning (1994) and Jarrow and Zhao (2006).

The above studies assumed the returns of risky assets are random variables and use the historical data to estimate their probability distribution. However, in the real financial market, there exists a lot of non-random factors such as expert opinions, political information and confidence level, they are often occurring with linguistic descriptions such as high risk, low degree and high level. In this situation, it is impossible for investors to get the precise value of the returns of risky assets which include lots of fuzzy uncertainty. As a powerful tool for describing the fuzzy uncertainty of the real event, the fuzzy set theory originally proposed by Zadeh (1965). Inspired by the idea of the fuzzy set theory, numerous researchers have applied it to study the fuzzy portfolio selection problem, they assumed the return of risky assets are fuzzy numbers and constructed the corresponding optimization model. Assuming the return of each risky asset is trapezoidal fuzzy number, Deng and Li (2012) proposed a fuzzy portfolio selection model with borrowing constraint. Using the turnover rates of risky assets to measure the liquidity of the portfolio, Barak et al. (2013) proposed a fuzzy mean-variance-skewness portfolio selection model with liquidity constraint and cardinality constraint. Chen et al. (2019) considered transaction costs, liquidity, buy-in thresholds and cardinality constraints, proposed a fuzzy mean-semivariance-entropy portfolio selection model, and designed a hybrid multiobjective bat algorithm to solve the proposed model. By incorporating fuzzy return rates and background risk, He and Lu (2021) proposed a fuzzy porfolio model with cardinality constraint, and designed an improved quantum-behaved particle swarm optimization algorithm to solve the model.

In the real investment process, investors often need to adjust the wealth allocated in assets. Therefore, it's reasonable to study the portfolio selection problem in multi-period setting. Sadjadi et al. (2011) proposed a multi-period fuzzy portfolio selection optimization model considering borrowing rate and lending rate. Using proportion entropy to measure the diversification of portfolio, Zhang et al. (2012) proposed a multi-period fuzzy mean-semi-variance-entropy portfolio selection optimization model with transaction cost. Zhang et al. (2014) proposed the concept of fuzzy semi-deviation to measure the risk of portfolio, constructed a multi-period fuzzy portfolio selection optimization model with diversification and boundary constraints. Considering the investment horizon may vary with different assets, Guo et al. (2016) proposed a multi-period fuzzy portfolio selection optimization model with different investment horizon and total risk constraint, where the objective is to maximize the final expected return of the portfolio. Mohebbi and Najafi (2018) used scenario tree to characterize the uncertainty of financial market, introduced some realistic constraints such as cardinality, liquidity and boundary constraint, then proposed a multi-period fuzzy portfolio selection optimization model with transaction cost. Yang et al. (2021) used expected value and semi-deviation to measure the return and risk of the single asset, respectively, introduced a diversification constraint involving risk-free asset, and proposed a multi-period fuzzy portfolio selection optimization model with the objectives of maximizing the final expected wealth and minimizing the cumulative risk.

In practice, investors are usually bounded rational due to the cognitive factors such as sentiment, confidence level and risk attitude. Using the value function to characterize decision makers' behavioral preferences such as loss aversion and reference dependence, Kahneman and Tversky (1979) proposed the prospect theory. Inspired by this pioneering work, numerous researchers introduced the prospect theory to the portfolio selection problem, and studied the influences of investors' behavioral preferences in the investment decision. Using the linear value function to characterize investors' loss aversion preference, Fortin and Hlouskova (2011) proposed a portfolio selection optimization model with the objective of maximizing the loss aversion utility. Considering investors' subjective preferences in the mean-variance framework, Fulga (2016) proposed a loss aversion based downside risk measure, and constructed a portfolio selection optimization model with the objectives of maximizing the expected return and minimizing the downside risk. Song et al. (2017) considered investors' loss aversion preferences in the continuous-time investment and consumption process, introduced the downside consumption constraint, proposed a continuous portfolio selection optimization model with maximizing the total discounted S-shaped utility from consumption. Wang et al. (2018) considered investors' loss aversion preferences in dynamic scenario, introduced the conditional value at risk (CVaR) constraint, and constructed a portfolio selection model with the objective of maximizing the loss aversion utility. Liu and Zhang (2021) considered the case that investors' loss aversion degrees are relevant to the excess return, introduced the cardinality and boundary constraints, proposed a multi-period fuzzy portfolio selection optimization model with the objective of maximizing the time vary loss aversion utility.

There are three points that encourage us to conduct this study. First, investors are often provided information with fuzzy uncertainty when making optimal decisions in multi-period investment process. Second, considering the bounded-rational psychological states of investors under uncertainty, we introduce the prospect theory to reflect their loss aversion. Third, due to the fact that investors' attitude toward risk vary with the different loss aversion, we propose an alternative methodology based on the perceived value for measuring risk, namely perceived risk. We summarize the main contributions of this study to the current literature as follows:

- A multi-period fuzzy portfolio optimization model considering investors' loss aversion is proposed.
- A new method is used to transform the portfolio return rate into perceived value by the value function in prospect theory.
- A risk measure is proposed to characterize investors' loss aversion based on the perceived value.
- A multiple particle swarm optimization algorithm is designed to solve the proposed model.

The remainder of this paper is organized as follows. Section 2 introduces some preliminaries. Section 3 establishes a multi-period fuzzy portfolio optimization model considering investors' loss aversion. Section 4 designs a multiple particle swarm optimization algorithm to solve the proposed model. Section 5 conducts a real case study to illustrate the effectiveness of the proposed model and algorithm. Section 6 draws the conclusions of this paper.

2 Preliminaries

In this section, we introduce some preliminaries, which will be used in the following discussion.

2.1 Fuzzy number

Definition 1 (Zadeh, 1965). A fuzzy number $\tilde{A}$ is said to be an LR-power fuzzy number if its membership function has the following form

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{a-x}{\alpha}\right), & a-\alpha < x < a, \\ 1, & a \le x \le b, \\ R\left(\frac{x-b}{\beta}\right), & b < x < b+\beta, \\ 0, & otherwise, \end{cases} \tag{1}$$

where $\alpha, \beta \ge 0$ represent the left and right spreads, respectively; $L, R : [0,1] \to [0,1]$ are the left and right functions, $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$. Denote it by $\tilde{A} = (a, b, \alpha, \beta)_{LR}$.

Lemma 1 (Dubois and Prade, 1980). Let $\tilde{A}_1 = (a_1, b_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (a_2, b_2, \alpha_2, \beta_2)$ are two trapezoidal fuzzy numbers, and l be a real number. Then

$$\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2), \tag{2}$$

$$l\tilde{A}_1 = \begin{cases} (la_1, lb_1, l\alpha_1, l\beta_1), & l \ge 0, \\ (lb_1, la_1, -l\beta_1, -l\alpha_1), & l < 0. \end{cases} \tag{3}$$

Definition 2 (Carlsson and Fullér, 2001). Let $\tilde{A}$ be a fuzzy number with membership function $\mu_{\tilde{A}}(\cdot)$, $\gamma \in (0,1]$ be a real number. Define the $\gamma-$level set of $\tilde{A}$ by

$$[\tilde{A}]^{\gamma} = \{x \in \mathrm{R} : \mu_{\tilde{A}}(x) \ge \gamma\}. \tag{4}$$

Particularly, if $\tilde{A}$ is an LR-power fuzzy number, its $\gamma-$level set is an inverval, denote it by $[\tilde{A}]^{\gamma} = [\underline{a}(\gamma), \bar{a}(\gamma)]$.

Definition 3 (Carlsson and Fullér, 2001). Let $\tilde{A}$ be a fuzzy number with $\gamma-$level set $[\tilde{A}]^{\gamma} = [\underline{a}(\gamma), \bar{a}(\gamma)]$, $\gamma \in (0,1]$. Define its possibilistic mean value and lower possibilistic semi-variance by

$$E(\tilde{A}) = \int_0^1 \gamma(\underline{a}(\gamma) + \bar{a}(\gamma))d\gamma, \tag{5}$$

$$SV(\tilde{A}) = 2\int_0^1 \gamma(E(\tilde{A}) - \underline{a}(\gamma))^2 d\gamma. \tag{6}$$

Definition 4 (Zadeh, 1965). Let $\tilde{A}$ be a fuzzy number with membership function $\mu_{\tilde{A}}(\cdot)$, $\phi : \mathrm{R} \to \mathrm{R}$ be a function. Define a fuzzy number $\tilde{B}$ and its membership function by

$$\mu_{\tilde{B}}(y) = \max_{\{x:\ \phi(x)=y\}} \mu_{\tilde{A}}(x). \tag{7}$$

2.2 Prospect theory

Prospect theory, initially proposed by Kahneman and Tversky (1979), is a descriptive theory for decision makers' bounded rational behaviors under uncertainty. In this pioneering theory, there are three fundamental principles, which describe decision makers' behavioral preferences. The first one is reference dependence, by using a presetting wealth level, decision makers divide the wealth result into two areas, i.e., gain area and loss area, which reflect whether the wealth result is better or worse than the presetting wealth level. The second one is loss aversion, decision makers put a higher value on the loss than on the gain when assessing the wealth result. Third one is diminishing sensitivity, decision makers are risk-averse in the area of gain, while are risk-seeking in the area of loss.

Under the assumption that the decision maker is more focus on the relative wealth amount of gain or loss than the absolute amount of the final wealth, prospect theory captured the decision maker's behavioral preferences by the value function, which is as follows:

$$v(r) = \begin{cases} (r - r_{\mathrm{b}})^{\delta_1}, & r \ge r_{\mathrm{b}}, \\ -\theta(r_{\mathrm{b}} - r)^{\delta_2}, & r < r_{\mathrm{b}}, \end{cases} \tag{8}$$

where r represents the final wealth, r_{b} is the parameter that represents the presetting wealth level, which reflects the decision makers' reference dependence, $0 < \delta_1, \delta_2 < 1$ are the parameters that represent the decision makers' decreasing sensitivity degree in gain area and loss area, respectively, $\theta > 1$ is the parameter that represents the decision makers' loss aversion degree. A general case of the value function is shown in Fig. 1.

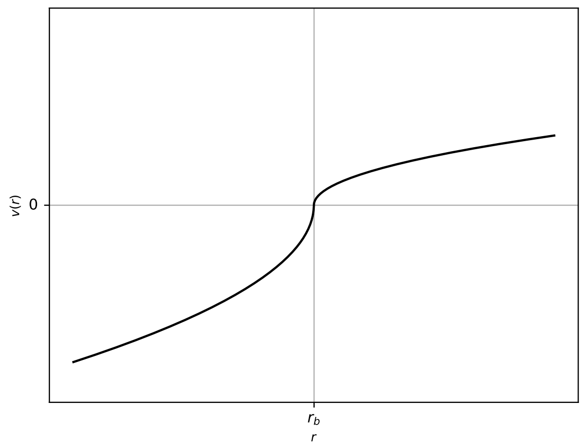


Fig. 1 The value function $v(r)$

For the influence of loss aversion, different investors may have different risk attitudes toward the same uncertain return of a portfolio. Therefore, an adjustable risk measure should be developed to suit varying loss aversion. In order to incorporate the loss aversion of investors, we define the following perceived risk based on the perceived value.

Definition 5 Let $\tilde{A}$ be a fuzzy number, its $\gamma-$level set is $[\tilde{A}]^{\gamma} = [\underline{a}(\gamma), \bar{a}(\gamma)]$, $\gamma \in (0,1]$. Denote the perceived value of $\tilde{A}$ by $\tilde{V} = v(\tilde{A})$ with $\gamma-$level set $[\tilde{V}]^{\gamma} = [\underline{q}(\gamma), \bar{q}(\gamma)]$, $\gamma \in (0,1]$. Define the perceived risk of $\tilde{A}$ by

$$PR\left(\tilde{V}\right) = \int_0^1 \gamma \left(\min\left\{\underline{q}(\gamma), 0\right\}\right)^2 d\gamma. \tag{9}$$

3 Modeling

In this section, we aim to construct a multi-period fuzzy portfolio selection optimization model with investors' loss aversion.

3.1 Problem description and notations

We consider a portfolio optimization problem with several risky assets. Assume that the investor with initial wealth W_0 gets into the financial market at the beginning of the first period horizon, he/she intends to invest the wealth among n risky assets and get the final wealth after T periods. He/she can adjust the proportion of wealth invested in each risky asset at the beginning of each period. We formulate the portfolio optimization model under fuzzy uncertainty, in which the return rate of each risky asset is regarded as trapezoidal fuzzy number. We consider investors' loss aversion, which means the investor usually presets the reference level of return rate according to his/her subjective judgement, and puts a higher value on the loss than on the gain. In fact, we only care about whether portfolio selection is affected by his/her loss aversion under the fuzzy uncertainty environment. Specifically, we characterize investors' loss aversion through two aspects: one is transforming the portfolio return rate into the perceived value by the value function in prospect theory; the other is using the perceived risk measure to capture investors' attitude toward risk under different loss aversion degrees.

We introduce the following notations for convenient discussion.

$\tilde{r}_{t,i}$: the fuzzy return rate of asset i during period t, which is represented by a trapezoidal fuzzy number, i.e., $\tilde{r}_{t,i} = (a_{t,i}, b_{t,i}, \alpha_{t,i}, \beta_{t,i})$;

$x_{t,i}$: the investment proportion of asset i during period t;

$\boldsymbol{x_t}$: the portfolio of period t, which is represented by $\boldsymbol{x_t} = (x_{t,1}, x_{t,2}, \cdots, x_{t,n})$;

$\mathbf{x}$: the investment strategy for T periods, where $\mathbf{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_T)$;

c^+: the transaction cost rate of purchasing risky asset;

c^-: the transaction cost rate of selling risky asset.

Transaction cost, which is paid by the investor when adjusting the portfolio, has a significant impact on the investment decision making. The investor need to pay transaction cost when purchasing risky assets or selling risky assets. Denote the adjustment of the purchasing and selling proportion of risky asset i at period t by $\Delta x^+_{t,i} = \max\{x_{t,i} - x_{t-1,i}, 0\}$ and $\Delta x^-_{t,i} = \max\{x_{t-1,i} - x_{t,i}, 0\}$, respectively. Then the amount of transaction cost paid at period t is

$$c_t = \sum_{i=1}^{n} \left(c^+ \Delta x^+_{t,i} + c^- \Delta x^-_{t,i}\right). \tag{10}$$

Then the return rate of the portfolio at period t is

$$\tilde{R}_t = \sum_{i=1}^{n} x_{t,i}\tilde{r}_{t,i} - c_t. \tag{11}$$

Denote $a_t = \sum_{i=1}^{n} x_{t,i}a_{t,i} - c_t,\ b_t = \sum_{i=1}^{n} x_{t,i}b_{t,i} - c_t,\ \alpha_t = \sum_{i=1}^{n} x_{t,i}\alpha_{t,i},\ \beta_t = \sum_{i=1}^{n} x_{t,i}\beta_{t,i}$. Then the return rate of the portfolio at period t is represented by

$$\tilde{R}_t = (a_t, b_t, \alpha_t, \beta_t)\,. \tag{12}$$

The expected wealth of the portfolio at the end of period t is

$$\begin{aligned} W_t =& W_{t-1}(1 + E(\tilde{R}_t)) \\ =& W_{t-1}\left(1 + \frac{a_t + b_t}{2} + \frac{\beta_t - \alpha_t}{6}\right. \\ &\left. - \sum_{i=1}^{n}\left(c_{t,i}^{+}\Delta x_{t,i}^{+} + c_{t,i}^{-}\Delta x_{t,i}^{-}\right)\right), \end{aligned} \tag{13}$$

and the terminal cumulative wealth of the investment strategy is

$$\begin{aligned} CW_T =& W_0\prod_{t=1}^{T}\left(1 + \frac{a_t + b_t}{2} + \frac{\beta_t - \alpha_t}{6}\right. \\ &\left. - \sum_{i=1}^{n}\left(c_{t,i}^{+}\Delta x_{t,i}^{+} + c_{t,i}^{-}\Delta x_{t,i}^{-}\right)\right). \end{aligned} \tag{14}$$

The semi-variance of the portfolio at period t is

$$SV(\tilde{R}_t) = \left(\frac{b_t - a_t}{2} + \frac{\alpha_t + \beta_t}{6}\right)^2 + \frac{\alpha_t^2}{18}, \tag{15}$$

and the terminal cumulative semi-variance of the investment strategy is

$$\begin{aligned} CR_T &= \sum_{t=1}^{T} SV(\tilde{R}_t) \\ &= \sum_{t=1}^{T}\left(\frac{b_t - a_t}{2} + \frac{\alpha_t + \beta_t}{6}\right)^2 + \frac{\alpha_t^2}{18}. \end{aligned} \tag{16}$$

3.2 Objective function

Most fuzzy portfolio optimization models are proposed with the objectives of maximizing the terminal cumulative wealth CW_T and minimizing the terminal cumulative semi-variance of the investment strategy CR_T. The objectives of maximizing CW_T and minimizing CR_T can't reflect investors' loss aversion during the investment process. However, the investor's portfolio decision making may be significantly affected by the loss aversion. Specifically, investor's psychological perceived value toward the portfolio may be vary with different loss aversion degree, then he/she may make different portfolio selection due to the varying satisfaction degree. Therefore, we consider the objectives of maximizing the terminal cumulative expected perceived value $CEPV_T$ and minimizing the terminal cumulative perceived risk CPR_T to reflect the loss aversion of different investors, instead of taking the maximization of CW_T and the minimization of CR_T.

Given a return rate $\tilde{R}_t$, we generate its expected perceived value by the following two steps. Firstly, let $\tilde{V}_t$ be the fuzzy number, which is transformed by $\tilde{R}_t$ using the value function $v(\cdot)$ introduced in Section 2.2. Denote it by $\tilde{V}_t = v(\tilde{R}_t)$, which is called the perceived value of $\tilde{R}_t$. Since $\tilde{R}_t$ is a fuzzy number, from Definition 4, the perceived value $\tilde{V}_t$ is also a fuzzy number, its membership function can be represented by

$$\mu_{\tilde{V}_t}(x) = \begin{cases} \mu_{\tilde{R}_t}\left(r_{\mathrm{b}} + x^{\frac{1}{\delta_1}}\right), & x \ge 0, \\ \mu_{\tilde{R}_t}\left(r_{\mathrm{b}} - \left(-\frac{x}{\theta}\right)^{\frac{1}{\delta_2}}\right), & x < 0. \end{cases} \tag{17}$$

Then, from Definition 3 and the above membership function, we can calculate the expected value of the perceived value $\tilde{V}_t$, denote it by EPV_t, which represents the expected perceived value of the portfolio at period t. We only discuss the most common cases while $r_{\mathrm{b}} \in [a_t - \alpha_t, b_t + \beta_t]$, where the expected perceived value EPV_t can be calculated as follows

(1) If $a_t - \alpha_t \le r_{\mathrm{b}} < a_t$, we have

$$\begin{aligned} EPV_t = & \frac{(b_t + \beta_t - r_{\mathrm{b}})^{\delta_1+2}}{\beta_t^2(\delta_1^2 + 3\delta_1 + 2)} \\ & - \frac{\theta[(r_{\mathrm{b}} - a_t + \alpha_t)^{\delta_2+2}]}{\alpha_t^2(\delta_2^2 + 3\delta_2 + 2)} \\ & + \frac{(a_t - r_{\mathrm{b}})^{\delta_1+1}(r_{\mathrm{b}} - a_t + 2\alpha_t + \alpha_t\delta_1)}{\alpha_t^2(\delta_1^2 + 3\delta_1 + 2)} \\ & - \frac{(b_t - r_{\mathrm{b}})^{\delta_1+1}(b_{\mathrm{N}} - r_{\mathrm{b}} + 2\beta_t + \beta_t\delta_1)}{\beta_t^2(\delta_1^2 + 3\delta_1 + 2)}. \end{aligned} \tag{18}$$

(2) If $a_t \le r_{\mathrm{b}} \le b_t$, we have

$$\begin{aligned} EPV_t = & \frac{(b_t + \beta_t - r_{\mathrm{b}})^{\delta_1+2}}{\beta_t^2(\delta_1^2 + 3\delta_1 + 2)} \\ & - \frac{\theta(r_{\mathrm{b}} - a_t + \alpha_t)^{\delta_2+2}}{\alpha_t^2(\delta_2^2 + 3\delta_2 + 2)} \\ & - \frac{(b_t - r_{\mathrm{b}})^{\delta_1+1}(b_{\mathrm{N}} - r_{\mathrm{b}} + 2\beta_t + \beta_t\delta_1)}{\beta_t^2(\delta_1^2 + 3\delta_1 + 2)} \\ & + \frac{\theta(r_{\mathrm{b}} - a_t)^{\delta_2+1}(r_{\mathrm{b}} - a_t + 2\alpha_t + \alpha_t\delta_2)}{\alpha_t^2(\delta_2^2 + 3\delta_2 + 2)}. \end{aligned} \tag{19}$$

(3) If $b_t < r_{\mathrm{b}} \le b_t + \beta_t$, we have

$$
\begin{aligned}
EPV_t = & \frac{(b_t+\beta_t-r_{\mathrm{b}})^{\delta_1+2}}{\beta_t^2(\delta_1^2+3\delta_1+2)} \\
& - \frac{\theta(r_{\mathrm{b}}-a_t+\alpha_t)^{\delta_2+2}}{\alpha_t^2(\delta_2^2+3\delta_2+2)} \\
& + \frac{\theta(r_{\mathrm{b}}-a_t)^{\delta_2+1}(r_{\mathrm{b}}-a_t+2\alpha_t+\alpha_t\delta_2)}{\alpha_t^2(\delta_2^2+3\delta_2+2)} \\
& - \frac{\theta(r_{\mathrm{b}}-b_t)^{\delta_2+1}(b_t+2\beta_t-r_{\mathrm{b}}+\beta_t\delta_2)}{\beta_t^2(\delta_2^2+3\delta_2+2)}.
\end{aligned}
\tag{20}
$$

According to the above discussion, the terminal cumulative expected perceived value of the investment strategy is

$$
CEPV_T = \sum_{t=1}^{T} EPV_t. \tag{21}
$$

From Definition 5, we can calculate the perceived risk of the portfolio at period t by
(1) If $r_{\mathrm{b}} \in (a_t-\alpha_t,\ a_t]$, we have

$$
PR_t\left(\tilde{V}_t\right) = \frac{\theta^2(r_{\mathrm{b}}-a_t+\alpha_t)^{2\delta_2+2}}{\alpha_t^2(4\delta_2^2+6\delta_2+2)}. \tag{22}
$$

(2) If $r_{\mathrm{b}} \in (a_t,\ +\infty]$, we have

$$
\begin{aligned}
PR_t\left(\tilde{V}_t\right) = & \frac{\theta^2[(r_{\mathrm{b}}-a_t+\alpha_t)^{2\delta_2+2}}{\alpha_t^2(4\delta_2^2+6\delta_2+2)} \\
& - \frac{(r_{\mathrm{b}}-a_t)^{2\delta_2+1}(r_{\mathrm{b}}-a_t+2\alpha_t+2\alpha_t\delta_2)]}{\alpha_t^2(4\delta_2^2+6\delta_2+2)}.
\end{aligned}
\tag{23}
$$

Then, the terminal cumulative perceived risk of the investment strategy is

$$
CPR_T = \sum_{t=1}^{T} PR_t\left(\tilde{V}_t\right). \tag{24}
$$

We use an example to explain the relation between the loss aversion degree and the investor's decision preference. Consider two portfolios with trapezoidal fuzzy return rates $\tilde{R}_1$ and $\tilde{R}_2$, where $\tilde{R}_1 = (0.2,\ 0.3,\ 0.3,\ 0.6)$ and $\tilde{R}_2 = (0.3,\ 0.4,\ 0.5,\ 0.1)$, as shown in Fig. 2. Then, set $\delta_1 = \delta_2 = 0.88$, $r_{\mathrm{b}} = 0.20$, and plot the expected perceived value and perceived risk of the two portfolios with different θ, respectively, as shown in Fig. 3. We can see from Fig. 3(a) that the expected perceived value of Portfolio 1 is higher than that of Portfolio 2 when $\theta < 1.75$, while it is lower than that of Portfolio 2 when $\theta > 1.75$, and from Fig. 3(b) that the perceived risk of Portfolio 1 is always lower than that of Portfolio 2. Then, we find that the investor prefers Portfolio 1 to Portfolio 2 when $\theta < 1.75$, while he/she may have different selections when $\theta > 1.75$. This indicates that the investor's attitude toward portfolios varies with the loss aversion degree θ.

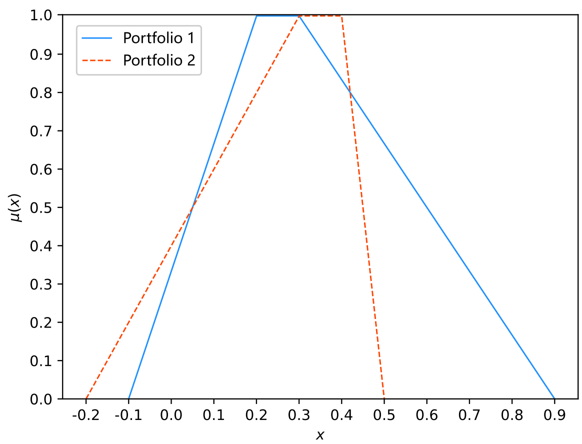


Fig. 2 The membership functions of the two portfolios' fuzzy return rates

To sum up, we can characterize different investors' loss aversion θ. The investors with higher loss aversion degree get lower perceived value and higher perceived risk from the fuzzy return rates of the portfolio. Therefore, by considering the objectives of maximizing the terminal cumulative expected perceived value $CEPV_T$ and minimizing the terminal cumulative perceived risk CPR_T, investors can choose the appropriate parameter θ according to their loss aversion degree, and generate the optimal investment strategy.

3.3 Realistic constraints

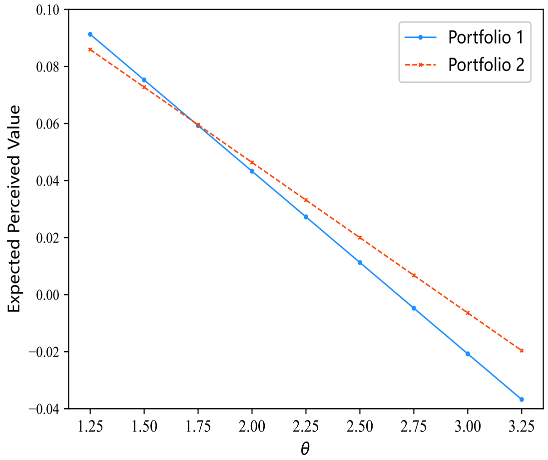
To control the non-systematic risk borrowed from the wealth over-concentration, investors usually more willing to take the diversify portfolio than the centralize portfolio. In this study, we use the proportion entropy to measure the portfolio's diversification. Then, the diversification constraint at period t can be represented by

$$
-\sum_{i=1}^{n} x_{t,i} \ln x_{t,i} \ge e. \tag{25}
$$

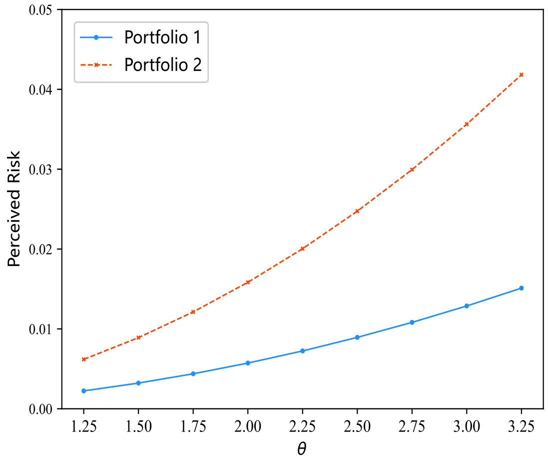
We consider the case of not short selling proportion. Then the boundary constraint can be represented by

$$
x_{t,i} \ge 0,\ i = 1, 2, \cdots, n, t = 1, 2, \cdots, T. \tag{26}
$$

In addition, the budget constraint at period t can be represented by



(a) Expected perceived value



(b) Perceived risk

Fig. 3 The expected perceived value and perceived risk of the two portfolios with different θ

$$\sum_{i=1}^{n} x_{t,i} = 1. \tag{27}$$

3.4 Portfolio optimization model

Assume that the investor aims to maximize the terminal cumulative expected perceived value and to minimize the terminal cumulative perceived risk. Assume the short-selling is not allowed in the whole investment process and the investor is self-financing. Meanwhile, assume the investor requires the portfolio to satisfy the diversification degree at each period. Then, the multi-period portfolio optimization model can be established as

$$P_1 \begin{cases} \max\ CEPV_T \\ \min\ CPR_T \\ \text{s.t.} \\ \sum_{i=1}^{n} x_{t,i} = 1, \\ -\sum_{i=1}^{n} x_{t,i} \ln x_{t,i} \geq e, \\ x_{t,i} \geq 0,\ i = 1, 2, \cdots, n, t = 1, 2, \cdots, T. \end{cases}$$

Considering that P_1 is a bi-objective model, we transform it into the following single objective model by using the weighted max-min fuzzy programing method proposed by Lin (2004)

$$P_2 \begin{cases} \max\ \lambda \\ \text{s.t.} \\ \dfrac{CEPV_T - CEPV^-}{CEPV^+ - CEPV^-} \geq \omega\lambda, \\ \dfrac{CPR^- - CPR_T}{CPR^- - CPR^+} \geq (1-\omega)\lambda, \\ \sum_{i=1}^{n} x_{t,i} = 1, \\ -\sum_{i=1}^{n} x_{t,i} \ln x_{t,i} \geq e, \\ x_{t,i} \geq 0,\ i = 1, 2, \cdots, n, t = 1, 2, \cdots, T, \end{cases}$$

where $CEPV^+$ and $CEPV^-$ are the ideal solution and the anti-ideal solution of the cumulative expected perceived value, respectively; CPR^+ and CPR^- are the ideal solution and the anti-ideal solution of the cumulative perceived risk, respectively; ω and $1-\omega$ are the objective weight of $CEPV_T$ and the objective weight of CPR_T, respectively. We can obtain a pareto optimal solution of model P_1 by solving the above model P_2 under a certain value of ω. Hence, the pareto frontier of model P_1 can be obtained by solving model P_2 with ω ranging over $[0, 1]$.

4 Multiple particle swarm optimization

This section designs a multiple particle swarm optimization (MPSO) to solve a generic form of model P_2, which is as follows

$$P_3 \begin{cases} \max f(\mathbf{x}) \\ \text{s.t.} \\ g_k(\mathbf{x}) \le 0, k = 1, 2, \cdots, m, \\ \sum_{i=1}^{n} x_{t,i} = 1, t = 1, 2, \cdots, T, \\ x_{t,i} \ge 0, \ i = 1, 2, \cdots, n, t = 1, 2, \cdots, T. \end{cases}$$

Particle swarm optimization (PSO), based on the idea of the collaborative process of the flock of birds searching for food, is a heuristic algorithm proposed by Kennedy and Eberhart (1995). In PSO, a swarm is formed by a group of particles, each particle searches for the optimal solution and exchanges the information with other particles in the search space. The global optimal position of the current swarm is obtained by comparing all of the particles, and the personal optimal position of each particle is represented by its historical optimal position. The above two positions provide the social learning information and self-learning information for each particle in the swarm to adjust its position in the iterative process. The social learning ability and self-learning ability can improve the swarm's global exploration ability and the particle's local exploitation ability, respectively. Since it is difficult to combine the two kinds of abilities, PSO may suffer some deficiencies, such as parameter sensitivity and premature convergence. Then, some researchers, such as Niu et at. (2007) and Ma et at. (2018), introduced multi-swarm cooperative scheme to improve the performance of PSO.

Next, we propose a multiple particle swarm optimization, in which the population is divided into a leader swarm and several subsidiary swarms. The self-learning rates and social learning rates vary with different swarms. When the particle is in the subsidiary swarm, it adjusts its position based on the optimal particle information of the subsidiary swarm, while each particle of the leader swarm adjusts its position based on the optimal particle information of the leader swarm and all subsidiary swarms. Through the above collaborative approach, the leader swarm can obtain additional reference information from the subsidiary swarms, thus reducing the possibility of converging to a local optimal solution. In addition, we introduce chaos initialization, which can help to expand search space and improve population stability.

4.1 Encoding and decoding

A solution $\mathbf{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_T)$ of model P_3 is encoded by an nT-dimension position vector, which is $\mathbf{P} = (p_{1,1}, \cdots, p_{1,n}, \cdots, p_{T,1}, \cdots, p_{T,n})$ that represents a particle, where $p_{t,i} \in [0, 1]$. Define the search space by

$$SP = \{\mathbf{P} = (p_{1,1}, \cdots, p_{1,n}, \cdots, p_{T,1}, \cdots, p_{T,n}) : \\ 0 \le p_{t,i} \le 1, t = 1, 2, \cdots, T, i = 1, 2, \cdots, n\}. \tag{28}$$

For a position vector $\mathbf{P} \in SP$, the actual investment proportion of asset i at period t is decoded by

$$x_{t,i} = \frac{p_{t,i}}{\sum_{i=1}^{n} p_{t,i}}, \ t = 1, 2 \cdots, T. \tag{29}$$

It can be seen that the corresponding solution x obtained by any particle in the search space satisfies constraint $\sum_{i=1}^{n} x_{t,i} = 1$ and constraint $x_{t,i} \ge 0$.

4.2 Chaos initialization

Denote the number of the swarms by L, the number of particles in each swarm by NP, where the swarms $1, 2, \cdots, L-1$ are subsidiary swarms, and the swarm L is the leader swarm. Denote the maximum number of iterations by $G_{\max}$. Denote the upper limit for the velocity of the particles by $v_{\max}$. Denote the maximum and minimum of the inertia weight are $w_{\max}$ and $w_{\min}$, respectively.

In this subsection, we use the following chaos mapping method to intialize each population

$$p_{t,i}^{s+1} = 4p_{t,i}^{s}(1 - p_{t,i}^{s}). \tag{30}$$

Specifically, in the search space, we randomly generate a particle $P^0 = (p_{1,1}^0, \cdots, p_{1,n}^0, \cdots, p_{T,1}^0, \cdots, p_{T,n}^0)$, then the above chaos mapping method is used to generate a new particle, and the same operation is performed on the newly generated particle to continue generating the next new particle, repeating this process $NP-1$ times to generate NP particles.

4.3 Fitness

We adapt the following penalty function to handle the constraint $g_k(\mathbf{x}) \le 0$. For the k-th constraint, the penalty value is

$$p_k = \max\{g_k(\mathbf{x}), \ 0\}. \tag{31}$$

The total penalty value of the solution $\mathbf{x}$ is

$$p(\mathbf{x}) = \sum_{k=1}^{m} p_k. \tag{32}$$

If $\mathbf{x}$ satisfies the constraint $g_k(\mathbf{x}) \leq 0$, $p(\mathbf{x}) = 0$; otherwise, $p(\mathbf{x}) > 0$.

We design the following fitness function to evaluate each particle in the search space

$$F(\mathbf{P}) = \exp\left[f(\mathbf{x}) - M \cdot p(\mathbf{x})\right]. \tag{33}$$

where $f(\mathbf{x})$ is the objective function value of the solution $\mathbf{x}$, M is a positive number that sufficiently large. It can be seen that in a series of particles that satisfy the constraint $g_k(\mathbf{x}) \leq 0$, the higher the objective function value of a particle, the higher its fitness; while if a particle is not satisfying the constraint $g_k(\mathbf{x}) \leq 0$, its fitness is lower than any particle that satisfies the constraint $g_k(\mathbf{x}) \leq 0$.

4.4 Update process

Denote the position and velocity of particle j of swarm l in the current generation g by $P_j^l(g)$ and $V_j^l(g)$, respectively. Denote the best previous position of particle j, which represents the personal optimal position, by $Pbest_j^l(g)$, i.e.,

$$F(Pbest_j^l(g)) = \max_{1 \leq g' \leq g} \left\{F(P_j^l(g'))\right\}. \tag{34}$$

For the global optimal position of subsidiary swarm $l(l = 1, \cdots, L-1)$, denote it by $Gbest^l(g)$, i.e.,

$$F(Gbest^l(g)) = \max_{1 \leq j \leq NP} \left\{F(Pbest_j^l(g))\right\}. \tag{35}$$

For the global optimal position of leader swarm L, denote it by $Gbest^L(g)$, i.e.,

$$\begin{aligned} F(Gbest^L(g)) &= \max_{\substack{1 \leq j \leq NP \\ 1 \leq l \leq L}} \left\{F(Pbest_j^l(g))\right\} \\ &= \max_{\substack{1 \leq j \leq NP \\ 1 \leq l \leq L-1}} \left\{F(Gbest^l(g)), F(Pbest_j^L(g))\right\}. \end{aligned} \tag{36}$$

Then, the following equations are used to update the velocity and position of each particle at each iteration

$$\begin{cases} V_j^l(g+1) = wV_j^l(g) + c_1^l r_1 \left[Pbest_j^L(g) - P_j^l(g)\right] \\ \qquad\qquad + c_2^l r_2 \left[Gbest^l(g) - P_j^l(g)\right], \\ P_j^l(g+1) = j^l(g) + V_j^l(g+1), \end{cases} \tag{37}$$

where $w = w_{\max} - (w_{\max} - w_{\min}) \cdot g / G_{\max}$ is the inertia weight proposed by Tripathi (2007); c_1^l and c_2^l are the self-learning rate and social learning rate of swarm l, respectively; r_1 and r_2 are two numbers that randomly generated from interval $[0, 1]$.

4.5 Main procedure

The main procedure of the MPSO is summarized as follows

Step 1 Set parameters used in the algorithm including L, NP, $v_{\max}$, $w_{\max}$, $w_{\min}$, c_1^l, c_2^l, $G_{\max}$.

Step 2 Let $g = 1$, and generate a series of particles for each swarm, where the initial positions of the particles in each swarm are generated by the chaos mapping method, and the velocity of each particle is randomly generated on interval $[-v_{\max}, v_{\max}]$.

Step 3 Calculate the fitness of the position of each particle, and update the personal optimal position and the global optimal position.

Step 4 Update the velocity of each particle. If an element of the velocity matrices is excluded from the interval $[-v_{\max}, v_{\max}]$, replace it by the closer interval endpoint.

Step 5 Update the position of each particle. If an element of the position vector is excluded from interval $[0, 1]$, replace it by the closer interval endpoint.

Step 6 If $g = G_{\max}$, quit the iteration and report corresponding solution decoded by $Gbest^L(g)$; otherwise, let $g \leftarrow g + 1$, and return to Step 3.

5 Real case studies

In this section, we give a real case study to illustrate the practicability and effectiveness of the proposed model and algorithm. We assume the investor plans to invest his/her wealth in five stocks from Shanghai Stock Exchange, i.e., A_1(600004), A_2(600008), A_3(600011), A_4(600017), A_5(600023). We collect the weekly historical data of the above stocks over six years, i.e., January 1st, 2016 to December 31st, 2021, and divide it into three periods evenly. We adopt the method proposed by Mehlawat (2016) to estimate the weekly fuzzy return rates of the stocks at each period, as shown in Table 1.

Assume the initial wealth is $W_0 = 10000$, and the transaction cost rates of purchasing and selling the risky assets are $c_{t,i}^+ = 0.003$ and $c_{t,i}^- = 0.004$, respectively. Assume the reference level of the return rate is $r_{\mathrm{b}} = 0.02$, the decreasing sensitivity degree is $\delta_1 = \delta_2 = 0.5$, the loss aversion degree is $\theta = 1.1$, and the diversification level is $e = 1.13$ except sensitivity analysis.

In the proposed algorithm MPSO, the parameters are set as follows: the total number of swarms is $L = 6$; the number of particles in each swarm is $NP = 300$; the upper velocity of each particle is $v_{\max} = 3$; the maximum and minimum inertia weights are $w_{\max} = 0.7$ and $w_{\min} = 0.3$, respectively; the self-learning rate and social learning rate are $c_1^l = 3$, 2.5, 2, 1.5, 1, 2 and $c_2^l =$

Table 1 The fuzzy return rates of the five risky assets on the three periods

Period	Assets	Fuzzy weekly return rates
$t = 1$	A_1	(-0.0120, 0.0532, 0.5055, 0.3671)
	A_2	(-0.0286, 0.2359, 0.6412, 0.6707)
	A_3	(-0.0169, 0.0379, 0.3550, 0.3552)
	A_4	(-0.0192, 0.0508, 0.3971, 0.3781)
	A_5	(-0.0128, 0.0308, 0.3854, 0.3429)
$t = 2$	A_1	(-0.0286, 0.0716, 0.4328, 0.3838)
	A_2	(-0.0239, 0.0492, 0.3851, 0.3787)
	A_3	(-0.0214, 0.0665, 0.3594, 0.3958)
	A_4	(-0.0242, 0.0402, 0.3633, 0.3551)
	A_5	(-0.0178, 0.0535, 0.3389, 0.3752)
$t = 3$	A_1	(-0.0337, 0.0736, 0.3579, 0.4066)
	A_2	(-0.0192, 0.0491, 0.3697, 0.3669)
	A_3	(-0.0335, 0.1801, 0.3809, 0.5849)
	A_4	(-0.0141, 0.0710, 0.3536, 0.4090)
	A_5	(-0.0138, 0.0506, 0.3553, 0.3737)

1, 1.5, 2, 2.5, 3, 2, respectively; the maximum iteration number is $G_{\max} = 500$.

5.1 Algorithm testing

To test the performance of the proposed MPSO, we adapt it to solve the programming model of form P_4. Specifically, we take model P_2 as an example by setting the objective weight $\omega = 0.5$, and run the MPSO four times. The convergence process of the MPSO is presented in Fig. 4. The convergence processes of the PSOs corresponding to the subsidiary swarms are also presented for convenient comparison.

From Fig. 4, we can see that the proposed algorithm MPSO gets better performance than that of the PSOs in the four dependent tests. This means that gathering the information from other PSOs can effectively help the MPSO to reduce the possibility of premature convergence. Therefore, we will adapt the MPSO to solve the portfolio optimization problems.

5.2 Solving process

In this subsection, we introduce how to get the efficient frontier of model P_1 by using the designed algorithm MPSO to solve model P_2. As mentioned in the above section, we set the objective weight ω ranging over $[0,\ 1]$ with the interval 0.1, and solve model P_2 under different ω. Then, we can obtain a series of efficient solutions, which form the efficient frontier of model P_1, as shown in Fig. 5. From Fig. 5, we can see that there is a positive correlation between cumulative expected perceived value and cumulative perceived risk. Specifically, as the given cumulative perceived risk level increases, the optimal strategy has higher cumulative perceived value. This indicates that the greater the given cumulative perceived risk level, the investor has a higher tolerate level toward risk, and pays more attention to the perceived value. Therefore, our proposed model can provide the suitable investment strategies for investors with respect to their personal preferences.

5.3 Comparison results

In order to illustrate the effectiveness of the proposed model P_1 in reflecting the investors' loss aversion, we compare it with a classical form of multi-period fuzzy portfolio model, which is as follows

$$\text{MSV}\begin{cases}\max\ CW_T\\ \min\ CR_T\\ \text{s.t.}\\ \displaystyle\sum_{i=1}^{n} x_{t,i} = 1,\\ \displaystyle -\sum_{i=1}^{n} x_{t,i}\ln x_{t,i} \geq e,\\ x_{t,i} \geq 0,\ i = 1,2,\cdots,n, t = 1,2,\cdots,T.\end{cases}$$

To evaluate the excess return per unit downside risk, Keating and Shadwick (2002) proposed the Omega Ratio. Here, we propose a similar ratio, named Perceived Ratio, to measure the perceived value per unit perceived risk, which is defined as follows

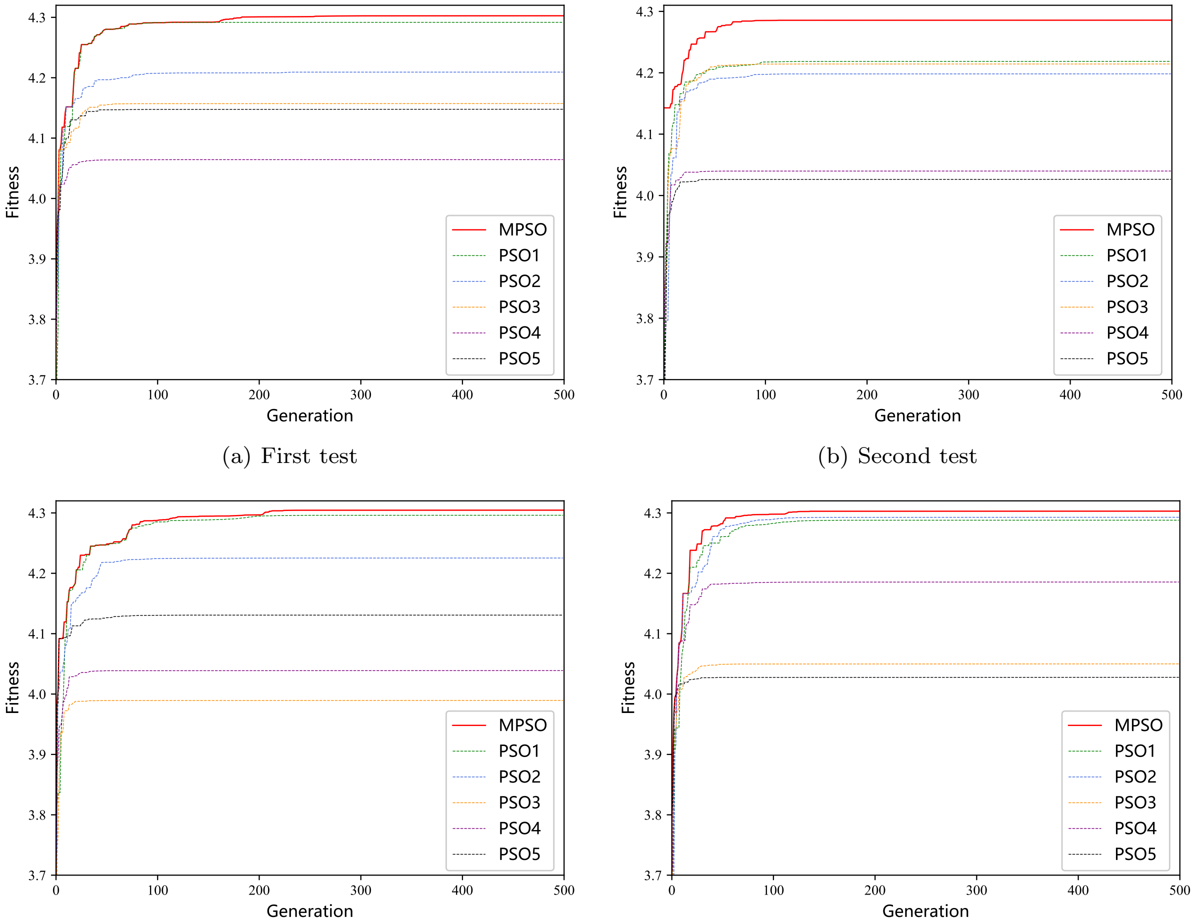
$$\text{PR}(\mathbf{x}) = \frac{CEPV(\mathbf{x})}{CPR(\mathbf{x})}. \tag{38}$$

where $\mathbf{x}$ represents the investment strategy, $CEPV(\mathbf{x})$ and $CPR(\mathbf{x})$ represent the terminal cumulative expected perceived value and terminal cumulative perceived risk corresponding to the investment strategy $\mathbf{x}$, respectively.

To measure the differences between two efficient frontiers, Fulga (2016) proposed a dissimilarity index by comparing each pair of efficient portfolios. Inspired by this idea, we aim to assess the performance of the efficient frontier based on the above Perceived Ratio (PR), and define the Perceived Ratio of Frontier (PRF) as follows

$$\text{PRF} = \frac{1}{J}\sum_{j=1}^{J}\text{PR}(\mathbf{x}_j^*),\ \ \mathbf{x}_j^* \in \boldsymbol{D}. \tag{39}$$

where $\mathbf{x}_j^*$ represents one of the investment strategies in the efficient frontier; $\boldsymbol{D}$ represents the set of all of the



(a) First test (b) Second test

(c) Third test (d) Fourth test

Fig. 4 The performances of the six algorithms in the four tests

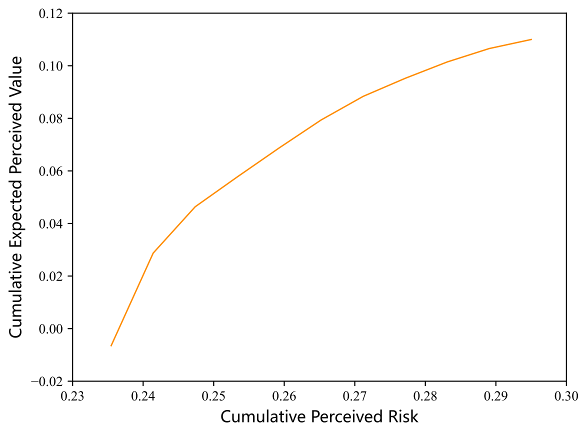


Fig. 5 The efficient frontier of the proposed model

investment strategies in the efficient frontier; J represents the number of investment strategies selected from the efficient frontier.

Next, we use the PRF to compare the efficient portfolios that created by model P_1 and MSV. Specifically, given the selected number J ranged from [10, 90] with the interval of 10, we use MPSO to solve the proposed model P_1 and model MSV, and compare the PRF values of the above two models with respect to different J, as shown in the following Fig. 6.

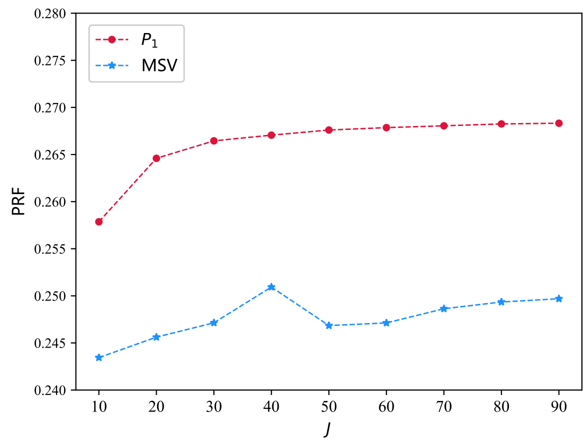


Fig. 6 The performances of the two models under different J

From Fig. 6, we can see that for the selected number J, the PRF value of our proposed model P_1 is higher than that of the model MSV, which indicates that it performs better than the model MSV. The main reason is that our proposed model P_1 incorporates the investors' loss aversion into portfolio decisions. In summary, the proposed model has a good performance in reflecting the investors' subject preferences and providing more reasonable investment strategies.

5.4 Analysis of investors' loss aversion

This subsection discusses the effect of investors' loss aversion on investment decision through the following aspects. First, we consider three cases of the loss aversion degree, i.e., $\theta = 1.1,\ 1.2,\ 1.3$. By using the MPSO to solve the optimization problems, we obtain the efficient frontiers of model P_1 under different loss aversions in the $CEPV - CPR$ space, as shown in Fig. 7. From Fig. 7, we can see that the loss aversion has a significant effect on the efficient frontier in the $CEPV - CPR$ space. Specifically, as the given loss aversion degree increases, the efficient frontier obtained by model P_1 moves right down. The main reason is that the investor with higher loss aversion degree is more pessimistic about the future returns of the risky assets, and thus gets a lower cumulative expected perceived value and a higher cumulative perceived risk from the investment strategy. The indicates that an investor with high loss aversion degree may be more conservative than the one with low loss aversion degree, and thus he/she may not invest wealth in a risky asset when its risk level is excessing the maximum risk level that he/she can tolerate.

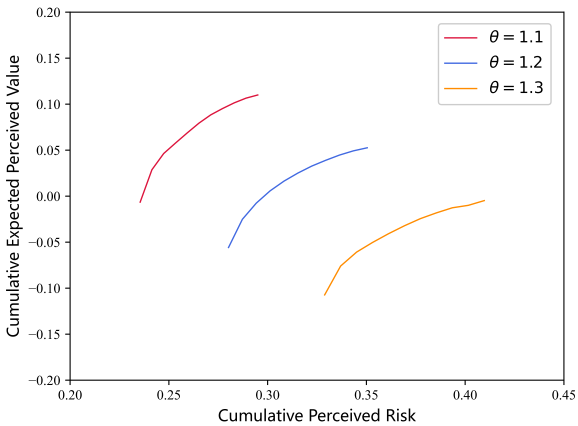


Fig. 7 The efficient frontiers of model P_1 under different loss aversions in the CEPV-CPR space

Next, we analyse the effect of the loss aversion on the optimal investment strategy with respect to the cumulative wealth (CW) and cumulative risk (CR). Specifically, we set the objective weight $\omega = 0.5$, and the loss aversion degree θ ranged from $[1.1,\ 5.1]$ with the interval of 0.5, respectively, then obtain the optimal investment strategies of model P_1 by solving model P_2 under the above different parameter settings, and the cumulative wealth and cumulative risk of these investment strategies, as shown in Fig. 8. From Fig. 8, we find that the loss aversion has an effect on the investment strategy. As the given loss aversion degree θ increases, the investor intends to select the investment strategy of lower cumulative wealth and lower cumulative risk, which indicates that as the higher the given loss aversion degree, the investor tends to be more conservative. From the above analysis, it is meaningful to incorporate investors' loss aversion into the portfolio selection model. Therefore, investors can determine the parameter θ according to their loss aversion degree in the practical investment management.

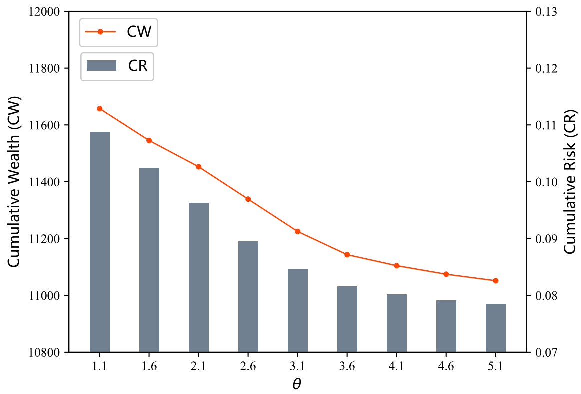


Fig. 8 The optimal strategies obtained by model P_2 under different loss aversion degree θ

5.5 Analysis of diversification

In this subsection, we analyse the effect of the investor's diversification requirement on investment decision. We consider three cases of the diversification level, i.e., $e = 0.93,\ 1.13,\ 1.33$. By using the MPSO to solve the optimization problems, we obtain the efficient frontiers of model P_1 under different diversification requirement, as shown in Fig. 9. From Fig. 9, we can see that the diversification level has a significant effect on the efficient frontier. Specifically, in the $CEPV - CPR$ space, as the given diversification level increases, the efficient frontier obtained by model P_1 gets shorter and moves right down. The main reason is that an efficient solution to the model with high diversification level may be dominated by the one with low diversification level, and thus it cannot be efficient solution to the latter. This indicates that as the higher the diversification level, the investor is more willing to reduce the non-systematic risk rather than increasing the excess return. Therefore, investors can determine e according to their diversification requirement in the practical investment management.

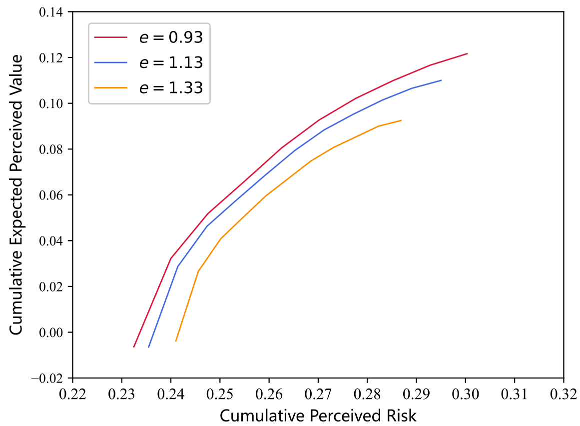


Fig. 9 The efficient frontiers of model P_1 under different diversification requirements

6 Conclusion

This paper addressed the multi-period portfolio selection problem considering investors' loss aversion in fuzzy environment. Firstly, the return rate of each risky asset is described by a trapezoidal fuzzy number, which captures the uncertainty in financial market. Next, by employing the value function in prospect theory, we transformed the portfolio return rate into the perceived value, and used the perceived risk to measure the portfolio's risk. Then, we proposed a multi-period fuzzy portfolio selection model with investors' loss aversion. Moreover, in order to solve the proposed model, we designed a multiple particle swarm optimization with respect to its specific situation. Finally, we illustrated the effectiveness of the designed algorithm and proposed model through a real case. The results show that the proposed model could provide decision support for investors with different loss aversion degrees.

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Availability of data and materials

The datasets generated during and analysed during the current study are available from the corresponding author on reasonable request.

Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

References

Barak S, Abessi M, Modarres M (2013) Fuzzy turnover rate chance constraints portfolio model. Eur J Oper Res 228(1):141–147

Best MJ, Hlouskova J (2000) The efficient frontier for bounded assets. Math Methods Oper Res 52(2):195–212

Carlsson C, Fullér R (2001) On possibilistic mean value and variance of fuzzy numbers. Fuzzy Sets and Syst 122(2):315–326

Chen W, Xu W (2019) A hybrid multiobjective bat algorithm for fuzzy portfolio optimization with real-world constraints. Int J Fuzzy Syst 21:291–307

Chow KV, Denning KC (1994) On variance and lower partial moment betas the equivalence of systematic risk measures. J Bus Financ Account 21(2):231–241

Deng X, Li R (2012) A portfolio selection model with borrowing constraint based on possibility theory. Appl Soft Comput 12(2):754–758

Dubois D, Prade H (1980) Fuzzy sets and systems: theory and applications. Academic press

Fortin I, Hlouskova J (2011) Optimal asset allocation under linear loss aversion. J Bank Financ 35(11):2974–2990

Fulga C (2016) Portfolio optimization under loss aversion. Eur J Oper Res 251(1):310–322

Guo S, Yu L, Li X, Kar S (2016) Fuzzy multi-period portfolio selection with different investment horizons. Eur J Oper Res 254(3):1026–1035

Harlow WV, Rao RK (1989) Asset pricing in a generalized mean-lower partial moment framework: Theory and evidence. J Financ Quant Anal 24(3):285–311

He G, Lu Xl (2021) An improved qpso algorithm and its application in fuzzy portfolio model with constraints. Soft Comput 25:7695–7706

Jarrow R, Zhao F (2006) Downside loss aversion and portfolio management. Manag Sci 52(4):558–566

Kahneman D, Tversky A (1979) Prospect theory: An analysis of decision under risk. Econometrica 47(2):263–292

Keating C, Shadwick WF (2002) A universal performance measure. J Perform Meas 6(3):59–84

Kennedy J, Eberhart R (1995) Particle swarm optimization. In: Proceedings of ICNN'95-international conference on neural networks, IEEE, vol 4, pp 1942–1948

Lin CC (2004) A weighted max–min model for fuzzy goal programming. Fuzzy Sets Syst 142(3):407–420

Liu YJ, Zhang WG (2021) Fuzzy multi-period portfolio selection model with time-varying loss aversion. J Oper Res Soc 72(4):935–949

Ma K, Hu S, Yang J, Xu X, Guan X (2018) Appliances scheduling via cooperative multi-swarm pso under day-ahead prices and photovoltaic generation. Appl Soft Comput 62:504–513

Markowitz H (1952) Portfolio selection. J Financ 7(1):77–91

Markowitz H (1959) Portfolio selection: efficient diversification of investments. Wiley

Mehlawat MK (2016) Credibilistic mean-entropy models for multi-period portfolio selection with multi-choice aspiration levels. Inf Sci 345:9–26

Merton RC (1972) An analytic derivation of the efficient portfolio frontier. J Financ Quant Anal 7(4):1851–1872

Mohebbi N, Najafi AA (2018) Credibilistic multi-period portfolio optimization based on scenario tree. Physica A 492:1302–1316

Niu B, Zhu Y, He X, Wu H (2007) Mcpso: A multi-swarm cooperative particle swarm optimizer. Appl Math Comput 185(2):1050–1062

Perold AF (1984) Large-scale portfolio optimization. Manag Sci 30(10):1143–1160

Sadjadi SJ, Seyedhosseini SM, Hassanlou K (2011) Fuzzy multi period portfolio selection with different rates for borrowing and lending. Appl Soft Comput 11(4):3821–3826

Sharpe WF (1964) Capital asset prices: A theory of market equilibrium under conditions of risk. J Financ 19(3):425–442

Song J, Bi X, Li R, Zhang S (2017) Optimal consumption and portfolio selection problems under loss aversion with downside consumption constraints. Appl Math Comput 299:80–94

Tripathi PK, Bandyopadhyay S, Pal SK (2007) Multi-objective particle swarm optimization with time variant inertia and acceleration coefficients. Inf Sci 177(22):5033–5049

Wang J, Zhou M, Guo X, Qi L (2018) Multiperiod asset allocation considering dynamic loss aversion behavior of investors. IEEE Trans Comput Soc Syst 6(1):73–81

Yang X, Liu W, Chen S, Zhang Y (2021) A multi-period fuzzy mean-minimax risk portfolio model with investor's risk attitude. Soft Comput 25:2949–2963

Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353

Zhang WG, Liu YJ, Xu WJ (2012) A possibilistic mean-semivariance-entropy model for multi-period portfolio selection with transaction costs. Eur J Oper Res 222(2):341–349

Zhang WG, Liu YJ, Xu WJ (2014) A new fuzzy programming approach for multi-period portfolio optimization with return demand and risk control. Fuzzy Sets Syst 246:107–126