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# A new approach to decomposition of a bivariate rank dependent index using recentered influence function regression

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**Abstract:**

Decomposition of a bivariate rank dependent index, such as the concentration index, is commonly used to explain socioeconomic inequalities in health. We introduce a new decomposition technique based on the recentered influence function that yields the marginal effects of covariates on the bivariate rank dependent index. This technique is simple to estimate and interpret. We show that our method, compared to the current standard procedure, relies on the imposition of fewer conditions and is therefore preferable both in a descriptive setting and in the estimation of causal effects given a suitable identification strategy. In an empirical application using Swedish Twin Registry data, we illustrate that this result bears out in practice. The two methods yield contradicting results due to differences in the conditions imposed. Using a within twin pair fixed effects identification strategy, our new method finds no evidence of a causal effect of education on income-related health inequality.

# 1. Introduction

Socioeconomic differences in health are well documented across the western world (Deaton, A.S. 2003; Mackenbach et al. 2008, 2015). This awareness has led to a rapidly growing interest in the measurement and analysis of socioeconomic related inequality in health. In terms of measurement, the dominant family of measures are the various versions of the concentration index (CI) – a family of bivariate rank dependent indices that relate an individual's level of health to her socioeconomic rank (Wagstaff et al. 1991; Fleurbaey and Schokkaert 2009). Policymakers' and researchers' interest in health inequality also extends to explaining and understanding the underlying causes of the observed socioeconomic related health inequality.

One way to examine what determines socioeconomic related health inequality is to decompose a measure of inequality into a function of its (potential) causes. The dominant decomposition procedure to decompose a bivariate rank dependent index is the technique developed by Wagstaff et al. (2003) (WDW, onwards).[1] The interpretation of such an exercise is (at least) two-fold: it may be interpreted as how much of the index is due to cause *X*, what we will refer to as *percentagewise contributions*; or it may be interpreted as how much the index will change due to a shift in the distribution of cause *X*, what we will call *marginal contributions*. Subsequent research has used the WDW decomposition method extensively as a tool to analyse the potential sources of the socioeconomic related health inequality using data from different countries and various health measures, income definitions, and

[1] Gravelle (2003) is acknowledged for developing the same method although the explicit aim of his paper was not to decompose, but to standardise, the concentration index. The resulting methodology is nevertheless the same as that of WDW decomposition.

determinants (e.g. Leu and Schellhorn, 2004; Gomez and Nicholas, 2005; Lauridsen et al. 2007; Hosseinpoor et al. 2006; McGrail et al. 2009; Morasae et al. 2012).

Within the field of labour economics, Fortin et al. (2011) have criticised existing decomposition procedures for being unclear as to what the parameters of interest are and the conditions necessary to uncover these parameters. This criticism is equally applicable to the area of health inequality and in particular the WDW decomposition method. The health inequality literature has also highlighted that the WDW decomposition approach relies upon a set of implicit and restrictive conditions (Van Doorslaer et al. 2004; Erreygers and Kessels 2013; Gerdtham et al. 2015). To the best of our knowledge, all the conditions that underpin WDW decomposition have never been stated explicitly in one place, either in its application or otherwise. We set out the conditions imposed by the WDW decomposition method, highlight that these conditions are often not met in common empirical applications and derive an alternative decomposition method that requires less stringent conditions to be imposed in order to derive the parameters of interest.

In order to help contextualise the conditions the WDW decomposition method imposes note that: first, a rank dependent index (I) equals the *covariance* between health and fractional rank multiplied by a weighting function; and second, the WDW decomposition method is based on a linear *health* regression. The conditions imposed by WDW decomposition are:

I. Health can be modelled as a function linear in variables X and an error term.
II. Exogeneity: The errors from the health regression have zero conditional mean.
III. The determinants of health do not determine rank (rank ignorability).

IV. The determinants of health do not determine the weighting function (weighting function ignorability).

If all the conditions above hold, WDW decomposition identifies both percentagewise and marginal contributions yielding results of great empirical interest.[2] However, these conditions are unlikely to hold and instead WDW decomposition is generally viewed as a "simple descriptive accounting exercise" based on some correlations from an OLS regression (Gerdtham et al. 2015). That is, condition II – which OLS requires for causal interpretation – is not seen as a necessary condition. WDW decomposition is therefore generally thought of as yielding descriptive percentagewise contributions. Even as an accounting exercise however, this still requires the results to be interpreted in light of conditions I, III & IV that are in empirical practice often not reasonable to impose and this muddies the interpretation of the results.

Condition I requires health to be modelled as a function linear in variables, yet there are very few health outcomes that can be validly modelled this way, thus violating the condition. It is common to find non-linear health functions: outcomes may be categorical (Underweight, normal, overweight, obese), censored at zero (doctor visits) or two-part decisions (quantity smoked) all of which are non-linear. However, relaxing the linearity assumption is not easily achievable within the WDW decomposition framework.[3] The restrictiveness of *rank ignorability* (Condition III)

[2] It is worth noting that percentagewise contributions is a global parameter and requires the assumption that under large changes in the covariate there will be no (unaccounted for) general equilibrium effects.

[3] Van Doorslaer et al. (2004) relax this assumption for the health measure - number of doctor visits. However, non-linear estimates need to be translated back to the linear setting for WDW decomposition to work. To do this the authors had to choose which marginal effects to use, which requires untestable assumptions to be made and the resulting decomposition performed in this way is also still only a local approximation. It should also be noted that the decomposition requires the underlying health regression

has been pointed out by Erreygers and Kessels (2013), who label the WDW decomposition approach as a *univariate* decomposition (of a bivariate index) because it only decomposes one part of the covariance. Ignoring the association between the covariates and rank means that for any causal explanation of changes in covariates the income rank is assumed to remain the same even after the change. The assumption of *weighting function ignorability* is similarly restrictive because the weighting function can include a random variable that in turn can be affected by changes in the covariates. For example the concentration index is the absolute concentration index weighted by the inverse of the mean of health. This weighting function will by design be correlated with our covariates, as our covariates are predictors of health. However, w*eighting function ignorability* requires the assumption that the weighting function is unaffected by a change in these covariates.[4] In most health based empirical applications, one or more of these three conditions are violated, and therefore the usefulness of WDW decomposition can be questioned even as a descriptive exercise: the violation of these conditions encumbers both interpretation of the results and its translation into policy.

Contributing to this literature, we derive and empirically illustrate an alternative regression based decomposition method for rank dependent indices that does not necessitate these restrictive conditions. Our object of interest is the effect of a small shift in the distribution of a covariate on the level of inequality: that is, the marginal effect on the bivariate rank dependent index. This is less ambitious than estimating percentagewise contributions as in a WDW decomposition, but this new method is

to be linear in variables, not just in parameters, in order to allow clear interpretation of the percentagewise contributions.

[4] That the weighting function is not decomposed also explains why the percentagewise contributions in the WDW decomposition are the same for all forms of I.

more likely both in theory and in practice to yield our parameter of interest. This alternative approach uses a regression of a recentered influence function (RIF). The RIF is a concept that originates from the robustness literature of statistics. It is a first order linear approximation of the rank dependent index that yields a vector of each individual's (recentered) influence on the index. We decompose the index by regressing the RIF of a chosen form of bivariate rank dependent index on a set of covariates yielding our object of interest, the marginal effects of the covariates on the inequality index.[5] Firpo et al. (2007, 2009) introduced the concept of RIF regression and applied this approach in an income inequality context, estimating and decomposing RIFs for univariate measures such as the variance and the Gini index. Highlighting the potential use of this method, Essama-Nssah and Lambert (2012) calculated the RIFs for a number of different inequality and poverty measures. In a key step forward for the health inequality literature, we show that RIFs can also be calculated for *bivariate* inequality measures. We derive the RIF for a general bivariate rank dependent index and also the RIFs for some familiar versions such as the concentration index and adjustments suggested by Erreygers (2009) and Wagstaff (2005) and directly compare this approach to that of the WDW decomposition approach.

A key advantage of the RIF regression approach in comparison to WDW decomposition is that the method captures the impact of the covariates on the full statistic – remember that the rank dependent index is a weighted covariance of *both* health *and* rank. It therefore addresses Erreygers and Kessels' (2013) criticism of the WDW decomposition method. Another key advantage of RIF regression decomposition is that, unlike WDW decomposition, it is not conditional on a

[5] marginal is interpreted here as the effect of a small change in the covariate(s)

functional form for health linear in variables. In fact, it allows for any possible form of regression function including non-linear methods. Importantly, RIF based decomposition is straightforward to estimate and the results are familiar in their interpretation: Assuming a linear relationship means the RIF is the dependent variable in an OLS regression whose coefficients equal our parameters of interest: the marginal effect of covariates $X$ on the rank dependent index. This is analogous to the interpretation of OLS regression of a random variable. Indeed, a RIF decomposition of the mean (assuming a linear function of the dependent variable) is simply OLS (Firpo et al. 2009). The less stringent conditions imposed by RIF regression of a bivariate rank dependent index make it a preferable descriptive decomposition tool in comparison to WDW decomposition, In addition, it is also well suited to policy evaluation. RIF regression allows the effect of a policy to be evaluated across a wide range of statistics, highlighting its potential in the field of program evaluation.

From a program evaluation viewpoint another advantage of RIF decomposition is that it can distinguish the effect of a covariate on the different forms of a bivariate rank dependent index that exist in the literature. Given the lack of consensus as to which index is preferred, and therefore the value judgements that these indices represent, it is important that any decomposition analysis is able to encompass as broad a view as possible. We show that this is important in our empirical example where the choice of index has bearing on whether education has an association. Our empirical illustration also highlights that the restrictive conditions imposed by the WDW decomposition method may be misleading not just in theory but in practice too. Having illustrated the method we go one step further and attempt to isolate the causal effect of education on socioeconomic related heath inequality using a twin differencing strategy. Our estimates using data from the Swedish Twin Registry suggest there is no causal effect of education on any common choice of bivariate rank based measure of health inequality.

To help the reader understand why RIF regression based decomposition is a useful step forward for the analysis of bivariate rank dependent indices we present in the next section a brief description of rank dependent indices, the WDW decomposition method and the conditions WDW decomposition imposes. In section 3 we introduce the RIF concept and derive the RIF for a general bivariate rank dependent index, before we describe RIF decomposition in section 4. In section 5 we present an empirical example of a RIF decomposition, alongside results obtained by the WDW decomposition approach, highlighting the differences in interpretation and how RIF decomposition is well suited to causal effect analysis. Finally, in section 6 we discuss the relative merits of this new approach, before concluding in section 7.

## 2. Preliminaries

*A rank dependent index*

Let us define $H \in [0, +\infty)$ as a random variable of health with mean denoted as $\mu_H$ and rank each individual by a random variable for socioeconomic status, *Y*. The CDF of *Y*, $F_Y$, corresponds to the fractional rank for each individual and by definition has mean 1/2. A general form for a rank dependent index is then given by:

(1) $$I(H, F_Y) = \omega_I(H) AC(H, F_Y),$$

where $\omega_I(H)$ is a weighting function specific to a particular form of rank dependent index, and the absolute concentration index (AC) is given by twice the covariance between $H$ and $F_Y$:

(2) $$AC(H, F_Y) = 2cov(H, F_Y),$$

We refer to this as the absolute concentration index as it is invariant to the addition or subtraction of an equal amount of health for all individuals in the population.[6] The relative counterpart is the standard concentration index (CI), which is invariant to equi-proportional changes in health. The weighting functions for these common forms of rank dependent index are:

Absolute concentration index:

(3) $$\omega_{AC}(H) = 1$$

Concentration index:

(4) $$\omega_{CI}(H) = \frac{1}{\mu_H}$$

Different choices of weighting function imply different value judgements, in this case a preference for absolute or relative inequality. The choice of index, and therefore the choice of weighting function, is more complex when the health variable of interest has both an upper and lower bound denoted as $b_H$ and as $a_H$ respectively, i.e. $H \in [a_H, b_H]$ (Wagstaff 2005; Erreygers, 2009; Erreygers and van Ourti 2011; Kjellsson and Gerdtham 2013a,b; Kjellsson et al. 2015). For such a variable, health can be represented as both attainments $(H - a_H)$ and shortfalls $(b_H - H)$, and the

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[6] In the literature the absolute concentration index is sometimes called the generalised concentration index, although it is not a generalisation of the concentration index. We label it the absolute concentration index because it is an absolute measure of socioeconomic-related health inequality (it is not affected by the addition or subtraction of a certain amount of health).

choice of which affects the value of the concentration index. One set of indices adjusted for bounded variables assures that the level of inequality is the same irrespective of this representation. The weighting functions for these rank dependent indices are:

Erreygers index:

(5) $$\omega_{EI}(H) = \frac{4}{b_H - a_H}$$

Wagstaff index:

(6) $$\omega_{WI}(H) = \frac{b_H - a_H}{(b_H - \mu_H)(\mu_H - a_H)}$$

The Erreygers Index (EI) is an absolute index adjusted for a bounded variable, whereas the underlying value judgment of Wagstaff Index (WI) is more complex (Wagstaff 2005; Kjellsson and Gerdtham 2013a,b; Allanson and Petrie 2014). We may also define a concentration index that is invariant to either proportional changes in attainment or shortfalls. Following Kjellsson et al. (2015), we denote these as:

Attainment-relative concentration index (ARCI)

(7) $$\omega_{AR}(H) = \frac{1}{(\mu_H - a_H)}$$

Shortfall-relative concentration index[7] (SRCI)

(8) $$\omega_{SR}(H) = \frac{1}{(b_H - \mu_H)}$$

There exists no actual consensus as to which index is preferred, but the literature stresses that any choice of index represents a value judgement (Allanson and Petrie 2014; Kjellsson et al. 2015). Given this lack of consensus it is arguably important that any decomposition analysis is able to encompass as broad a view as possible.

*The standard decomposition*

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[7] An index using this weighting function is equivalent to the applying the (attainment-relative) concentration index representing the health variable in terms of shortfalls, or ill health.

The leading decomposition method applied to I is the WDW decomposition method based on a linear regression of health. Assuming health, represented by $h$, a n x 1 vector, is observed alongside covariates *X*, and that health can be expressed as a linear in variables model in *X,* yields the following regression equation:

$$(9) \qquad h = \alpha + X'\beta + e,$$

where *X* is a *k x n* matrix*,* $\alpha$ is an intercept, $\beta$ is a *k x 1* vector of regression coefficients, and *e* is a *n x 1* vector of error terms. Following Wagstaff et al. (2003), I can then be decomposed by substituting equation (7) into (1), yielding the following formula:

$$(10) \qquad I(H, F_Y) = \omega_I(H) \sum_{k=1}^{K} \beta_k \, 2cov(X_k, F_Y) + \omega_I(H) 2cov(e, F_Y) \,,$$

where $\beta_k$ is the regression coefficient corresponding to the $k^{th}$ regressor from the linear regression equation (7), $2cov(X_k, F_Y)$ is the absolute concentration index of the $k^{th}$ covariate $X_k$ and $2cov(e, F_y)$ is the absolute concentration index of *e*. The first part of the WDW decomposition formula, given by equation (10), expresses the change in $I(H, F_Y)$ predicted by a change in either $cov(X_k, F_Y)$ or $\beta_k$. The first part of equation (10) has also been used to express I as the proportion explained by *X,* "the explained part" (percentage wise contributions), plus the second part of equation (10), as "the unexplained part".

The WDW decomposition method shows that a bivariate rank dependent index is decomposable under certain conditions. However, in empirical practice we are often not willing to accept conditions I-IV or some combination thereof. For example, holding rank constant results in a health focussed decomposition, but as equations (1) and (2) highlight, I captures the weighted *covariance* between health and socioeconomic rank. Therefore, it is the weighted *covariance* between health and socioeconomic rank that should be decomposed, not just health. WDW decomposition also conditions on exogeneity and that the functional form for health is linear in variables. Equation (10) also shows that the weighting function is not decomposed, even though variants of I such as the CI and the WI include a function of health (the mean) in the weighting function. As we discussed in the introduction, these conditions are often restrictive leaving us with results that are hard to interpret – even in a non-causal descriptive setting. In the next section we derive a completely different approach to regression-based decomposition of a bivariate rank dependent index that requires less stringent conditions and is therefore more likely to uncover the parameters of interest.

## 3. The RIF for a general bivariate rank dependent index

The RIF is derived from the influence function (IF) and has the same properties as the IF with the singular exception that the RIF has a different expected value to that of the IF. The IF originates from the robustness literature of statistics, where Hampel (1974) introduced the concept with the original purpose to explore how various statistics are affected (or influenced) by particular observations, hence the name, influence function. A relevant example to the topic of this paper is that of Monti (1991) who assessed the robustness of the Gini using an IF. She found that observations at the extremes of the (income) distribution have much greater influence on the value of the index than other observations towards the middle of the distribution. Firpo et al. (2009) developed the concept of the RIF, RIF regression and hence RIF decomposition. In this section we introduce the concept of the IF and RIF and derive the RIF for a general *bivariate* rank dependent index.

*The influence function and the recentered influence function*

The influence function is a specific form of a directional derivative (or Gâteaux derivative). A directional derivative is used to find the influence of a shift in a distribution, from $F$ towards a new distribution $G_h$, on a statistic. The general term for a statistic, such as the mean, variance or the Gini for example, is a functional, $v(F)$ where $F$ is a probability measure for which $v(F)$ is defined. Let $G_h$, the distribution we are moving towards, be a mixture probability measure of $F$:

(11) $$G_h = (1-\varepsilon)F + \varepsilon\delta_h,$$

where $\delta_h$ is a cumulative distribution function for a probability measure that puts mass 1 at $h$:

$$\delta_h(l) = \begin{cases} 0 & if \;\; l < h \\ 1 & if \; l \geq h \end{cases},$$

where $l$ is a drawing from $H$ and $\varepsilon \in (0,1)$ is a probability, or a weight, representing the relative change in the population through the addition of $\delta_h$. The IF of $v(F)$ finds the *limiting* influence of a small location shift of $F$ towards $G_h$:

(12) $$IF(h; v) = \frac{dv\big((1-\varepsilon)F + \varepsilon\delta_h\big)}{d\varepsilon}\bigg|_{\varepsilon=0} = \; lim_{\varepsilon\to 0}\frac{v(G_h)-v(F)}{\varepsilon},$$

if this limit is defined for every point $h \in \mathbb{R}$, where $\mathbb{R}$ is the real line.[8] Intuitively speaking, the IF captures the (limiting) influence of an *individual* observation on the functional $v(F)$ (Wilcox, 2005). In practice, calculating an IF yields an influence function value for each individual in the sample.

Having defined the IF we can now define the RIF. We can think of the RIF in two ways: one is as a linear approximation of the functional and the other is as a minor transformation of the IF. In terms of the former, the RIF consists of the first two leading terms of a Von Mises linear approximation. It may be helpful to think of a Von Mises linear approximation as the equivalent to a Taylor series expansion of a non-linear function, but instead for a functional. In terms of the latter, the RIF is obtained from the IF by adding back the original functional $v(F)$:

(13) $$RIF(h; v) = v(F) + IF(h; v),$$

An important property of an IF is that its expectation is zero (e.g. an observation equal to the mean has no influence on the mean) (Monti 1991). The minor transformation of the IF into a RIF recenters the IF so that its expectation is equal to the original distributional statistic $v(F)$ (Firpo et al. (2009).[9]

(14) $$E[RIF(h; v)] = \int_{-\infty}^{\infty} RIF(h; v) \cdot dF = v(F)$$

*The RIF for a general (bivariate) rank dependent index*

[8] Another way of checking whether the IF exists is to check if the functional is continuous (has no jumps or spikes) and the differential is bounded.

[9] This characteristic implies that the mean value of the RIF *is* equal to the statistic. This facilitates the interpretation of the intercept term in RIF regression as the intercept is interpreted as the (unweighted) functional value for the reference group. This is important when considering Oaxaca-blinder type decompositions as it is desirable to be able to identify and interpret the intercept and changes in the intercept.

The *RIF* for a univariate rank dependent index, e.g., the Gini, has been derived and applied in previous work (Monti, 1991; Essama-Nssah and Lambert, 2012, Firpo et al. 2007). We expand this to a general bivariate rank dependent index for socioeconomic related health inequality in proposition 1, leaving the proof to **Appendix A**.

***Proposition 1****: Let $I(H,F_Y) = \omega_I(H)AC(H,F_Y)$ be a general rank dependent index, the AC be defined as $AC(H,F_Y) = 2cov(H,F_Y)$ and $F_{H,F_Y}$ be the joint CDF of H and $F_Y$ with corresponding pdf denoted as $f_{H,F_Y}$. Then the RIF for $I(H,F_Y)$ is given by:*

$$RIF(h,F_Y(y);I) = I(H,F_Y) + IF(h;\omega_I) * AC(H,F_Y) + \omega_I(H) * IF(h,F_Y(y);AC),$$

*where $IF(h;\omega_I)$ denotes the IF of the weighting function for I and*
$IF(h,F_Y(y);AC) = -2AC + \mu_H - h + 2hF_Y(y) - 2\int^{y}\int^{+\infty} hf_{H,F_Y}\,dhdF_Y(z)$
*denotes the IF for AC.*

*Proposition 1* shows that for any general rank dependent index the RIF of I is a function of the weighting function, AC, and their respective IFs. As the IF for the AC is given in the proposition, the RIF of any I follows from calculating the IF of the weighting function for the particular I in question and then slotting this into the formula for the RIF given in proposition 1. Corollary 1 presents the RIF for the common forms of I that appear in the health inequality literature, again leaving the proof to **Appendix A**.[10]

**Corollary 1:** *The RIFs for the* AC, EI, CI, ARCI, SRCI *and the* WI *are given by:*

$$RIF(h,F_Y(y);\mathrm{AC}) = \mathrm{AC} + IF(h,F_Y(y);\mathrm{AC})$$
$$RIF(h,F_Y(y);\mathrm{EI}) = EI + \frac{4}{b_H - a_H} IF(h,F_Y(y);\mathrm{AC})$$

[10] The formula for the RIF for the CI is very similar to the RIF for the univariate Gini. Indeed we show in Appendix B that if we derive the RIF for the univariate Gini from the covariance formula, as we have done in the proof of proposition 1, this is the same as presented in Firpo et al. (2007) where the RIF for the Gini has been derived from a formula for the Lorenz curve.

$$RIF(h, F_Y(y); \mathrm{CI}) = CI + \frac{(\mu_H - h)}{\mu_H^2} * \mathrm{AC} + \frac{1}{\mu_H} IF(h, F_Y(y); \mathrm{AC})$$

$$RIF(h, F_Y(y); AR\mathrm{CI}) = ARCI + \frac{(\mu_H - h)}{(\mu_H - a_H)^2} \mathrm{AC} + \frac{1}{\mu_H - a_H} IF(h, F_Y(y); \mathrm{AC})$$

$$RIF(h, F_Y(y); SR\mathrm{CI}) = SRCI + \frac{(-\mu_H + h)}{(b_H - \mu_H)^2} * \mathrm{AC} + \frac{1}{b_H - \mu_H} IF(h, F_Y(y); \mathrm{AC})$$

$$RIF(h, F_Y(y); \mathrm{WI}) = WI + \frac{-(b_H - a_H)[(b_H + a_H - 2\mu_H)(h - \mu_H)]}{((b_H - \mu_H)(\mu_H - a_H))^2} \mathrm{AC} + \frac{b_H - a_H}{(b_H - \mu_H)(\mu_H - a_H)} IF(h, F_Y(y); \mathrm{AC}).$$

The RIF formulas appear more complicated than the formulas for the original index. They are however just a linearisation of the statistic and in practice are straightforward to estimate. The empirical estimator of the population RIF for I is estimated using sample data:

(15)

$$\widehat{RIF}(h, F_Y(y); I) = \hat{I}(H, F_y) + \widehat{IF}(h; \omega_I) * \widehat{AC}(H, F_y) + \widehat{\omega_I}(H)\left[-\widehat{AC}(H, F_y) + \widehat{\mu_H} - h + 2h\widehat{F_Y}(y) - 2\int_0^y \int_0^{+\infty} h\widehat{f_{H,F_Y}}\, dh d\hat{F}_Y(z)\right]$$

To empirically estimate the RIF, the data of *N* observations is first ordered using the ranking variable, *Y*, so that $y_1 \leq y_2 \leq ... \leq y_i \leq ... \leq y_N$. Then estimates of $\hat{I}(H, F_Y)$, $\widehat{AC}(H, F_Y)$, $\widehat{\omega_I}(H)$, and $\widehat{\mu_H}$ are obtained using the formulas in Section 2. The estimate of the $\widehat{F_Y}$ and the absolute concentration curve coordinate, $\int_0^y \int_0^{+\infty} h\widehat{f_{H,F_Y}}\, dh d\hat{F}_Y(z)$ can be calculated as follows:

(16) $$\widehat{F_Y} = \frac{\sum_{j=1}^{i} 1}{N}$$

(17) $$\int_0^y \int_0^{+\infty} h\widehat{f_{H,F_Y}}\, dh d\hat{F}_Y(z) = \frac{\sum_{j=1}^{i} h_j}{N}$$

where the orderings of the $i$ values of $h$ are considered in the numerators.[11] For decomposition analysis (RIF regression) the empirical RIF is then used as a dependent variable in a regression. We now turn to the concept of RIF regression.

# 4. RIF regression decomposition

RIF regression is a method that allows us to decompose a RIF of any functional into a function of the sources of its variation, the covariates, *X*. Our focus is decomposition of I and hence RIF of I regression decomposition. RIF regression is based on one of the key insights in Firpo et al. (2009). They show that the law of iterated expectations can be applied to a RIF allowing us to express I as a conditional expectation of the RIF given *X*. Firpo et al. (2009) further identify two parameters of interest and show how RIF regression, the estimation of the conditional expectation of the RIF, can be used to obtain them. These parameters are the marginal effect of covariate $X_k$ on a functional (evaluated at a particular part of the distribution); and the average partial effect.[12] That is, RIF regression estimates *marginal contributions* (not percentagewise contributions as WDW decomposition results are often presented). In this section we show how these parameters are calculated for I. We first consider the marginal effect of covariates *X* on I, holding the conditional distribution of I given *X* constant. The definition of an IF given by equation (12) states that an IF is the limiting influence of a small shift in the distribution of $H$, $F_H$, towards a new distribution, $G_h$, on the functional. Now the interest lies in the limiting influence on the (R)IF of a small shift

[11]The absolute concentration curve is a mapping of cumulative health and fractional rank (Wagstaff et al. 2003).

[12] We use the term marginal to mean the effect of small changes

in the distribution of the determining covariates.[13] We denote the original distribution of covariates as $F_X$ and the new distribution of $X$ after an infinitesimal shift in the distribution as $G_X$. The effect of an infinitesimal shift in the distribution of $X$ on the functional is obtained by integrating up the conditional expectation of the RIF with respect to a shift in the distribution of covariates, *d($F_X$ – $G_X$ ) whilst holding the conditional distribution given X constant* (Firpo et al. 2009):

(18) $$\lim_{\varepsilon \to 0} \frac{v(G_h) - v(F_H)}{\varepsilon} = \int_{-\infty}^{+\infty} E[RIF(h; v)|X = x] \cdot d(G_X - F_X)(x).$$

This is the marginal effect of a shift in the distribution of $X$ on the functional assuming the conditional distribution given $X$ does not change. Using the RIF for AC as an example (as it is the simplest from of I and therefore helps illustration), we obtain the marginal effects for AC by using regression methods to estimate the following conditional expectation:

(19) $$E\left[-AC + \mu_H - h + 2hF_Y - 2\int_0^y \int_0^{+\infty} hf_{h,F_Y}\, dhdF_Y(z)\,|X = x\right] = E[\lambda(X, \epsilon)|X = x]$$

where $\lambda(X, \epsilon)$ is a function of covariates $X$ and an error term $\epsilon$. The marginal effect with respect to $X_k$ is given by the partial derivative of our regression estimates of (18):

(20) $$\frac{dE[RIF(h, F_Y(y); AC)|X=x]}{dX_k} = \frac{dE[\lambda(X,\epsilon)|X=x]}{dX_k}$$

The potential choice of regression methods we could use to estimate the marginal effects is limitless but the eventual choice will depend on the form one is willing to assume for the function $\lambda(.)$. The second parameter of interest Firpo et al. (2009)

[13] The phrase a small shift in the distribution of the determining covariates can mean both or either a location sift in the distribution or a change in the shape in the distribution.

highlight is the average I partial effect (AIPE). This captures the response of I to small location shifts in a continuous covariate (unconditional on the other covariates), and the response of I to marginal changes in the conditional distribution of a binary covariate *given* the other covariates. The vector of average I partial effects (AIPE), denoted as $\gamma(I)$, is just a vector of average partial derivatives. For the AC, $\gamma(AC)$may be expressed as:

(21) $$\gamma(AC) = \int_{-\infty}^{\infty} \frac{dE[RIF(h,F_Y(y);AC)|X=x]}{dx} \cdot dF(x)$$

Assuming a linear in parameters functional form for the regression model for the RIF of AC, we may rewrite equation (19) as:

*(22)* $$E[RIF(h, F_Y(y); \mathrm{AC})|X = x] = E[X'\psi + \mu|X = x]$$

To be clear, assuming a linear functional form for the AC involves the assumption that the sum of: health, the product of health and fractional rank, and the individuals position on the absolute concentration curve $\int_0^y \int_0^{+\infty} hf_{H,F_Y} \, dhdF_Y(z)$ can be modeled as linear in parameters.[14] As is the case for standard OLS, linearity implies that the marginal effects are constant along the distribution of *X* and the derivative of equation (22) with respect to the $k^{th}$ covariate $X_k$ equals the coefficient $\psi_k$:

(23) $$\frac{dE[RIF(h,F_Y(y);AC)|X=x]}{dX_k} = \frac{dE[X'\psi+\mu|X=x]}{dX_k} = \psi_k$$

which also means that the AIPE equals $\psi_k$:

(24) $$\gamma_k(AC) = \int_{-\infty}^{\infty} \frac{dE[RIF(h,F_Y(y);AC)|X=x]}{dX_k} \cdot dF(x) = \psi_k$$

[14]The absolute concentration curve is a mapping of cumulative health and fractional rank (Wagstaff et al. 2003).

Under the linearity assumption, RIF regression is optimally estimated using OLS. We refer to this estimator as RIF-I-OLS. The procedure first involves estimating the empirical RIF, as we outlined in the final part of section three. This yields empirical estimates of each individual's recentered influence on I. Then, using the empirical RIF as the dependent variable in an OLS regression we yield the AIPEs. Although other regression methods are possible, we illustrate in the next section RIF regression of I using OLS because it is both simple and attractive from an operational perspective. Even though we are restricting our illustration to a linear in parameters functional form, this still allows for a fairly flexible functional form by inclusion of non-linear or higher order transformations of the covariates.

# 5. An empirical illustration of WDW decomposition and *RIF-I-OLS*

In this section we aim to empirically illustrate what the RIF function is, and how WDW decomposition and RIF-I-OLS compare in their interpretation. We also show how RIF-I-OLS is both a well-suited method for determining the causal effect of a covariate on I given a suitable identification strategy, and preferable to WDW decomposition even as a descriptive decomposition method when no causal inference can be made. The illustrative example presented here focuses on the effect of education on income-related health inequality and uses data on monozygotic ("identical") twins. The data is a replica of the data used in Gerdtham et al. (2015). Performing a WDW decomposition, Gerdtham et al. (2015) find education to be significantly associated with a higher level of health and to significantly contribute to the level of inequality, but that this all but disappears when controlling for family and genetic fixed effects common to twin pairs using a twins differencing strategy. To see if these results hold subject to a theoretically less restrictive decomposition method, we extend the analysis by decomposing income-related health inequality using RIF-I-OLS across twins. As in Gerdtham et al. (2015), we first apply a naïve selection on observables identification strategy using OLS and then a twin fixed effects identification strategy.[15] We use the latter primarily to illustrate the difference in the interpretation of the results of the two methods. First, however, we introduce the data and illustrate the empirical RIF of I (focusing on EI in particular).

*Data material*

The data used in this empirical example is a subset from the Swedish Twin Registry consisting of respondents that took part in a telephone interview, including a question on self-assessed health, called Screening Across the Lifespan Twin study (SALT) conducted between the years 1998–2002. The final sample size includes 3,328 twin pairs born between the years 1931 to 1958. The survey data is matched with registers from Statistics Sweden on annual taxable gross income (income from earnings, own business, parental leave benefits, unemployment insurance and sickness benefits) and education level. Register data should have relatively small measurement error, which is very important as measurement errors are magnified when differencing between twins, as we do here in the final part of this section. Income is measured as an average of gross income over ages 35-39 years.[16] The education variable is measured as years of schooling and ranges between 8 and 20 years of schooling.[17] To obtain a health measure appropriate for a rank dependent index, we cardinalise the categorical self-

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15 We refer the reader to Gerdtham et al. (2015) for both an up-to-date discussion on the merits of twin design based studies in revealing the treatment effect of education and for more detailed discussion of the dataset and the twin based fixed effects methodology.

16 This point is discussed further in Gerdtham et al. (2015).

17 Years of schooling is imputed from register data using the highest educational degree obtained in the year 1990 as outlined in the appendix in Gerdtham et al. (2015).

rated health measure using a linear algorithm from Burström et al. (2013) (model 3, supplementary table 8) that transforms self-rated health to a time trade-off (TTO) quality of life utility value (See **table I**).[18] Summary statistics are also presented in **table I**.[19]

**Table I – Variable descriptions, 1st moments and algorithm weights**

| Variable | Description | Mean | Algorithm weight |
|---|---|---|---|
| Health | Health utility from TTO algorithm | 0.916 | |
| Health1 | 1 = Very Good Health (self assessed) | 0.379 | (reference) |
| Health2 | 1 = Good Health (self assessed) | 0.37 | -0.0315 |
| Health3 | 1 = Fair Health (self assessed) | 0.169 | -0.1414 |
| Health4 | 1 = Poor Health (self assessed) | 0.064 | -0.3189 |
| Health5 | 1 = Very Poor Health (self assessed) | 0.018 | -0.4817 |
| Age4044 | 1= aged between 40 and 44 years | 0.083 | 0.0109 |
| Age4554 | 1= aged between 45 and 54 years | 0.427 | 0.0179 |
| Age5564 | 1= aged between 55 and 64 years | 0.449 | 0.0235 |
| Age6567 | 1= aged between 65 and 67 years | 0.042 | 0.0193 |
| Female | 1 = female, 0=male | 0.551 | 0.0058 |
| Schooling | Number of years in education | 11.571 | |
| Income | Gross income (35-39 years)* | 199,145 | |
| Constant | | 1 | 0.9589 |

Notes: * Income is in 2010 prices, SEK.

*Empirical estimation of the RIF*

[18] Health economists often value health states of people by the TTO method where respondents value quality of life in relation to length of live; respondents are asked to imagine living in a given state of health for (typically) ten years, and then to state the shorter amount of time in full health which makes them indifferent between the two options (Drummond et al. 2005). Reference categories are very good self rated health, age 18-24 years and female.

[19] Gerdtham et al. (2015) show that the Swedish Twin Registry data used here is fairly representative of Sweden's population more widely, which otherwise may be a concern for twin based datasets.

The EI for estimated health utility scores ranked by income is 0.03 (**table II**) indicating that higher health utility is more concentrated amongst the rich. The empirical RIF for EI of health utility score ranked by income is calculated as explained in **Section 3** and the result is shown in a scatter plot in **Figure 1**. Each scatter point in **Figure 1** is an individual's recentered influence value plotted against their income rank. If an individual were to be removed from the sample, the influence on the statistic would be minus that individual's RIF value weighted by the inverse of the sample size. The figure shows that those at the extreme ends of the income distribution have greatest influence on the EI. This is similar to the findings in Monti (1991) for the Gini (a univariate rank dependent index): individuals whose income value is at the extremes of the income distribution have greatest influence on the Gini. As the EI is a bivariate index, health, in addition to the ranking variable, affects the degree of influence an individual has on EI. In this particular example those with very poor health (black squares) *and* income levels at the extreme ends of the distribution are the ones with the greatest influence on EI. This result is important to note for researchers and policy makers. Researchers estimating a rank dependent index as a measure of socioeconomic related health inequality need to be sure that the observations with the largest influence on the statistic are not miss-codings. Policy makers may want to focus attention towards those individuals they can help with most influence on inequality – the extreme poor with poor health in this instance.

**Figure 1 – Scatter plot of individual RIF of EI values plotted against individual's fractional income rank**

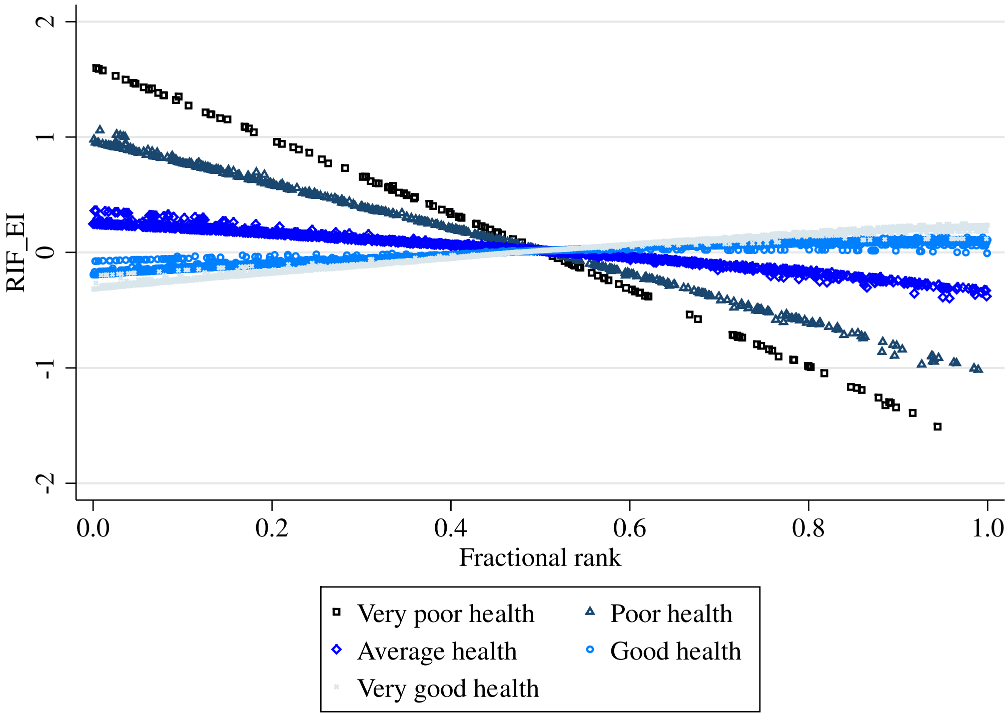


Figure notes: Each scatter point represents an individual's recentered influence on EI plotted against their fractional income rank by health value.

*Interpretation of RIF decomposition and comparison with WDW decomposition*

To provide more information of the characteristics of the individuals that are influencing the statistic, either positively or negatively and to a greater or lesser extent, we may plot the RIF against another variable or turn to the RIF regression method.[20] **Table II** reports descriptive decomposition results of WDW decomposition of E, and RIF-EI-OLS decomposition, in addition to results for RIF-I-OLS for AC, CI, and WI alongside standard mean regression. In a descriptive RIF-I-OLS decomposition the estimated coefficients $\hat{\psi}$ may be interpreted as an association between the covariate and the influence on I, providing valuable information as to which groups of individuals influence the inequality index. If we (naively) assume the error term, $\epsilon$, and covariates, $X$, are independent having controlled for selection on observables then the RIF-I-OLS parameter $\hat{\psi}$ identifies the (causal) marginal effects

[20] One could for example plot a Lowess curve of the RIF and explanatory variable to visually assess a potential relationship and any functional form assumption. We did this for education but there was no real relationship by years of education and therefore do not report the results here.

of a shift in the distribution of *X* on I. Thus, interpretation of $\hat{\psi}$ is similar to the interpretation of the coefficients in standard mean regression (the results of which are shown in column (1) of **Table II**). Indeed, RIF decomposition of the mean of health, assuming a health function linear in parameters, is standard OLS (Firpo et al. 2009).

In the decomposition analysis, years of schooling enters the model as an explanatory variable alongside age, gender and interview year dummies (because each twin was not necessarily interviewed at the same time). The coefficient estimates from RIF-EI-OLS in column (3) of **table II** suggest, if interpreted as the average I partial effect, that if we made a marginal increase in the distribution of the number of years of education in the population, this would have no discernible effect on EI. There also appears to be no age profile regarding EI.

Importantly and in contrast to the contribution estimates of WDW decomposition, the RIF-I-OLS identifies the effect of the covariates *X* on the full statistic. That is, the parameter estimate $\hat{\psi}$ captures *both* the effect of the covariates on AC (which is two times the covariance of health and fractional rank) *and* the effect of the covariates on the weighting function $\omega_I(h)$. The parameter estimates $\hat{\psi}$ presented in Columns 2-6 of **table II** also vary between rank dependent indices depending on the weighting function. Education is found to be significantly associated with the RIF of WI and SRCI, but not with the RIF of AC, EI, or ARCI. That is, more educated individuals have larger influence on the inequality index when measured as WI and SRCI, but not when the AC, EI, or ARCI are considered. This highlights an important issue. The differences in weighting functions, and hence value judgements, among the inequality indices can also lead to important differences in the decomposition results. In this particular example the judgement of whether to consider attainment relative inequality or shortfall relative inequality has bearing on whether education has a potential impact.[21] It is worth noting that it is possible to identify the effect of a particular weighting function by comparing decomposition results for I with decomposition results for AC: AC has a constant weighting function, so any differences in the (standardised) decomposition results compared to those for the AC will be due to the weighting function.

The last two columns in **Table II** report the results from WDW decomposition of E. The interpretation is different to any standard form of mean decomposition and to RIF-I-OLS decomposition: The procedure summarises I as a summation of the contribution of each covariate, where these are the covariate-rank covariances weighted by a linear health-covariate correlation. Following the standard practice, we report the WDW decomposition results as contributions from the covariates in levels and percentagewise contributions of the total index. The results suggest that about 29% of the income-related inequalities in health is due to income-related inequalities in education. The contribution is statistically significant suggesting that eliminating income-related inequalities in education might reduce the EI of health, *assuming no change in the ranking variable and a linear health function*.[22] As the procedure ignores the potential impact of the covariates on the weighting function $\omega_I(h)$, the percentagewise "contributions" are the same no matter the choice of I (only levels vary with the weighting function). That is, WDW decomposition of any inequality measure decomposes an absolute index (such as EI or AC) as it implicitly assumes a

[21] Note that the results divide the indices into two groups. On the one hand EI, AC, ARCI, and on the other hand WI and SRCI. This is a consequence of the high mean of the health utility index.
[22] In the case of EI, the weighting function is a constant and therefore the condition that the weighting function is constant is not binding in this case. However WDW decomposition of CI and WI would also assume that the weighting function is a constant, which it is not.

constant weighting function. Bearing that in mind, the results of the two methods are contradicting: WDW decomposition finds a significant contribution due to education whereas RIF for the EI or AC finds no significant effect of education. The WDW result is conditional on the health utility function being linear and education having no impact on the ranking variable. We can possibly ignore the first former but not the latter; the body of evidence showing a causal effect of education on income is now well accepted (Card 1999). The results of the WDW decomposition appear to be misleading in this example and this is likely to be because of the stringent conditions it imposes.

**Table II – RIF-I-OLS and WDW decomposition estimates of effect of education, age and gender on rank dependent index of health**

| | MZ OLS estimation | | | | | | WDW - OLS | |
|---|---|---|---|---|---|---|---|---|
| | RIF-mean-OLS (1) | RIF-AC-OLS (2) | RIF-EI-OLS (3) | RIF-ARCI-OLS (4) | RIF-SRCI-OLS (5) | RIF-WI-OLS (6) | Contribution (7) | % contribution (8) |
| Years Schooling | 0.005*** | 0.000 | 0.001 | 0.000 | 0.009*** | 0.010*** | 0.008*** | 0.282*** |
| | (0.000) | (0.000) | (0.001) | (0.000) | (0.003) | (0.003) | (0.001) | (0.035) |
| Age | -0.000* | 0.000 | 0.001 | 0.000 | 0.002 | 0.003 | 0.001 | 0.022 |
| | (0.000) | (0.000) | (0.001) | (0.000) | (0.002) | (0.002) | (0.000) | (0.014) |
| Male | 0.009*** | 0.003*** | 0.014*** | 0.004** | 0.051*** | 0.054*** | 0.006*** | 0.212*** |
| | (0.002) | (0.001) | (0.005) | (0.001) | (0.016) | (0.017) | (0.002) | (0.050) |
| Constant | 0.930*** | -0.016 | -0.066 | -0.018 | -0.181 | -0.199 | | |
| | (0.018) | (0.015) | (0.058) | (0.016) | (0.167) | (0.183) | | |
| Statistic | Mean of *h* | AC | EI | ARCI | SRCI | WI | EI | |
| Mean of RIF | 0.916 | 0.007 | 0.030 | 0.008 | 0.089 | 0.098 | 0.030 | |
| Observations | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 |
| WTP FE | NO | NO | NO | NO | NO | NO | NO | NO |

Table notes: Each column represents a separate decomposition. Column 1 is simply OLS of the health variable, because the RIF of the mean is *h* and RIF regression of the mean assuming linearity in parameters is optimally estimated using OLS. All decompositions control for year of interview fixed effects. Robust standard errors in parenthesis for RIF-mean-OLS and bootstrap standard errors in parenthesis for RIF-I-OLS, 999 repetitions with replacement. Bootstrap standard errors are calculated by bootstrapping the whole procedure (Both for RIF and WDW procedures). Testing null of the coefficient/contributions/% contribution: *** p<0.01, ** p<0.05, * p<0.1.

*The causal effect of education on income-related health inequality*

In the previous section, our identification of the average partial effects did not use twin fixed effects but instead (naïvely) relied on selection on observables to satisfy the assumption that the errors are independent of the covariates. To highlight the importance of causal inference in decomposition analysis we now apply a twins differencing strategy that allows unobserved heterogeneity common between twins to be differenced out. That is, we control for factors such as innate ability and early life factors common to both twins, which may invalidate the exogeneity assumption and yield biased parameter estimates. In the case of income-related health inequality the concern is specifically that this unobserved heterogeneity may be correlated with education and the weighted *covariance* of health and income rank.

To formally derive the within twin pair fixed effect decomposition, we denote the RIF values of the *j*th twin pair, $RIF(h, F_Y(y); I)_{1\mathrm{j}}$ and $RIF(h, F_Y(y); I)_{2\mathrm{j}}$. Further, we let $u_j$ denote unobserved factors that vary between twin pairs but not within pairs, such as genetic characteristics and certain early life environmental factors and $e_{1j}$ and $e_{2j}$ denote unobserved factors specific to each twin. Assuming a linear functional form for the *RIF*, we may write these as:

(25) $$RIF(h, F_Y(y); I)_{1\mathrm{j}} = X_{1j}^{\prime}\psi + u_j + e_{1j}$$

(26) $$RIF(h, F_Y(y); I)_{2\mathrm{j}} = X_{2j}^{\prime}\psi + u_j + e_{2j}$$

where $X_{1j}$ is a k x n matrix of covariates for the first twin in the twin pair *j*, $X_{2j}$ is for the second twin in the twin pair and $\psi$ is a k x 1 vector of marginal effects. Taking the difference yields the WTP estimator:

(27) $$RIF(h, F_Y(y); I)_{1j} - RIF(h, F_Y(y); I)_{2j} = \left(X_{1j} - X_{2j}\right)' \psi_{WTP} + e_{1j} - e_{2j}$$

where $\psi_{WTP}$ is the Within-Twin-Pair estimator of the effect of education. The unobserved factors that are common to both twins such as genetic or environmental form captured by $u_j$ will be differenced out of the equation yielding an unbiased OLS-estimator of $\hat{\psi}$ (given that these are the only sources of unobserved heterogeneity)[23]. Applying the WTP approach to the RIF of EI using OLS yields the RIF-EI-FE estimator.

**Table III** reports the WTP results for EI, AC, CI, and WI alongside standard mean fixed effects regression and WDW decomposition. The results for the RIF-I-FE decomposition suggest that if we made a marginal increase in the number of years of education in the population, this would have no discernible effect on any measure of I, nor the mean. It therefore appears that either education has no effect on income-related health inequality, or possibly better put: the variation in education that exists under an extensive egalitarian education system cannot explain the observed income-related health inequality in Sweden.[24]

[23] For a further discussion of potential sources of unobserved heterogeneity see Gerdtham et al. (2015).

[24] Gerdtham et al. (2015) show that the variation between twins is still fairly representative of the variation in education of the wider population and it is therefore not a reduction in variation that is causing this finding.

**Table III – RIF-I-FE and WDW-WTP decomposition estimates of effect of education on rank dependent index of health**

| | MZ WTP estimation | | | | | | WDW - WTP | |
|---|---|---|---|---|---|---|---|---|
| | RIF-mean-OLS | RIF-AC-OLS | RIF-EI-OLS | RIF-ARCI-OLS | RIF-SRCI-OLS | RIF-WI-OLS | Contribution | % contribution |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Years Schooling | 0.001 | -0.000 | -0.002 | -0.000 | -0.004 | -0.004 | 0.001 | 0.05 |
| | (0.001) | (0.001) | (0.002) | (0.001) | (0.009) | (0.007) | (0.002) | (0.058) |
| Constant | 0.930*** | 0.020 | 0.081 | 0.022 | 0.256 | 0.277 | | |
| | (0.034) | (0.029) | (0.117) | (0.032) | (0.452) | (0.401) | | |
| Statistic | mean of *h* | AC | EI | ARCI | SRCI | WI | EI | |
| Mean of RIF | 0.916 | 0.007 | 0.030 | 0.008 | 0.089 | 0.098 | 0.030 | |
| Observations | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 | 6,656 |
| WTP FE | YES | YES | YES | YES | YES | YES | YES | YES |

Table notes: Each column represents a separate decomposition. Column 1 is simply OLS with FE of the health variable. All decompositions control for year of interview fixed effects. Robust standard errors in parenthesis for RIF-mean-FE and bootstrap standard errors in parenthesis for RIF-I-FE, 999 repetitions with replacement. Bootstrap standard errors are calculated by bootstrapping the whole procedure (Both for RIF and WDW procedures). Testing null of the coefficient/contributions/% contribution: *** p<0.01, ** p<0.05, * p<0.1.

# 6. Discussion

Having introduced and illustrated both the WDW decomposition and RIF-I-OLS decomposition, we now compare the two approaches by summarising the underlying conditions and differences in interpretation. For clarity, we start by giving a side-by-side comparison of the conditions of the two approaches.

***WDW conditions:***

I. Health can be modelled as a function linear in variables X and an error term.
II. Exogeneity: The errors from the health regression have zero conditional mean.
III. The determinants of health do not determine rank.
IV. The determinants of health do not determine the weighting function.

***RIF-I-OLS Conditions:***

I. $\text{RIF}(h, F_Y(y); \text{I})$ can be modelled as a linear in parameters function of $X$ and an error term.
II. Exogeneity: The errors from the RIF regression have zero conditional mean.
III. I is differentiable and the differential is bounded.

It is clear from the comparison that RIF-I-OLS imposes fewer conditions than WDW decomposition. The third condition for RIF-I-OLS holds as shown in the proof. The first condition for RIF-I-OLS is not necessary, is testable and any form of regression method can be used including non-linear and semi-nonparametric methods. Still, OLS permits very flexible formulation through inclusion of interactions and higher order polynomials, which is not possible in the WDW framework. Exogeneity is of huge importance for causal inference and is common to both methods – but both methods may be used as descriptive exercises without this assumption. The conditions of

WDW decomposition, as discussed in the introduction are often restrictive, and RIF-I-OLS is therefore more likely to uncover the (causal) parameters of interest and is easier to interpret even as a descriptive exercise.

The different conditions of the decomposition methods also affect the interpretation of the parameters as illustrated in the empirical example. The results in Tables II and III highlight how education's effect on income-related health inequality as estimated by RIF-I-OLS can be shown alongside its effect on mean health in a consistent manner. Similar to mean OLS regression coefficients, the RIF-I-OLS coefficients should be interpreted as how a marginal change in a covariate, e.g. education, influences the inequality index – or if the exogeneity assumption must be relaxed, how the influence on the index varies by education. Under some restrictive assumptions, the WDW decomposition yields an interpretation in terms of both marginal and percentagewise (global) contributions. That is, WDW decomposition has ambitious claims on explaining inequality globally by dividing the index into contributions from the covariates potentially causing inequality. Relaxing the exogeneity assumption reduces these claims making WDW decomposition a descriptive exercise. Even descriptively, however, interpretation is blurred when the conditions of a functional form for health linear in variables, no change in either rank or the weighting function do not hold in practice. In this situation it is not clear what WDW decomposition based descriptive contributions measure. Thus, even if the interest of the decomposition is reduced to descriptively highlighting potentially important covariates that may drive inequality, RIF-I-OLS is preferable to WDW decomposition in most empirical applications because it imposes fewer implausible conditions.

As a result of not imposing weighting function ignorability, RIF-I-OLS, has an additional benefit in that it allows the analyst to assess the impact of covariates on different forms of I. RIF-I-OLS includes the impact of the covariates on the weighting function and therefore the importance of the covariates may differ between particular indices– in the illustrative example based on the full sample (not WTP), education had no association with the AC, EI and ARCI, but a significant association with WI and SRCI. RIF-I-OLS allows researchers to explore how the policy impacts on the level of inequality and how this differs depending on the particular value judgment and hence particular inequality index policy makers sympathise with.

There is a limitation to the RIF approach. As RIF-I-OLS estimates are a first-order approximation of the effect of $X$ on $I(H, F_Y)$, the marginal effects is a local effect estimate of a small change in $X$.[25] That RIF-I-OLS is a local estimate implies that it should only be considered for relatively small changes. It is therefore not reasonable to calculate percentagewise contributions using RIF regression. The usefulness of a local estimate should, however, be placed into the larger context of the overall aims of decomposition analysis. Fleurbaey and Schokkaert (2011) convincingly make the case for a structural model approach to be used for analysing fair and unfair inequalities in health. As a road map for the health inequality literature this may very well be the goal or ideal we should be aiming for. However, we are often unable to specify a complete structural model of health but it may still be of huge interest how a policy change (which is most often a marginal one) impacts both average health and health inequality. RIF of I regression allows this reduced form type of analysis to be made

[25] The marginal effect estimate assumes that the joint conditional distribution of h and $F_Y$ given X remains the same after a small change in the distribution of X (similarly for mean OLS where it is assumed that the conditional distribution is not affected by changes in X, i.e. what economist usually refer to as no general equilibrium effects).

without the need for restrictive assumptions making it a useful addition to a health economist's toolkit.

## 7. Conclusion

In this paper we have illustrated that the conditions required by WDW decomposition do not hold in most common applications and therefore the estimated parameters are not easily interpreted. This makes causal analysis difficult. It is also a concern even when WDW decomposition is interpreted as a descriptive accounting exercise. We have introduced an alternative rank dependent index decomposition method that requires less stringent conditions, and is therefore more likely to yield the parameters of interest. This alternative is based on a RIF regression. We have extended these concepts from a univariate setting to a general bivariate rank dependent index, providing a method that yields the marginal effect of a shift in the distribution of X on the inequality index and has strong links to the program evaluation literature. This new decomposition approach is simple to estimate and the interpretation resembles that of standard conditional mean analysis. Supported by our application using the Swedish Twin Registry, we also claim that even as descriptive exercise the RIF champions the WDW decomposition: the discrepancy between the results of the two methods is plausibly explained by the conditions imposed by WDW decomposition being too restrictive in this empirical example. In an attempt to illustrate RIF-I-OLS's close link to the program evaluation literature, we use linear WTP fixed effects and find little evidence that (twin differences) in education causally impact income-related health inequality in Sweden.

Finally, it is worth noting that the usefulness of the RIF regression goes beyond the estimation of marginal effects using OLS. One can for example use instrumental variables techniques, or control functions as per Rothe (2010), to solve endogeneity issues. RIF regression also allows Oaxaca-blinder type decompositions of between group/time differences to be decomposed for statistics other than the mean (Firpo et al. 2011). We have not discussed these in any great detail but they highlight the potential of our suggested decomposition method and its applicability to a wide range of empirical questions.

**Appendix A – Derivation of the *RIF* for a general rank dependent index (*I*), the IF for the AC and the RIFs for AC, EI, CI, ARCI, SRCI and WI.**

***Proposition 1**: Let $I(H, F_Y) = \omega_I(H)AC(H, F_Y)$ be a general rank dependent index, the AC be defined as $AC(H, F_Y) = 2cov(H, F_Y)$ and $F_{H,F_Y}$ be the joint CDF of H and $F_Y$ with corresponding pdf denoted as $f_{H,F_Y}$. Then the RIF for $I(H, F_Y)$ is given by:*

$$RIF(h, F_Y(y); I) = I(H, F_Y) + IF(h; \omega_I) * AC(H, F_Y) + \omega_I(H) * IF(h, F_Y(y); AC),$$

*where $IF(h; \omega_I)$ denotes the IF of the weighting function for I and*
$IF(h, F_Y(y); AC) = -2AC + \mu_H - h + 2hF_Y(y) - 2\int^y \int^{+\infty} hf_{H,F_Y}\, dhdF_Y(z)$
*denotes the IF for AC.*

Proof: To show $RIF(h, F_Y(y); I) = I(H, F_Y) + IF(h; \omega_I) * AC(H, F_Y) + \omega_I(H) * IF(h, F_Y(y); AC)$, we first apply the definition of the IF given by (12) to I yielding:

A(1) $\quad IF(h, F_Y(y); I) = \frac{d}{d\varepsilon}[\omega_I(H)AC(H, F_Y)]|_{\varepsilon=0}$

Applying the product rule to A(1) yields:

A(2) $\quad IF(h, F_Y(y); I) = IF(h; \omega_I) * AC(H, F_Y) + \omega_I(H) * IF(h, F_Y(y); I)$

As per equation (13), adding $I(H, F_Y)$ to A(2) yields the RIF for $I(H, F_Y)$.

To show that

$IF(h, F_Y(y); AC) = -2AC + \mu_H - h + 2hF_Y(y) - 2\int^y \int^{+\infty} hf_{H,F_Y}\, dhdF_Y(z)$, we first note that the absolute concentration index can be written as:

A(3) $\quad AC(H, F_Y) = 2cov(H, F_Y) = 2\int hF_Y dF_{H,F_Y} - 2\int hdF_{H,\infty} \int F_Y dF_{\infty,F_Y}$

Equation A(3) states that AC is a functional of the joint probability distribution $F_{H,F_Y}$ and the probability distribution $F_Y$. Let $G_{h,F_Y(y)}$ be a bivariate distribution function obtained by an infinitesimal contamination of $F_{H,F_Y}$ in both *h* and $F_Y(y)$, A(4) $G_{h,F_Y(y)} = (1-\varepsilon)F_{H,F_Y} + \varepsilon\delta_{h,F_Y(y)}$

We define $\delta_{h,F_Y(y)}$ as a joint cumulative distribution function for a joint probability measure that gives mass 1 to ($h$, $F_Y(y)$) jointly:

A(5) $\delta_{h,F_Y(y)}(l,r) = \begin{cases} 0 & if\ \ l < h\ or\ \ r < F_Y(y) \\ 1 & if\ \ l \geq h\ and\ r \geq F_Y(y) \end{cases}$

where *l* and *r* are drawings from $H$ and $F_Y$ respectively. Note that if this were a covariance of two random variables, *H* and *Y* say, then we could go directly to deriving the IF. However, the ranking variable, $F_Y$, is not a random variable but a function of a random variable. This function is also affected by the infinitesimal contamination. Let $G_y$ also be a distribution function obtained by an infinitesimal contamination of $F_Y$ in *y*:

A(6) $G_y = (1-\varepsilon)F_Y + \varepsilon\delta_y$

where $\delta_y$ is defined a cumulative distribution function for a probability measure that gives mass 1 to y:

A(7) $\delta_y(p) = \begin{cases} 0 & if\ \ p < y \\ 1 & if\ \ p \geq y \end{cases};$

where *p* is a drawing from *Y*. Then, note that the IF of AC, which we can define as a function of two probability measures, $AC\left(F_{H,F_Y}, F_Y\right)$, is the Gâteaux derivative of AC in the direction of the distribution functions $G_{h,F_y(y)}$ and $G_y$:

A(8) $IF(h, F_Y(y); AC) = \ lim_{\varepsilon\to 0} \frac{AC(G_{h,F_y(y)}, G_y) - AC\left(F_{H,F_Y}, F_Y\right)}{\varepsilon}$

if this limit is defined for every point $h \in \mathbb{R}$ and $y \in \mathbb{R}$, where $\mathbb{R}$ denotes the real line. By substitution for $AC(G_{h,F_y}, G_y)$ and $AC(F_{H,F_Y}, F_Y)$ in the formula for the $IF$ we have:

A(9) $\quad IF(h, F_Y(y); AC) = \ lim_{\varepsilon\to 0} \frac{2\int hG_y dG_{h,F_Y(y)} - \int hdG_{h,\infty}\int G_y dG_{\infty,F_Y(y)} - cov(H,F_Y)}{\varepsilon}$

Substituting A(4) and A(6) into A(9) yields:

A(10) $IF(h, F_Y(y); AC) = \ lim_{\varepsilon\to 0} 2\big[\int h((1-\varepsilon)F_Y + \varepsilon\delta_y)d((1-\varepsilon)F_{H,F_Y} + \varepsilon\delta_{h,F_Y(y)}) - \int hd((1-\varepsilon)F_{H,\infty} + \varepsilon\delta_{h,\infty}) \int((1-\varepsilon)F_Y + \varepsilon\delta_y)d((1-\varepsilon)F_{\infty,F_Y} + \varepsilon\delta_{\infty,F_Y(y)}) - cov(H, F_Y)\big]/\varepsilon$

Which after taking the limit and re-arranging yields:

A(11) $IF(h, F_Y(y); AC) = 2\Big[-2\big(\int hF_Y dF_{H,F_Y} - \int hdF_{H,\infty}\int F_Y dF_{\infty,F_Y}\big) + \int hdF_{H,\infty}\int F_Y dF_{\infty,F_Y} - \int hd\delta_{h,\infty}\int F_Y dF_{\infty,F_Y} + \int hF_Y d\delta_{h,F_Y(y)} - \int hdF_{H,\infty}\int F_Y d\delta_{\infty,F_Y(y)} + \int h\delta_y dF_{H,F_Y} - \int hdF_{H,\infty}\int \delta_y dF_{\infty,F_y}\Big]$

Term by term A(11) is equal to:

A(12) $-2\int hF_Y dF_{H,F_Y} + 2\int hdF_{H,\infty}\int F_Y dF_{\infty,F_Y} = -AC$,

A(13) $\int hdF_{H,\infty}\int F_Y dF_{\infty,F_Y} = \frac{\mu_H}{2}$,

A(14) $-\int hd\delta_{h,\infty}\int F_Y dF_{\infty,F_Y} = -\frac{h}{2}$,

A(15) $\int hF_Y d\delta_{h,F_Y(y)} = hF_y(y)$, A(16) $-\int hdF_{H,\infty}\int F_Y d\delta_{\infty,F_Y(y)} = -\mu_H F_y(y),$

A(17) $-\int hdF_{H,\infty}\int \delta_y dF_{\infty,F_Y} = -\mu_H \int^{+\infty}\int^{+\infty}\delta_y f_{\infty,F_Y}\, dhdF_Y(y) =$

$-\mu_H \int_y^{+\infty}\int^{+\infty} 1 f_{\infty,F_Y}\, dhdF_Y(z) = -\mu_H \int^{+\infty}\int^{+\infty} 1 f_{\infty,F_Y}\, dhdF_Y(y) +$

$\mu_H \int^{y}\int^{+\infty} 1 f_{\infty,F_Y}\, dhdF_Y(y) = -\mu_H + \mu_H F_Y(y),$

A(18) $\int h\delta_y dF_{H,F_Y} = \int^{+\infty}\int^{+\infty} h\delta_y f_{H,F_Y}\, dhdF_Y(y) = \int_y^{+\infty}\int^{+\infty} hf_{H,F_Y}\, dhdF_Y(z) =$

$\int^{+\infty}\int^{+\infty} hf_{H,F_Y}\, dhdF_Y(y) - \int^{y}\int^{+\infty} hf_{H,F_Y}\, dhdF_Y(z) =$

$\mu_H - \int^{y}\int^{+\infty} hf_{H,F_Y}\, dhdF_Y(z).$

Together these yield:

A(19) $IF(h, F_Y(y); AC) = -2AC + \mu_H - h + 2hF_Y(y) -$

$2\int^{y}\int^{+\infty} hf_{H,F_Y}\, dhdF_Y(z),$

This completes the proof.

**Corollary 1:** *The RIFs for the* AC, EI, CI, ARCI, SRCI *and the* WI *are given by:*

$$RIF(h, F_Y(y); \text{AC}) = \text{AC} + IF(h, F_Y(y); \text{AC})$$
$$RIF(h, F_Y(y); \text{EI}) = EI + \frac{4}{b_H - a_H} IF(h, F_Y(y); \text{AC})$$
$$RIF(h, F_Y(y); \text{CI}) = CI + \frac{(\mu_H - h)}{\mu_H^2} * \text{AC} + \frac{1}{\mu_H} IF(h, F_Y(y); \text{AC})$$
$$RIF(h, F_Y(y); AR\text{CI}) = ARCI + \frac{(\mu_H - h)}{(\mu_H - a_H)^2}\text{AC} + \frac{1}{\mu_H - a_H} IF(h, F_Y(y); \text{AC})$$
$$RIF(h, F_Y(y); SR\text{CI}) = SRCI + \frac{(-\mu_H + h)}{(b_H - \mu_H)^2} * \text{AC} + \frac{1}{b_H - \mu_H} IF(h, F_Y(y); \text{AC})$$
$$RIF(h, F_Y(y); \text{WI}) = WI + \frac{-(b_H - a_H)[(b_H + a_H - 2\mu_H)(h - \mu_H)]}{((b_H - \mu_H)(\mu_H - a_H))^2}\text{AC} + \frac{b_H - a_H}{(b_H - \mu_H)(\mu_H - a_H)} IF(h, F_Y(y); \text{AC})$$

*Proof:* To show the result of *Corollary 1*, *Proposition 1* states the IFs for the weighting functions for the AC, EI, CI, ARCI, SRCI and WI need to be calculated. The weighting functions for both the AC and EI are constants, therefore the IFs for their weighting functions will be zero and we can plug in straight away the functions we need into the formula for the RIF of I. The IF for the CI weighting function is:

A(20) $IF(h; \omega_{CI}) = \frac{d}{d\varepsilon} \frac{1}{[(1-\varepsilon)\int hdF_H + \varepsilon h]} - \frac{1}{-\int hdF_H} |_{\varepsilon=0}.$

Differentiating A(20), taking the limit with respect to $\varepsilon$ $\varepsilon$ and noting that $\int hdF_H = \mu_H$ gives us:

A(21) $IF(h; \omega_{CI}) = \frac{\int hdF_H - h}{\int hdF_H \int hdF_H} = \frac{(\mu_H - h)}{\mu_H^2}$

Substituting A(21) into the formula for the RIF for I yields the RIF for CI:

A(22) $RIF(h, F_Y(y); \text{CI}) = CI + \frac{(\mu_H - h)}{\mu_H^2} * \text{AC} + \frac{1}{\mu_H} IF(h, F_Y(y); \text{AC})$

The IF for the ARCI weighting function is:

A(23) $IF(h; \omega_{ARCI}) = \frac{d}{d\varepsilon} \frac{1}{[(1-\varepsilon)\int (h-a_H)dF_H + \varepsilon(h-a_H)]} - \frac{1}{-\int (h-a_H)dF_H} |_{\varepsilon=0}$

Differentiating A(23) and taking the limit with respect to $\varepsilon$ gives us:

A(24) $IF(h; \omega_{ARCI}) = \frac{\int (h-a_H)dF_H - (h-a_H)}{\int (h-a_H)dF_H \int (h-a_H)dF_H} = \frac{(\mu_H - h)}{(\mu_H - a_H)^2}$

Substituting A(24) into the formula for the RIF for I yields the RIF for ARCI:

A(25) $RIF(h, F_Y(y); ARCI) = ARCI + \frac{(\mu_H - h)}{(\mu_H - a_H)^2} \text{AC} + \frac{1}{\mu_H - a_H} IF(h, F_Y(y); \text{AC})$

Following a similar argument as for ARCI, the IF for the SRCI is given by:

A(26) $IF(h;\omega_{SRCI}) = \frac{\int (b_H - h)dF_H - (b_H - h)}{\int (b_H - h)dF_H \int (b_H - h)dF_H} = \frac{(-\mu_H + h)}{(b_H - \mu_H)^2}$

Substituting A(26) into the formula for the RIF for I yields the RIF for SRCI:

A(27) $RIF(h, F_Y(y); SRCI) = SRCI + \frac{(-\mu_H + h)}{(b_H - \mu_H)^2} * \text{AC} + \frac{1}{b_H - \mu_H} IF(h, F_Y(y); \text{AC}).$

The IF for the WI weighting function is given by:

A(28) $IF(h;\omega_{WI}) =$

$$\frac{d}{d\varepsilon}\left[\frac{b_H - a_H}{(b_H - \int hd((1-\varepsilon)F_H + \varepsilon\delta))(\int hd((1-\varepsilon)F_H + \varepsilon\delta) - a_H)} - \frac{b_H - a_H}{(b_H - \mu_H)(\mu_H - a_H)}\right]|_{\varepsilon=0}$$

Expanding gives us:

A(29) $IF(h;\omega_{WI}) =$

$$\frac{d}{d\varepsilon}\left[\frac{b_H - a_H}{(b_H(1-\varepsilon)\mu_H + b_H\varepsilon h - b_H a_H - (1-\varepsilon)^2\mu_H^2 - (1-\varepsilon)\varepsilon h\mu_H + (1-\varepsilon)a_H\mu_H - (1-\varepsilon)\varepsilon h\mu_H - \varepsilon^2 h^2 + \varepsilon a_H h)} - \frac{b_H - a_H}{(b_H - \mu_H)(\mu_H - a_H)}\right]|_{\varepsilon=0}$$

Differentiating with respect to $\varepsilon$ and taking the limit w.r.t $\varepsilon$ yields:

A(30) $IF(h;\omega_{WI}) = \frac{-(b_H - a_H)[(b_H + a_H - 2\mu_H)(h - \mu_H)]}{((b_H - \mu_H)(\mu_H - a_H))^2}$

Substituting A(30) into the formula for the *RIF* for *I* yields the *RIF* for WI:

A(31) $RIF(h, F_Y(y); \text{WI}) =$

$$WI + \frac{-(b_H - a_H)[(b_H + a_H - 2\mu_H)(h - \mu_H)]}{((b_H - \mu_H)(\mu_H - a_H))^2}\text{AC} + \frac{b_H - a_H}{(b_H - \mu_H)(\mu_H - a_H)} IF(h, F_Y(y); \text{AC}),$$

This completes the proof.

**Appendix B - Linking proposition 1 and corollary 1 to Essama-Nssah and Lambert (2012) and Firpo et al. (2007)**

The (*R*)IF for a univariate rank dependent index, the Gini index (a measure of the concentration of one variable), has been derived in Essama-Nssah and Lambert (2012) and Monti (1991) and reported in Firpo et al. (2007). If a univariate setting is assumed, where individuals are ranked by health instead of income (i.e. $F_H$ is substituted for $F_Y$), our derivation of the RIF of the concentration index coincides with previous derivations of the (R)IF of the Gini. As Essama-Nssah and Lambert (2012) show that their result is the same as shown in Firpo et al. (2007), we only need to link our results to the latter.

The IF for the AC is given by proposition 1:

(B1) $IF(h, F_Y(y); AC) = -2AC + \mu_H - h + 2hF_Y(y) - 2\int^{y}\int^{+\infty} hf_{H,F_Y}\, dhdF_Y(z)$

If in deriving (B1) we had used $F_H$ as the ranking variable instead of $F_Y$ we would have got the IF for the absolute Gini index (AG):

(B2) $IF(h, F_Y(y); AG) =$

$$-2AG + \mu_H - h + 2hF_H(h) - 2\int^{h}\int^{+\infty} hf_{H,F_H}\, dhdF_H(z).$$

Similarly to how the RIF of CI was derived in **Appendix A,** we find the RIF of the Gini index (GI) equals:

(B3) $RIF(h; \mathrm{GI}) = -\frac{h-2\mu_H}{\mu_H}\mathrm{GI} + \frac{1}{\mu_H} IF(h; \mathrm{AG}).$

Rearranging yields

(B4) $RIF(h;\mathrm{GI}) =$

$$-\frac{h-2\mu_H}{\mu_H}\mathrm{GI} + \frac{1}{\mu_H}\left[-2\mathrm{AG} + \mu_H - h + 2hF_H - 2\int^{h}\int^{+\infty} hf_{H,F_z}\, dhdF_z\right] =$$

$$-\frac{h-2\mu_H}{\mu_H}\mathrm{GI} + -2\mathrm{GI} + 1 - \frac{h}{\mu_H} + \frac{2}{\mu_H}hF_H - \frac{2}{\mu_H}\int^{h}\int^{+\infty} hf_{H,F_z}\, dhdF_z$$

Note: Firpo et al. (2007) denote the Lorenz ordinate as:

(B5) $\frac{1}{\mu_H}\int^{h}\int^{+\infty} hf_{H,F_z}\, dhdF_z = \frac{1}{\mu_H}q(\alpha, F_H)$

Where $\alpha$ is the fractional rank. Firpo et al. (2007) also denote the area under the Lorenz curve as:

(B6) $R(F_H) = \int_0^1 q(\alpha, F_H)\, d\alpha$

The Gini index equals the area between the line of equality and the Lorenz curve:

(B7) $\mathrm{GI} = 1 - 2R(F_H)$

Substituting for (B6)-(B8) into (B5) yields:

(B8) $RIF(h;\mathrm{GI}) = -\frac{(h-2\mu_H)(1-2R(F_H))}{\mu_H} - 2 + 4R(F_H) + 1 - \frac{h}{\mu_H} + \frac{2}{\mu_H}(hF_H -$

$q(\alpha, F_H))]$

Which after re-arranging yields the expression presented in Firpo et al. (2007):

(B9) $RIF(h;\mathrm{GI}) = 1 + \frac{2hR(F_H)}{\mu_H} - \frac{2}{\mu_H}(h(1 - F_H) + q(\alpha, F_H))]$

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