A New Approach for Improved Evaluation of Sommerfeld Integral Tails for PEC-terminated Single Layered Media

Shaun D. Walker[1], Deb Chatterjee*[1], and Michael S. Kluskens[2]

[1] CSEE Department, University of Missouri Kansas City (UMKC), 570-F Flarsheim Hall, 5110 Rockhill Road, KS, 64110, USA.
E-mail: chatd@umkc.edu

[2] Code 5314, Radar Division, Naval Research Laboratory, Washington, DC 20375-5336, USA.

Introduction

Sommerfeld integrals [1] appearing in the Green's function for layered media pose significant challenges in computation of the tail part of the integrand when the observer and source locations have large lateral separations with the observer close to the interface(s) [2]-[5]. Analytical [3],[4] and novel numerical [5] methods have been utilized to circumvent such problems, and is the subject of this paper. The purpose of this investigation is to obtain improved analytical forms for closed-form evaluation of the Sommerfeld integral tail that are more general compared to [3],[4].

The limitation of the results presented here are that only the G_{zx} component of a HED located at the interface of air-dielectric are included. The results are, however, significant because electrically thick substrates are considered in this paper. The poles of the integrand for such cases are investigated following the algorithm in [6]. Following the detailed studies in [7] for antennas layered media, the results here are related to the z-component of far and near fields due to a HED located at the interface for observation point moving from inside the substrate to outside (air).

In what follows, the algorithm for real-axis integration of Sommerfeld integral is presented. The main results for behavior of the integrand with and without pole singularities are included.

Problem Formulation and Analyis

The z-component of the electric field for a x-oriented HED, shown in Fig. 1, of current moment magnitude p_x is $E_z^m = -\jmath\eta_0 k_0 G_{zx}^m p_x$. For observation point in air and substrate, $m = 0, 1$, respectively. The Green's function

$$G_{zx}^m = \frac{\jmath \cos\phi}{2\pi k_0^2} \int_0^{+\infty} \xi^2 J_1(\rho\xi) \overline{\mathcal{F}}_{zx}^m(\xi) d\xi. \quad (1)$$

contains the Sommerfeld integral whose evaluation along the $\Re e(\xi)$ axis is the principal subject of this paper. In (1) the integrand

$$\begin{aligned}\overline{\mathcal{F}}_{zx}^m(\xi) &= \exp(-\jmath\kappa_0|z|)\frac{\kappa_1 \sin(\kappa_1 d)}{\mathcal{D}_{\mathrm{TM}}(\xi)}, \text{ when point P is in air } m = 0 \text{ in Fig. 1},\\ &= \frac{-\epsilon_{r1}\kappa_0}{\mathcal{D}_{\mathrm{TM}}(\xi)}\cos[\kappa_1(d - |z|)], \text{ when point P is inside substrate } m = 1 \text{ in Fig. 1;}\end{aligned}$$

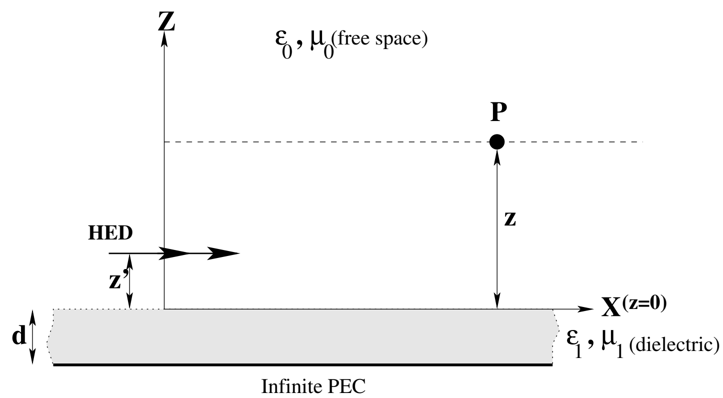

Figure 1: A $\hat{\mathbf{x}}$ oriented Hertzian electric dipole (HED) radiating on the air-dielectric interface of a single-layer media terminated by a PEC ground plane.

$$\mathcal{D}_{\mathrm{TM}} \;=\; \epsilon_{r1}\kappa_0\cos(\kappa_1 d)+\kappa_1\sin(\kappa_1 d). \tag{2}$$

In (2) the z-directed propagation wavenumbers

$$\kappa_m=\begin{cases}\sqrt{\mathrm{k}_m^2-\xi^2}, & \text{when } \mathrm{k}_m\geq\xi,\\ -\jmath\sqrt{\xi^2-\mathrm{k}_m^2}, & \text{when } \mathrm{k}_m\leq\xi.\end{cases} \tag{3}$$

In (3), $\mathrm{k}_m=\sqrt{\epsilon_m\mu_m}$. Real axis integration involves determination of a “breakpoint”, ξ_{A}, such that one can derive the following sequence of steps:

$$\begin{aligned}
\mathrm{G}_{zx}^m \;&=\; \mathtt{C}_0\int\limits_0^{+\infty}\xi^2\mathrm{J}_1(\rho\xi)\overline{\mathcal{F}}_{zx}^m(\xi)d\xi\\
&\approx\; \mathtt{C}_0\left\{\int\limits_0^{\xi_{\mathrm{A}}}\xi^2\mathrm{J}_1(\rho\xi)\overline{\mathcal{F}}_{zx}^m(\xi)d\xi+\int\limits_{\xi_{\mathrm{A}}}^{+\infty}\xi^2\mathrm{J}_1(\rho\xi)\overline{\mathcal{F}}_{zx}^{m\infty}(\xi\geq\xi_{\mathrm{A}})d\xi\right\}\\
&\approx\; \mathtt{C}_0\left\{\int\limits_0^{\xi_{\mathrm{A}}}\xi^2\mathrm{J}_1(\rho\xi)[\overline{\mathcal{F}}_{zx}^m(\xi)-\overline{\mathcal{F}}_{zx}^{m\infty}(\xi\geq\xi_{\mathrm{A}})]d\xi+\rho z\frac{e^{-\jmath\mathrm{k}_m r}}{r^5}(3+3\jmath\mathrm{k}_m r-(\mathrm{k}_m r)^2)\right\}.
\end{aligned} \tag{4}$$

The tail part of the Sommerfeld integral $\xi_{\mathrm{A}}\leq\xi\leq\infty$, defined by the second integral of the second line in (4), can thus be evaluated in closed form. To date, the best result for the closed form evaluation of the Sommerfeld tail is given [4, Eq. (A4)], since that result does not approximate the Bessel function by its trigonometric forms as in [3]. The details of the derivation, and the algorithm for locating the poles ξ_p, defined by $\mathcal{D}_{\mathrm{TM}}(\xi=\xi_p)=0$ can be found [8].

Results and Discussion

The results of this section reflect the “smoothness” of the integrand in (4) when the effects of the poles are subtracted out. The assumption here is that the poles are of the first order and lie on the proper Riemann sheet. The residues at these poles are calculated and then utilized to form a smooth integrand as in (4). The details are available in [2]. The poles, for a single layer substrate, lie in the strip $\mathrm{k}_0 \leq \xi \leq \Re e(\mathrm{k}_1)$, where $\mathrm{k}_1 = \mathrm{k}_0\sqrt{\epsilon_{r1}}$. Here $\mu_1 = \mu_0$. The results refer to $\rho = 10\lambda$,$|\epsilon_{r1}| = 9.8$,$\tan\delta = 0$,$\mathrm{d} = \lambda$,$\phi = 0°$. Figs. 2 and 3 show the smooth behavior

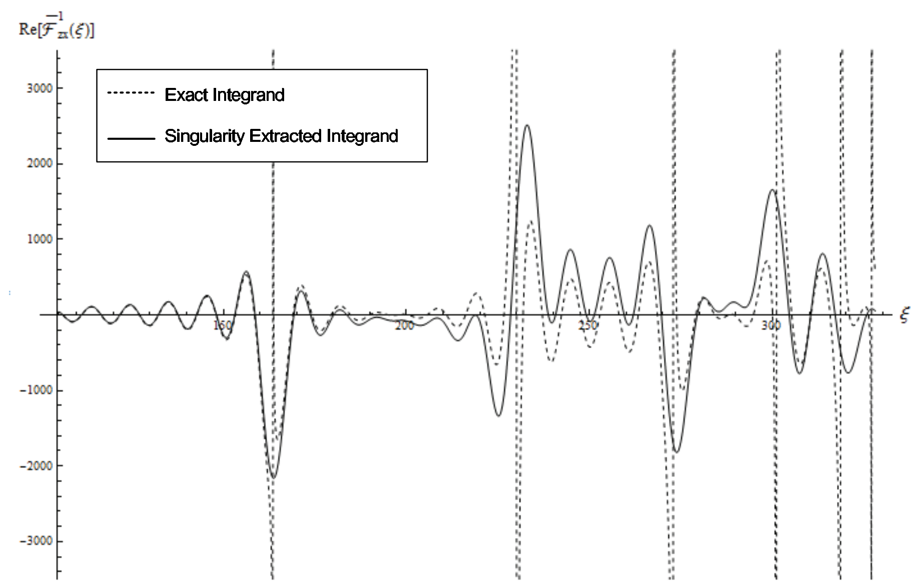

Figure 2: Here $\mathrm{z} = -\frac{\lambda}{2}$ and the observation point is inside ($m = 1$) the substrate.

of the integrand in the when the effects of the pole singularities are subtracted out. This observation facilitates the direct numerical integration for G_{zx} in (4). (Additional results from [7] and [8] will be presented at the time of the conference.)

Summary

A new, closed-form analytical result for evaluating the Sommerfeld integral tail for single-layer PEC-backed substrates is included. The numerical results demonstrate the smoothness of the integrand when pole singularities are subtracted out, and hence facilitate the application of real-axis numerical integration.

References

[1] P. B. Katehi and N. G. Alexopoulos, "Real Axis Integration of Sommerfeld Integrals," *Jour. Math. Phys.*,vol. 24, no. 3, pp. 527-533, March 1983.

[2] J. R. Mosig and R. C. Hall and F. Gardiol, "Numerical Analysis of Microstrip Patch Antennas," (chapter 8) in *Handbook of Microstrip Antennas* (vol. 1), J. R. James and P. S. Hall (eds.). London, UK: IEE Press (Peter Peregrinus), 1989.

[3] K. A. Michalski, "Extrapolation Methods for Sommerfeld Integral Tails,"*IEEE Trans. Antennas. Propagat.*, vol. 46, no. 10, pp. 1405-1418, October 1998.

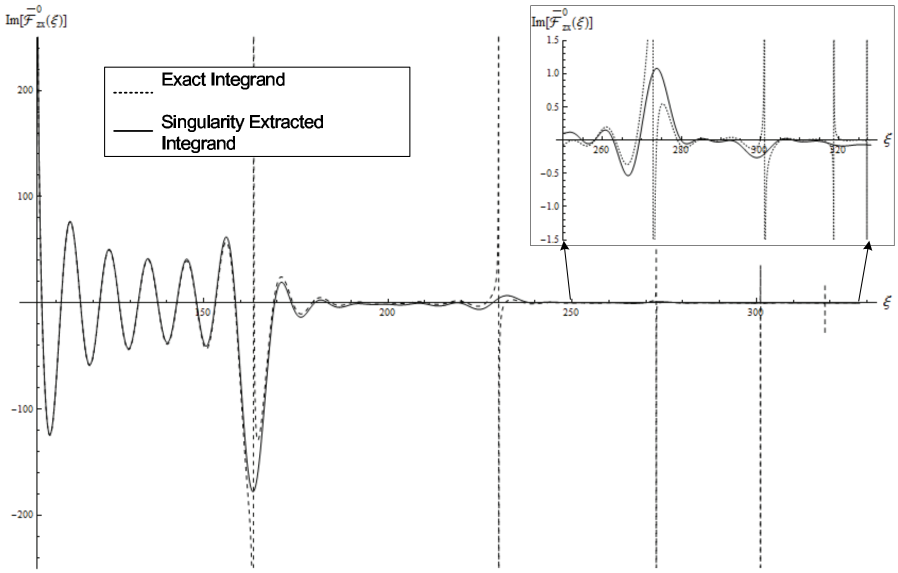

Figure 3: z $= +\frac{\lambda}{2}$, and the observation point is outside ($m = 0$) the substrate.

[4] Y. Ge and K. P. Esselle, "New Closed-Form Green's Functions for Microstrip Structures - Theory and Results," *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 6, pp. 1556-1560, June 2002.

[5] M. Yuan and T. K. Sarkar, "Computation of Sommerfeld Integral Tails Using the Matrix Pencil Method," *IEEE Trans. Antennas. Propagat.*, vol. 54, no. 4, pp. 1358-1362, April 2006.

[6] L. Tsang and B. Wu, "Electromagnetic Fields of Hertzian Dipoles in Layered Media of Moderate Thickness Including the Effects of All Modes," *IEEE Antennas and Wireless Prop. Lett.*(AWPL), vol. 6, pp. 316-319, 2007.

[7] M. S. Kluskens, unpublished notes, Radar Division, NRL, Washington, DC, 2008.

[8] D. Chatterjee and S. D. Walker, "Study of Sommerfeld and Phase Integral Approaches for Green's Functions for PEC-terminated Inhomogeneous Media," Naval Research Laboratory, Washington, DC, USA, technical memorandum NRL/MR/5310-10-9240, December 2009.