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A New Approach for Designing Orthogonal Wavelets for Multicarrier Applications

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Abstract – **The Daubechies, coiflet and symlet wavelets, with properties of orthogonal wavelets are suitable for multicarrier transmission over band-limited channels. It has been shown that similar wavelets can be constructed by Lagrange approximation interpolation. In this work and using established wavelet design algorithms, it is shown that ideal filters can be approximated to construct new orthogonal wavelets. These new wavelets, in terms of BER behave slightly better than the wavelets mentioned above, and much better than biorthogonal wavelets, in multipath channels with additive white Gaussian noise (AWGN). It is shown that the construction, which uses a simple simultaneous solution to obtain the wavelet filters from the ideal filters based on established wavelet design algorithms, is simple and can easily be reproduced.**

Keywords – ***Orthogonal wavelets; finite impulse response filter; FIR; multicarrier system; simultaneous solution; ideal filter;***

I. Introduction

Wavelets have been pursued in the design of multicarrier systems (MCS), even though it was initially feared that they might not be optimal in that application [1-3]. This was due to the authors only studying biorthogonal wavelets, which lack the necessary orthogonality for signal transmissions. Another class of wavelets, orthogonal wavelets, was not considered, and now it has been realised that they can be used in the design of MCSs with better BER than the biorthogonal wavelets [4]. Examples of well-known orthogonal wavelets are Daubechies, coiflet, symlet and Haar wavelets. Detailed performance analyses have been shown in [5] for discrete and wavelet packets in MCSs. Orthogonal wavelets permit multicarrier signals to be transmitted without a cyclic prefix, and, in fact, they have been adopted in the design of marketed MCSs such as the IEEE 901 HD Panasonic system for power line communications [6]. Meanwhile, [7] has shown that filters used in constructing the Daubechies wavelets can be reproduced by the Lagrange approximation. So, MCSs can be designed using existing wavelets, or by constructing new, orthogonal wavelets.

There are two different ways by which wavelets have been constructed, namely by changing the basis functions of the parent scaling function or by constructing new filters [8]. Using these filters, new wavelets are constructed by "lifting" one filter into another [9-13] or by approximation of a related ideal filter. Such approximation has been performed using the Lagrange approximation to construct orthogonal wavelets in [7] whose mother behaviour must be as discussed in [14]. Following the general algorithm discussed in [8] for designing any type of wavelet, we report a design that can construct wavelets with similar properties to other orthogonal wavelets such as Daubechies, symlet, coiflets and Haar wavelets. By the definitions of the algorithms in [8], the applied filters were approximated following band-limited conditions and the results obtained by a simple simultaneous solution.

The construction of new wavelets for specific applications has been reported [8, 14], presenting a general algorithm for designing mother wavelets $\psi(t)$ to match a signal of interest such that the family of wavelets $2^{-(j/2)}\psi(2^{-j}t-\tau)$ forms the orthogonal basis of a square-integrable family: here j is the scale parameter and τ the shift parameter. In band-limited systems, j characterises and limits the available narrow bands.

It is common knowledge that a band-limited scaling function that generates an orthonormal multi-resolution analysis (MRA) also gives rise to a band-limited wavelet. So, for band-limited multicarrier applications, finite impulse response (FIR) filters approximated from band-limited criteria can be used in the construction of new wavelets [8]. Besides the Lagrange solution for constructing orthogonal wavelets reported in [7], the design of orthogonal wavelets from FIR filters by solving the results of ideal filters approximation using band-limited conditions simultaneously is shown in this work. These are linear phase filters derived and approximated to FIR conditions, and are maximally flat. They are distortion-free, with few, if any, side-lobes so that the narrowband interference accruing from non-convergence to zero (faster decay) leads to higher performance when compared to some known base wavelets. This property explains, for example, their good BER and PAPR performances. Other methods for designing multicarrier filters are described in [15, 16] and [17].

Wavelets constructed from other filters have been discussed elsewhere, such as the raised cosine functions [18, 19]. Depending on the symmetry and periodicity, these filters construct wavelets for uniquely different operations. Using raised cosine functions in the construction of wavelets has been well discussed in [19, 20]. Instead of raised cosine filters [20], maximally flat filters based on the Lagrange approximation have been reported to yield orthogonal wavelets equivalent in coefficients and properties to the Daubechies wavelets [7]. By treating the maximally flat approximations in a different way and according to band-limited constraints, type I FIR filters following some approximations and solutions can be used in the design of new wavelets. We report the case of a fourth order ideal filter, approximated under band-limited constraints, with the results solved simultaneously. These filters constitute the filter banks of the new wavelets reported in this paper.

In Section II the preliminary idea of a wavelet multicarrier system is discussed, with FIR filters discussed in Section III. The proposed filter is set out in Section IV, simulation and results are in Section V, with the conclusions in the final section.

II. Preliminary Knowledge

Consider an input signal $s(t)$, that modulates the transforming function, or scaling function, $\varphi(t)$. There are narrowband functions $\psi(t)$ derivable from $\varphi(t)$, which are orthogonal wavelets useful in the design of multicarrier systems. By the Fourier relation and Parseval's theory, the signal for band-limited case can be periodic with β, $-2\pi \leq \beta \leq 2\pi$, so that if $\psi_{l,m}(t)$ belongs to a set of orthonormal functions, then;

$$\int \psi_{l,m}(t)\psi_{k,n}(t)dt = \delta(l-k)\delta(m-n) \quad (1)$$

where $\delta(\bullet)$ is a Dirac delta. Equation (1) defines a simple orthogonality condition between daughter wavelets. Since $\psi(t)$ is a decomposition of $\varphi(t)$, we can express the relationship of the input signal with $\varphi(t)$ in discrete form as [21];

$$S_{DWT} = \sum_{m=0}^{M-1} s[m]\varphi_m(t) \quad (2)$$

where M is the length of the characteristic filter. $s[m]$ is the discrete equivalent of $s(t)$. The mother wavelet has a clear relation to the filters;

$$\psi(t) = \sqrt{2}\sum_m g(m)\varphi(2t-m) \quad (3)$$

where $g(m)$ is a high-pass filter (HPF) and can be directly derived or constructed from a low-pass filter (LPF) that belongs to the parent scaling function as;

$$\varphi(t) = \sqrt{2}\sum_m h(m)\varphi(2t-m) \quad (4)$$

where $h(m)$ is the LPF and $\varphi(t)$ is the scaling function. Thus, as an alternative to modulating the input symbols by the sinusoids, these half-band filters can be used.

III. Finite Impulse Response (FIR) Filters

Filter coefficients used in the design of wavelet filter banks relate to Laurent polynomials [22], Chebyshev polynomials [23] or the Lagrange half-band functions shown in [7] derived as approximations of Chebyshev polynomials. These filters must have linear phase [24, 25] and finite length to be finitely supported [26]: examples are FIR filters. This finite support derives from approximating an ideal filter response which leads to the realization of a new filter, with the number of approximations related to the number of ripples which the filter exhibits. Maximally flat filters obtain their approximations without these ripples. If these ripples (or side-lobes) do not converge to zero, i.e. decay quickly, two known problems are caused; namely ISI and high peak to average power ratio (PAPR).

For a distortion-free data transmission, these filters must have linear phase [27, 28] and a narrowband characteristic to be suitable for multicarrier transmission. Thus, filters used in the design of wavelets must have FIR properties and linear phase. For zero-ISI transmission, the filters must satisfy the Nyquist criterion: linear phase FIR filters are a perfect example [28]. For distortion-free transmission, maximally flat FIR linear phase filters are required [27]. These filters must be symmetric and of even order to be used in the design of low-pass filters.

In this work, we adopt the maximally-flat design method for distortion-free FIR linear phase design. Other types of filters exist, most commonly the opposite of the linear phase FIR filters called non-linear phase filters [29]. A very pronounced example is the minimum phase filter. Its group delay is not constant and so, the phase undergoes some phase distortion. FIR filters with non-linear phase delay cause ISI [18]. FIR filters of the Nyquist criterion can guarantee freedom from ISI and ICI in data transmission [28, 30]. They require that the stop-band attenuation be as high as possible [30].

IV. Transmission FIR Filter

For distortion-free communications, FIR filters with constant group delay are preferred, that is linear phase FIR filters. Let the frequency response of a linear system $h(n) = Ax(n-k)$ be;

$$H(\omega) = AX(\omega)e^{-j\omega k} \quad (5)$$

If the input signal has amplitude (A), the properties needed for the filter formulation are;

$$H(\omega) = Ae^{-j\omega k} \quad (6)$$

where $|H(\omega)| = A$ is the amplitude and $\theta(\omega) = -k\omega$ is the phase. In distortion-free transmission, the amplitude is constant and the phase response is linear. Only FIR filters have these properties if stability and causality is required. This property will be exploited for band-limited multicarrier system design in the following sections. An FIR filter is defined as;

$$H(\omega) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n} \quad (7)$$

where k in this equation 5 is equivalent to n in Equation 7 in defining the unique discrete filter coefficients. A linear-phase filter can be designed using a symmetry condition for the impulse response;

$$h(n) = h(M-1-n) \quad (8)$$

Thus, $h(n)$ represents the impulse response of the linear phase filter which repeats according to the symmetry condition in Equation 8. It is most times preferred as mirroring.

A. General Case for Symmetric Filters with Even Order M

The general case for symmetric filters is defined by [31] as;

$$H(\omega) = H_r(\omega)e^{-j\omega(M-1)/2} \tag{9}$$

where

$$H_r(\omega) = 2\sum_{n=0}^{M/2-1} h(n)\cos\omega(\frac{M-1}{2} - n) \tag{10}$$

and

$$\theta(\omega) = \begin{cases} -\omega\left(\dfrac{M-1}{2}\right), & H_r(\omega) > 0 \\ -\left(\dfrac{M-1}{2}\right) + \pi, & H_r(\omega) < 0 \end{cases} \tag{11}$$

where M is the filter length. Equation 9 represents the case of an ideal filter, and Equation 11 represents the phase.

B. Approximation of the Ideal Filter and The Simultaneous Solution

We choose an ideal filter of order 4, as an example, so by Equations 9 and 10, we set $M = 4$;

$$H(\omega) = h(0) + h(1)e^{-jw} + h(2)e^{-j2w} + h(3)e^{-j3w} \tag{12}$$

From the symmetry condition defined in Equation 8,

$$h(0) = h(3)$$

and

$$h(1) = h(2)$$

$$H(\omega) = e^{-j3w/2}\begin{Bmatrix} h(0)e^{j3w} + h(1)e^{jw/2}\ldots \\ + h(2)e^{-jw/2} + h(3)e^{-j3w/2} \end{Bmatrix} \tag{13}$$

By trigonometric identities, Equation 13 reduces to;

$$H(\omega) = e^{-j3\omega/2}\{2h(0)\cos(3\omega/2) + 2h(1)\cos(\omega/2)\} \tag{14}$$

For a signal band-limited at [-π π], Nyquist zero-ISI sampling criteria requires that the signal be sampled between $\omega = 0$, and $\omega = \pi/2$. Thus, at $\omega = 0$,

$$H(0) = 1 \tag{15}$$

$$2h(0) + 2h(1) = 1$$

Since we are considering half-band filters, at $\omega = \pi/2$,

$$\begin{aligned} H(\pi/2) &= \frac{1}{2} \\ 2h(0).(\frac{-1}{\sqrt{2}}) + 2h(1).(\frac{1}{\sqrt{2}}) &= \frac{1}{2} \end{aligned} \tag{16}$$

Solving Equations 15 and 16 simultaneously for $h(0)$ and $h(1)$,

$$\begin{aligned} h(0) &= 1/4\left(1 - \frac{\sqrt{2}}{2}\right) \\ h(1) &= 1/4\left(1 + \frac{\sqrt{2}}{2}\right) \end{aligned} \tag{17}$$

Thus, the effective filter coefficients needed for the linear phase maximally flat FIR (low-pass filter) filter design are given as;

$$h = \begin{bmatrix} h(0) & h(1) & h(1) & -h(0) \end{bmatrix} \tag{18}$$

Solving for higher orders than the example above can yield closer to the ideal frequency response of the filter but this would require more run-time. Meanwhile, using Equation 18, the filter can be plotted as in Figure 1.

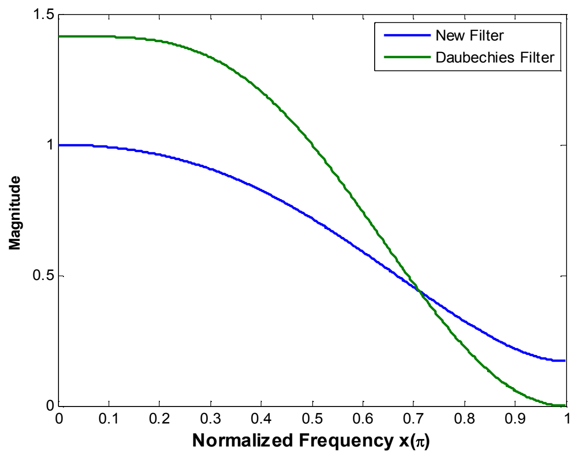

Figure 1: New filter vs. Daubechies filter (db2)

V. The Proposed Wavelet in Multicarrier System

For a first simple test, we compare the proposed wavelet with other base wavelets (db2, bior3.1, bior3.3, bior3.5) in multicarrier systems built from their respective filters. In the first case, the system model is described and in the second case, the channel model with equalization followed is also described.

A. The Proposed Wavelet Filter in Multicarrier System Model Over AWGN Channels Only

The model involves a multicarrier BPSK system modulated by the wavelet packet transform as shown in Figure 2.

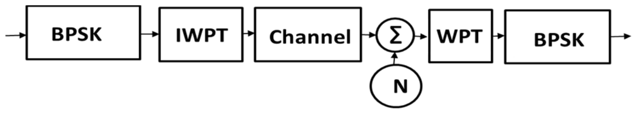

Figure 2: WPT-Modulation with AWGN channel

Using a BPSK mapping scheme with 256000 symbols, the input symbols were wavelet-modulated and passed through an ideal channel, with just the addition of AWGN, indicated by "**N**" in Figure 2.

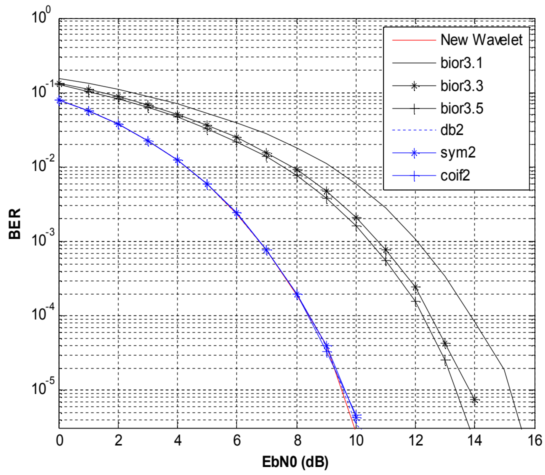

Figure 3: BER compared for the proposed and other wavelets over AWGN only

In the receiver, the received symbols were wavelet-demodulated and demapped. No form of coding was applied. Figure 3 shows that the proposed wavelet and db2, sym2, coif2 (which are orthogonal wavelets) performed alike. However, it is also shown that the proposed wavelet clearly outperforms the biorthogonal wavelets by up to 5 dB. It is common knowledge that biorthogonal wavelets filters lack the required reconstruction ability for orthogonal transmission, hence the poor BER performance.

B. *System Model Over Multipath Channels with Channel Equalization*

The investigation was further extended to a multipath environment, using the parameters as above for a Rayleigh multipath channel with AWGN and frequency domain equalization (FDE), as shown in Figure 4.

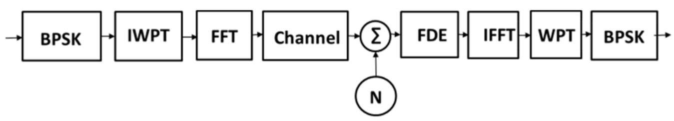

Figure 4: WPT-Modulation with a channel

Traditionally, the Rayleigh law describes a change in the amplitude magnitude of a signal as it traverses multipath. For instance, if there are K paths, the Rayleigh law shows that [32, 33]:

$$q(t,\tau) = \sum_{k=0}^{K-1} \alpha_k \,\delta(t-\tau_k).e^{j\theta_k}, \; k = 0,1,\cdots,K-2,K-1 \tag{19}$$

where $q(t,\tau)$ is the channel impulse response of the k^{th} multipath with θ_k phase which influences signal of α_k amplitude. For frequency selective fading, the channel transfer function of $q(t,\tau)$ can be written as:

$$Q(t,f) = \sum_{k=0}^{K-1} \alpha_k \,\delta(t-\tau_k).e^{-j\theta_k}, \; k = 0,1,\cdots,K-2,K-1 \tag{20}$$

If the wavelet-transformed signal is $s(t)$, then an FFT will be required to obtain the frequency domain equivalent; $S(f) \overset{FT}{\Leftrightarrow} s(t)$, with FT as the Fourier transform. The wavelet does not operate with a cyclic prefix (CP), so the convolution with the channel transfer function becomes;

$$Y(f) = Q(t,f) \otimes S(f) + Z(f) \tag{21}$$

Equation 21 is the received signal in the frequency domain with $Z(f)$ as the AWGN. If $Q(t, f) = 1$, Equation 21 can also be used to model an AWGN channel only. In the receiver, the equalization follows as:

$$\begin{aligned} R(f) &= \frac{Q(t,f)^{*}.Y(f)}{\left\|Q(t,f)\right\|^2} \\ &= \frac{Q(t,f)^{*}.Q(t,f)\otimes S(f)}{\left\|Q(t,f)\right\|^2} + \frac{Q(t,f)^{*}.Z(f)}{\left\|Q(t,f)\right\|^2} \end{aligned} \tag{22}$$

where $\otimes$ and $(.)^*$ are convolution and conjugation operators respectively, and $\left\|\cdot\right\|$ is the absolute operator. If at a certain time $\left\|Q(t,f)\right\| = 0$, then Equation 22 can be modified to include error correction parameter as, ε thus;

$$\begin{aligned} R(f) &= \frac{Q(t,f)^{*}Y(f)}{\left\|Q(t,f)\right\|^2 + \varepsilon} \\ &= \frac{Q(t,f)^{*}.Q(t,f)\otimes S(f)}{\left\|Q(t,f)\right\|^2 + \varepsilon} + \frac{Q(t,f)^{*}.Z(f)}{\left\|Q(t,f)\right\|^2 + \varepsilon}, \forall 0 \le \varepsilon \le 1 \end{aligned}$$

… (23)

This is the FDE equalization. $R(f)$ is the frequency domain content of the received signal after equalization. The received signals were transformed into the wavelet domain by the inverse FFT (IFFT) before demodulating by the forward WPT and demapping using BPSK. Figure 5 shows that, in terms of BER, the proposed wavelet slightly outperforms db2 (and other observed orthogonal wavelets) but strongly outperforms all observed biorthogonal wavelets by up to 5 dB.

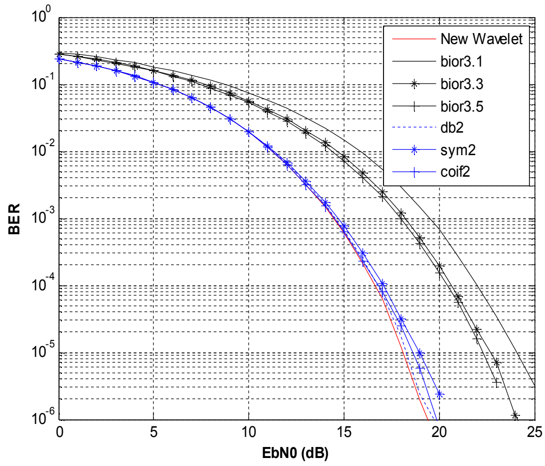

Figure 5: BER of the proposed and other wavelets compared over multipath channel with AWGN

It is interesting to note that even though the proposed method has fewer filter coefficients, which reduces processing time, the resulting wavelets can do well in multipath environments where the orthogonality loss could lead to severe distortion and poor BER performance.

VI. Conclusion

In addition to the Lagrange approximation method for orthogonal wavelets like Daubechies, coiflets, symlets, and Haar, a new way of designing orthogonal wavelets for multicarrier system applications has been presented. Using a filter approximation for band-limited conditions, new filters have been constructed. The approximations were solved simultaneously to obtain the required filters. These filters were then used to construct wavelets used in the modulation of a multicarrier system. These wavelets have been constructed from FIR filters, which were approximated from FIR linear phase filters. With such distortion-free filters, the proposed wavelet has good properties for multicarrier transmission over multipath where loss of signal orthogonality would tend to produce information distortion and poor BER. In such simulation conditions, the new wavelet was compared with other base wavelets, orthogonal wavelets db2, sym2, coif2 and biorthogonal wavelets bior3.1, bior3.3, bio3.5. Results show that the proposed wavelet had a similar performance to orthogonal wavelets in AWGN channels, but was better than the biorthogonal wavelets. Over a multipath channel, the proposed wavelet slightly outperformed db2 and clearly outperformed the other wavelets. Higher order approximations to construct sub-member wavelets using the studied simultaneous approach just like all other known orthogonal wavelets is possible but the consequent system would incur more run-time.

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