# A Multivariate Complexity Analysis of Lobbying in Multiple Referenda

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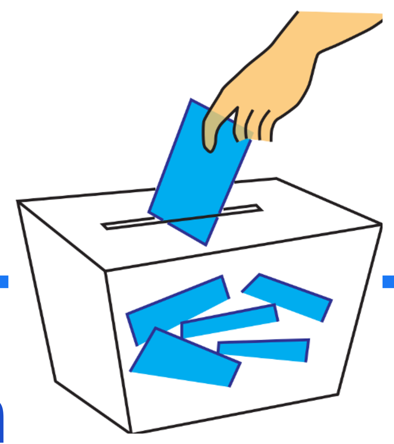


## Multi-Referenda Election

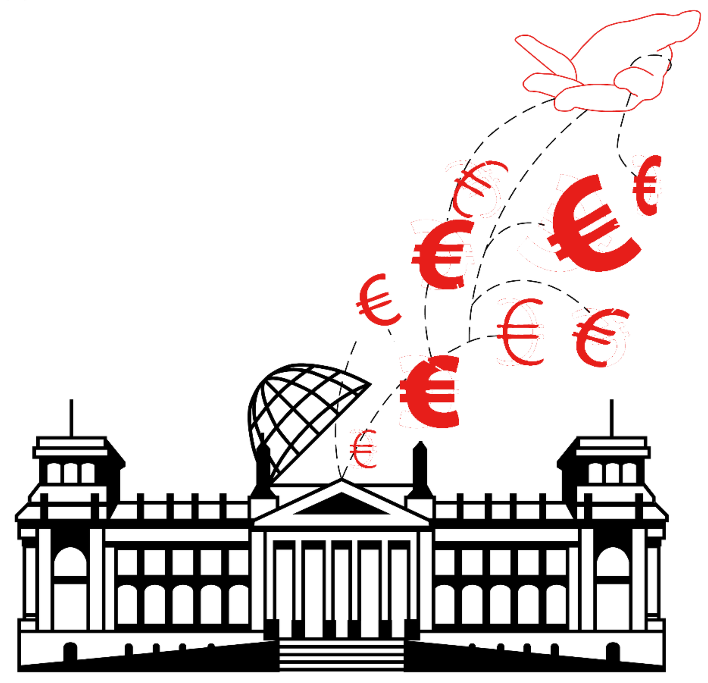
**Input:** Voters deciding ✔ or ✘ on multiple issues.
**Result:** An issue has value ✔ if the strict majority of voters decide ✔.

| Issues: | Emissions trading | Nuclear power | Tax raise |
|---|---|---|---|
| Voter 1 | ✘ | ✘ | ✔ |
| Voter 2 | ✔ | ✘ | ✔ |
| Voter 3 | ✘ | ✔ | ✘ |
| Voter 4 | ✔ | ✘ | ✘ |
| Voter 5 | ✘ | ✔ | ✘ |
| **Result** | ✘ | ✘ | ✘ |

Parameters: **Max. distance to majority:** 1
**Max. # ✔s per voter:** 2

Lobby at most $k = 2$ voters such that all issues get ✔?

$\Longrightarrow$



## Lobbying

**Input:** A multi-referenda election and a number $k$.
**?:** Can one lobby at most $k$ voters such that all issues get ✔?

| Issues: | Emissions trading | Nuclear power | Tax raise |
|---|---|---|---|
| Voter 1 | ✔ | ✔ | ✔ |
| Voter 2 | ✔ | ✘ | ✔ |
| Voter 3 | ✘ | ✔ | ✘ |
| Voter 4 | ✔ | ✔ | ✔ |
| Voter 5 | ✘ | ✔ | ✘ |
| **Result** | ✔ | ✔ | ✔ |

Parameters: **# lobbied voters:** 1

## Central Question:

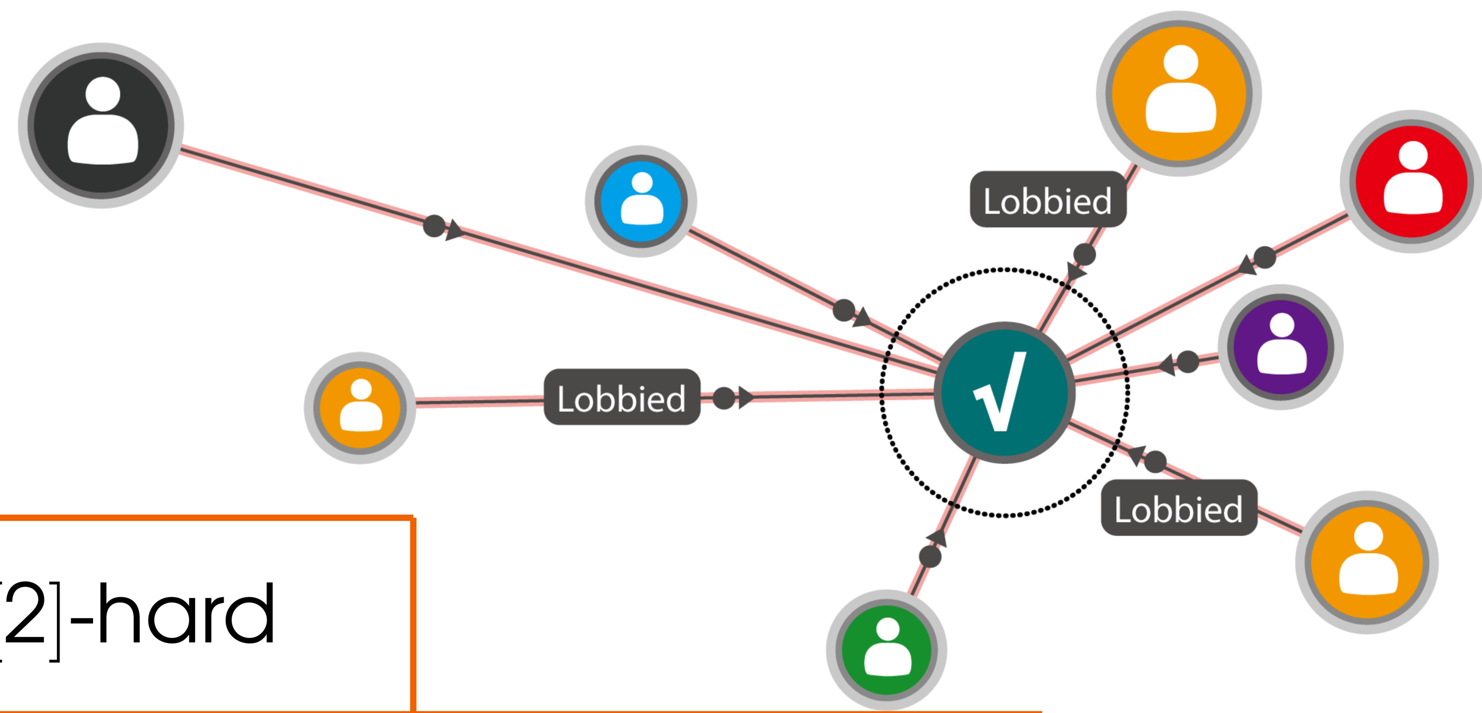
How do natural parameters influence the computational complexity of Lobbying?
$\longrightarrow$ Analyze this by means of tools from **Parameterized Complexity**!

## Central Conclusions:

- Lobbying with low budget is hard. [Christian et al., Review of Economic Design'07]
- **Having only a few issues makes Lobbying easy. Our greedy algorithm is optimal for up to $4$ issues.**
- **Effective preprocessing for Lobbying is hard.**
- **Lobbying nay-sayers is hard.**

## Known & New Results

| Parameter 1 / Parameter 2 | # issues | # lobbied voters | Max. distance to majority | Max. # ✔s per voter |
|---|---|---|---|---|
| **# issues** | FPT + No polynomial-size problem kernel | | | |
| **# lobbied voters** | FPT + No polynomial-size problem kernel | W[2]-complete | | |
| **Max. distance to majority** | FPT + No polynomial-size problem kernel | W[2]-hard | W[2]-hard | |
| **Max. # ✔s per voter** | FPT + No polynomial-size problem kernel | No polynomial-size problem kernel | No polynomial-size problem kernel | NP-hard |




### Further Results (New Models)

- **Restricted Lobbying:**
  Can only change a limited number $t'$ of issues per voter.
  - NP-hard already for four issues.
  - FPT wrt. $(t', k)$.
- **Partial Lobbying:**
  Not the full list of issues needs to be disapproved.
  - FPT wrt. $(g, k)$.

### Complexity Theory in a Nutshell

- Problem $x$ is **FPT** wrt. parameters $(k, t)$
  ≙ Solvable in $f(k,t)|x|^c$ time.
  ($c$ being a constant)
- **No polynomial-size problem kernel** indicates limits of preprocessing.
- **W(2)-hard** wrt. parameters $(k, t)$ means an **FPT** algorithm is unlikely to exist.

### Related Work

- First studied by Christian et al.
  [Review of Economic Design'07]
- Closely related to combinatorial markets in multi-agent systems.
  [Sandholm et al., AAMAS'02]
- Also related to Judgment Aggregation.
  [Baumeister et al., ADT'11]