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A new approach for ranking of intuitionistic fuzzy numbers using a centroid concept

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Abstract Ranking of intuitionistic fuzzy numbers is a difficult task. Many methods have been proposed for ranking of intuitionistic fuzzy numbers. In this paper we have ranked both trapezoidal intuitionistic fuzzy numbers and triangular intuitionistic fuzzy numbers using the centroid concept. Some of the properties of the ranking function have been studied. Also, comparative examples are given to show the effectiveness of the proposed method.

Keywords Intuitionistic fuzzy set · Trapezoidal intuitionistic fuzzy number · Triangular intuitionistic fuzzy number · Ranking of trapezoidal intuitionistic fuzzy number · Centroid of an intuitionistic fuzzy number

Introduction

The fuzzy sets [29] were extended by Atanassov [4] to develop the intuitionistic fuzzy sets by including non-membership function which is useful to express vagueness more accurately as compared to fuzzy sets. Fuzzy numbers [1] are special kind of fuzzy sets which are of importance in solving fuzzy linear programming problems. An important issue in fuzzy set theory is ranking of uncertainty numbers. When numerical values are represented in uncertain nature termed as fuzzy numbers, a comparison of these numerical values is not easy. Several methods have been proposed in literature to rank fuzzy numbers. As a generalization of fuzzy numbers, an intuitionistic fuzzy number (IFN) seems to fit more suitably to describe uncertainty. After this, many research works have been carried out in defining and studying interesting properties of various types of intuitionistic fuzzy numbers. Recently, the research on IFN's has received high attention, since it is more suitable for solving intuitionistic fuzzy linear programming problems. Many ranking methods for ordering of IFNs have been introduced in the literature.

Grzegorzewski [7, 8] treated IFNs as two families of metrics and developed a ranking method for IFNs. Mitchell [11] proposed a ranking method to order triangular intuitionistic fuzzy numbers (TIFNs) by accepting a statistical viewpoint and interpreting each IFN as ensemble of ordinary fuzzy numbers. Ranking of TIFN on the basis of value index to ambiguity index is proposed by Li [9] and solved a multiattribute decision-making problem. Dubey et al. [6] extended the definitions given by Li [9] to the newly defined TIFNs. Thereafter, a ranking function was proposed to solve a class of linear programming problems. A ranking function based on score function was proposed and the same used to solve IFLP, in which the data parameters are TIFNs. In the past, Nayagam et al. [14] introduced TIFNs and proposed a method to rank them.

Nehi [15] proposed a new ranking method, in which the membership function and non-membership function of IFNs are treated as fuzzy quantities. In the same way, Li [10] defined the value and ambiguity index for TIFN which is similar to those of Delgado et al. [5]. These are then used to define the value index and the ambiguity index for TIFN. Using this concept, a method based on ratio ranking is developed for ranking TIFNs. The limitations and shortcomings of some of the existing ranking were overcome by a new ranking approach by modifying an existing ranking approach proposed for comparing IF numbers by Amit

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Kumar [2]. With the help of the proposed ranking approach, a new method is proposed to find the optimal solution of unbalanced minimum cost flow (MCF) problems, in which all the parameters are represented by IF numbers. The values and ambiguities of the membership degree and the non-membership degree for trapezoidal intuitionistic fuzzy number are defined as well as the value index- and ambiguity index-based ranking approach given by Rezvani [18]. In 2011, Nayagam et al. [13] defined new intuitionistic fuzzy scoring method for the intuitionistic fuzzy number in which hesitation is greater than membership fuzzy number. Similarly, in the intuitionistic fuzzy number, the hesitation is less than the membership fuzzy number. This new method includes the concept of both membership and non-membership function of an intuitionistic fuzzy number. By this defined method, the problems involving hesitation can be easily studied. Also in that paper, the defined intuitionistic fuzzy scoring method was applied to the clustering problem.

In Peng et al. [17], defined the concepts of canonical intuitionistic fuzzy numbers and fuzzy cut sets, and the relation between generalized fuzzy numbers and canonical intuitionistic fuzzy numbers were studied. Next, the concept of center index and radius index of canonical intuitionistic fuzzy numbers, based on fuzzy cut sets are introduced, and the ranking index with the degree of optimism of the decision maker for canonical intuitionistic fuzzy numbers is defined. Then a new ranking method based on the ranking index is developed. The properties of values index and ambiguity index of TIFNs are studied and a compromise ratio ranking method for TIFNs is developed based on the value index and ambiguity index of TIFNs. Using the above ranking, MADM problems in which the ratings of alternatives on attributes are expressed as TIFNs are solved by the extended additive weighted method in [30] by Zhang et al. In [20], a novel approach for ranking triangular intuitionistic fuzzy numbers (TIFNs) is obtained by converting each TIFN to two related triangular fuzzy numbers (TFNs) based on their membership functions and non-membership functions. Then, a new defuzzification for the obtained TFNs using their values and ambiguities was suggested by Salahshour [20]. The average ranking index is introduced to find out order relations between two TIFNs by Seikh et al. in [23]. A method is described to approximate a TIFN to a nearly approximated interval number. Applying this result and using interval arithmetic, a bound unconstrained optimization problem is solved whose coefficients are fixed TIFNs.

Based on the possibility degree formula, Wei et al. [27] gave a possibility degree method to rank intuitionistic fuzzy numbers, which is used to rank the alternatives in multi-criteria decision-making problems. Cosine similarity measure method was extended for ranking alternatives, and then a practical example of the developed approach was given to select the investment alternatives by Ye [28].

The basic arithmetic operations of generalized triangular intuitionistic fuzzy numbers (GTIFNs) and the notion of (α,β)-cut sets were defined by Seikh et al. [24]. Also a nearest interval approximation method is described to approximate a GTIFN to a nearest interval number. Moreover, the average ranking index is introduced to find out order relations between two GTIFNs. Intuitionistic trapezoidal fuzzy weighted arithmetic averaging operator and weighted geometric averaging operator were proposed by Wang et al. in [25]. The expected values, score function, and the accuracy function of intuitionistic trapezoidal fuzzy numbers are also defined. By comparing the score function and the accuracy function values of integrated fuzzy numbers, a ranking of the whole alternative set was attained. Nagoorgani et al. [12] introduced a ranking technique for TIFN using α,β-cut, score function and accuracy function. The method is validated by applying the concept to solve the intuitionistic fuzzy variable linear programming problem.

In this paper, we introduce a new approach to rank intuitionistic fuzzy numbers which is easy to handle. The rest of the paper is organized as follows: Sect. 2 gives some basic definitions and notations of intuitionistic fuzzy sets and intuitionistic fuzzy numbers. In Sect. 3, a new approach for ranking trapezoidal and triangular intuitionistic fuzzy numbers is introduced and some properties of ranking functions are investigated. In Sect. 4, some numerical examples are illustrated to prove the advantage of the proposed paper. In Sect. 5 a comparative study is given. The last section gives the conclusion and future work.

Preliminaries

This section introduces some definitions and basic concepts related to intuitionistic fuzzy sets and intuitionistic fuzzy numbers.

Definition 2.1 [4] An IFS A in X is given by $A = \{(x, \mu_A(x), \nu_A(x), x \in X)\}$, where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ define, respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X, 0 \le \mu_A(x) + \nu_A(x) \le 1$. Obviously, every fuzzy set has the form $\{(x, \mu_A(x), \mu_{A^c}(x)), x \in X\}$.

For each IFS A in X, we will call $\Pi_A(x) = 1 - \mu(x) - \nu(x)$ the intuitionistic fuzzy index of x in A. It is obvious that $0 \le \Pi_A(x) \le 1$, for all $x \in X$.

Definition 2.2 [15] An intuitionistic fuzzy set (IFS) $A = \{(x, \mu_A(x), \nu_A(x), x \in X)\}$ is called IF-normal, if there exist at least two points $x_0.x_1 \in$ X such that $\mu_A(x_0) = 1$, $\nu_A(x_1) = 1$. It is easily seen that given intuitionistic fuzzy set A is IF-normal, if there is at least one point it surely belongs to A, and at least one point does not belong to A.

Definition 2.3 [15] An intuitionistic fuzzy set (IFS) $A = \{(x, \mu_A(x), \nu_A(x), x \in X)\}$ of the real line is called IF-convex, if $\forall x_1, x_2 \in \mathrm{R}, \forall \lambda \in [0, 1]$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \Lambda \mu_A(x_2)$$
$$\nu_A(\lambda x_1 + (1-\lambda)x_2) \geq \nu_A(x_1) \Lambda \nu_A(x_2)$$

Definition 2.4 [15] An IFS $A = \{(x, \mu_A(x), \nu_A(x),\ x \in X)\}$ of the real line is called an intuitionistic fuzzy number (IFN) if:

(i) A is IF-normal,
(ii) A is IF-convex,
(iii) μ_A is upper semi-continuous and ν_A is lower semi continuous,
(iv) *Supp* $\mathrm{A} = \{(\mathrm{x} \in \mathrm{X}/\nu_{\mathrm{A}}(x) < 1)\}$ is bounded.

Definition 2.5 [15] A trapezoidal intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ is denoted by $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$. In this case, we have

$$\mu_A(x) = \begin{cases} 0; & \mathrm{x} < \mathrm{a}_1 \\ \dfrac{\mathrm{x} - \mathrm{a}_1}{\mathrm{a}_2 - a_1}; & a_1 \leq x \leq a_2 \\ 1; & \mathrm{a}_2 \leq x \leq a_3 \\ \dfrac{x - a_4}{a_3 - a_4}; & a_3 \leq x \leq a_4 \\ 0; & a_4 < \mathrm{x} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0; & \mathrm{x} < \mathrm{b}_1 \\ \dfrac{\mathrm{x} - \mathrm{b}_2}{\mathrm{b}_1 - b_2}; & b_1 \leq x \leq b_2 \\ 0; & \mathrm{b}_2 \leq x \leq b_3 \\ \dfrac{x - b_4}{b_3 - b_4}; & b_3 \leq x \leq b_4 \\ 1; & \mathrm{b}_4 < x \end{cases}$$

In the above definition, if we let $b_2 = b_3$ (and hence $a_2 = a_3$), then we will get a triangular intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq (b_2 = a_2 = a_3 = b_3) \leq a_4 \leq b_4$ denoted by $A = (b_1, a_1, b_2, a_4, b_4)$.

Definition 2.6 [16] An intuitionistic fuzzy number (IFN) A in R is said to be a symmetric trapezoidal intuitionistic fuzzy number if there exists real numbers a_1, a_2, h, h' where $a_1 \leq a_2, h \leq h'$ and $h, h' > 0$ such that the membership and non-membership functions are as follows:

$$\mu_A(x) = \begin{cases} \dfrac{x - (a_1 - h)}{h}; & x \in [a_1 - h, a_1] \\ 1; & x \in [a_1, a_2] \\ \dfrac{a_2 + h - x}{h}; & x \in [a_2, a_2 + h] \\ 0; & \text{otherwise} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} \dfrac{(a_1 - x)}{h'}; & x \in [a_1 - h', a_1] \\ 0; & x \in [a_1, a_2] \\ \dfrac{x - a_2}{h}; & x \in [a_2, a_2 + h'] \\ 1; & \text{otherwise} \end{cases},$$

where $A = [a_1, a_2, h, h; a_1, a_2, h', h']$.

Centroid-based approach for ranking of intuitionistic fuzzy numbers

In this section, we derive the centroid point of the trapezoidal intuitionistic fuzzy number and triangular intuitionistic fuzzy numbers.

The method of ranking trapezoidal intuitionistic fuzzy numbers with centroid index uses the geometric center of a trapezoidal intuitionistic fuzzy number. The geometric center corresponds to $\tilde{x}(A)$ value on the horizontal axis and $\tilde{y}(A)$ value on the vertical axis.

Consider a trapezoidal intuitionistic fuzzy number of the form $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$, whose membership function can be defined as follows:

$$\mu_A = \begin{cases} 0, & x < a_1 \\ f_A^L(x), & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ f_A^R(x), & a_3 \leq x \leq a_4 \\ 0, & a_4 \leq x \end{cases},$$

and non-membership can generally be defined as

$$\nu_A = \begin{cases} 0, & x < b_1 \\ g_A^L(x), & b_1 \leq x \leq b_2 \\ 0, & \mathrm{b}_2 \leq x \leq b_3 \\ g_A^R(x), & b_3 \leq x \leq b_4 \\ 1, & b_4 \leq x \end{cases},$$

where $f_A^L : R \to [0, 1]$, $f_A^R : R \to [0, 1]$, $g_A^L : R \to [0, 1]$ and $g_A^R : R \to [0, 1]$, called the sides of an intuitionistic fuzzy number, where f_A^L and g_A^R are non-decreasing and f_A^R, g_A^L are non-increasing. Therefore, the inverse functions of. f_A^L, f_A^R, g_A^L and g_A^R exist which are also of the same nature. Let $h_A^L : [0, 1] \to R$, $h_A^R : [0, 1] \to R, k_A^L : [0, 1] \to R$ and $k_A^L : [0, 1] \to R$ be the inverse functions of f_A^L, f_A^R, g_A^L and g_A^R, respectively. Then, h_A^L,h_A^R,k_A^L and k_A^L should be integrable on R. In the case of the above-defined trapezoidal intuitionistic fuzzy number, the above inverse functions can be analytically expressed as follows:

$$h_A^L\ (y) = a_1 + (a_2 - a_1)y, \quad 0 \le y \le 1,$$
$$h_A^R\ (y) = a_4 + (a_3 - a_4)y, \quad 0 \le y \le 1,$$
$$k_A^L\ (y) = b_2 + (b_1 - b_2)y, \quad 0 \le y \le 1,$$
$$k_A^R\ (y) = b_3 + (b_4 - b_3)y, \quad 0 \le y \le 1.$$

The diagrammatic representations are shown Figs. 1 and 2.

The centroid point $(\tilde{x}(A), \tilde{y}(A))$ of the trapezoidal intuitionistic fuzzy numbers $\tilde{A}$ is determined as follows:

$$\tilde{x}_\mu(A) = \frac{\int_{a_1}^{a_2} x f_A^L(x)\mathrm{d}x + \int_{a_2}^{a_3} x\mathrm{d}x + \int_{a_3}^{a_4} x f_A^R(x)\mathrm{d}x}{\int_{a_1}^{a_2} f_A^L(x)\mathrm{d}x + \int_{a_2}^{a_3} \mathrm{d}x + \int_{a_3}^{a_4} f_A^R(x)\mathrm{d}x},$$

$$= \frac{\int_{a_1}^{a_2} \frac{x^2 - xa_1}{a_2 - a_1}\mathrm{d}x + \int_{a_2}^{a_3} x\mathrm{d}x + \int_{a_3}^{a_4} \frac{x^2 - a_4 x}{a_3 - a_4}\mathrm{d}x}{\int_{a_1}^{a_2} \frac{x - a_1}{a_2 - a_1}\mathrm{d}x + \int_{a_2}^{a_3} \mathrm{d}x + \int_{a_3}^{a_4} \frac{x - a_4}{a_3 - a_4}\mathrm{d}x},$$

$$= \frac{\frac{1}{a_2 - a_1}\left[\frac{x^3}{3} - \frac{x^2 a_1}{2}\right]_{a_1}^{a_2} + \left[\frac{x^2}{2}\right]_{a_2}^{a_3} + \frac{1}{a_3 - a_4}\left[\frac{x^3}{3} - \frac{a_4 x^2}{2}\right]_{a_3}^{a_4}}{\frac{1}{a_2 - a_1}\left[\frac{x^2}{2} - a_1 x\right]_{a_1}^{a_2} + [x]_{a_2}^{a_3} + \frac{1}{a_3 - a_4}\left[\frac{x^2}{2} - x\right]_{a_3}^{a_4}},$$

$$= \frac{\left(\frac{a_2^2 + a_1 a_2 + a_1^2}{3}\right) - \frac{a_1}{2}[a_2 + a_1] + \left[\frac{a_3^2 - a_2^2}{2}\right] + \left[\frac{-a_4^2 - a_3 a_4 - a_3^2}{3}\right] - \frac{a_4}{2}[a_4 + a_3]}{\frac{a_2 + a_1}{2} - a_1 + a_3 - a_2 - \frac{a_4 - a_3}{2} + a_4},$$

$$\tilde{x}_\mu(A) = \frac{1}{3}\left[\frac{a_3^2 + a_4^2 - a_1^2 - a_2^2 - a_1 a_2 + a_3 a_4}{a_4 + a_3 - a_2 - a_1}\right].$$

Similarly,

$$\tilde{x}_\upsilon(A)7 = \frac{\int_{b_1}^{b_2} x\ g_A^L(x)\mathrm{d}x + \int_{b_2}^{b_3} x\mathrm{d}x + \int_{b_3}^{b_4} x g_A^R(x)\mathrm{d}x}{\int_{b_1}^{b_2} g_A^L(x)\mathrm{d}x + \int_{b_2}^{b_3} \mathrm{d}x + \int_{b_3}^{b_4} g_A^R(x)\mathrm{d}x}$$
$$= \frac{\int_{b_1}^{b_2} \frac{x^2 - xb_2}{b_1 - b_2}\mathrm{d}x + \int_{b_2}^{b_3} x\mathrm{d}x + \int_{b_3}^{b_4} \frac{x^2 - xb_3}{b_4 - b_3}\mathrm{d}x}{\int_{b_1}^{b_2} \frac{x - b_2}{b_1 - b_2}\mathrm{d}x + \int_{b_2}^{b_3} \mathrm{d}x + \int_{b_3}^{b_4} \frac{x - b_3}{b_4 - b_3}\mathrm{d}x},$$

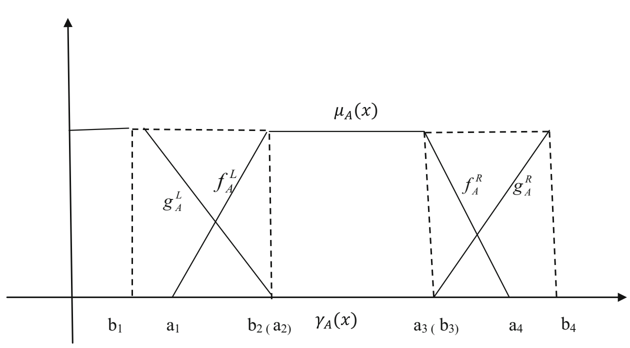

Fig. 1 TrIFN with non-decreasing and non-increasing functions

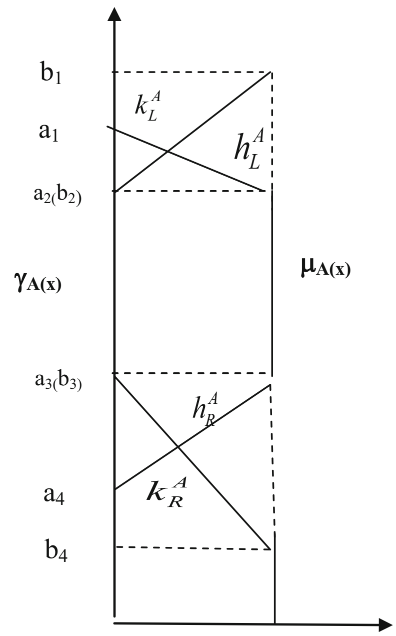

Fig. 2 TrIFN with inverse non-decreasing and non-increasing functions

$$= \frac{\frac{1}{b_1 - b_2}\left[\frac{x^3}{3} - b_2\frac{x^2}{2}\right]_{b_1}^{b_2} + \left[\frac{x^2}{2}\right]_{b_2}^{b_3} + \frac{1}{b_4 - b_3}\left[\frac{x^3}{3} - b_3\frac{x^2}{2}\right]_{b_3}^{b_4}}{\frac{1}{b_1 - b_2}\left[\frac{x^2}{2} - b_2 x\right]_{b_1}^{b_2} + [x]_{b_2}^{b_3} + \frac{1}{b_4 - b_3}\left[\frac{x^2}{2} - b_3 x\right]_{b_3}^{b_4}},$$

$$\tilde{x}_\upsilon(A) = \frac{1}{3}\left[\frac{2b_4^2 - 2b_1^2 + 2b_2^2 + 2b_3^2 + b_1 b_2 - b_3 b_4}{b_3 + b_4 - b_1 - b_2}\right].$$

Next,

$$\tilde{y}_\mu(A) = \frac{\int_0^1 y h_A^L(y)\mathrm{d}y - \int_0^1 y h_A^R(y)\mathrm{d}y}{\int_0^1 h_A^L(y)\mathrm{d}y - \int_0^1 h_A^R(y)\mathrm{d}y},$$

$$\tilde{y}_\mu(A) = \frac{\int_0^1 (a_2 y^2 + a_1 y - a_1 y^2)\mathrm{d}y - \int_0^1 (a_3 y^2 + a_4 y - a_4 y^2)\mathrm{d}y}{\int_0^1 (a_2 y + a_1 - a_1 y)dy - \int_0^1 (a_3 y + a_4 - a_4 y)\mathrm{d}y},$$

$$= \frac{\left(a_2\left[\frac{y^3}{3}\right] + a_1\left[\frac{y^2}{2}\right] - a_1\left[\frac{y^3}{3}\right]\right)_0^1 - \left(a_3\left[\frac{y^3}{3}\right] + a_4\left[\frac{y^2}{2}\right] - a_4\left[\frac{y^3}{3}\right]\right)_0^1}{\left(a_2\left[\frac{y^2}{2}\right] + a_1 y - a_1\left[\frac{y^2}{2}\right]\right)_0^1 - \left(a_3\left[\frac{y^2}{2}\right] + a_4 y - a_4\left[\frac{y^2}{2}\right]\right)_0^1},$$

$$= \frac{\left(\left[\frac{a_2}{3}\right] + \left[\frac{a_1}{2}\right] - \left[\frac{a_1}{3}\right]\right) - \left(\left[\frac{a_3}{3}\right] + \left[\frac{a_4}{2}\right] - \left[\frac{a_4}{3}\right]\right)}{\left(\left[\frac{a_2}{2}\right] + a_1 - \left[\frac{a_1}{2}\right]\right) - \left(\left[\frac{a_3}{2}\right] + a_4 - \left[\frac{a_4}{2}\right]\right)},$$

$$= \frac{1}{3}\left(\frac{2a_2 + 3a_1 - 2a_1 - 2a_3 - 3a_4 + 2a_4}{a_2 + 2a_1 - a_1 - a_3 - 2a_4 + a_4}\right),$$

$$\tilde{y}_\mu(A) = \frac{1}{3}\left(\frac{a_1 + 2a_2 - 2a_3 - a_4}{a_1 + a_2 - a_3 - a_4}\right),$$

$$\tilde{y}_v(A) = \frac{\int_0^1 y\ k_A^L(y)\mathrm{d}y - \int_0^1 y k_A^R(y)\mathrm{d}y}{\int_0^1 k_A^L(y)\mathrm{d}y - \int_0^1 k_A^R(y)\mathrm{d}y},$$

$$\tilde{y}_{\upsilon}(A)=\frac{\int_0^1 (b_1y^2+b_2y-b_2y^2)\mathrm{d}y-\int_0^1 (b_4y^2+b_3y-b_3y^2)\mathrm{d}y}{\int_0^1 (b_1y+b_2-b_2y)\mathrm{d}y-\int_0^1 (b_4y+b_3-b_3y)\mathrm{d}y},$$

$$=\frac{\left(b_1\left[\frac{y^3}{3}\right]+b_2\left[\frac{y^2}{2}\right]-b_2\left[\frac{y^3}{3}\right]\right)_0^1-\left(b_4\left[\frac{y^3}{3}\right]+b_3\left[\frac{y^2}{2}\right]-b_3\left[\frac{y^3}{3}\right]\right)_0^1}{\left(b_1\left[\frac{y^2}{2}\right]+b_2y-b_2\left[\frac{y^2}{2}\right]\right)_0^1-\left(b_4\left[\frac{y^2}{2}\right]+b_3y-b_3\left[\frac{y^2}{2}\right]\right)_0^1},$$

$$=\frac{\left(\left[\frac{b_1}{3}\right]+\left[\frac{b_2}{2}\right]-\left[\frac{b_2}{3}\right]\right)-\left(\left[\frac{b_4}{3}\right]+\left[\frac{b_3}{2}\right]-\left[\frac{b_3}{3}\right]\right)}{\left(\left[\frac{b_1}{2}\right]+b_2-\left[\frac{b_2}{2}\right]\right)-\left(\left[\frac{b_4}{2}\right]+b_3-\left[\frac{b_3}{2}\right]\right)},$$

$$=\frac{1}{3}\left(\frac{2b_1+3b_2-2b_2-2b_4-3b_3+2b_3}{b_1+2b_2-b_2-b_4-2b_3+b_3}\right),$$

$$\tilde{y}_{\upsilon}(A)=\frac{1}{3}\left(\frac{2b_1+b_2-b_3-2b_4}{b_1+b_2-b_3-b_4}\right).$$

Then, $(\tilde{x}_\mu(A),\tilde{y}_\mu(A));(\tilde{x}_\upsilon(A),\tilde{y}_\upsilon(A))$ gives the centroid of the trapezoidal intuitionistic fuzzy number.

The above relations can be reduced to get the centroid point of the triangular intuitionistic fuzzy numbers, as they are a special case of trapezoidal intuitionistic fuzzy numbers with $b_2 = a_2 = a_3 = b_3$. Its centroid can be determined by

$$\tilde{x}_\mu(A)=\frac{1}{3}\left[\frac{a_4^2-a_1^2+a_2(a_4-a_1)}{a_4-a_1}\right]=\frac{a_1+a_2+a_4}{3};$$

$$\tilde{x}_\upsilon(A)=\frac{1}{3}\left[\frac{2(b_4^2-b_1^2)+b_2(b_1-b_4)}{b_1-b_4}\right]=\frac{2b_1-b_2+2b_4}{3};$$

$\tilde{y}_\mu(A)=\frac{1}{3}$ and

$\tilde{y}_\upsilon(A)=\frac{2}{3}$.

Theorem 3.1 *Let* $A=(b_1,a_1,b_2,a_4,b_4)$ and $B=(b'_1,a'_1,b'_2,a'_4,b'_4)$ *be two triangular intuitionistic fuzzy numbers. Then the following equation is valid:* $\tilde{x}_\mu(A+B)=\tilde{x}_\mu(A)+\tilde{x}_\mu(B)$

Proof We have $A+B=(b_1+b'_1,a_1+a'_1,b_2+b'_2,a_4+a'_4,b_4+b'_4)$.

Now,

$$\tilde{x}_\mu(A+B)=\frac{(a_1+a'_1)+(a_2+a'_2)+(a_4+a'_{4)}}{3}$$
$$=\frac{a_1+a_2+a_4}{3}+\frac{a'_1+a'_2+a'_4}{3},$$
$$\tilde{x}_\mu(A+B)=\tilde{x}_\mu(\mathrm{A})+\tilde{x}_\mu(\mathrm{B}).$$

Theorem 3.2 *Let* $A=(b_1,a_1,b_2,a_4,b_4)$ and $B=(b'_1,a'_1,b'_2,a'_4,b'_4)$ *be two triangular intuitionistic fuzzy numbers. Then,* $\tilde{x}_v(A+B)=\tilde{x}_v(A)+\tilde{x}_v(B)$.

Proof We have $A+B=(b_1+b'_1,a_1+a'_1,b_2+b'_2,a_4+a'_4,b_4+b'_4)$.

Now,

$$\tilde{x}_\upsilon(A+B)=\frac{2(b_1+b'_1)-(b_2+b'_2)+2(b_4+b'_{4)}}{3}$$
$$=\frac{2b_1-b_2+2b_4}{3}+\frac{2b'_1-b'_2+2b'_4}{3},$$
$$\tilde{x}_\upsilon(A+B)=\tilde{x}_\upsilon(\mathrm{A})+\tilde{x}_\upsilon(\mathrm{B}).$$

Theorem 3.3 *Let* $A=(b_1,a_1,b_2,a_4,b_4)$ and $B=(b'_1,a'_1,b'_2,a'_4,b'_4)$ *be two triangular intuitionistic fuzzy numbers. Then*

$$\tilde{x}(A+B)=\tilde{x}(A)+\tilde{x}(B)+\tilde{x}_v(\mathrm{A})*\tilde{x}_\mu(\mathrm{B})+\tilde{x}_\mu(\mathrm{A})*\tilde{x}_v(\mathrm{B})$$

Proof We have

$$\tilde{x}(A+B)=\tilde{x}_\mu(A+B)*\tilde{x}_\upsilon(A+B)$$
$$=[\tilde{x}_\mu(A)+\tilde{x}_\mu(B)]*[\tilde{x}_\upsilon(A)+\tilde{x}_\upsilon(B)]$$
$$=\left[\frac{a_1+a_2+a_4}{3}+\frac{a'_1+a'_2+a'_4}{3}\right]*\left[\frac{2b_1-b_2+2b_4}{3}+\frac{2b'_1-b'_2+2b'_4}{3}\right]$$
$$=\frac{\begin{array}{c}\times(2a_1b_1+2a_2b_1+2a_4b_1-a_1b_2-a_2b_2-a_4b_2+2a_1b_4+2a_2b_4+2a_4b_4)\\+(2a'_1b'_1+2a'_2b'_1+2a'_4b'_1-a'_1b'_2-a'_2b'_2-a'_4b'_2+2a'_1b'_4+2a'_2b'_4+2a'_4b'_4)\\+(a'_1+a'_2+a'_4)*(2b_1-b_2+2b_4)+(a_1+a_2+a_4)*(2b'_1-b'_2+2b'_4)\end{array}}{9},$$
$$\tilde{x}(A+B)=\tilde{x}(A)+\tilde{x}(B)+\tilde{x}_\upsilon(\mathrm{A})*\tilde{x}_\mu(\mathrm{B})+\tilde{x}_\mu(\mathrm{A})*\tilde{x}_\upsilon(\mathrm{B}).$$

Theorem 3.4 *Let* $A=(b_1,a_1,b_2,a_4,b_4)$ and $B=(b'_1,a'_1,b'_2,a'_4,b'_4)$ *be two triangular intuitionistic fuzzy numbers. Then the following equation is valid:* $\tilde{y}(A+B)=\tilde{y}(A)+\tilde{y}(B)+2*\tilde{y}(A)$.

Proof Consider

$$\tilde{y}(A+B)=\tilde{y}_\mu(A+B)*\tilde{y}_\upsilon(A+B)$$
$$=[\tilde{y}_\mu(A)+\tilde{y}_\mu(B)]*[\tilde{y}_\upsilon(A)+\tilde{y}_\upsilon(B)]$$
$$=\left[\frac{2}{3}\right]*\left[\frac{4}{3}\right]=\frac{8}{9}$$
$$=\frac{2}{9}+\frac{2}{9}+2*\frac{2}{9}$$
$$=\tilde{y}(A)+\tilde{y}(B)+2*\tilde{y}(A).$$

Remark 3.5 The result of the above theorem can also be simplified as

$\tilde{y}(A+B)=4*\tilde{y}(A)$ (or) $\tilde{y}(A+B)=4*\tilde{y}(B)$,

where $\tilde{y}(A)=\tilde{y}(B)$ in case of triangular intuitionistic fuzzy number.

Theorem 3.6 *The centroids of the symmetric trapezoidal intuitionistic fuzzy number are given by the following relations:*

$$\tilde{x}_\mu\ (\mathrm{A}) = \frac{1}{2}\left[\frac{a_1 h + a_2 h + a_2^2 - a_1^2}{a_2 - a_1}\right],$$
$$\tilde{x}_\upsilon\ (\mathrm{A}) = \frac{a_1 + a_2}{2},$$
$$\tilde{y}_\mu\ (\mathrm{A}) = \frac{1}{3}\left[\frac{3a_1 - 3a_2 - 2h}{2a_1 - 2a_2 - 2h}\right],$$
$$\tilde{y}_\upsilon\ (\mathrm{A}) = \frac{1}{3}\left[\frac{3a_1 - 3a_2 - 4h'}{2a_1 - 2a_2 - 2h'}\right].$$

The proof of the result is similar to the one as derived earlier for trapezoidal intuitionistic fuzzy numbers.

Definition 3.7 The ranking function of the trapezoidal (triangular) intuitionistic fuzzy number A is defined by

$$\Re(A) = \sqrt{\frac{1}{2}\left(\left[\tilde{x}_\mu(A) - \tilde{y}_\mu(A)\right]^2 + \left[\tilde{x}_\upsilon(A) - \tilde{y}_\upsilon(A)\right]^2\right)},$$

which is the Euclidean distance.

It is obvious that the proposed ranking function $\Re$ satisfies the properties A1;A2;A3;A4;A5 and A6 of [23]. We list these properties below for the completeness of the section. Let S be the set of fuzzy quantities, and M be an ordering approach.

A1: For an arbitrary finite subset A of S,$\tilde{a} \in A$, $\tilde{a} \succeq \tilde{a}$ by M on A.

A2: For an arbitrary finite subset A of S and $(\tilde{a}, \tilde{b}) \in A^2, \tilde{a} \succeq \tilde{b}$ and $\tilde{b} \succeq \tilde{a}$ by M on A, we should have $\tilde{a} \approx \tilde{b}$ by M on A.

A3: For an arbitrary finite subset A of S and $(\tilde{a}, \tilde{b}, \tilde{c}) \in A^3, \tilde{a} \succeq \tilde{b}$ and $\tilde{b} \succeq \tilde{c}$ by M on A, we should have $\tilde{a} \succeq \tilde{c}$ by M on A.

A4: For an arbitrary finite subset A of S and $\left(\tilde{a}, \tilde{b}\right) \in A^2, \inf supp(\tilde{a}) \succcurlyeq \sup supp(\tilde{b})$, we should have $\tilde{a} \succeq \tilde{b}$ by M on A.

A4′: For an arbitrary finite subset A of S and $\left(\tilde{a}, \tilde{b}\right) \in A^2, \inf supp(\tilde{a}) \succ \sup supp(\tilde{b})$, we should have $\tilde{a} \succ \tilde{b}$ by M on A.

A5: Let S and S′ be two arbitrary finite sets of fuzzy quantities in which M can be applied and $\tilde{a}$ and $\tilde{b}$ are in $S \cap S'$. We obtain the ranking order $\tilde{a} \succ \tilde{b}$ by M on S′ iff $\tilde{a} \succ \tilde{b}$ by M on S.

A6: Let $\tilde{a}, \tilde{b}, \tilde{a} + \tilde{c}, \tilde{b} + \tilde{c}$ be elements of S. If $\tilde{a} \geq \tilde{b}$ by M on $\left\{\tilde{a}, \tilde{b}\right\}$, then $\tilde{a} + \tilde{c} \succeq \tilde{b} + \tilde{c}$ by M on $\left\{\tilde{a} + \tilde{c}, \tilde{b} + \tilde{c}\right\}$.

A6′: Let $\tilde{a}, \tilde{b}, \tilde{a} + \tilde{c}, \tilde{b} + \tilde{c}$ be elements of S. If $\tilde{a} \succ \tilde{b}$ by M on $\left\{\tilde{a}, \tilde{b}\right\}$, then $\tilde{a} + \tilde{c} \succ \tilde{b} + \tilde{c}$ by M on $\left\{\tilde{a} + \tilde{c}, \tilde{b} + \tilde{c}\right\}$.

A7: Let $\tilde{a}, \tilde{b}, \tilde{a}\tilde{c}, \tilde{b}\tilde{c}$ be elements of S. If $\tilde{a} > \tilde{b}$ by M on $\left\{\tilde{a}, \tilde{b}\right\}$, then $\tilde{a}\tilde{c} \succeq \tilde{b}\tilde{c}$ by M on $\{\tilde{a}\tilde{c},\ \tilde{b}\tilde{c}\}$

It is easy to verify that $\Re(A)$ satisfies the axioms A1–A3,A5 and A7. We focus on verifying axioms A4 and A6.

Theorem 3.8 *Let A and B be two TrIFNs; if* $a_{A1} > a_{B4}$ *and* $b_{A1} > b_{B4}$, *then* $A \succ B$.

Proof *It is known that*

$x_\mu(A) > a_{A1}$ and $y_\mu(A) > a_{A1}$,
$x_\mu(B) < a_{B4}$ and $y_\mu(A) < a_{B4}$,
from $a_{A1} > a_{B4}$,
$\Re(A) > \Re(B)$.
Similarly,
$x_\upsilon(A) > b_{A1}$ and $y_\upsilon(A) > b_{A1}$,
$x_\upsilon(B) < b_{B4}$ and $y_\upsilon(B) < b_{B4}$,
from $b_{A1} > b_{B4}$,
$\Re(A) > \Re(B)$.
Therefore, $\Re(A) > \Re(B) \Rightarrow A \succ B$.

Theorem 3.9 *Let A and B be two TrIFNs, then* $\Re(A + C) > \Re(B + C) \Rightarrow A + C \succ B + C$.

Proof

$\Re(A + B) = \Re(A) + \Re(B)$,
similarly,
$\Re(B + C) = \Re(B) + \Re(C)$.
Therefore, if $A \succ B$,
$\Re(A + C) > \Re(B + C) \Rightarrow A + C \succ B + C$.

Algorithm 3.10 The approach of ranking any two trapezoidal (triangular) intuitionistic fuzzy numbers:

$A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and
$B = \left(b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4'\right)$,

(respectively, $A = (b_1, a_1, b_2, a_4, b_4)$ and $B = \left(b_1', a_1', b_2', a_4', b_4'\right)$,) developed is summarized as follows:

Step 1: Compute $\tilde{x}_\mu(A), \tilde{x}_v(A), \tilde{y}_\mu(A)$ and $\tilde{y}_v(A)$.
Step 2: Compute $\tilde{x}_\mu(B), \tilde{x}_v(B), \tilde{y}_\mu(B)$ and $\tilde{y}_v(B)$.
Step 3: Evaluate $\Re(A)$.
Step 4: Evaluate $\Re(B)$
Step 5: Calculate $\Re(A)$ and $\Re$(B). Then,

(a) $A \prec B$ if and only if $\Re$(A) < $\Re$(B).
(b) $A \succ B$ if and only if $\Re$(A) > $\Re$(B).
(c) $A \approx B$ if and only if $\Re$(A) = $\Re$(B).

Numerical examples

This section gives some numerical examples of the proposed method.

Examples 4.1 Consider two TIFNs $A = (0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6)$ and $B = (0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5)$.

Then, using the proposed method we get

$$\tilde{x}_\mu(A) = 0.9; \tilde{x}_\upsilon(A) = 0.9; \tilde{y}_\mu(A) = 0.39; \tilde{y}_\upsilon(A) = 0.57,$$
$$\tilde{x}_\mu(B) = 0.8; \tilde{x}_\upsilon(B) = 0.8; \tilde{y}_\mu(B) = 0.39; \tilde{y}_\upsilon(B) = 0.57,$$
$$\Re(A) = \sqrt{\frac{1}{2}\left(\left[\tilde{x}_\mu(A) - \tilde{y}_\mu(A)\right]^2 + [\tilde{x}_\upsilon(A) - \tilde{y}_\upsilon(A)]^2\right)} = 0.56,$$
$$\Re(B) = \sqrt{\frac{1}{2}\left(\left[\tilde{x}_\mu(B) - \tilde{y}_\mu(B)\right]^2 + [\tilde{x}_\upsilon(B) - \tilde{y}_\upsilon(B)]^2\right)} = 0.44,$$
$$\Re(A) > \Re(B) \Rightarrow A \succ B.$$

Example 4.2 Consider two triangular intuitionistic fuzzy numbers $A = (0.2,0.4,0.6,0.8,1)$ and $B = (0.1,0.3,0.5,0.7,0.9)$, then

$$\tilde{x}_\mu(A) = \frac{a_1 + a_2 + a_4}{3} = 0.6;\ \tilde{x}_\mu(B) = 0.5,$$
$$\tilde{x}_\upsilon(A) = \frac{2b_1 - b_2 + 2b_4}{3} = 0.6; \tilde{x}_\upsilon(B) = 0.5,$$
$$\tilde{y}_\mu(A) = \frac{1}{3} = 0.33 = \tilde{y}_\mu(B),$$
$$\tilde{y}_\upsilon(A) = \frac{2}{3} = 0.67 = \tilde{y}_\upsilon(B),$$
$$\Re(A) = \sqrt{\frac{1}{2}\left(\left[\tilde{x}_\mu(A) - \tilde{y}_\mu(A)\right]^2 + [\tilde{x}_\upsilon(A) - \tilde{y}_\upsilon(A)]^2\right)} = 0.27,$$
$$\Re(B) = \sqrt{\frac{1}{2}\left(\left[\tilde{x}_\mu(B) - \tilde{y}_\mu(B)\right]^2 + [\tilde{x}_\upsilon(B) - \tilde{y}_\upsilon(B)]^2\right)} = 0.20,$$
$$\Re(A) > \Re(B) \Rightarrow A \succ B.$$

Example 4.3 Consider two symmetric trapezoidal intuitionistic fuzzy numbers $A = (25,23,1,1;25,23,3,3)$ and $B = (5,7,2,2;5,7,4,4)$, then

$$\tilde{x}_\mu\ (A) = \frac{1}{2}\left[\frac{a_1h + a_2h + a_2^2 - a_1^2}{a_2 - a_1}\right] = 12,$$
$$\tilde{x}_v\ (A) = \frac{a_1 + a_2}{2} = 24,$$
$$\tilde{y}_\mu\ (A) = \frac{1}{3}\left[\frac{3a_1 - 3a_2 - 2h}{2a_1 - 2a_2 - 2h}\right] = 0.67,$$
$$\tilde{y}_v\ (A) = \frac{1}{3}\left[\frac{3a_1 - 3a_2 - 4h'}{2a_1 - 2a_2 - 2h'}\right] = 1.$$
$$\tilde{x}_\mu\ (B) = \frac{1}{2}\left[\frac{a_1h + a_2h + a_2^2 - a_1^2}{a_2 - a_1}\right] = 12,$$
$$\tilde{x}_\upsilon\ (B) = \frac{a_1 + a_2}{2} = 6,$$
$$\tilde{y}_\mu\ (B) = \frac{1}{3}\left[\frac{3a_1 - 3a_2 - 2h}{2a_1 - 2a_2 - 2h}\right] = 0.42,$$
$$\tilde{y}_\upsilon\ (B) = \frac{1}{3}\left[\frac{3a_1 - 3a_2 - 4h'}{2a_1 - 2a_2 - 2h'}\right] = 0.61,$$
$$\Re(A) = \sqrt{\frac{1}{2}\left(\left[\tilde{x}_\mu(A) - \tilde{y}_\mu(A)\right]^2 + [\tilde{x}_\upsilon(A) - \tilde{y}_\upsilon(A)]^2\right)} = 18.24,$$
$$\Re(B) = \sqrt{\frac{1}{2}\left(\left[\tilde{x}_\mu(B) - \tilde{y}_\mu(B)\right]^2 + [\tilde{x}_\upsilon(B) - \tilde{y}_\upsilon(B)]^2\right)} = 9.03.$$

Comparative study

A comparative study between the proposed method and other existing methods in the literature is Table 1.

Conclusion

In this paper, a new ranking method based on the centroid is proposed for TrIFNs and TIFNs. The comparative results are given for the justification of the proposed method with the existing ranking. The advantages of the proposed ranking method can be pointed out as follows:

Table 1 Comparative Examples of the proposed ranking process with the existing ranking process

S. no.	Examples	Wei's process	Wang and Zhong's process	Rezvani's process	Li's process	Dubey and Mehra's process	Satyajit's process	Sagaya's process	Proposed method
1.	$A_1 = [(0.57,0.73,0.83);0.73,0.2]$ $A_2 = [(0.58,0.74,0.819);0.72,0.2]$	$A_1 \approx A_2$	$A_1 \succ A_2$	$A_1 \prec A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \approx A_2$	$A_1 \succ A_2$
2.	$A_1 = [(-9,1.5,3);0.6,0.2]$ $A_2 = [(-9,1.5,3);0.7,0.3]$	$A_1 \prec A_2$	$A_1 \prec A_2$	$A_1 \approx A_2$	$A_1 \approx A_2$	$A_1 \prec A_2$	$A_1 \prec A_2$	$A_1 \approx A_2$	$A_1 \prec A_2$
3.	$A_1 = [(3,4,5);0.8,0.2]$ $A_2 = [(6,8,10);0.4,0.6]$	$A_1 \approx A_2$	$A_1 \succ A_2$	$A_1 \prec A_2$	$A_1 \approx A_2$	$A_1 \approx A_2$	$A_1 \prec A_2$	$A_1 \prec A_2$	$A_1 \succ A_2$
4.	$A_1 = [(1,2,3);0.6,0.4]$ $A_2 = [(2,4,6);0.3,0.7]$	$A_1 \approx A_2$	$A_1 \succ A_2$	$A_1 \prec A_2$	$A_1 \approx A_2$	$A_1 \approx A_2$	$A_1 \prec A_2$	$A_1 \prec A_2$	$A_1 \prec A_2$
5.	$A_1 = [(4,5.5,6,8);1,0]$ $A_2 = [(3.5,5,7,7.5);1,0]$	$A_1 \succ A_2$	$A_1 \prec A_2$	$A_1 \approx A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \prec A_2$	$A_1 \succ A_2$
6.	$A_1 = [(0.55,0.75,0.8,0.9);0.5,0.5]$ $A_2 = [(0.5,0.7,0.85,0.95);0.5,0.5]$	$A_1 \approx A_2$	$A_1 \approx A_2$	$A_1 \approx A_2$	$A_1 \approx A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$

1. The method proposed is time consuming compared to the existing methods.
2. The proposed ranking method can be applied for both TrIFNs and TIFNs, which reflects the uncertainty suitably.
3. The proposed ranking technique based on centroid is flexible to the researchers in the ranking index of their attitudinal analysis.

Using this ranking, many decision-making and optimization problems of uncertain nature can be solved. In future, we will use the proposed ranking technique to solve intuitionistic fuzzy linear programming and multi-criteria decision-making problems.

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