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# A NEW ANALYTICAL APPROACH TO NONLINEAR VIBRATION OF AN ELECTRICAL MACHINE

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In this paper, the nonlinear dynamic behaviour of an electrical machine exhibiting nonlinear vibration is investigated using a new analytical technique, namely Optimal Homotopy Asymptotic Method. This study provides an effective and easy to apply procedure which is independent on whether or not there exist small parameters in the considered nonlinear equation, different from perturbation methods, which require the existence of the small parameter. The approximate analytic solution is in very good agreement with the numerical simulations results, which prove the reliability of the method.

*Key words:* Nonlinear vibration, Optimal Homotopy Asymptotic Method, Electrical machine.

## 1. INTRODUCTION

Electrical machines are widely used in engineering applications and industry due to their reliability. They are dynamical systems encountering dynamical phenomena which can be detrimental to the system. From engineering point of view it is very important to predict the nonlinear dynamic behaviour of complex dynamical systems, such as the electrical machines. This is a significant stage in the design process, before the machine is exploited in real conditions, avoiding in this way undesired dynamical phenomena which could damage the system. Basically, the electric machines share the same dynamical problems with classical rotor systems, having specific sources of excitation, which lead to nonlinear vibration occurrence.

The main sources of dynamic problems are the unbalanced forces of the rotor [1], [2], bad bearings or nonlinear bearings [3], [4], mechanical looseness, misalignments, other electrical and mechanical faults which generate nonlinear vibration in the system. These problems are usually solved by numerical simulations [5], experimental investigations [6], [7] or by analytical developments [8], [9].

In general, the nonlinear vibration problems are usually solved using perturbation methods, which are the most used analytical techniques. Some of the most used methods are the Lindstedt-Poincare method [10], the Krylov-Bogoliubov-Mitropolsky method [11], [12] the Adomian decomposition method [13] and other perturbation method [14]. Unfortunately, as it is well-known, the perturbation methods have their limitations since they are based on the existence of a small parameter and especially in strongly nonlinear systems these classical methods fail. Therefore scientists are continuously concerned in developing new analytical techniques which aim at surmounting these limitations.

Recently, new powerful analytical tools were developed, such as the Variational Iteration Method [15], [16], [17], [18], [19], [20], Homotopy Analysis Method [21], [22], Homotopy Perturbation Method [23], [24], [25], [26] the parameter-expanding method [27], in an attempt to obtain effective analytical tools, valid for any strongly nonlinear problems.

In this paper, a new analytical procedure, namely Optimal Homotopy Asymptotic Method is employed in order to study the problem of nonlinear vibrations of an electric machine. The investigated electrical machine is considered to be supported by nonlinear bearings and the assumption made in development of the mathematical model is that these bearings are characterised by nonlinear stiffness of Duffing type. In the

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same time, the entire dynamical system is subjected to a parametric excitation caused by an axial thrust and a forcing excitation caused by an unbalanced force of the rotor, which is obviously harmonically shaped. In these conditions, the dynamical behaviour of the investigated electrical machine will be governed by the following second-order strongly nonlinear differential equation:

$$m\ddot{x} + k_1(1 - q\sin\omega_2 t)x + k_2 x^3 = f\sin\omega_1 t\,;\ x(0) = A\,;\ \dot{x}(0) = 0\,, \tag{1}$$

which can be written in the more convenient form:

$$\ddot{x} + \omega^2 x - \alpha x\sin\omega_2 t + \beta x^3 - \gamma\sin\omega_1 t = 0\,;\ \ x(0) = A\,;\ \dot{x}(0) = 0 \tag{2}$$

where $\omega^2 = \frac{k_1}{m}$, $\alpha = \frac{k_1 q}{m}$, $\beta = \frac{k_2}{m}$, $\gamma = \frac{f}{m}$, the dot denotes derivative with respect to time and A is the amplitude of the oscillations. Note that it is unnecessary to assume the existence of any small or large parameter in Eq.(2).

The main purpose of the present paper is to use the Optimal Homotopy Asymptotic Method (OHAM) for obtaining solutions of strongly nonlinear vibration of the electrical rotating machinery under study.

## 2. BASIC IDEA OF OHAM [28], [29]

The Eq.(2) describes a system oscillating with an unknown period T. We switch to a scalar time $\tau = 2\pi t / T = \Omega t$ . Under the transformation

$$\tau = \Omega t \tag{3}$$

the original Eq.(2) becomes

$$\Omega^2 x'' + \omega^2 x - \alpha x\sin\frac{\omega_2}{\Omega}\tau + \beta x^3 - \gamma\sin\frac{\omega_1}{\Omega}\tau = 0 \tag{4}$$

where the prime denotes the derivative with respect to τ.

By the homotopy technique, we construct a homotopy in a more general form:

$$H(\phi(\tau,p), h(\tau,p)) = (1-p)L(\phi(\tau,p)) - h(\tau,p)N[\phi(\tau,p),\Omega(\lambda,p)] = 0 \tag{5}$$

where L is a linear operator:

$$L(\phi(\tau,p)) = \Omega_0^2\left[\frac{\partial^2\phi(\tau,p)}{\partial\tau^2} + \phi(\tau,p)\right] \tag{6}$$

while N is a nonlinear operator:

$$N[\phi(\tau,p),\Omega(\lambda,p)] = \Omega^2(p)\frac{\partial^2\phi(\tau,p)}{\partial\tau^2} + (\omega^2+\lambda)\phi(\tau,p) - \alpha\phi(\tau,p)\sin\frac{\omega_2}{\Omega}\tau + \\ +\beta\phi^3(\tau,p) - \gamma\sin\frac{\omega_1}{\Omega}\tau - p\lambda\phi(\tau,p) \tag{7}$$

where $p \in [0,1]$ is the embedding parameter, h(τ,p) is an auxiliary function such as h(τ,0)=0, $h(\tau,p) \neq 0$ for $p \neq 0$, λ is an arbitrary parameter. From Eqs.(2) and (3) we obtain the initial conditions

$$\phi(0,p) = A\,,\ \left.\frac{\partial\phi(\tau,p)}{\partial\tau}\right|_{\tau=0} = 0 \tag{8}$$

Obviously when p=0 and p=1, it holds:

$$\phi(\tau,0) = x_0(\tau)\,,\ \phi(\tau,1) = x(\tau)\,,\ \Omega(0) = \Omega_0\,,\ \Omega(1) = \Omega \tag{9}$$

where $x_0(\tau)$ is an initial guess of $x(\tau)$. Therefore, as the embedding parameter p increases from 0 to 1, $\phi(\tau, p)$ varies from the initial guess $x_0(\tau)$ to the solution $x(\tau)$, so does $\Omega(p)$ from the initial guess $\Omega_0$ to the exact frequency $\Omega$.

Expanding $\phi(\tau, p)$, $\Omega(p)$ in series with respect to the parameter p, one has respectively:

$$\phi(\tau, p) = x_0(\tau) + p x_1(\tau) + p^2 x_2(\tau) + .... \tag{10}$$

$$\Omega(p) = \Omega_0 + p\Omega_1 + p^2\Omega_2 + .... \tag{11}$$

If the initial guess $x_0(\tau)$ and the auxiliary function $h(\tau,p)$ are properly chosen so that the above series converges at p=1, one has

$$x(\tau) = x_0(\tau) + x_1(\tau) + x_2(\tau) + ... \tag{12}$$

$$\Omega = \Omega_0 + \Omega_1 + \Omega_2 + .... \tag{13}$$

Notice that series (10) and (11) contain the auxiliary function $h(\tau,p)$ which determines their convergence regions.

The results at the m th-order approximations are given by:

$$\tilde{x}(\tau) = x_0(\tau) + x_1(\tau) + ... + x_m(\tau) \tag{14}$$

$$\tilde{\Omega} = \Omega_0 + \Omega_1 + ... + \Omega_m \tag{15}$$

We propose that the auxiliary function $h(\tau,p)$ to be of the form:

$$h(\tau, p) = p\left[C_1 f_1(\tau) + C_2 f_2(\tau) + ... + C_k f_k(\tau)\right] \tag{16}$$

where $C_1$, $C_2$,….$C_k$ are constants and $f_1(\tau)$, $f_2(\tau)$,… $f_k(\tau)$ are functions depending on variable $\tau$, k being a fixed arbitrary number.

Substituting Eqs.(12) and (13) into Eq.(7) yields:

$$\begin{aligned} N(\phi,\Omega) &= N_0(x_0,\Omega_0,\lambda) + pN_1(x_0,x_1,\Omega_0,\Omega_1,\lambda) + \\ &+ p^2 N_2(x_0,x_1,x_2,\Omega_0,\Omega_1,\Omega_2,\lambda) + ... \end{aligned} \tag{17}$$

If we substitute Eqs.(17) and (16) into Eq.(5) and equate the coefficients of various powers of p equal to zero, we obtain the following linear equations:

$$L(x_0) = 0, \; x_0(0) = A, \; \dot{x}(0) = 0 \tag{18}$$

$$\begin{aligned} & L(x_i) - L(x_{i-1}) - \left[C_1 f_1(\tau) + C_2 f_2(\tau) + ... + C_k f_k(\tau)\right] N_{i-1}(x_0, x_1, ... x_{i-1}, \Omega_0, ... \Omega_{i-1}, \lambda) = 0 \\ & i = 1,2,...m \;\; , \;\; x_i(0) = 0 \;\; , \;\; x'_i(0) = 0 \end{aligned} \tag{19}$$

Note that $\Omega_k$ can be determined avoiding the presence of secular terms in the left-hand side of Eq.(19).

The frequency $\Omega$ depends on the arbitrary parameter $\lambda$ and we can apply the so-called "principle of minimal sensitivity" [30] in order to fix the value of $\lambda$. We do this by imposing that

$$\frac{d\Omega}{d\lambda} = 0 \tag{20}$$

At this moment, m th-order approximation given by Eq.(14) depends on the parameters $C_1$, $C_2$,…,$C_m$. The constants $C_i$ can be identified via various ways, for example: collocation method, Galerkin method, least square method etc.

It must be highlighted that our procedure contains the auxiliary function $h(\tau,p)$ which provides us with a simple way to adjust and optimally control the convergence region and rate of solution series. Note that instead of an infinite series, the OHAM searches for only few terms (mostly three terms).

## 3. APPLICATION OF OHAM TO THE INVESTIGATION OF NONLINEAR VIBRATION OF THE CONSIDERED ELECTRICAL MACHINE

The validity of the proposed procedure is illustrated for the electrical machine whose dynamic behaviour is governed by Eq.(1).

Form Eq.(18) it is obtained the following solution:

$$x_0(\tau) = A\cos\tau \tag{21}$$

For i=1 into Eqs.(17) and (19), we obtain:

$$N_0(x_0,\Omega_0,\lambda) = \Omega_0^2 x_0'' + (\omega^2+\lambda)x_0 - \alpha x_0 \sin\frac{\omega_2}{\Omega}\tau + \beta x_0^3 - \gamma\sin\frac{\omega_1}{\Omega}\tau \tag{22}$$

Substituting Eq.(21) into Eq.(22) it is obtained:

$$\begin{aligned} N_0(x_0,\Omega_0,\lambda) &= A\left(-\Omega_0^2+\omega^2+\lambda+\frac{3}{4}\beta A^2\right)\cos\tau + \frac{\beta A^3}{4}\cos 3\tau - \\ &-\frac{1}{2}\alpha A\left[\sin\left(\frac{\omega_2}{\Omega}+1\right)\tau + \sin\left(\frac{\omega_2}{\Omega}-1\right)\tau\right] - \gamma\sin\frac{\omega_1}{\Omega}\tau \end{aligned} \tag{23}$$

If we choose k=3 and

$$f_1(\tau)=1,\ \ f_2(\tau)=2\cos 2\tau,\ \ f_3(\tau)=2\cos 4\tau \tag{24}$$

then Eq.(19) becomes for i=1:

$$\begin{aligned} &\Omega_0^2(x_1''+x_1) = A(C_1+C_2)(-\Omega_0^2+\omega^2+\lambda\frac{3}{4}\beta A^2)\cos\tau + \frac{1}{4}C_1\beta A^3\cos 3\tau + [\frac{1}{4}C_2\beta A^3 - \\ &-C_2A(-\Omega^2+\omega^2+\lambda+\frac{3}{4}\beta A^2)]\cos 5\tau - \frac{1}{4}C_2\beta A^3\cos 7\tau - \frac{1}{2}(C_1+C_2)\alpha A[\sin\left(\frac{\omega_2}{\Omega}+1\right)\tau + \\ &+\sin\left(\frac{\omega_2}{\Omega}-1\right)\tau] + \frac{1}{2}C_2\alpha A[\sin\left(\frac{\omega_2}{\Omega}+5\right)\tau + \sin\left(\frac{\omega_2}{\Omega}-5\right)\tau] - C_1\gamma\sin\frac{\omega_1}{\Omega}\tau - C_2\gamma[\sin\left(\frac{\omega_1}{\Omega}+2\right)\tau + \\ &+\sin\left(\frac{\omega_1}{\Omega}-2\right)\tau - \sin\left(\frac{\omega_1}{\Omega}+4\right)\tau - \sin\left(\frac{\omega_1}{\Omega}-4\right)\tau] \end{aligned} \tag{25}$$

Avoiding the presence of a secular term needs:

$$\Omega_0^2 = \omega^2 + \lambda + \frac{3}{4}\beta A^2 \tag{26}$$

With this requirement, the solution of Eq.(25) is

$$x_1(\tau) = M\cos\tau + N\cos 3\tau + P\cos 5\tau + Q\cos 7\tau + R\sin\tau + \ldots \tag{27}$$

where

$$M = \frac{C_1\beta A^3}{32\Omega_0^2} + \frac{C_2\beta A^3}{192\Omega_0^2};\ N = -\frac{C_1\beta A^3}{32\Omega_0^2};\ P = -\frac{C_2\beta A^3}{96\Omega_0^2};\ Q = \frac{C_2\beta A^3}{192\Omega_0^2}$$

$$\begin{aligned} R &= \frac{(C_1+C_2)\alpha A\Omega(2\Omega^2-\omega_2^2)}{\Omega_0^2(\omega_2^2-4\Omega^2)\omega_2} + \frac{C_2\alpha A\Omega\omega_2(\omega_2^2-26\Omega^2)}{\Omega_0^2(576\Omega^4-52\Omega^2\omega_2^2+\omega_2^4)} + \frac{C_1\gamma\Omega\omega_1}{\Omega_0^2(\Omega^2-\omega_1^2)} + \\ &+\frac{2C_2\gamma\Omega\omega_1(5\Omega^2-\omega_1^2)}{\Omega_0^2(9\Omega^4-10\Omega^2\omega_1^2+\omega_1^4)} + \frac{2C_2\gamma\Omega\omega_1(\omega_1^2-17\Omega^2)}{\Omega_0^2(225\Omega^4-34\Omega^2\omega_1^2+\omega_1^4)} \end{aligned} \tag{28}$$

Substituting Eqs.(21) and (27) into Eq.(19), we obtain the following equation:

$$\Omega_0^2(x_2''+x_2)=\cos\tau[\frac{3}{4}(C_1+C_2)\beta A^2(2M+N)-A(C_1+C_2)(2\Omega_0\Omega_1+\lambda)- \\ -\frac{(C_1+C_2)^2\alpha^2\Omega^2 A}{2\Omega_0^2(\omega_2^2-4\Omega^2)}-25C_2P\Omega_0^2-C_2(\omega^2+\lambda)P-\frac{3}{4}C_2\beta A^2(N+Q+2P)]+ \\ +\frac{3}{4}\beta A^2C_2R\sin\tau+N.T. \tag{29}$$

where N.T. means the other nonresonant terms.

No secular term in $x_2(\tau)$ requires that

$$\Omega_1=\frac{(3C_1+C_2)\beta^2A^4}{256\Omega_0^3}-\frac{\lambda}{2\Omega_0}+\frac{(C_1+C_2)\alpha^2\Omega^2}{4\Omega_0^3(4\Omega^2-\omega_2^2)}-\frac{25C_2^2\beta A^2}{192\Omega_0(C_1+C_2)}+ \\ +\frac{(\omega^2+\lambda)C_2^2\beta A^2}{192\Omega_0^3(C_1+C_2)}+\frac{3(2C_1+C_2)\beta^2A^4C_2}{512\Omega_0^3(C_1+C_2)} \tag{30}$$

$$R=0 \tag{31}$$

From Eqs.(26) and (30) we obtain the frequency in the form:

$$\Omega=\Omega_0+\Omega_1 \tag{32}$$

The parameter λ can be determined applying the "principle of minimal sensitivity". From Eq.(20), we obtain the following condition:

$$\lambda\Omega_0^2(C_1+C_2)-\frac{3\beta^2A^4(6C_1^2+14C_1C_2+5C_2^2)}{256}- \\ -\frac{3(C_1+C_2)^2\alpha^2\Omega^2}{2(4\Omega^2-\omega_2^2)}+\frac{C_2^2\beta A^2(29\Omega_0^2-3\omega^2-3\lambda)}{96}=0 \tag{33}$$

By means of Eqs.(30) and (32), Eq.(31) becomes:

$$\Omega=\Omega_0-\frac{\lambda}{3\Omega_0}-\frac{23C_2^2\beta A^2}{288\Omega_0(C_1+C_2)} \tag{34}$$

The Eq.(31) can be written as:

$$\frac{(C_1+C_2)\alpha A(2\Omega^2-\omega_2^2)}{\omega_2(\omega_2^2-4\Omega^2)}+\frac{C_2\alpha A\omega_2(\omega_2^2-26\Omega^2)}{576\Omega^4-52\Omega^2\omega_2^2+\omega_2^4}+\frac{C_1\gamma\omega_1}{\Omega^2-\omega_1^2}+ \\ +\frac{2C_2\gamma\omega_1(5\Omega^2-\omega_1^2)}{9\Omega^4-10\Omega^2\omega_1^2+\omega_1^4}+\frac{2C_2\gamma\omega_1(\omega_1^2-17\Omega^2)}{225\Omega^4-34\Omega^2\omega_1^2+\omega_1^4}=0 \tag{35}$$

The first order approximate solution is

$$\tilde{x}(\tau)=x_0(\tau)+x_1(\tau)$$

or by means of Eqs.(21), (27) and (3):

$$\tilde{x}(\tau)=\left(A+\frac{C_1\beta A^3}{32\Omega_0^2}+\frac{C_2\beta A^3}{192\Omega_0^2}\right)\cos\Omega t-\frac{C_1\beta A^3}{32\Omega_0^2}\cos 3\Omega t-\frac{C_2\beta A^3}{96\Omega_0^2}\cos 5\Omega t+\frac{C_2\beta A^3}{192\Omega_0^2}\cos 7\Omega t+$$
$$+\frac{(C_1+C_2)\alpha A\Omega^2}{2\Omega_0^2\omega_2(\omega_2+2\Omega)}\sin(\omega_2+\Omega)t+\frac{(C_1+C_2)\alpha A\Omega^2}{2\Omega_0^2\omega_2(\omega_2-2\Omega)}\sin(\omega_2-\Omega)t-$$
$$-\frac{C_2\alpha A\Omega^2}{2\Omega_0^2(24\Omega^2+10\Omega\omega_2+\omega_2^2)}\sin(\omega_2+5\Omega)t-\frac{C_2\alpha A\Omega^2}{2\Omega_0^2(24\Omega^2-10\Omega\omega_2+\omega_2^2)}\sin(\omega_2-5\Omega)t- \quad (36)$$
$$-\frac{C_1\gamma\Omega^2}{\Omega_0^2(\Omega^2-\omega_1^2)}\sin\omega_1 t+\frac{C_2\gamma\Omega^2}{\Omega_0^2(3\Omega^2+4\Omega\omega_1+\omega_1^2)}\sin(\omega_1+2\Omega)t+$$
$$+\frac{C_2\gamma\Omega^2}{\Omega_0^2(3\Omega^2-4\Omega\omega_1+\omega_1^2)}\sin(\omega_1-2\Omega)t-\frac{C_2\gamma\Omega^2}{\Omega_0^2(15\Omega^2+8\Omega\omega_1+\omega_1^2)}\sin(\omega_1+4\Omega)t-$$
$$-\frac{C_2\gamma\Omega^2}{\Omega_0^2(15\Omega^2-8\Omega\omega_1+\omega_1^2)}\sin(\omega_1-4\Omega)t$$

The constants $\lambda$, $\Omega_0$, $\Omega$, $C_1$ and $C_2$ can be determined from Eqs.(26), (33), (34), (35) and by means of the residual , which reads:

$$R(t,\lambda,\Omega_0,\Omega,C_1,C_2)=\ddot{\tilde{x}}+\omega^2\tilde{x}-\alpha\tilde{x}\sin\omega_2 t+\beta\tilde{x}^3-\gamma\sin\omega_1 t$$

The last condition can be written with collocation method:

$$R(\frac{\pi}{6},\lambda,\Omega_0,\Omega,C_1,C_2)=0 \quad (37)$$

Finally, five equations with five unknowns are obtained.

In the case when $\omega_1$=1.1, $\omega_2$=1.5, $\omega$=1.58, $\alpha$=2.75, $\beta$=12.5, $\gamma$=0.2, we obtain:

$\lambda$=-1.1120201913228436, $\Omega_0$=3.282055275793561, $\Omega$=3.3941604317791465,
$C_1$=-0.0007293171916262496, $C_2$=0.000714849572157505

Fig.1 shows the comparison between the approximate solution and the numerical solution obtained by a fourth-order Runge-Kutta method.

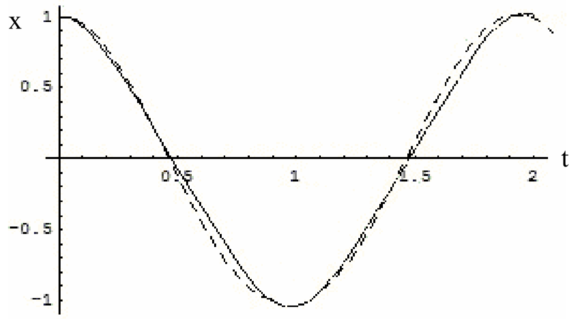


Figure 1 Comparison of the approximate solution with the numerical solution:
_______ numerical solution; _ _ _ _ _ approximate solution

It can be seen that the solution obtained by our procedure is nearly identical with that given by the numerical method.

## 4. CONCLUSIONS

In the present study, an analytical model for an electrical machine has been developed to obtain the nonlinear vibration response due to nonlinear stiffness. The system is parametrically excited by an axial thrust and at the same time a forcing excitation caused by an unbalanced force of the rotor is acting on the system. The mathematical model takes into account the sources of nonlinearity and the corresponding equation of motion is solved using the Optimal Homotopy Asymptotic Method to graphically obtain the time history of nonlinear response. The proposed procedure is valid even if the nonlinear equation does not contain any small or large parameter. The OHAM provide us with a simple way to optimally control and adjust the convergence of the solution series and can give good approximations in few terms. The convergence of the approximate solution series given by OHAM is determined by the auxiliary function $h(\tau,p)$. The obtained approximate analytical solution is in very good agreement with the numerical simulation results, which proves the validity of the method.

This paper shows one step in the attempt to develop a new nonlinear analytical technique, which is valid in the absence of a small or large parameter.

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