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A Multivariate Latent Variable Model for Mixed Continuous and Ordinal Responses

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Abstract: A joint model for multivariate mixed ordinal and continuous responses is presented. In this model the ordinal responses are intercorrelated and also are dependent on the continuous responses. The likelihood is found and modified Pearson residuals, where the correlation between multivariate responses can be taken into account, are presented to find abnormal observations. The model is applied to a medical data, obtained from an observational study on women, where the correlated responses are the ordinal response of osteoporosis of the spine and continuous responses are body mass index and waist. The effect of some covariates on all responses are investigated simultaneously.

Key words: Mixed data . ordinal and continuous responses . body mass index . osteoporosis of the spine . general location model

INTRODUCTION

Some biomedical, psychological, health and economic sciences data include some correlated discrete and continuous outcomes. The first example is the analysis of development toxicity endpoints when the relationship between fetal weight and malformation in live fetus is an important statistical issue [1]. The second example is in the study of the maternal smoking effect on respiratory illness in children where we have a continuous measure of pulmonary function and an ordinal measure of chronic symptoms in children. The third example is on the simultaneous effect of type of accommodation on body mass index and waist as continuous responses and osteoporosis of the spine as ordinal response (vide, our application in Section 3). For the first example, separate analysis of the categorical or the continuous responses can not properly assess the effect of dose on fetal weight and malformations simultaneously. For the second example, separate analysis can not assess the effect of maternal smoking on all the responses simultaneously. In the third example, separate analysis can not assess the effect of type of accommodation on body mass index, waist and osteoporosis of the spine. Furthermore, separate analysis give biased estimates for the parameters and misleading inference [2]. Consequently, we need to consider a method in which these variables can be modelled jointly.

For joint modelling of responses, one method is to use the general location model of Olkin and Tate [3], where the joint distribution of the continuous and categorical variables is decomposed into a marginal multinomial distribution for the categorical variables and a conditional multivariate normal distribution for the continuous variables, given the categorical variables (for a mixed poisson and continuous responses where Olkin and Tate's [3] method is used see Yang *et al*. [4]). A second method for joint modelling is to decompose the joint distribution as a multivariate marginal distribution for the continuous responses and a conditional distribution for categorical variables given the continuous variables. Cox and Wermuth [5] empirically examined the choice between these two methods. The third method uses simultaneous modelling of categorical and continuous variables to take into account the association between the responses by the correlation between errors in the model for responses. For more details of this approach see, for example, Heckman [6] in which a general model for simultaneously analyzing two mixed correlated responses is introduced and Catalano and Ryan [1] who extend and used the model for a cluster of discrete and continuous outcomes (vide also, Fitzmaurice and Laird [7] and Fitzmaurice and Laird [8]). All the above references consider correlated nominal and continuous responses.

The aim of this paper is to use and extend an approach similar to that of Heckman [6], for jointly modelling of a nominal and a continuous variable, to joint modelling of multivariate ordinal and continuous outcomes. The model is described in terms of a correlated multivariate normal distribution for the underlying latent variables of ordinal responses and

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continuous responses. Some modified Pearson residuals are also presented to detect outliers where the correlations between responses are also taken into account.

In Section 2, the model and modified Pearson residuals are given. In Section 3, the model is used on a medical data set where osteoporosis, body mass index (BMI) and waist are correlated responses in an observational study on women. In this model osteoporosis of the spine is an ordinal response, BMI and waist are continuous responses and age, the amount of total body calcium (Ca), job status (employee or housekeeper) and type of accommodation (house or apartment) are explanatory variables. We shall investigate the effects of these explanatory variables on responses simultaneously. In Section 4, the paper concludes with some remarks.

MODEL AND MODIFIED RESIDUALS

Model and likelihood: Suppose the vector of response for ith individual is:

$$H_i = (Y_{i1}, \ldots, Y_{iM_1}, Z_{i(M_1+1)}, \ldots, Z_{iM})'$$

where Y_{im} for m=1,…,M_1, are ordinal responses each with c_m levels and Z_{im} for m=M_1+1,…,M are continuous responses. All responses are correlated. Let Y^*_{im} for m=1,…,M_1 denote the underlying random variable of the ordinal response of ith individual and mth outcomes with c_m levels. Define

$$Y_{im} = \begin{cases} 1 & Y^*_{im} < \theta_{1m}, \\ j+1 & \theta_{mj} \le Y^*_{im} \le \theta_{m(j+1)},\ j=1,\ldots,c_m-2, \\ c_m & Y^*_{im} > \theta_{m(c_m-1)}, \end{cases}$$

where $\theta_m = (\theta_{m1}, \ldots, \theta_{m(c_{m-1})})$ are the vector of cutpoints parameters for m=1,…,M_1. The joint model takes the form:

$$Y^*_{im} = X_{im}'\beta_m + \varepsilon_{im}, \qquad m=1,\ldots,M_1 \qquad (1a)$$

$$Z_{im} = X_{im}'\beta_m + \varepsilon_{im}, \qquad m = M_1+1,\ldots,M \qquad (1b)$$

where X_{im} are vectors of explanatory variables for ith individual and mth outcomes.

It is assumed that $E(\varepsilon_{im}) = 0$. The covariance matrix of the vector of errors $[(\varepsilon_{i1}, \ldots, \varepsilon_{iM_1}, \varepsilon_{i(M_1+1)}, \ldots, \varepsilon_{iM})']$ is Σ. For example, when M_1=1 and M=3,

$$\Sigma = \begin{pmatrix} 1 & \sigma_1\rho_{12} & \sigma_2\rho_{13} \\ \sigma_1\rho_{21} & \sigma_1^2 & \sigma_1\sigma_2\rho_{23} \\ \sigma_2\rho_{31} & \sigma_1\sigma_2\rho_{32} & \sigma_2^2 \end{pmatrix}$$

The vector of parameters β_m, for m=1,…,M, Σ and the vector of parameters θ_m for m=1,…,M_1 should be estimated. The vector, β_m for m=M_1+1,…,M, includes an intercept parameter but β_m, for m=1,…,M_1, due to having cutpoints parameters, is assumed not to include any intercept. In this model any multivariate distribution can be assumed for the errors in the model. Here, a multivariate normal distribution is used. The likelihood for this model is given in the appendix.

Residual and goodness-of-fit: The quantities consider in residual analysis help to identify poorly fitting observations that are not well explained by the model. Start with using theoretical form, involving $E(Y|X_{im})$, for m=1,…,M_1, $E(Z|X_{im})$ for m=M_1,…,M and

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

where Y is the vector of ordinal response, Z is the vector of continuous response, Σ_{11}=Var(Y), Σ_{22}=Var(Z) and $\Sigma_{12}=\Sigma_{21}'$=Cov(Z,Y) are theoretical parameters, rather than their predicted values to define residuals. The Pearson residuals for a M-M_1 dimensional vector of continuous responses can take the form:

$$r^P_{iz} = \Sigma_{22}^{-\frac{1}{2}}[Z_i - E(Z_i \mid X_{i(M1+1)}, \ldots, X_{iM})] \qquad (2a)$$

and that for a M_1 dimensional vector of ordinal response can take the form:

$$r^P_{iy} = \Sigma_{11}^{-\frac{1}{2}}[Y_i - E(Y_i \mid X_{i1}, \ldots, X_{iM_1})] \qquad (2b)$$

where $Z_i = (Z_{i(M_1+1)}, \ldots, Z_{iM})'$ and $Y_i = (Y_{i1}, \ldots, Y_{M_1})'$. For example, when M_1=1, M=2 and c_1=3 we have:

$$E(Y_{i1} \mid X_{i}) = 3 - (P_1 + P_2)$$

$$Var(Y_{i1} \mid X_{i}) = 3\,P_1 + P_2 - (P_1 + P_2)^2$$

$$E(Z_{i2} \mid U_{i2}) = X_{i2}'\beta_2$$

where $P_1 = \Phi(\theta_1 - X_{i1}'\beta_1)$ and $P_2 = \Phi(\theta_2 - X_{i1}'\beta_1)$. The estimated Pearson residuals can be found by using the maximum likelihood estimates of the parameters, obtained by system (1), in (2a) and (2b). However, the residuals in equations (2a) and (2b) do not take into account the correlation between responses. The following modified Pearson residuals consider this correlation between responses:

$$r_i^P = \hat{\Sigma}^{-\frac{1}{2}}(K_i - \hat{\mu}_i) \tag{3}$$

Where,

$$K_i = (Z_{i'}, Y_{i'})'$$

$$\hat{\mu}_i = (\hat{E}(Z_{i'} | X_{i(M_1+1)}, \ldots, X_{iM}), \hat{E}(Y_{i'} | X_{i1}, \ldots, X_{iM_1}))'$$

and

$$\hat{\Sigma} = \begin{pmatrix} \hat{V}ar(Z) & \hat{C}ov(Z,Y) \\ \hat{C}ov(Z,Y) & \hat{V}ar(Y) \end{pmatrix}$$

For example, when M_1=1, M=2, c_1=3 and ρ is orrelation between Y_1 and Z_2, we have

$$\begin{aligned} Cov(Z_2, Y_1) &= E(Z_2Y_1) - E(Z_2)E(Y_1) \\ &= E(Z_2E(Y_1|Z_2)) - E(Z_2)E(Y_1) \\ &= \int z(\sum_y yP(Y_1 = y | Z_2 = z))f_{Z_2}(z)dz - E(Z_2)E(Y_1) \\ &= \int z\, E(Y_1|z)f_{Z_2}(z)dz - E(Z_2)E(Y_1) \\ &= \int z[3 - (N_1(z) + N_2(z))]f_{Z_2}(z)dz - E(Z_2)E(Y_1) \end{aligned} \tag{4}$$

Where,

$$N_1(z) = \Phi\left(\frac{\theta_1 - (\beta_1'X_{i1} + \frac{\rho}{\sigma}(z - \beta_1'X_{i1}))}{\sqrt{1-\rho^2}}\right)$$

and

$$N_2(z) = \Phi\left(\frac{\theta_2 - (\beta_1'X_{i1} + \frac{\rho}{\sigma}(z - \beta_1'X_{i1}))}{\sqrt{1-\rho^2}}\right)$$

In (3), in the case of Σ_{12}=ρ=0, $N_1(z)=P_1$ and $N_2(z)=P_2$ and so Cov(Z,y)=0 Hence, in this case components of r_i^P give residuals in (2a) and (2b). The Pearson residual for ith observation is based on the Pearson goodness-of-fit statistics

$$\chi^2 = \sum_{i=1}^{n} \chi_p^2(K_i, \hat{\mu}_i)$$

with the following ith component

$$\chi_p^2(K_i, \hat{\mu}_i) = (K_i - \hat{\mu}_i)'\hat{\Sigma}^{-1}(K_i - \hat{\mu}_i)$$

One may use a Cholesky decomposition for finding the square root of $\hat{\Sigma}$ in (3) and the function integrate in R to numerically calculate the integral given in (4).

APPLICATION

In this section, we use the joint model in equation (1a) and (1b) for the medical data set describe in the following subsection.

Data: The medical data set is obtained from an observational study on women in the Taleghani hospital of Tehran, Iran. These data record status of osteoporosis of the spine as an ordinal response and BMI and waist as continuous responses for 163 patients. Osteoporosis of the spine is a disease of bone in which the bone mineral density (BMD) is reduced, bone microarchitecture is disrupted and the amount and variaty of non-collagenous proteins in bone is altered. BMI is a statistical measure of the weight of body mass index. A person body mass index may be accurately calculated using any of the formulas such as:

$$BMI = \frac{W}{H^2}$$

where, W is weight and H is height. These three variables, osteoporosis of the spine, BMI and waist are endogenous correlated variables and they have to be modelled simultaneously. Explanatory variables which affect these variables are: (1) amount of total body calcium (Ca), (2) job status (Job, employee or housekeeper), (3) type of the accommodation (Ta, house or apartment) and (4) age.

Descriptive statistics (mean and standard deviation for continuous responses and frequency or precetage for ordinal response) are given in Table 1. Y_1 is osteoporosis

Table 1: Descriptive statistics for medical data

		No.	Mean	SD
Z_2:BMI		163	29.357	10.806
Z_3: Waist		163	96.990	4.781
Y_1:Osteoporosis of the spine	Levels	No.	Percentage	
	None	59	0.362	
	Mild	65	0.399	
	Severe	39	0.239	

of the spine of individual as an ordinal response with 3 levels. These levels defined as 1: individual hasn't osteoporosis of the spine (None), 2: individual has mild osteoporosis of the spine (Mild), 3: individual has severe osteoporosis of the spine (Severe). $Z = (Z_2, Z_3)'$ is the vector of continuous response where Z_2 is the BMI of individual and Z_3 is the waist of individual.

Table 1 shows less Precentage for severe osteoporosis than those of none and mild levels. The vector of explanatory variable is X=(Job, Age, Ta, Ca).

Models for medical data: For comparative purposes, two models are considered. The first model (model I) does not consider the correlation between all responses. This model is

$$Y_1^* = \beta_{11}\text{Job} + \beta_{12}\text{Age} + \beta_{13}\text{Ta} + \beta_{14}\text{Ca} + \varepsilon_1 \quad (3a)$$

$$Z_2 = \beta_{20} + \beta_{21}\text{Job} + \beta_{22}\text{Age} + \beta_{23}\text{Ta} + \beta_{24}\text{Ca} + \varepsilon_2 \quad (3b)$$

$$Z_3 = \beta_{30} + \beta_{31}\text{Job} + \beta_{32}\text{Age} + \beta_{33}\text{Ta} + \beta_{34}\text{Ca} + \varepsilon_3 \quad (3c)$$

The covariance matrix of the vector of errors (ε_1, ε_2, ε_3)′ for this model is $\Sigma_{\text{Ind}} = \text{diag}\{1, \sigma_1^2, \sigma_2^2\}$. The second model (model II) uses model I and takes into account the correlation between three errors. For this model covariance matrix is:

$$\Sigma = \begin{pmatrix} 1 & \sigma_1\rho_{12} & \sigma_2\rho_{13} \\ \sigma_1\rho_{21} & \sigma_1^2 & \sigma_1\sigma_2\rho_{23} \\ \sigma_2\rho_{31} & \sigma_1\sigma_2\rho_{32} & \sigma_2^2 \end{pmatrix}$$

Here, a multivariate normal distribution with correlation ρ_{12}, ρ_{13} and ρ_{23} are assumed for the errors and these parameters should be also estimated.

RESULTS

Results of using two models are given in Table 2. Model (I) shows a weak significant effect of age on BMI, a weak significant effect of Ta on waist and a weak significant effect of Ca on the ordinal response. From these effects we can inferred that the older the patient the less the BMI, people who live in apartment have more waist than of that people who live in a house and the more the amount of calcium of the body of the patient the higher is the probability of low value of osteoporosis of the spine. Model (II) gives the same results as model (I). To compare model (II) and model (I) we have deviance =123.318 with two d.f. (P-value < 0.001). So one may preffered model (II). For model (II) correlation parameters ρ_{23} and ρ_{12} are strongly significant. They show a positive correlation between BMI and waist ($\hat{\rho}_{23}$=0.715) and it shows a negative correlation between BMI and osteoporosis of the spine ($\hat{\rho}_{12}$=-0.210). The estimated variance of BMI and waist ($\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$) obtained by model (II) are less than those of model (I). A consequence of estimating correlation parameters by model (II) is that the estimated standard errors of constant parameters in models for continuous responses are reduced in comparing them with results obtained by model I. Using residuals defined in equation (3), we have not found any abnormal observation.

Table 2: Results using two models for medical data (**: Significant at %5 level, *: Significant at %10 level)

	Model I		Model II	
Parameter	Est.	SE	Est.	SE
β_{11}(Job)	-0.570	0.515	-0.548	0.515
β_{12}(Age)	0.002	0.013	0.002	0.021
β_{13}(Ta)	0.006	0.181	0.005	0.153
β_{14}(Ca)	0.211*	0.123	0.211*	0.126
θ_1	1.207	1.472	1.237	1.889
θ_2	2.284	1.477	2.315	1.894
β_{20}(Const)	35.523**	6.239	35.518**	5.689
β_{21}(Job)	1.734	2.168	1.734	2.520
β_{22}(Age)	-0.101 *	0.055	-0.101 *	0.054
β_{23}(Ta)	0.991	0.770	0.990	0.774
β_{24}(Ca)	-0.283	0.517	-0.282	0.368
β_{30} (Const)	86.151**	14.153	86.151**	9.664
β_{31}(Job)	1.653	4.918	1.652	6.498
β_{32}(Age)	0.127	0.125	0.126	0.127
β_{33}(Ta)	3.001 *	1.744	3.000*	1.746
β_{34} (Ca)	0.010	0.173	0.009	0.179
$\sigma_1^2(z_1)$	22.647**	0.259	21.949**	0.259
$\sigma_2^2(z_2)$	116.524**	0.588	112.949**	0.588
ρ_{12}	-	-	-0.210**	0.084
ρ_{13}	-	-	-0.101	0.086
ρ_{23}	-	-	0.715 **	0.038
-loglike	1273.044		1211.385	

DISCUSSION

In this paper a multivariate latent variable model is presented for simultaneously modelling of ordinal and continuous correlated responses. Some modified residual are also presented to detect outliers. We assume a multivariate normal distribution for errors in the model. However, any other multivariate distribution such as t or logistic can also be used. Binary responses are a special case of ordinal responses. So, our model can also be used for mixed binary and continuous responses. For correlated nominal, ordinal and continuous responses Deleon and Carriere [9] have developed a model by extending general location model. However, the kind of Scientific question they can answer is different with what our model can do (vide Section 1). Generalization of our model for nominal, ordinal and continuous responses is an ongoing research on our part.

Appendix: Likelihood for mixed model of multivariate ordinal and continuous responses

Suppose $Y_i=(Y_{i1},\ldots,Y_{iM_1})'$ and $Z_i=(Z_{i(M_1+1)},\ldots,Z_{iM})'$. Let $\eta=(\beta_1',\ldots,\beta_M',\theta_1',\ldots,\theta_{M_1}')'$. The likelihood function of model (1) is

$$L(\eta,\Sigma|z,y,x)=\prod_{i=1}^{n} f(z_i,y_i|x_i^{(1)},x_i^{(2)},\eta,\Sigma)=\prod_{i=1}^{n} f(Y_i=y_i|z_i,x_i^{(1)},\beta_1,\ldots\beta_M,\Sigma,\theta_1,\ldots,\theta_{M_1})f(z_i|x_i^{(2)},\beta_{M_1+1},\ldots,\beta_M)$$

$$=\prod_{i=1}^{n} P(Y_{i1}=y_{i1},\ldots,Y_{iM_1}=y_{iM_1}|z_i,x_i^{(1)},\beta_1,\ldots\beta_{M_1},\Sigma,\theta_1,\ldots,\theta_{M_1})f(z_i|x_i^{(2)},\beta_{M_1+1},\ldots\beta_M)=\prod_{i=1}^{n} f(z_i|x_i^{(2)},\beta_{M_1+1},\ldots,\beta_M)$$

$$\times P(\theta_{1(y_{i1}-1)}\le Y_{i1}^*\le\theta_{1y_{i1}},\ldots,\theta_{M_1(y_{iM_1}-1)}\le Y_{iM_1}^*\le\theta_{M_1(y_{iM_1})}|z_i,x_i^{(1)},\beta_1,\ldots,\beta_{M_1},\Sigma,\theta_1,\ldots,\theta_{M_1})$$

Where,

$$\theta_{m0}=-\infty,\ \theta_{mc_m}=+\infty,\ x_i^{(1)}=(x_{i1},\ldots,x_{iM})'\ \text{and}\ x_i^{(2)}=(x_{i(M_1+1)},\ldots,x_{iM})'.$$

To find a better from for the likelihood, define $I_{A_{\theta_{y_{im}}}}=I(\theta_{y_{im-1}}<Y_{im}^*<\theta_{y_{im}})$

where I(.)is the indicator function. So, $P(\theta_{y_{im-1}}<Y_{im}^*<\theta_{y_{im}})=E(I_{A_{\theta_{y_{im}}}})$

Thus, the likelihood would be,

$$\prod_{i=1}^{n}[f(z_i|x_i^{(2)},\beta_{M_1+1},\ldots,\beta_M)E(I_{i1}=1|x_i^{(1)},\theta_1)$$
$$\prod_{m=2}^{M_1}E(I_{A_{\theta_{y_{im}}}}=1|I_{A_{\theta_{y_{i1}}}}=1,\ldots,I_{A_{\theta_{y_{i(m-1)}}}}=1,z_i,x_i^{(1)},\beta_1,\ldots,\beta_{M_1},\Sigma,\theta_1,\ldots,\theta_{M_1})]$$

Now, the expectation in the right hand can be, approximated as (vide [10]),

$$E\left(I_{A_{\theta_{y_{im}}}}=1|I_{A_{\theta_{y_{i1}}}}=1,\ldots,I_{A_{\theta_{y_{i(m-1)}}}}=1,z_i,x_i^{(1)},\beta_1,\ldots,\beta_{M_1},\Sigma,\theta_1,\ldots,\theta_{M_1}\right)$$
$$=E(I_{A_{\theta_{y_{im}}}})+\Omega_{21}^{(im)}\Omega_{m-1}^{-1}(1-E(I_{A_{\theta_{y_{i1}}}}),\ldots,1-E(I_{A_{\theta_{y_{i(M_1-1)}}}}))'$$

where $\Omega_{21}^{(im)}$ is a row vector consisting of the entries

$Cov(I_{A_{\theta_{y_{im}}}},I_{A_{\theta_{y_{iM_1}}}})=E(I_{A_{\theta_{y_{im}}}}I_{A_{\theta_{y_{iM_1}}}})-E(I_{A_{\theta_{y_{im}}}})E(I_{A_{\theta_{y_{iM_1}}}}),m=1,\ldots,M_1$, Ω_{m-1} is$(M_1-1)\times(M_1-1)$ matrix with (h,l) element, h, l=1,..., (M_1-1).

Let Φ be the cumulative standard normal distribution, hence

$$L(\eta,\Sigma|z,y,x)=\prod_{i=1}^{n}[f(z_i|x_i^{(2)},\beta_{M_1+1},\ldots,\beta_M)\left(\Phi\left(\frac{\theta_{1y_i}-\mu_{Y_{i1}^*|z_{i1},z_{i2}}}{(Var(Y_{i1}^*|z_{i1},z_{i2}))^{1/2}}\right)-\Phi\left(\frac{\theta_{1(y_i-1)}-\mu_{Y_{i1}^*|z_{i1}z_{i2}}}{(Var(Y_{i1}^*|z_{i1},z_{i2}))^{1/2}}\right)\right)$$
$$\prod_{m=2}^{M_1}E(I_{A_{\theta_{y_{im}}}}=1|I_{A_{\theta_{y_{i1}}}}=1,\ldots,I_{A_{\theta_{y_{i(m-1)}}}}=1,z_i,x_i^{(1)},\beta_1,\ldots,\beta_{M_1},\Sigma,\theta_1,\ldots,\theta_{M_1})]$$

Where,

$$\mu_{Y_i^* | z_{i1} z_{i2}} = \beta'_1 x_{i1} + (\rho_{12}\sigma_1 \quad \rho_{13}\sigma_2)\begin{pmatrix} \sigma_1^2 & \rho_{23}\sigma_1\sigma_2 \\ \rho_{23}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}^{-1}\begin{pmatrix} z_{i2} - \beta'_2 x_{i2} \\ z_{i3} - \beta'_3 x_{i3} \end{pmatrix}$$

$$\mathrm{Var}(Y_i^* | z_{i1}, z_{i2}) = 1 - (\rho_{12}\sigma_1 \quad \rho_{13}\sigma_2)\begin{pmatrix} \sigma_1^2 & \rho_{23}\sigma_1\sigma_2 \\ \rho_{23}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}^{-1}\begin{pmatrix} \rho_{12}\sigma_1 \\ \rho_{13}\sigma_2 \end{pmatrix}$$

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