A Narrowband High Frequency Distributed Power Transformer Model for Partial Discharge Location

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***Abstract*--This paper utilizes a narrowband high frequency distributed transformer model to estimate partial discharge location. Here a narrow band of frequencies within the frequency response of the system, that exhibit resonant pole behavior, is specifically targeted. It is proposed that the observed response is due to the interaction of residual winding inductance with capacitance to ground. This physical phenomenon is inherently distributed; hence regions within the frequency response related to this interaction will be dependent upon the input location. With this premise an algorithm that estimates the location of the partial discharge by iteratively comparing the proposed model at various locations within the winding with the observed partial discharge frequency response is implemented. The algorithm was tested on a single phase of a 66kV/25MVA interleaved transformer winding where the partial discharge was injected via an oil immersed point-plane 7.5kV source.**

***Index Terms*-- Power Transformer, frequency, partial discharge, fault location, modeling, state space methods, simulation, transient response**

I. Introduction

PARTIAL discharges within an insulation system over a period of time can have catastrophic consequences [1]. In order to mitigate such circumstances it is important to regularly monitor the degree and severity of partial discharge (PD) activity.

Partial discharge monitoring tools generally monitor the magnitude, relative phase location (w.r.t mains frequency) and frequency of occurrence. The results of which provide an indication of the insulation system health with particular emphasis on the magnitude which is generally expressed in picocoulombs. A shortcoming of this type of monitoring is that these signals are recorded externally and have possibly undergone significant levels of attenuation during their transmission through the winding system of the transformer. Knowledge of the source location of a partial discharge would be invaluable not only for transformer maintenance but also for more realistic estimates of the true partial discharge magnitude and the corresponding insulation system health [1-3].

A partial discharge can be considered to be a brief exchange of charge within the insulation system. This discharge itself can be modeled as an ideal current impulse. By determining the frequency response of the measured response to a partial discharge, an approximation of the transfer function between the PD location and the measurement point is obtained. Incorporating this with an accurate model of the system can provide an estimate of the partial discharge site [4].

The paper presented extends on the author's work presented in [5] via improvements in the model and its implementation. Further improvements are made with the addition of a cost function which automates the estimate of the PD location. The overall approach is a significant step forward from previous results.

II. Theory

A. *The Distributed Transformer Model*

Most partial discharge localization techniques use a distributed model approach. This is generally in the form of a high order lumped parameter model, commonly an RLC ladder network [6, 7]. Difficulties arise with these models since they are only suitable for frequencies up to a few hundred kHz [8]. This is clearly a limitation since partial discharges are high frequency in nature and as such much of the high energy regions within their signature are not fully utilized. This results in a lower monitoring sensitivity than could actually be achieved. However, to target these higher frequencies, consideration is required for effects such as traveling waves and the influence of measurement equipment on the results [8].

For frequencies above a few hundred kHz one of two approaches to transformer modeling is generally undertaken. The first approach is an area that has seen significant research activity of late particularly in the transient over voltage area. This is MLTL or Multiconductor Transmission Line model. The premise of MLTL is that each entry of a winding be considered as multiple parallel transmission lines. The incidence of a high frequency signal at the junction will be coupled into all of the parallel paths [9]. The second approach

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is to model a system with circuit elements that dimensionally represent areas of the system not greater than one tenth of the wavelength of the maximum frequency being analyzed [10, 11]. This approach ensures that traveling wave effects will be negligible. One difficulty that arises in this approach is the mathematical size of the model that is required to be generated as authors choose to model a ladder section as a physical turn within the winding [8].

Both techniques mentioned above rely on a physically accurate model. Akbari et al [12] has described the search for a model that is functional at high frequencies as being an unsolved problem since the transformer winding is inherently complex with a non-linear and frequency dependent nature.

To overcome the problems highlighted, the current paper targets a narrow band of frequencies over which the non-linear and frequency dependent characteristics will not have a significant impact. The model used here takes advantage of a high frequency resonant pole response that is present after the capacitive region of an interleaved transformer winding frequency response (Figure 1). These features were originally highlighted by James et al [6]. The area of interest is Region C where this resonant peak was suggested by James et al [6] to be due to residual inductance within the winding. In Mitchell et al [5] a novel approach to PD location was proposed where this narrow frequency band was targeted since it is generated by an interaction that is distributive by nature, hence the response will be reflective of the input location.

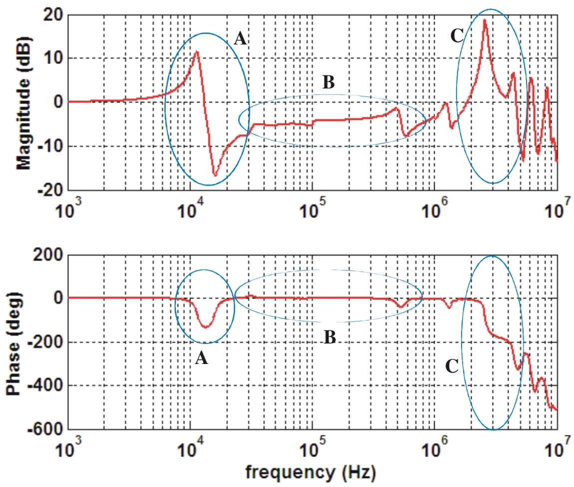

Fig. 1. Frequency Response from Bushing to Neutral
Region A – Self Resonant Frequency
Region B – Capacitive Region
Region C – Resonant Pole Region

The proposed model in [5] has been modified here to include the 33pF PD injection capacitor C_{EX} (Figure 2). This subtle inclusion improves the phase relationship between the model and the injected PD response.

The rationale behind this model is discussed in [13] where, to a fast transient burst, the first few turns of an interleaved transformer winding will appear like a series capacitance C_s. The remaining turns are then considered in series with each other *and* the shunt capacitance to ground resulting in C_G. Utilizing this argument and introducing James et al [6] proposal, i.e. that residual inductances are significant at higher frequencies, the circuit in Figure 2 was proposed [5]. Note the frequency dependent resistance. This is necessary to represent a combination of losses due to dielectric loss and proximity and eddy current effects. Traveling wave effects are ignored since an upper frequency bound of 5MHz is used. This ensures that the relevant dimensions of each disc pair in Figure 2 (of the order of a few metres) is less than 10% of the wavelength of the highest frequency.

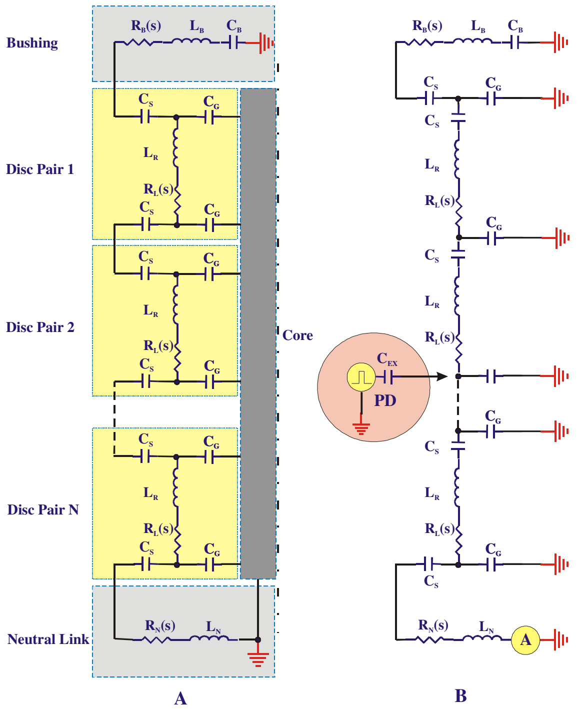

Fig. 2. High Frequency Distributed Model
A: High Frequency Model, B: Simplified Model
v_G = Shunt Capacitor voltage, v_S = Series Capacitor voltage,
v_X = External Capacitor voltage, i_L = Inductor current

B. Partial Discharge Location

By considering a partial discharge to approximate a current impulse, the recorded partial discharge signal will provide the impulse response, hence the system transfer function, between the PD inception point and that of the measurement location [4]. The state space representation of the circuit shown in Figure 2B is given in (1). It follows from the model given in [5] with fundamental changes due to the addition of the injection capacitor C_{EX}:

$$\dot{x} = Ax(t) + Bu(t)$$
$$y = Cx(t) \qquad (1)$$

Where:

$$x = \left[v_{G(1)}(t) \cdots v_{G(n+1)}(t) \; v_{S(1)}(t) \cdots v_{S(n+1)}(t) \; v_X(t) \; i_{L(1)}(t) \cdots i_{L(n+1)}(t) \right]^T$$

$$A=\begin{bmatrix} A_{GG} & A_{GS} & A_{GX} & A_{GL} \\ A_{SG} & A_{SS} & A_{SX} & A_{SL} \\ A_{XG} & A_{XS} & A_{XX} & A_{XL} \\ A_{LG} & A_{LS} & A_{LX} & A_{LL} \end{bmatrix}$$

$$A_{GL}=\begin{bmatrix} \frac{-1}{C_B} & 0 & \cdots & 0 \\ \frac{1}{C_G} & \frac{-1}{C_G} & 0 & \ddots \\ 0 & \ddots & \ddots & \ddots \\ \vdots & \ddots & \frac{1}{C_G} & \frac{-1}{C_G} \end{bmatrix}$$

$$A_{SL}=\begin{bmatrix} \frac{1}{C_S} & 0 & \cdots & 0 \\ 0 & \frac{1}{C_S} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{C_S} \end{bmatrix}$$

$$A_{LG}=\begin{bmatrix} \frac{1}{L_B} & \frac{-1}{L_B} & 0 & \cdots & 0 \\ 0 & \frac{1}{L_R} & \frac{-1}{L_R} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & 0 & \frac{1}{L_R} & \frac{-1}{L_R} \\ 0 & \cdots & \cdots & 0 & \frac{1}{L_N} \end{bmatrix}$$

$$A_{LS}=\begin{bmatrix} \frac{-1}{L_B} & 0 & \cdots & 0 \\ 0 & \frac{-1}{L_R} & \ddots & \ddots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ddots & 0 & \frac{-1}{L_N} \end{bmatrix}$$

$$A_{LL}=\begin{bmatrix} \frac{-R_B}{L_B} & 0 & \cdots & 0 \\ 0 & \frac{-R_L}{L_R} & \ddots & \ddots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ddots & 0 & \frac{-R_N}{L_N} \end{bmatrix}$$

$$A_{GG}=A_{GS}=A_{GX}=0$$
$$A_{SG}=A_{SS}=A_{SX}=0$$
$$A_{XG}=A_{XS}=A_{XX}=A_{XL}=0$$
$$A_{LX}=0$$

$$B=\begin{bmatrix} 0_{(1)} & \cdots & 0_{(k-1)} & \frac{-1}{C_{Gk}} & 0_{(k+1)} & \cdots & 0_{(n+1)} & 0_{(1)} & \cdots & 0_{(n+1)} & \frac{1}{C_{EX}} & 0_{(1)} & \cdots & 0_{(n+1)} \end{bmatrix}^T$$

and $C=\begin{bmatrix} 0 & \cdots & 0 & 1_{(3n+4)} \end{bmatrix}$

The transfer function can be found by taking the Laplace Transform of Equation (1):

$$H(s)=\frac{Y(s)}{U(s)}=C(sI-A)^{-1}B \qquad (2)$$

Since a partial discharge is considered to approximate an impulse the input U(s) can be considered to be unity.

Note that the PD injection location can be shifted in the model by changing the node k of N nodes in matrix B.

III. Experimental Results

A. *The Test Configuration*

Testing was conducted on a single phase of a 66kV 25MVA transformer. The primary winding is separated from the core and secondary winding. A split aluminum cylinder is used to represent the core, such that winding to core capacitive relationships are maintained. The winding is interleaved and has 80 turns per disc pair and 19 disc pairs in total. One end of the winding is terminated to ground and the other end is terminated via a 73kV Micafil Bushing. The bushing capacitance to ground is 100pF.

The injection test equipment consisted of an oil-immersed needle-plate electrode with a pressboard sandwiched in between. With the application of 7.5kV, steady streams of partial discharges were generated in the order of ~100pC. Partial discharges were injected into various locations throughout the winding. A broadband current probe and matching amplifier (DC-100MHz) were utilized in conjunction with a digital oscilloscope to record the data. The experimental setup is depicted in Figure 3.

The technique proposed in this paper only requires measurement at one location. This presents two options, the neutral or the DDF tap on the bushing (if present). Figures 4 and 5 show the digitally filtered time responses when recorded at the neutral and the bushing DDF tap. From a practical viewpoint, the DDF point may be a more convenient measurement location. However, for the demonstration of the technique proposed in this paper, the results obtained from the neutral location will be used due to the higher signal to noise ratio.

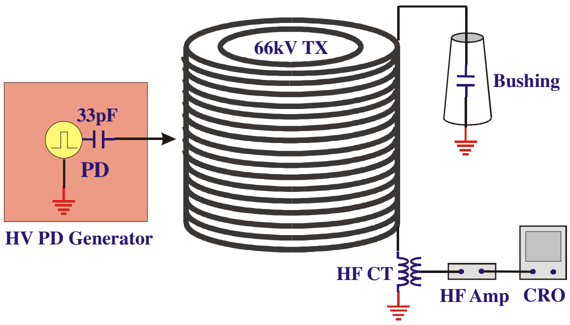

Fig. 3. Test Configuration

B. *Confirmation of the Model*

The Partial Discharge location technique utilized in this paper relies on two important properties. The first is that a PD will approximate an impulse such that the resulting frequency response observed at the neutral will, over the frequency range of interest, reflect the transfer function of that system. The second property is that there exists within the frequency response a region which has resonant pole behavior. The latter property is important since this response is inherently distributive and hence quite useful for PD location [5]. Figure 6 plots the model over the targeted frequency regions against the FFT response of a PD injected into the bushing end of the winding (Signal data as per Figure 4). It is clear that both of these properties are satisfied. The resonant pole model over the narrow band of interest closely resembles the frequency response generated from a PD injected at the bushing, and by extension, the transfer function generated by the Vector Analyzer in Figure 1. Two areas of variation between the model and the PD 'impulse response' are the gain, and to a lesser extent, the phase offset. These variations are primarily due to the non-ideal nature of the 'impulse'. The theory relies on the fact that the impulse in the frequency domain will be equivalent to unity (Equation 2).

In practice this is not the case and will result in spectrum coloring with magnitude variation to be expected. In addition, pre-impulse samples used in the Fourier transform are observed as a time delay which results in an accumulating phase error with increased frequency. The algorithm proposed in this paper automatically compensates for magnitude and minor phase offset by looking at the overall response and adjusting as required.

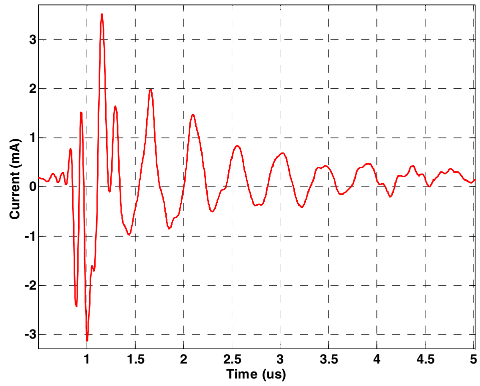

Fig. 4. PD Time Response at Neutral
Injection point at disc pair 1 (Bushing) of 19 with 4ns sampling interval

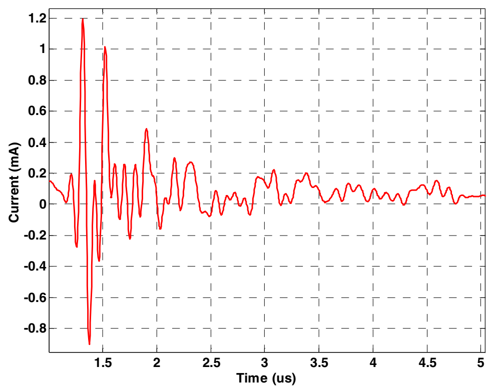

Fig. 5. PD Response at Bushing DDF Tap
Injection point at disc pair 1 (Bushing) of 19 with 4ns sampling interval

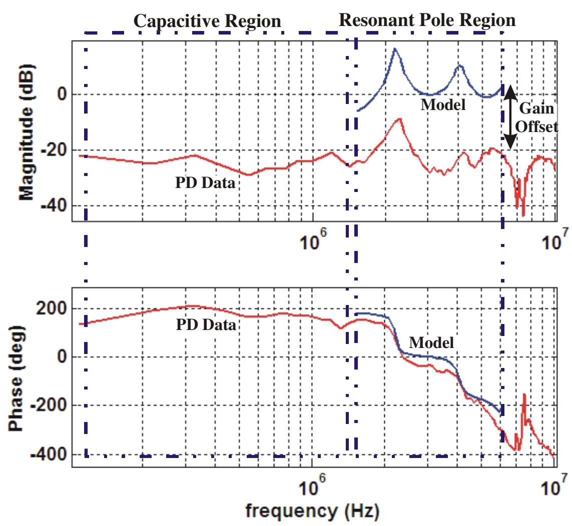

Fig. 6. PD Frequency Response when injected at the Bushing

C. PD Location Algorithm

Calibration of the model requires the acquisition of the bushing to neutral frequency response. This can be achieved via the injection of a calibration impulse and then performing an FFT or obtained directly using a Vector Analyzer. Once the frequency response is found the model parameters are required such that the resonant poles of the model and data response are aligned and have an appropriate level of damping. Once this step is completed PD location can begin.

Unlike the authors' proposal in [5] which relied on the zero location between the resonant poles within the magnitude response to estimate location, this paper extends on this work by comparing all frequency points within the frequency band with respect to both the magnitude and phase. This resulted in a more reliable and accurate outcome.

To compare the model and data waveforms a cost function (3) is used. The cost is calculated via the cumulative residual differences between corresponding complex data points such that the best possible match will have the lowest cost. The cost function is given by:

$$Cost = \left| \left[\log_{10}\left(\frac{\hat{H}}{H}\right) \right] \left[\log_{10}\left(\frac{\hat{H}}{H}\right) \right]^T \right| \qquad (3)$$

where:-

H = Observed Partial Discharge frequency response

$\hat{H}$ = Model frequency response

This function is applied to the PD frequency response with the injection model of every disc pair within the winding (in this instance, 19 disc pairs). Comparative data to model examples for various PD injection locations over the narrow target frequency band are shown in Figures 7 through to 9. Plots of the cost function are given in Figure 10. The complete results are listed in Table I and plotted in Figure 11.

The results are good and the associated minima for the various injection locations depicted in Figure 10, provides evidentiary support that the modeling approach taken is appropriate.

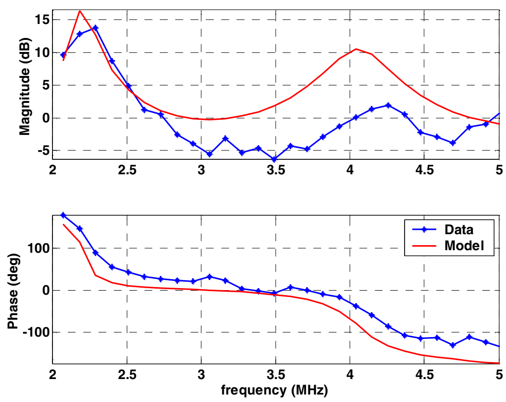

Fig. 7. Data versus Model for Disc Pair 1/19

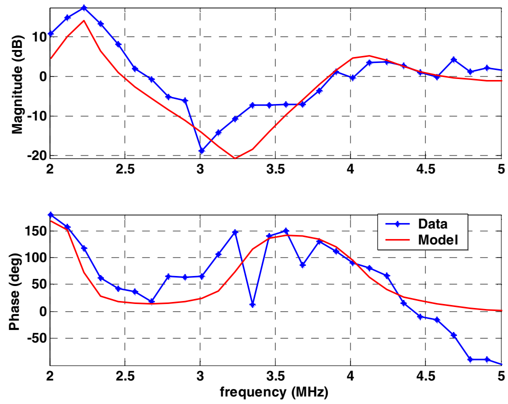

Fig. 8. Data versus Model for Disc Pair 9/19

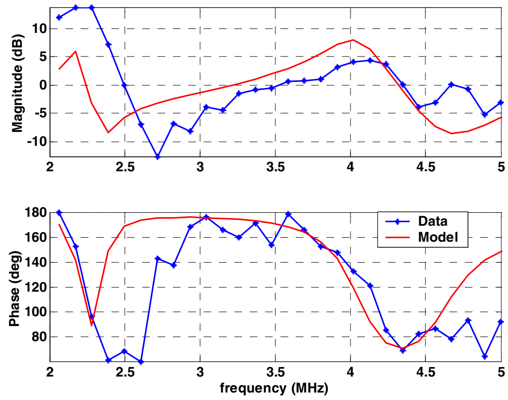

Fig. 9. Data versus Model for Disc Pair 17/19

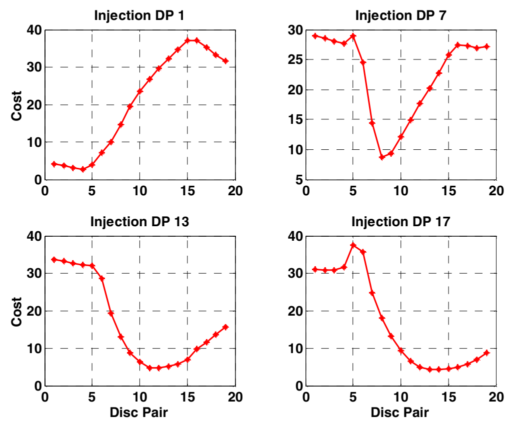

Fig. 10. Cost Function vs Winding Disc Pair (1-19)

TABLE I
PD INJECTION LOCATION VERSUS ALGORITHM ESTIMATE

PD Disc Pair	Est Disc Pair	Error (%)
***1/19	4/19	+16%
3/19	3/19	0%
5/19	7/19	+11%
7/19	8/19	+5%
9/19	9/19	0%
11/19	10/19	-5%
13/19	12/19	-5%
15/19	15/19	0%
17/19	13/19	-21%

*** Calibration node at bushing

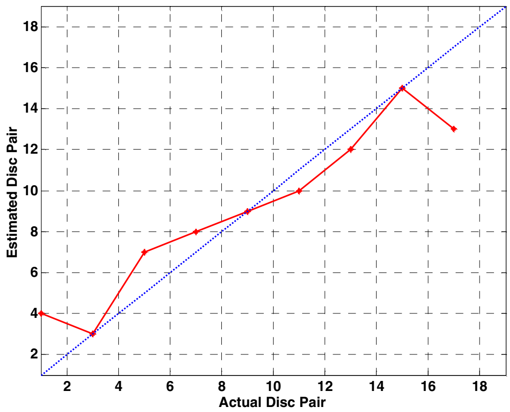

Fig. 11. Estimated versus actual PD injection location

IV. Conclusion

It has been proposed that the interaction within an interleaved power transformer winding of the residual inductance and the capacitance to ground is the physical phenomenon behind an observed resonant pole response. Since this interaction is inherently distributed, the response will be dependent upon the input location. As demonstrated in the paper the frequency response of a partial discharge will approximate the transfer function of the system with respect to the PD location. By implementing an algorithm that iteratively compares the proposed model at various locations within the winding with the observed frequency response, an estimate of the partial discharge location can be found. The results presented in this paper demonstrate the validity of this modeling approach.

V. Acknowledgment

The authors would like to thank the University of New South Wales, Australia, for access to their testing facilities.

VI. References

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VII. Biographies

Steven D Mitchell was born on 24th December 1969 in Newcastle Australia. He received his B.E degree (Hons) in electrical engineering from the University of Newcastle, Callaghan, NSW Australia in 1996. He is currently completing his PhD part time whilst working as a design engineer within the faculty of Engineering at the University of Newcastle. The PhD is focused on transformer fault diagnostics with research on partial discharge localization and frequency response analysis. Industrial experience was gained through work in heavy manufacturing and underground mine control/protection equipment design, between 1988 and 2000.

James Welsh was born in Maitland, Australia, in 1965. He received the B.E. degree (Hons. I) in electrical engineering from The University of Newcastle, Callaghan, NSW, Australia, in 1997. Dr. Welsh received his PhD in 2004, which studied ill-conditioning problems arising in system identification, from the same university. He gained industrial experience from 1981 to 1997 with Bloomfield Collieries, AES, and TYP Pty. Ltd., Australia. During the last several years, he has been actively involved in research projects which include, Powertrain Control, Model Predictive Control and System Identification Toolboxes, with the Centre for Complex Dynamic Systems and Control, the University of Newcastle. His research interests include auto-tuning, system identification, and process control. Dr Welsh is currently employed as a Lecturer in the School of Electrical Engineering and Computer Science at the University of Newcastle. He was awarded the University of Newcastle, Faculty of Engineering and Built Environment Prize for the Best International Journal Paper published by a postgraduate in 2003. In 1997, he won the Institute of Engineers Australia National Undergraduate Thesis Prize in Automation, Control, and Instrumentation.

Professor Richard H. Middleton was born on 10th December 1961 in Newcastle Australia. He received his B.Sc. (1983), B.Eng. (Hons-I)(1984) and Ph.D. (1987) from the University of Newcastle, Australia. He has had visiting appointments at the University of Illinois at Urbana-Champaign, the University of Michigan and the Hamilton Institute (National University of Ireland Maynooth). In 1991 he was awarded the Australian Telecommunications and Electronics Research Board Outstanding Young Investigator award. In 1994 he was awarded the Royal Society of New South Wales Edgeworth-David Medal, and the M.A. Sargent Award from the Electrical College of Engineers Australia in 2004. He has served as an associate editor of the IEEE Transactions on Automatic Control, the IEEE Transactions on Control System Technology, and Automatica, as Head of Department of Electrical and Computer Engineering at the University of Newcastle; as a panel member and sub panel chair for the Australian Research Council; as Vice President - Member Activities of the IEEE Control Systems Society, and as Director of the ARC Centre for Complex Dynamic Systems and Control. He was elected to the grade of Fellow of the IEEE starting 1999. He is currently a Research Professor at the Hamilton Institute, The National University of Ireland, Maynooth; a Senior Research Associate of the University of Newcastle; and Vice President (Conference Activities) of the IEEE Control System Society. His research interests include a broad range of Control Systems Theory and Applications.

B.T. (Toan) Phung (M'1987) received his Ph.D in electrical engineering in 1998 for his work on computer-based partial discharge detection and characterisation. He is currently a senior lecturer at the University of New South Wales, Australia. His main research interests are in high-voltage engineering and condition monitoring of power equipment. He is a member of the CIGRE Australian Panel D1.