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A NEW ADAPTIVE OBSERVER.(U)

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Kumpati S. Narendra and Lena S. Valavani

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Letter on file

A New Adaptive Observer

Kumpati S. Narendra and Lena S. Valavani

1. Introduction:

An adaptive observer is defined as one which estimates the state variables and parameters of an unknown stable linear time-invariant plant from its input-output data. During the period 1973-1975 several seemingly different versions of the adaptive observer [1-3] appeared in the control literature. Recently, however, it has been shown [5] that all these results can be derived in a unified manner. At the present time, there are two distinct approaches to the design of adaptive observers for a plant whose input output behavior can be represented by an n-th order differential equation. In the first approach, the observer is of the same order as the plant and is referred to as a minimal (order) observer. Using the second approach, a non-minimal observer of order (2n-1) is obtained. Minimal observers are considerably more difficult to synthesize than non-minimal observers and require the generation of additional signals for the stabilization of the adaptive loop [5]. However, they have the advantage of yielding simultaneously both parameter and state estimates of the plant. Non-minimal observers are considerably simpler in structure but the n state variables of the plant have to be estimated from the available (2n-1) state variables of the observer.

In this brief paper, ~~we propose~~ it is a new observer is proposed which appears to combine the advantages of the two types of observers described above. With this observer, the parameter estimates of the plant are directly obtained with a structure which is no more complex than that of a non-minimal observer [5] which is widely used at the present time. The parameter estimates are simultaneously used to determine directly the state estimates of the plant. Under certain conditions, ~~discussed in section 5,~~ the new observer has a faster rate of convergence than the observers known at present, which makes it particularly attractive for use in the control problem [6].

From a theoretical point of view, the lemmas in section 4 contain the principal ideas in this paper. A direct application of the lemmas results in a new prototype for an error model and this in turn forms the mathematical basis for the adaptive observer.

2. <u>The Problem</u>:

A plant P has an admissible input u(t) {a scalar valued piecewise continuous function} and a corresponding output y(t). A linear system $\{h^T, A_p, b_p\}$ is said to characterize this plant if every pair {u(t),y(t)} satisfies the equations

$$\begin{aligned} \dot{x}_p &= A_p x_p(t) + b_p u(t) \\ y(t) &= h^T x_p(t) \end{aligned} \tag{1}$$

where A_p is an (nxn) matrix and h and b_p are constant n-vectors. The identification problem may be *qualitatively defined as the problem* of constructing a suitable model which, when subjected to the same input u(t) as the plant, produces an output $y_m(t)$ which tends towards $y_p(t)$ asymptotically.

In the simplest case, we are interested in output identification in which $|e_1(t)| \to 0$, where $e_1(t) \triangleq y(t) - y_m(t)$, for a given input u(t). When the input is "sufficiently rich" this also implies transfer function identification. If the transfer function of the plant identified corresponds to a unique parametrization of the plant, we also have parameter identification.

We are interested here in a specific parametrization of the plant and, hence, in both the output and parameter identification problems.

The plant is described by equation (1) where A_p is a stable (nxn) matrix which is in output canonical form so that:

$$A_p = \left[a_p \left| \begin{array}{c} I \\ \hline 0 \end{array} \right. \right] \tag{2}$$

and $h^T = [1,0,0,\dots,0]$. The vectors a_p and b_p in equation (1) are the unknown parameters of the plant which have to be estimated.

If A_m is a known stable matrix in output canonical form such that

$$A_m = \left[a_m \left| \frac{I}{0} \right. \right] \tag{3}$$

equation (1) may be expressed as

$$\begin{aligned} \dot{x}_p &= A_m x_p + \theta_1 x_{1p} + b_p u \\ x_{1p} &= h^T x_p \quad \text{and} \quad \theta_1 = a_p - a_m \end{aligned} \tag{4}$$

To identify the parameter vectors a_p and b_p, we set up an observer which is described by the differential equation

$$\begin{aligned} \dot{x}_m &= A_m x_m + b_m u + d[\hat{\theta}_1^T v^1 + \hat{\theta}_2^T v^2] \\ x_{1m} &= h^T x_m \end{aligned} \tag{5}$$

where b_m and d are known constant vectors, $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ are n-dimensional adjustable parameter vectors which are the estimates of the parameter error vectors $\theta_1 = (a_p - a_m)$ and $\theta_2 = (b_p - b_m)$, and $v^1(t)$ and $v^2(t)$ are known input signals which can be generated using the input and output of the plant.

If $x_p(t) - x_m(t) \triangleq e(t)$, the state error vector and $h^T e(t) = e_1(t)$, the output error, we have from equations (4) and (5)

$$\begin{aligned} \dot{e} &= A_m e + \theta_1 x_1 + \theta_2 u - d[\hat{\theta}_1^T v^1 + \hat{\theta}_2^T v^2] \\ e_1 &= h^T e \end{aligned} \tag{6}$$

The problem now is to determine how $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ must be adjusted so that they evolve to θ_1 and θ_2 respectively and result in the output error $e_1(t)$ tending to zero as $t \to \infty$. As in all adaptive observers, the adaptive laws are expressed in terms of the time derivatives of $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$.

3. Error Equations for Minimal and Non-minimal Adaptive Observers:

Equation (6) describes the behaviour of the output error between plant and model in terms of the unknown parameter error vectors θ_1 and θ_2 and the adjustable parameters, $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$. Once the adaptive laws for updating $\dot{\hat{\theta}}_1(t)$ and $\dot{\hat{\theta}}_2(t)$ are specified, equation (6) can be written entirely in terms of the output and parameter errors of the overall system. The analysis of the stability behaviour of the error equations is obviously central to the understanding of the convergence properties of the adaptive observer. Asymptotic stability of the error equations, for example, implies that the states and parameters of the observer converge to those of the plant asymptotically.

Before proceeding to consider the error equations of the new adaptive observer, we shall discuss briefly in this section the error equations which have arisen in the past in the design of minimal and non-minimal observers.

As pointed out by Narendra and Kudva [5] and Anderson [7], an interesting prototype for error equations has the form [Figure 1]:

$$\begin{aligned} \dot{\varepsilon} &= A_m\varepsilon + d\theta^T(t)v(t) \\ \varepsilon_1 &= h^T\varepsilon \end{aligned} \tag{7}$$

Where $\varepsilon(t)$ is an n-vector of errors, $\theta(t)$ is an m-vector of parameter errors, $v(t)$ is an m-vector of piecewise continuous bounded time functions which are linearly independent on the semi-infinite interval, h and d are constant n-vectors, A_m is an (nxn) stable matrix and the triple $\{h^T, A_m, d\}$ is completely controllable and completely observable.

It has been shown that if the transfer function $h^T(sI-A_m)^{-1}d$ is strictly positive real then the adaptive law

$$\dot{\theta} = -\Gamma\varepsilon_1 v \qquad \Gamma = \Gamma^T > 0 \tag{8}$$

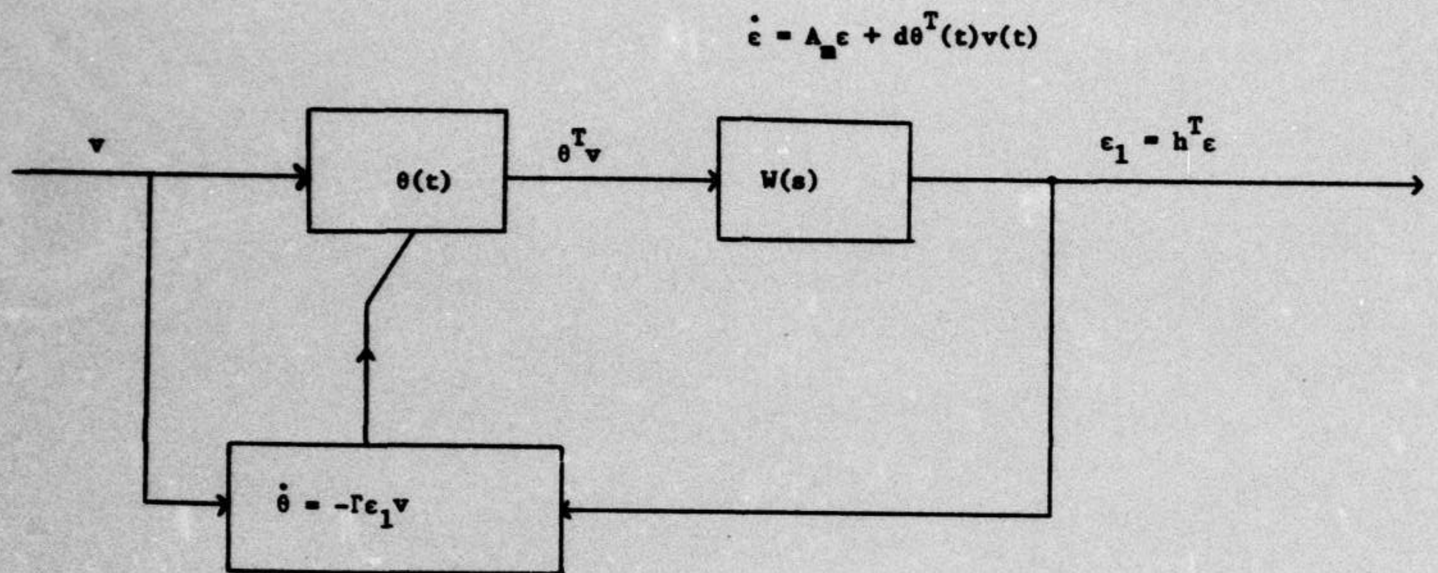

Figure 1

where Γ is any symmetric positive definite matrix, will result in a stable system in which $\epsilon(t)$ will tend to zero and $\theta(t)$ to some constant vector θ_0 as $t \to \infty$. The adaptive law (8) is obtained by selecting a function

$$V(\epsilon,\theta) = \frac{1}{2}[\epsilon^T P\epsilon + \theta^T\Gamma^{-1}\theta] \tag{9}$$

as a candidate for a Lyapunov function and choosing $\dot{\theta}(t)$ to make $\dot{V}(\epsilon,\theta) = \frac{dV}{dt}$ negative semidefinite. It has further been shown that, if the input v(t) is "sufficiently rich", the parameter error vector $\theta(t)$ will tend to zero [8].

The above simple but powerful result can be directly applied to the design problem of non-minimal observers. In the discussion that follows it is shown that it can also be related to an error differential equation (10) which arises in the design of minimal observers.

In the differential equation

$$\dot{e}(t) = A_m e(t) + \theta(t)z_1(t) \qquad e_1(t) = h^T e(t) \tag{10}$$

let e(t) be an n-vector, A_m an (nxn) stable constant matrix, $\theta(t)$ an n-vector of parameter errors and $z_1(t)$ a uniformly bounded scalar function of time. It is desired to adjust $\dot{\theta}(t)$ using only available input and output data (i.e. $e_1(t)$ and $z_1(t)$) such that $e_1(t)$ will tend to zero asymptotically.

The form of equation (10) does not lend itself easily to the generation of adaptive laws of the form

$$\dot{\theta} = f(e_1,z_1). \tag{11}$$

However, it has been shown [4] that vectors w(t) and v(t) exist such that

$$\dot{e} = A_m e + \theta(t)z_1(t) + w(t) \qquad e_1(t) = h^T e(t) \tag{12}$$

$$\dot{\epsilon} = A_m \epsilon + d\theta^T(t)v(t) \qquad \epsilon_1(t) = h^T \epsilon_1(t) \tag{13}$$

have the same outputs $e_1(t)$ and $\varepsilon_1(t)$. The vector $v(t)$ can be generated by a dynamical system whose input is $z_1(t)$ while $w(t)$ depends on both $e_1(t)$ and $v(t)$.

In view of the input-output equivalence of the two systems (12) and (13), the adaptive law for (12) is generated in an identical manner to that of (13) and is given by equation (8)

$$\dot{\theta} = -\Gamma\varepsilon_1(t)v = -\Gamma e_1(t)v \quad \Gamma = \Gamma^T > 0 \qquad (14)$$

Hence, a minimal order adaptive observer can be designed by generating suitable vectors $v(t)$ and $w(t)$, where $v(t)$ is used in the adaptive law and the vector $w(t)$ is used as an input to the observer. This in turn yields the well known form of the adaptive observer shown in Figure [2].

Equations (7) and (8) represent the form of the error equations for a non-minimal observer while equations (12) and (14) are the error equations for a minimal observer. In both cases the time derivative of the parameter error vector θ is adjusted so that in the limit $\theta(t)$ tends to zero. In equation (6) which is the error equation of the new adaptive observer, no provision is made to directly adjust the parameter error vectors θ_1 and θ_2. Instead, new adjustable parameter vectors $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ are introduced to estimate θ_1 and θ_2 and compensate for the effect on $e_1(t)$. It is this fact that distinguishes the new adaptive observer from those that are currently in existence.

The error equations which describe the behavior of the new observer are of the form:

$$\begin{aligned}
\dot{e} &= A_m e + \theta_1 x_{1p} + \theta_2 u - d[\hat{\theta}_1^T v^1 + \hat{\theta}_2^T v^2] \\
\dot{\hat{\theta}}_1 &= f_1[e_1, x_{1p}, u, v^1, v^2] \qquad (15) \\
\dot{\hat{\theta}}_2 &= f_2[e_1, x_{1p}, u, v^1, v^2]
\end{aligned}$$

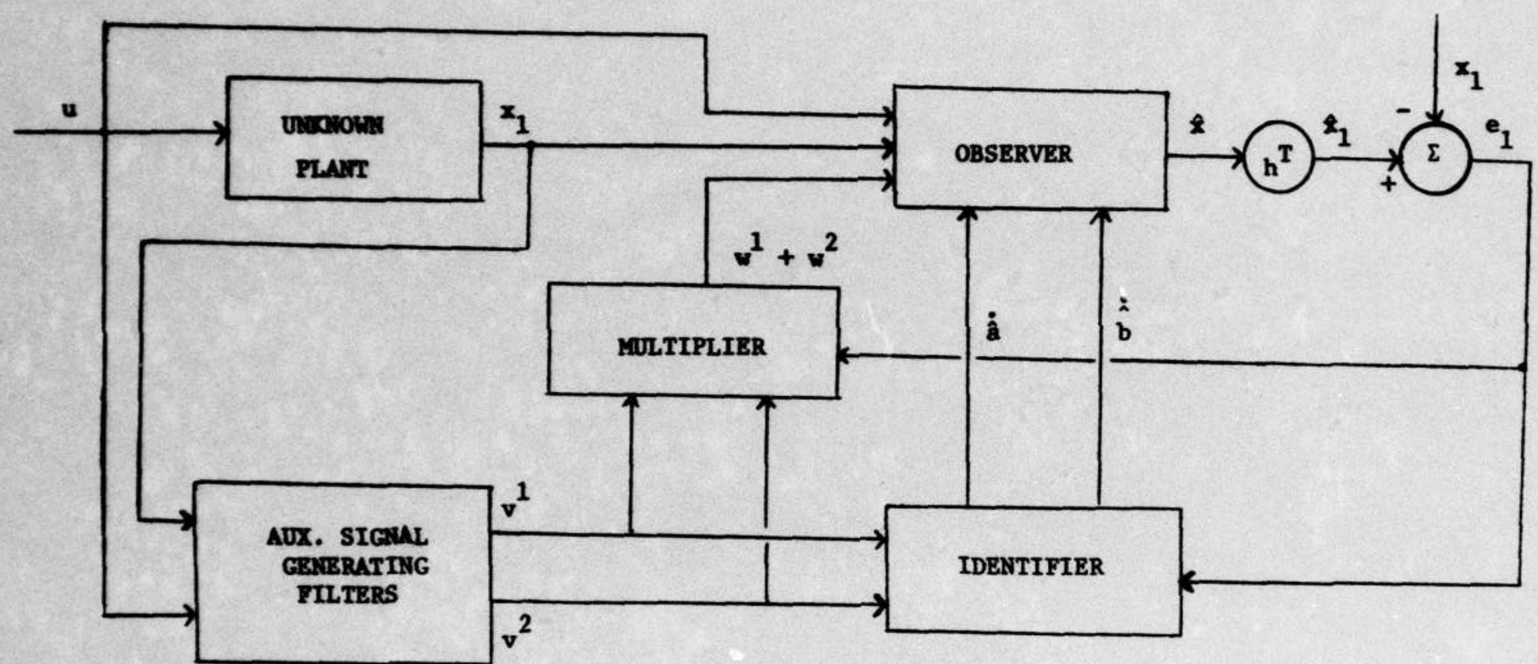

Figure 2 The Adaptive Observer

where f_1 and f_2 have to be suitably chosen so that $e_1(t) \to 0$, $\hat{\theta}_1(t) \to \theta_1$ and $\hat{\theta}_2(t) \to \theta_2$ as $t \to \infty$.

4. **Mathematical Preliminaries:**

The following lemmas are found to be directly related to the questions raised at the end of the last section and hence to the final structure of the adaptive observer.

Lemma 1: A time-invariant dynamical system

$$\dot{z} = Az + \phi u \qquad z_1(t) = h^T z(t) \tag{16}$$

where A is an (nxn) stable matrix in output canonical form, $h^T = [1,0,\ldots,0]$, $\phi^T = [\phi_1,\phi_2,\ldots,\phi_n]$ is zero state input-output equivalent to the dynamical system of order (2n-1)

$$\begin{aligned} \dot{w} &= Aw + d\phi^T v^1 \qquad w_1(t) = h^T w \\ \dot{v} &= Dv + bu(t) \\ v^1 &= L \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned} \tag{17}$$

where the (n-1 x n-1) matrix D is stable, v is an (n-1) dimensional vector, $b^T = [0,0,\ldots,0,1]$ $d^T = [1,d_2,d_3,\ldots,d_n]$ and the matrices D and L are defined as

$$D \triangleq \begin{bmatrix} 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & 0 & . & . \\ 0 & 0 & . & . & . & 1 \\ -1 & -d_2 & , & , & , & -d_n \end{bmatrix} \qquad L \triangleq \begin{bmatrix} 1 & -d_2 & -d_3 & . & . & . & -d_n \\ 0 & 1 & 0 & . & . & . & . \\ 0 & 0 & . & . & . & 1 & 0 \\ 0 & . & . & . & . & . & 1 \end{bmatrix}$$

Proof: The transfer function of (16) is

$$T_1(s) = \frac{\phi_1 s^{n-1} + \ldots + \phi_n}{s^n + a_1 s^{n-1} + \ldots + a_n}$$

and the transfer function of the system (17) is

$$T_2(s) = \frac{s^{n-1} + d_2 s^{n-2} + \ldots + d_n}{s^n + a_1 s^{n-1} + \ldots + a_n} \qquad \frac{\phi_1 s^{n-1} + \ldots + \phi_n}{s^{n-1} + \ldots + d_n}$$

Hence, for zero initial states the two systems have the same output corresponding to any input $u(t)$.

<u>Lemma 2</u>: In the dynamical system described by the differential equations

$$\begin{aligned} \dot{z} &= Ax + \phi u(t) - d[\hat{\phi}^T(t) v^1(t)] \\ \dot{\hat{\phi}} &= \Gamma z_1(t) v^1(t) \qquad \Gamma = \Gamma^T > 0 \\ z_1(t) &= h^T z(t) \end{aligned} \tag{18}$$

where the matrix A and the vectors ϕ,d, and $v^1(t)$ are as defined in lemma 1, Γ is any symmetric positive definite matrix and $h^T(sI-A)^{-1}d$ is strictly positive real, $\hat{\phi}(t) \to \phi$ if the input is sufficiently rich and the overall system is asymptotically stable.

When the input $u(t) \equiv 0$ equation (18) has the same form as equation (7) and hence $\|\hat{\phi}(t)\|$ is bounded.

Considering the differential equations

$$\dot{\xi}_1 = A\xi_1 + \phi u(t) \qquad \zeta_1 = h^T \xi_1 \tag{19}$$

and

$$\dot{\xi}_2 = A\xi_2 - d[\hat{\phi}^T(t) v^1(t)] \qquad \zeta_2 = h^T \xi_2 \tag{20}$$

we have $\xi_1 + \xi_2 = z$ and $\zeta_1 + \zeta_2 = z_1(t)$ in equation (18).

Equation (19) is input-output equivalent to

$$\dot{\xi}_3 = A\xi_3 + d\phi^T(t) v^1(t) \qquad \zeta_3 = h^T \xi_3 \tag{21}$$

Hence equation (18) has the same input-output behavior (for zero initial conditions) as

$$\dot{\eta} = A\eta + d[\{\phi-\hat{\phi}(t)\}^T v^1(t)]$$
$$\eta_1 = h^T\eta = z_1(t). \tag{22}$$

Hence, by the prototype discussed in section (3) if

$$\dot{\hat{\phi}} = \Gamma z_1(t)v^1(t) \tag{23}$$

and the input u(t) is sufficiently rich

$$\lim_{t\to\infty} \hat{\phi}(t) = \phi(t) \quad \text{and} \quad \lim_{t\to\infty} e_1(t) = 0.$$

5. The Adaptive Observer:

The results of section 4 can be directly applied to the adaptive observer error equations given in (15). In such a case we have

$$\dot{e} = A_m e + \theta_1 x_{1p} + \theta_2 u - d[\hat{\theta}_1^T v^1 + \hat{\theta}_2^T v^2]$$
$$e_1 = h^T e$$
$$\dot{\hat{\theta}}_1 = \Gamma_1 e_1(t)v^1(t) \tag{24}$$
$$\dot{\hat{\theta}}_2 = \Gamma_2 e_1(t)v^2(t) \qquad \Gamma_i = \Gamma_i^T > 0 \qquad i = 1,2$$

It is obvious that $v^1(t)$ and $v^2(t)$ can be generated using two identical systems with $x_{1p}(t)$ and u(t) as the inputs. If the 2n components of $[v^1(t),v^2(t)]^T$ are linearly independent and the input u(t) is sufficiently rich, the parameters $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ converge to the true parameter errors θ_1 and θ_2.

From the arguments of section 4, it is clear that as $t \to \infty$, $e_1(t) \to 0$. However, the other components of the error vector e(t) do not tend to zero indicating that $x_m(t)$ is not an estimate of $x_p(t)$. To obtain such an estimate $\hat{x}_p(t)$ of $x_p(t)$, the estimates of the parameter errors are used in equation (4) to yield

$$\dot{\hat{x}}_p(t) = A_m\hat{x}_p + b_m u + \hat{\theta}_1 x_{1p} + \hat{\theta}_2 u \tag{25}$$

The complete description of the observer is then given by the differential equations

$$\dot{x}_m = A_m x_m + b_m u + d[\hat{\theta}_1^T v^1 + \hat{\theta}_2^T v^2] \tag{26a}$$

$$x_{m1} = h^T x_m$$

$$\dot{\hat{x}}_p = A_m\hat{x}_p + b_m u + \hat{\theta}_1 x_{1p} + \hat{\theta}_2 u \tag{26b}$$

$$\dot{\hat{\theta}}_1 = -\Gamma_1 e_1 v^1$$

$$\dot{\hat{\theta}}_2 = -\Gamma_2 e_1 v^2 \tag{26c}$$

The resulting observer structure is shown in figure 3.

Comments on the New Adaptive Observer:

a) As seen from the lemmas in section 4, the approach used in the design of the new observer is to represent the unknown transfer function as the product of two transfer functions, one of which is stable and the other strictly positive real. Using such a representation, the prototype error model developed for the design of non-minimal observers can be directly used to estimate the unknown parameters.

b) The estimate $\hat{x}_p$ of the state of the plant is obtained by using the estimates of the unknown parameters in equation 26b. The error in the state estimate $\epsilon(t) \overset{\Delta}{=} x_p(t) - \hat{x}_p(t)$ satisfies the differential equation

$$\dot{\epsilon} = A_m\epsilon + [\theta_1 - \hat{\theta}_1(t)]x_1(t) + [\theta_2 - \hat{\theta}_2(t)]u(t)$$

and tends to zero as the parameter errors tend to zero.

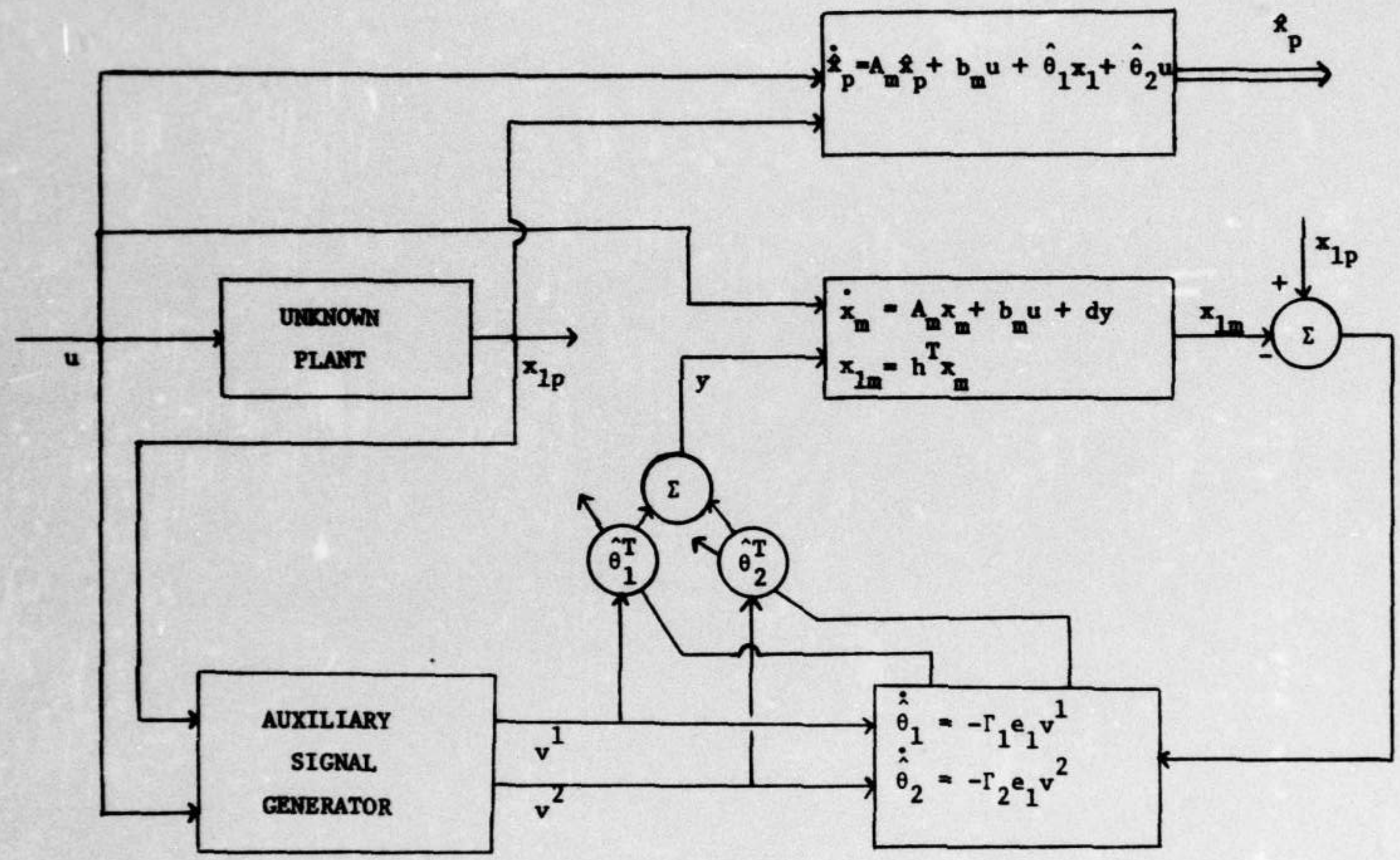

Figure 3 The New Adaptive Observer

c) The examples discussed in the next section indicate that the adaptive observer converges rapidly when the system is of low order or when either the poles or the zeros of the plant transfer function are known. However, when all the parameters of a high order plant are unknown the rate of convergence is found to be rather slow which may be attributed to the fact that the components of the vector $v^1(t)$ and $v^2(t)$ are linearly dependent. Faster schemes of convergence are currently being investigated.

d) The n-vectors $v^1(t)$ and $v^2(t)$ can be generated using identical dynamical systems with $x_1(t)$ and $u(t)$ as inputs. This implies that a single vector d has to be chosen such that $h^T(sI-A_m)^{-1}d$ is strictly positive real. In practice, for faster convergence, it may be desirable to use a correction signal of the form $d_1\hat{\theta}_1^T v^1 + d_2\hat{\theta}_2^T v^2$, instead of $d[\hat{\theta}_1^T v^1 + \hat{\theta}_2^T v^2]$. In this case the transfer functions $h^T(sI-A_m)^{-1}d_1$ and $h^T(sI-A_m)^{-1}d_2$ must be positive real.

6. Examples:

Several plants were identified using the new scheme on a digital computer. Three typical cases are included below. In the first case, the plant is of second order and has four unknown parameters. In the second and third cases, the plant is of third and fourth order respectively. In the first case the transfer function of the plant is shown and the evolution of the unknown parameters to their final values is indicated. The states of the plant and the corresponding estimates are also tabulated. For the other two cases the final values of the parameters are given.

Example 1:

$$w(s)\,[\text{Transfer function of the Plant}] = \frac{2(s+2)}{s^2+3.5s+2.5}$$

Time in Seconds	Plant Parameters $a_1 = 3.5$	$a_2 = 2.5$	$b_1 = 2$	$b_2 = 4$
	OBSERVER PARAMETERS			
2	3.06	2.97	2.90	3.99
4	3.53	2.28	1.86	3.99
6	3.51	2.35	2.02	3.98

The following table shows the states of the plant and the corresponding estimates at t = 2,4 and 6 seconds.

Time	x_{1p}	$\hat{x}_{1p}$	x_{2p}	$\hat{x}_{2p}$
2	1.69	2.12	3.69	5.65
4	- .14	- .13	- .47	- .14
6	- .87	- .87	-1.60	-1.47

Example 2: In this case the plant is of third order with a transfer function

$$w(s) = \frac{2(s^2+5s+6)}{s^3+8.3s^2+21.4s+16.8}$$

The unknown parameters are b_1 and a_1, a_2 and a_3. The parameter estimates given by the observer after twelve seconds are shown below:

$$b_1 = 2 \quad \hat{b}_1 = 1.9 \ ; \quad a_1 = 8.3 \quad \hat{a}_1 = 8.02 \ ;$$
$$a_2 = 21.4 \ \hat{a}_2 = 21.6 \ ; \quad a_3 = 16.8 \quad \hat{a}_3 = 17.2 \ .$$

Example 3: A fourth order plant has a transfer function

$$w(s) = \frac{20[s^3+2s^2+2s+1]}{s^4+12s^3+45s^2+60s+20}$$

where the zeros of the plant are known. The following estimates of the unknown parameters were obtained after 6 seconds.

$$a_1 = 12 \quad \hat{a}_1 = 11.98 \ ; \quad a_2 = 45 \quad \hat{a}_2 = 45.43$$
$$a_3 = 60 \quad \hat{a}_3 = 60.39 \quad \text{and} \quad a_4 = 20 \quad \hat{a}_4 = 20.19$$

In all cases a square wave input of amplitude 5 and frequency 5 radians/sec was used.

Conclusion:

A new structure for an adaptive observer is proposed in this paper. The unknown parameters of the system are continuously estimated in a stable feedback loop and are in turn used to generate the estimate of the state of the plant.

The overall system is found to be no more complex than the non-minimal observer currently in use.

When the plant can be represented by a differential equation of low order or when only the poles or zeros of its transfer function are unknown the observer parameters are found to converge very rapidly to their final values. For high order plants whose poles and zeros are not known convergence appears to depend critically on the auxiliary signals used in the adaptive laws.

The importance of the new adaptive observer lies in its potential application to the control problem.

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