## A New Approach for Merging Edge Line Segments

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Abstract $\square \mathrm{A}$ novel method for the linking or merging of edge line segments belonging to a single line is proposed; the segments can be non-overlapping or overlapping, either fully or partially. Explicit criteria for merging, based on proximity of segments and on directional difference are included in the process.

## I-INTRODUCTION

In many areas of computer vision it is often required to link or to merge line segments belonging to the same line. This may be required to reduce the number of entities having common characteristics in an image, thus building new entities with stronger characteristics (e.g., length, orientation).

The method herein proposed is quite versatile, being appropriate either for linking nonoverlapping segments, or for merging line segments partially or fully overlapping. The main advantage of the method derives from the consideration of the lengths of each segment as explicit weights for the definition of the orientation and placement of the resulting line.

## II - Overview

Several methods have been proposed to merge two segments which are close of each other and have similar directions. We shall briefly describe two methods, both analysed in [1].

The first method is due to Liu and Huang. The principle of the method consists in defining the resulting segment as the one that connects the ends of the given segments which are farther apart, as shown in Fig. 1.

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Fig. 1 - Principle of the Liu and Huang method. The resulting segment (length $l_{r}$ ) connects the most distant ends of the given segments (lengths $l_{i}$ and $l_{j}$ ).
There are three main drawbacks in the method:

- when the segments are totally overlapping, the result is the largest of them, thus cancelling the influence of the smaller one (see Fig. 2);
- when the segments are partially overlapping or non-overlapping, the resulting line can have a poor placement (see Fig. 3);
- the orientation and placement of the merged line is not influenced by the relative lengths of the given segments.


Fig. 2 - Merging of two totally overlapping segments by the Liu and Huang method.


Fig. 3 - Merging of two partially overlapping segments by the Liu and Huang method.

Hussien and Sridhar propose a different method in [1], consisting of the two following steps:

1) determine the lines perpendicular to the ends of the given segments which are farther apart;
2) define the resulting line as the one that passes by the midpoints of both segments and which ends are the intersections with the lines determined in 1) (see Fig. 4).


Fig. 4 - Principle of the method proposed by Hussien and Sridhar.

Better results can be obtained by the Hussien and Sridhar method, as confirmed by comparison of Figs. 1 and 4. However, some drawbacks must be pointed out:

- the method is not appropriate for partially overlapping segments (see Fig. 5);
- When the two segments are totally overlapping, the resulting line can be very wrong in length and in orientation (see Fig. 6);
- the orientation and placement of the merged line is not influenced by the relative lengths of the given segments.


Fig. 5 - Hussien and Sridhar method applied to partially overlapping segments (wrong orientation).


Fig. 6 - Hussien and Sridhar method applied to totally overlapping segments.

## III - Proposed Method

The two methods discussed in the previous section are not general in scope, that is, they are not tailored to address both the cases of nonoverlapping and of overlapping segments. On the other hand, none of the two includes the provision to weigh the segments according to their relative importance (length).

We propose a new method for merging line segments that overcomes the aforementioned problems. The principle of the method is best explained by the following five steps:

1) Define the coordinates of the centroid $\left(x_{G}, y_{G}\right)$ - of the set formed by the four endpoints, taking the respective segment lengths $-l_{i}$ and $l_{j}$ - as the point "masses", that is

$$
\begin{gathered}
x_{G}=\frac{l_{i}\left(a_{x}+b_{x}\right)+l_{j}\left(c_{x}+d_{x}\right)}{2\left(l_{i}+l_{j}\right)}, \\
y_{G}=\frac{l_{i}\left(a_{y}+b_{y}\right)+l_{j}\left(c_{y}+d_{y}\right)}{2\left(l_{i}+l_{j}\right)} .
\end{gathered}
$$

(Eqs. 1a and 1b)
where $(a, b)$ are the endpoints of segment $i$ and $(c, d)$ are the endpoints of segment $j$. The merged line will contain this centroid.
2) Define the orientation of the merged line as the weighted sum of the orientations of the given segments, taking for weights their respective lengths. The orientation of the merged line - $\theta_{r}$ - is thus given by

If $\left|\theta_{i}-\theta_{j}\right| \leq \frac{\pi}{2}$ then
$\theta_{r}=\frac{l_{i} \theta_{i}+l_{i} \theta_{j}}{l_{i}+l_{j}}$, else
$\theta_{r}=\frac{l_{i} \theta_{i}+l_{j}\left(\theta_{j}-\pi \frac{\theta_{j}}{\left|\theta_{j}\right|}\right)}{l_{i}+l_{j}}$.
(Eqs. 2a and 2b)
where $l_{k}$ and $\theta_{k}$ represent the length and direction of segment $k$, respectively.
3) Define a frame $\left(X_{G}, Y_{G}\right)$ centered on the centroid $\left(x_{G}, y_{G}\right)$ and having the $X_{G}$ axis parallel to the direction $\theta_{r}$ of the merged line.
4) Determine the coordinates of the endpoints $a$ , $b, c$ and $d$ of both segments in the frame $\left(X_{G}, Y_{G}\right)$. The coordinate transformation is a translation followed by a rotation :

$$
\begin{aligned}
& \delta_{X G}=\left(\delta_{y}-y_{G}\right) \sin \theta_{r}+\left(\delta_{x}-x_{G}\right) \cos \theta_{r}, \\
& \delta_{Y G}=\left(\delta_{y}-y_{G}\right) \cos \theta_{r}-\left(\delta_{x}-x_{G}\right) \sin \theta_{r} . \\
& \quad \text { (Eqs. 3a and b) }
\end{aligned}
$$

where $\left(\delta_{X G}, \delta_{Y G}\right)$ are the coordinates of the point $\delta$ in frame ( $X_{G}, Y_{G}$ ).
5) The two orthogonal projections over the axis $X_{G}$ of the four endpoints $a, b, c$ and $d$, which are farther apart, define the endpoints of the merged line.

The results of the proposed method are shown in Figs. 7, 8 and 9, as applied to non-overlapping, partially overlapping and totally overlapping segments, respectively.


Fig. 7 - Merging of two non-overlapping segments by the proposed method.


Fig. 8 - Merging of two partially overlapping segments by the proposed method.


Fig. 9 - Merging of two totally overlapping segments by the proposed method.

As shown in the previous figures, the method gives good results both for non-overlapping and for overlapping segments. The method also allows the simple integration of criteria for deciding the merging of segments based on their proximity and relative position. By analysing the projections of the four endpoints in frame ( $X_{G}, Y_{G}$ ), it is straightforward to determine how close the two segments are and what their relative position is. It suffices to compare the length $l_{r}$ of the resulting line with the lengths of the segments defined by the projections of the endpoints $a, b, c$ and $d$ on the frame ( $X_{G}, Y_{G}$ ). Three cases have to be considered:

## Case 1:

$$
\begin{equation*}
\text { If }\left|l_{r}\right| \geq\left(\left|\overline{a_{X_{G}} b_{X_{G}}}\right|+\left|\overline{c_{X_{G}} d_{X_{G}}}\right|\right) \tag{Eq.4}
\end{equation*}
$$

then there is no overlap of the segments. The criterion of proximity to test the condition for merging is:
$\left(\left|l_{r}\right|-\left|\overline{a_{X_{G}} b_{X_{G}}}\right|+\left|\overline{c_{X_{G}} d_{X_{G}}}\right|\right) \leq d_{M A X_{X G}}$
and $\left|\overline{\delta_{Y_{G}} \kappa_{Y_{G}}}\right| \leq d_{M A X_{Y G}}$,
(Eqs. 5a and 5b)
where $d_{M A X_{X G}}$ is the maximum distance, on the $X_{G}$ axis, allowed for merging, $\delta_{Y_{G}}$ and $\kappa_{Y_{G}}$ are the most distant projections of the endpoints on the $Y_{G}$ axis, and $d_{M A X_{Y G}}$ is the maximum distance, on the $Y_{G}$ axis, allowed for merging.

## Case 2:

$$
\begin{equation*}
\text { If }\left|l_{r}\right|=\left|\overline{a_{X_{G}} b_{X_{G}}}\right| \text { or }\left|l_{r}\right|=\left|\overline{c_{X_{G}} d_{X_{G}}}\right| \tag{Eq.6}
\end{equation*}
$$

then the segments are totally overlapping. The proximity criterion is given again by Eq. 5 b.

## Case 3:

If $\left|l_{r}\right|<\left(\left|\overline{a_{X_{G}} b_{X_{G}}}\right|+\left|\overline{c_{X_{G}} d_{X_{G}}}\right|\right)$
(Eq. 7)
and both Eqs. 6a and 6b are not verified, then the segments are partially overlapping. The proximity criterion is given again by Eq. 5b.

A detailed presentation of the method is given in [2] and [3]. The following section presents some experimental results.

## IV - Experiments and Results

The following experimental results were obtained for the well-known image "cameraman", where the segments for merging were extracted using a method combining edge tracking and polygonal approximation using a dynamic strip algorithm ${ }^{1}$

Figs. 12 and 13 show the original image and the result of polygonal approximation.


Fig. 12 - Original image "cameraman".


Fig. 13 - Image obtained by edge extraction and polygonal approximation.

In order to simplify the illustration of the method, only the segments longer than 20 pixels are retained for merging. The simplified image is shown in Fig. 14.


Fig. 14 - Input image for the merging process (simplification of Fig. 13).

Fig. 15 shows three images resulting from the merging of segments present in Fig. 14, using different criteria. For comparison, Fig. 16 displays the results in Fig. 15c) superimposed on the input image.

[^1]

Fig. 15 - Segment merging results considering different criteria.


Fig. 16 - Superposition of the images in Figs. 14 and 15c).

## V-Conclusions

We have presented a new method for merging line segments, that compares favourably with previous methods briefly reviewed. The method has a general scope, enabling the merging of segments either overlapping or not, and produces merged lines with good placement and orientation. The method embodies simple criteria to control the degree of merging according to specified limits in directional difference and in segment proximity. The placement and direction of the merged line are weighted by the lengths of the segments, thus assigning a stronger influence to longer segments. A few experimental results are shown to illustrate the quality of the results
achieved and the ability to control the degree of merging.

## Appendix <br> Polygonal Approximation Using Dynamic Strips

The basic idea of the method ([4], [3]) consists in approximating a given line by using strips of given width that may rotate around its starting point in order to cover the succeeding points. Each strip is defined by a critical line and two border lines, as shown in Fig. A1.


Fig. A1 - Definition of an approximation strip.
The critical line is defined by two reference points, the first and second points in the line to approximate (points $o$ and $a$ in Fig. A1). The border lines are parallel to the critical line at a distance $D$ to each side. The space between the border lines defines a strip. The line to be approximated is then followed point by point. When a point crosses one of the border lines (e.g.,
point $d$ in Fig. A1) the process stops. If that point can be covered by the strip by rotating it around the initial point without uncovering any of the previous points, this rotation is performed and the critical and border lines are updated (Fig. A2). If that is not the case, a line segment is completed by joining the first and last points covered by the strip (Fig. A3), and a new strip is initiated (Fig. A4).


Fig. A2 - Rotation of the approximation strip to include $d$.


Fig. A3 - Inclusion of point $d$ by rotation is not possible.


Fig. A4 - Definition of a new approximation strip.

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[^1]:    ${ }^{1}$ This method is briefly described in the appendix..

