A new approach to quantify power-law cross-correlation and its application to crude oil markets

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Abstract: We proposed a new method: Detrended Moving-average Cross-correlation Analysis (DMCA) to detect the power-law cross-correlation between two correlated non-stationary time series by combining Detrended Cross-Correlation Analysis (DCCA) and Detrended Moving Average (DMA). In order to compare the performance of DMCA and DCCA in the detection of cross-correlation, and to estimate the influence of periodic trend, we generate two cross-correlated time series $x(i)$ and $y(i)$ by a periodic two-component fractionally autoregressive integrated moving average (ARFIMA) process. Then we apply both methods to quantify the cross-correlations of the generated series, whose theoretical values are already known to us. By comparing the results we obtained, we find that the performance of this new approach is comparable to DCCA with less calculating amounts; our method can also reduce the impact of trends; furthermore, DMCA (for background and forward moving average case) outperforms DCCA in more accurate estimation when the analyzed times series are short in length. To provide an example, we also apply this new method to the time series of the real-world data from Brent and WTI crude oil spot markets, to investigate the complex cross-market correlation between these commodity markets. In all, our method is another practical choice to detect the cross-correlation between two short period non-stationary time series, and has potential application to real world problems.

Key Words: DMCA; DCCA; power-law cross-correlation; crude oil spot markets

1. Introduction

Empirical evidence supports the existence of fractals or multifractals in commodity or financial markets [1-6]. The existence of multifractality in the markets implies that the scaling geometry of the market patterns may be better described by a spectrum of scaling exponents. Many simultaneously recorded time series in various real world commodity or financial markets are found to exhibit spatial or temporal cross-correlations [3, 7-17]. Therefore, in recent years, many researchers attempted to quantify cross-correlations between real systems. Podobnik *et al.* analyze 1340 members of the NYSE Composite by using random matrix theory, and found the power-law magnitude cross-correlations as a collective mode [16]. Podobnik *et al.* also find long-range cross-correlations in absolute values of returns between Dow Jones and S&P500 by cross-correlation function [18]. Plerou *et al.* apply random matrix theory to analyze the cross-correlation matrix of price changes of the largest 1000 US stocks [11]. Richman and Moorman measure the similarity of two distinct physiological time series by means of cross approximation entropy and cross sample entropy [19]. Pincus and Kalman apply approximate entropy to analyze the financial data [8]. However, many previous methods which deal with cross-correlations are mainly based on the dubious assumption that both of the time series are stationary, but now various empirical studies in current literature show that many real world time series are non-stationary [6, 20, 21], which may lead to a spurious detection of auto- or cross-correlation.

Many scientists thereby incorporate the temporal and/or spatial factors and apply Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to investigate the cross-correlation between two non-stationary signals. For example, Jun *et al.* propose a detrended cross-correlation approach to quantify the correlations between positive and negative fluctuations in a single time series [22]. Based on the previous studies, Podobnik and Stanley propose Detrended Cross-Correlation Analysis (DCCA) to investigate power-law cross-correlations between two simultaneously recorded time series in the presence of nonstationarity [23]. Podobnik *et al.* then apply their new method to uncover long-range power-law cross-correlations in the random part of the underlying stochastic process [24] and the cross-correlation between volume change and price change [25]. He and Chen apply DCCA to investigate nonlinear dependency between characteristic commodity market

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quantities (variables), especially the relationships between trading volume and market price in many important agricultural commodity futures markets [3]. To unveil multifractal features of two cross-correlated nonstationary signals, Zhou proposes Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to combine Multifractal Detrended Fluctuation Analysis (MF-DFA) and DCCA approaches [26], while the latter can be also regarded as a generalization of Detrended Fluctuation Analysis (DFA). The advantage of DFA lies that it can uncover the scaling features by filtering out polynomial trends. Meanwhile, Detrended Moving Average (DMA), which can filter the trends by the moving average, is based on the Moving Averaging (MA) or mobile average technique proposed by Vandewalle and Ausloos [27, 28], and then further developed to quantify long-range auto-correlation in non-stationary fluctuation signals [29-31]. It is well known that the trends may have an influence on the detection of power-law correlation and lead to a crossover [24, 32]. Polynomial fitting can not entirely eliminate the effect of trends in time series. But DMA method can efface the trends by moving average, whose filter is continuous and able to adjust the fitting curve dynamically, by which the estimation accuracy may be increased. In this paper, therefore, we combine the DCCA and DMA, and then propose a new method, Detrended Moving-average Cross-correlation Analysis (DMCA), trying to incorporate the advantages of both DCCA and DMA.

2. Detrended Moving-average Cross-correlation Analysis (DMCA)

Suppose that there are two simultaneously recorded time series $x(i)$ and $y(i)$, $i = 1, 2, ..., L$, where L is the length of each time series. We calculate two integrated signals:

$$X(i) = \sum_{k=1}^{i} x(i), Y(i) = \sum_{k=1}^{i} y(i), i = 1, 2, ..., L. \tag{1}$$

For a window of size n, the moving average is given by:

$$\bar{X}_n(i) = \frac{1}{n} \sum_{k=[-(n-1)\theta]}^{[(n-1)(1-\theta)]} X(i-k), \bar{Y}_n(i) = \frac{1}{n} \sum_{k=[-(n-1)\theta]}^{[(n-1)(1-\theta)]} Y(i-k) \tag{2}$$

where θ is a position parameter ranging from 0 to 1. In this formulation of moving average (Eq.(2)), three special cases are considered, namely $\theta = 0$ (backward moving average, in which the filter is obtained by the past data points), $\theta = 0.5$ (centered moving average, in which the filter is obtained by the present data points) and $\theta = 1$ (forward moving average, in which the filter is obtained by the future data points) [31, 33]. Then the detrended covariance can be defined as:

$$F_{DMCA}^2(n) = \frac{1}{L-n+1} \sum_{i=n}^{L} (X(i) - \bar{X}_n(i))(Y(i) - \bar{Y}_n(i)) \tag{3}$$

If the power-law cross-correlation exists, the following scaling relationship can be observed:

$$F_{DMCA}(n) \propto n^{H_{DMCA}} \tag{4}$$

The Exponent H_{DMCA} can describe the power-law cross-correlation relationship between the two related time series. If $x(i)$ is identical to $y(i)$, this method degenerates into DMA.

As a comparison, let us briefly introduce the algorithm of DCCA [23]:

First, two integrated signals $X(i)$ and $Y(i)$ are calculated by Eq.(1). Then we divide both time series into $L - n$ overlapping boxes, each containing $n + 1$ values. For each box that starts at i and ends at $i + n$, the local trends are estimated by linear least-squares fits $\tilde{X}_i(k)$ and $\tilde{Y}_i(k)$. Then, the covariance of residuals can be given by

$$f_{DCCA}^2(n,i) = \frac{1}{n+1} \sum_{k=i}^{i+n} (X(k) - \tilde{X}_i(k))(Y(k) - \tilde{Y}_i(k)) \tag{5}$$

Then calculate the detrended covariance by summing over all overlapping $N - n$ boxes of size n,

$$F_{DCCA}^2(n,i) = \frac{1}{N-n}\sum_{i=1}^{N-n} f_{DCCA}^2(n,i) \tag{6}$$

If power-law cross-correlations do exist, the square root of the detrended covariance that grows with time window n will satisfy

$$F_{DCCA}(n,i) \propto n^{H_{DCCA}} \tag{7}$$

The power-law correlation can be quantified by the exponent H_{DCCA}.

3. Comparative Studies

In order to compare the performance of DMCA and DCCA in the detection of cross-correlation, and to estimate the influence of periodic trend, we generate two cross-correlated time series $x(i)$ and $y(i)$ by a periodic two-component fractionally autoregressive integrated moving average (ARFIMA) process [18, 24, 34-37]:

$$x_i = \left[W\sum_{i=1}^{+\infty} a_i(\rho_1)x_{i-j} + (1-W)\sum_{i=1}^{+\infty} a_i(\rho_2)y_{i-j}\right] + A_1 \sin\left(\frac{2\pi}{T_1}i\right) + \eta_i \tag{8}$$

$$y_i = \left[(1-W)\sum_{i=1}^{+\infty} a_i(\rho_1)x_{i-j} + W\sum_{i=1}^{+\infty} a_i(\rho_2)y_{i-j}\right] + A_2 \sin\left(\frac{2\pi}{T_2}i\right) + u_i \tag{9}$$

$$a_i(\rho) = \rho\frac{\Gamma(i-\rho)}{\Gamma(1-\rho)\Gamma(1+j)} \tag{10}$$

where η and u represent Gaussian noise, $\Gamma(x)$ denotes Gamma function, ρ is a parameter ranging from 0 to 0.5, $T_k(k=1,2)$ is a sinusoidal period, $A_k(k=1,2)$ is a sinusoidal amplitude, and W is a weight ranging from 0.5 to 1 which controls the strength of power-law cross-correlation between $x(i)$ and $y(i)$. If $W=1$ and $A_1 = A_2 = 0$, the two correlated time series (Eqs. (8) and (9)) decouple to two separate ARFIMA series, whose Hurst Exponent $H = 0.5 + \rho$ [34]. Podobnik and Stanley found both numerically [23] and analytically [24] that the cross-correlation exponent is equal to the average of individual Hurst exponents for two time series generated by separate ARFIMA process sharing with same random noise η, namely, $H_{xy} = (H_{xx} + H_{yy})/2$.

Our intention is to estimate the cross-correlation exponents of the generated time series whose theoretical values are already known, using both DMCA (under different parameters $\theta = 0, 1, 0.5$) and DCCA. By doing so, we can compare the actual performance of these two methods. As the real world data are often of different sizes, we generate four groups of time series by separate ARFIMA processes with same noise. The groups comprise of 2000, 4000, 10000, and 20000 data points (lengths); for each length of time series we generated 10 different pairs.

Fig. 1 shows the results of DMCA vs. DCCA for all those different lengths when $\rho_1 = 0.1$ and $\rho_2 = 0.4$, from which one can find that power-law cross-correlations can be clearly identified by both methods. We also calculated the averages of cross-correlation exponents (see Table 1), and the means of standard errors (see the results in parentheses) for each group. Compared with theoretical values of cross-correlation exponents between the ARFIMA series, it is interesting to note that the cross-correlation exponents are somewhat underestimated by both methods for most cases, and that the unfavorable deviation tends to increase for the results estimated by the DMCA if the theoretical exponent is large (e.g. when the theoretical value is 0.85); meanwhile, compared with results estimated by DCCA, for most case the exponents estimated by DMCA are slightly smaller. For example, $\bar{H}_{DCCA} = 0.7471$, $\bar{H}_{DMCA} = 0.7232$ (for $\theta = 0$), 0.7382 (for $\theta = 0.5$) and 0.7241 (for $\theta = 1$) when $\rho_1 = 0.2$, $\rho_2 = 0.3$ and $L = 20000$.

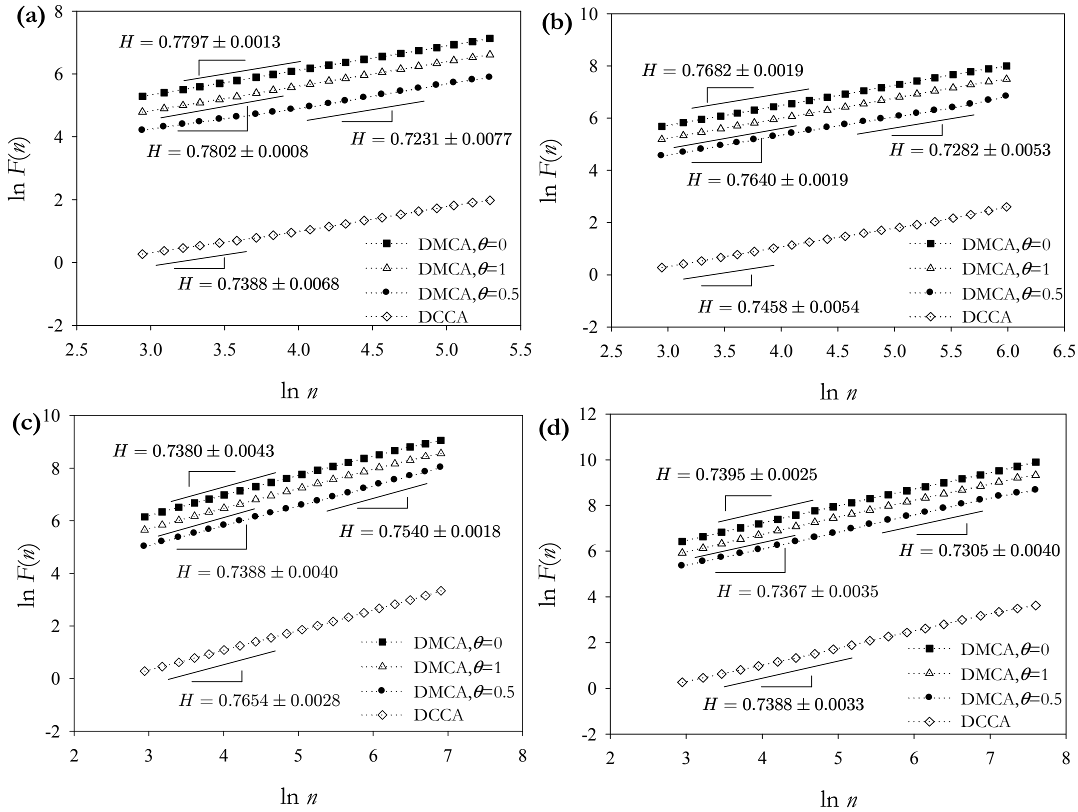

Fig. 1 The log-log plot for the square root of the detrended covariance *vs.* time scale n. Each time series is generated by separate ARFIMA process (namely $W = 0$) with same Gaussian noise, whose Hurst Exponent is equal to $\rho + 0.5$ and the cross-correlation exponent between each series $H_{xy} = (H_{xx} + H_{yy})/2$. In order to make clearer contrast among the different curves, some constants are subtracted from the original results. It is unnecessary to plot the results from all ten pairs; therefore we choose the results from one pair out of the ten when $\rho_1 = 0.1$ and $\rho_2 = 0.4$. For each panel, the lengths of data points are (a) $L = 2000$; (b) $L = 4000$; (c) $L = 10000$; (d) $L = 20000$, from which one can find that power-law cross-correlations are identified by both methods.

Table 1 The results of cross-correlation exponents estimated by DMCA and DCCA for time series generated by separate ARFIMA processes with same noise

		$\rho_1 = 0.1, \rho_2 = 0.4$	$\rho_1 = 0.2, \rho_2 = 0.3$	$\rho_1 = 0.1, \rho_2 = 0.2$	$\rho_1 = 0.3, \rho_2 = 0.4$
Theoretical Value		0.75	0.75	0.65	0.85
$L = 2000$	DCCA	0.7578(0.0058)	0.7396(0.0067)	0.6566(0.0048)	0.8039(0.0082)
	DMCA($\theta = 0$)	0.7519(0.0041)	0.7299(0.0047)	0.6331(0.0057)	0.7768(0.0046)
	DMCA($\theta = 0.5$)	0.7368(0.0073)	0.7188(0.0082)	0.6369(0.0063)	0.7794(0.0104)
	DMCA($\theta = 1$)	0.7559(0.0038)	0.7260(0.0045)	0.6404(0.0041)	0.7631(0.0064)
$L = 4000$	DCCA	0.7106(0.0053)	0.7149(0.0052)	0.6373(0.0051)	0.8309(0.0067)
	DMCA($\theta = 0$)	0.7206(0.0047)	0.7163(0.0048)	0.6480(0.0051)	0.8205(0.0043)
	DMCA($\theta = 0.5$)	0.6936(0.0060)	0.6979(0.0060)	0.6217(0.0059)	0.8148(0.0070)
	DMCA($\theta = 1$)	0.7188(0.0043)	0.7147(0.0046)	0.6508(0.0045)	0.8196(0.0043)
$L = 10000$	DCCA	0.7396(0.0037)	0.7458(0.0041)	0.6312(0.0047)	0.8447(0.0038)
	DMCA($\theta = 0$)	0.7190(0.0052)	0.7268(0.0047)	0.6222(0.0049)	0.7981(0.0061)
	DMCA($\theta = 0.5$)	0.7279(0.0041)	0.7339(0.0044)	0.6191(0.0054)	0.8324(0.0043)
	DMCA($\theta = 1$)	0.7191(0.0055)	0.7283(0.0041)	0.6198(0.0057)	0.7976(0.0064)
$L = 20000$	DCCA	0.7387(0.0035)	0.7471(0.0038)	0.6458(0.0042)	0.8419(0.0037)
	DMCA($\theta = 0$)	0.7264(0.0050)	0.7232(0.0052)	0.6222(0.0067)	0.8045(0.0057)
	DMCA($\theta = 0.5$)	0.7294(0.0040)	0.7382(0.0042)	0.6369(0.0051)	0.8320(0.0040)
	DMCA($\theta = 1$)	0.7292(0.0049)	0.7241(0.0052)	0.6215(0.0070)	0.8067(0.0055)

Note: For each length of time series, we generate 10 pairs. The results show the average of 10 cross-correlation exponents of those generated pairs; in parentheses there are the means of standard errors.

Then, what are the advantages of this new approach compared with DCCA?

In the real world, influenced by economic, seasonal, or some other factors, actual time series usually possess trends and are non-stationary; therefore, detrended approaches are usually applied to analyze those series. Polynomial fitting is thereby widely applied, but this approach sometimes can not completely efface the influence of trends, which may lead to crossovers [24, 32]. In our DMCA method, moving average is used to estimate trends. If the time series are long- or short-range correlated, the moving average for large (or small) window size n may contain long-term (or short-term) trend.

To further discuss this problem, we generated the time series by a periodic two-component ARFIMA process (see Eq.(8)). We chose $W = 0.5, A_1 = A_2 = 0.3, \rho_1 = \rho_2 = 0.4$ and $T_1 = T_2 = 1000$ or 500. As a comparison, we also generated a group of time series with no trend, namely $A_1 = A_2 = 0$. Each group consists of 2000, 4000, 10000, or 20000 data points; and for each length of time series we generated 10 pairs. The weight W controls the strength of power-law cross-correlations between two correlated time series; that is, when $W \neq 0$, each variable depends not only on its own past, but also on the historical values of the other variables. When $W = 0.5$, each time series is equally dependent on the past of the other; and the two time series are almost similar. The cross-correlation exponent approximates to the Hurst exponent, namely, $H_{xy} \approx H = \rho + 0.5 = 0.9$ [24].

In Fig. 2, in order to improve the readability and make clearer contrast among the different curves, we subtract some constants, i.e., 3 is subtracted from the original results when $L = 10000$, 6 when $L = 4000$, and 9 when $L = 2000$. From this figure, one can find that for both DMCA and DCCA methods, there are crossovers. The moving average approach can not completely eliminate the effect of trend.

From Table 2, interestingly, we find that the results of DCCA and DMCA (for $\theta = 0.5$, namely, centered moving average) are similar, while the exponents estimated by DMCA for $\theta = 0$ and $\theta = 1$ (i.e., backward and forward moving averages) are similar, too. Compared with DMCA (for backward and forward moving average cases), DCCA may have a disadvantage in dealing with the unfavorable influence of trend. For example, when $T = 1000$ and $L = 10000$, $\bar{H}_{DCCA} = 1.4569$, $\bar{H}_{DMCA} = 0.8304$ (for $\theta = 0$) and 0.8396 (for $\theta = 1$), while the theoretical value is only 0.9 (see Table 2). Because of the existence of crossovers, the cross-correlation exponents may be significantly overestimated by DCCA and DMCA (for $\theta = 0.5$), but slightly underestimated by DMCA (for $\theta = 0$ and $\theta = 1$). Meanwhile, please note that if the length of time series is short, DMCA (for $\theta = 0$ and $\theta = 1$) outperform DCCA and DMCA (for $\theta = 0.5$). But if the length is relatively longer, the results tend to be underestimated by DMCA (for $\theta = 0$ and $\theta = 1$), especially when $T = 500$ and $L = 20000$, $\bar{H}_{DMCA} = 0.6639$ (for $\theta = 0$) and 0.6666 (for $\theta = 1$). At this time, $\bar{H}_{DCCA} = 1.1861$ and $\bar{H}_{DMCA} = 1.1574$ (for $\theta = 0.5$) seem to be closer to the theoretical value 0.9. In short, if the length of time series is long enough, DCCA and DMCA (for $\theta = 0.5$) may perform better; but if the length is short, DMCA (for $\theta = 0$ and $\theta = 1$) may be a better choice. Furthermore, since the empirical time series are usually short in length, DMCA (for $\theta = 0$ and $\theta = 1$) may be more accurate, and thereby a better solution in practice, because for short time series, its estimations are closer to theoretical values.

Table 2 The results of DMCA and DCCA methods for the parameters $\rho = 0.4$ and $W = 0.5$

		$T = 1000$	$T = 500$	No Trend
Theoretical Value		0.9	0.9	0.9
$L = 2000$	DCCA	1.2707(0.0188)	1.3802(0.0262)	0.9808(0.0154)
	DMCA($\theta = 0$)	0.9892(0.0037)	0.9349(0.0102)	0.8225(0.0052)
	DMCA($\theta = 0.5$)	1.1796(0.0296)	1.3135(0.0307)	0.9447(0.0175)
	DMCA($\theta = 1$)	0.9830(0.0041)	0.9312(0.0097)	0.8227(0.0052)
$L = 4000$	DCCA	1.3771(0.0297)	1.4495(0.0221)	0.9985(0.0101)
	DMCA($\theta = 0$)	0.9557(0.0094)	0.8348(0.0278)	0.8754(0.0045)
	DMCA($\theta = 0.5$)	1.3414(0.0296)	1.4250(0.0230)	0.9780(0.0112)
	DMCA($\theta = 1$)	0.9525(0.0091)	0.8419(0.0256)	0.8778(0.0044)
$L = 10000$	DCCA	1.4569(0.0278)	1.3503(0.0372)	0.9473(0.0095)
	DMCA($\theta = 0$)	0.8304(0.0345)	0.6816(0.0404)	0.8609(0.0049)
	DMCA($\theta = 0.5$)	1.4246(0.0332)	1.3296(0.0410)	0.9283(0.0099)
	DMCA($\theta = 1$)	0.8396(0.0324)	0.6857(0.0401)	0.8616(0.0047)
$L = 20000$	DCCA	1.3767(0.0343)	1.1861(0.0575)	0.9491(0.0088)
	DMCA($\theta = 0$)	0.7292(0.0423)	0.6639(0.0322)	0.8522(0.0058)
	DMCA($\theta = 0.5$)	1.3616(0.0379)	1.1574(0.0637)	0.9391(0.0089)
	DMCA($\theta = 1$)	0.7319(0.0420)	0.6666(0.0320)	0.8524(0.0057)

Note: For each length of time series, we generate 10 pairs. The results show the average of 10 cross-correlation exponents of those generated pairs; in parentheses there are the means of standard errors.

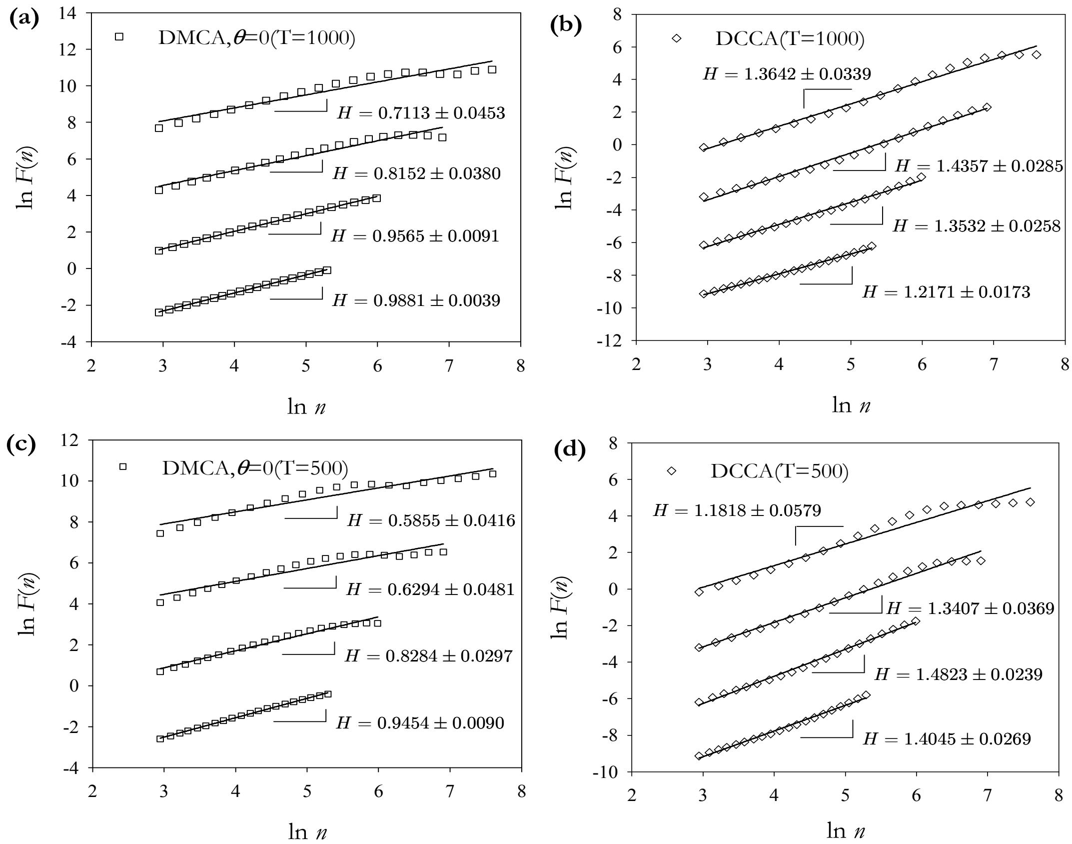

Fig. 2 The log-log plot for the square root of the detrended covariance *vs.* time scale n. It is unnecessary to plot the results from all ten pairs; therefore we choose the results from one pair out of the ten when the parameters $A = A_1 = A_2 = 0.3$, $\rho = \rho_1 = \rho_2 = 0.4$, $W = 0.5$, with the sinusoidal periods $T = 1000$ and $T = 500$, and for the DMCA method, we only show the results for the case $\theta = 0$ (the others, namely $\theta = 0.5$ (overestimated) and $\theta = 1$ (underestimated), are similar). In order to make clearer contrast among the different curves, some constants are subtracted from the results, namely, 3 is subtracted when $L = 10000$, 6 when $L = 4000$ and 9 when $L = 2000$. For each panel, from top to bottom, the lengths of each series are 2000, 4000, 10000 and 20000 respectively. One may find the crossovers, which cause the exponents underestimated by DMCA (for $\theta = 0$) and overestimated by DCCA.

Podobnik *et al.* propose a global detrending approach, which can effectively eliminate the influence of trend [24]. But one might criticize that they have known the format of the trend in advance. In real world, there are seasonal periods, economic periods, non-periodic cycles, or even complex trends, which we do not know the format or function before we analyzed the time series. Although the DMCA we propose can not complete eliminate all influence of trend, it can reduce the effect, which can identify the power-law cross-correlation given the presence of unfavorable trends in the real world.

Therefore, compared with the DCCA, this new approach may possess the following advantages:

First, it can reduce the amount of calculation. Compared with the polynomial fitting applied in the DCCA, the latter only needs simple moving average filtering. In addition, unlike the DCCA, there is no need to divide the boxes for the DMCA; while as for the DCCA, to make sure the accuracy, overlapping boxes are used, which may increase a huge amount of additional calculations.

Second, continuous adjusting filter is applied. For the DCCA, the polynomial trend between each box is discontinuous; but all of the filters are continuous in our method, for the moving average filter adjusts the fitting curve dynamically, which may increase the accuracy.

Third, the influence of trend is reduced. Although DMCA (for backward and forward moving average cases) can not completely eliminate the influence of the trend, and for a long period time series, the results may be underestimated; but for a short period, DMCA for backward and forward moving average cases can perform better than DCCA.

Four, it is a practical choice for real world data with short lengths. As we discussed earlier, DMCA using for backward and forward moving average filters outperforms DCCA in more accurate estimation when the analyzed times series are short in length.

4. Empirical Study

As an example, we apply DMCA to investigate the cross-correlation between some commodity markets. We proceed an empirical study on international crude oil spot markets, which according to our previous empirical findings, are found to be fractal and multifractal [5, 38, 39], and choose the daily price of WTI Spot Price FOB (Dollars per Barrel) and Europe Brent Spot Price FOB (Dollars per Barrel), from May 20th, 1987 to Jun. 29th, 2010, which consist of 5775 simultaneously recorded data points (data source: US Energy Information Administration, http://tonto.eia.doe.gov/dnav/pet/pet_pri_spt_s1_d.htm).

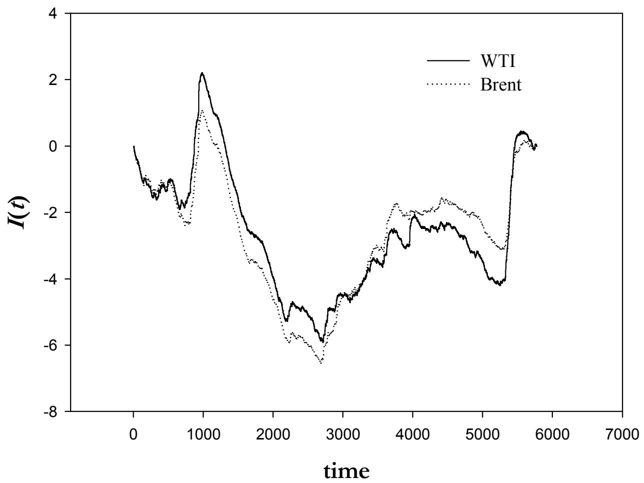

Fig. 3 The integrated profiles of absolute logarithmic returns for WTI and Brent. It is clear that the two markets are highly correlated.

Let $p(t)$ to be the price of crude oil. Then the daily price return $r(t)$ is calculated as its logarithmic difference, $r(t) = \ln(p(t+\Delta t) - \ln(p(t))$, where $\Delta t = 1$. To get better understanding of the data, we perform summary statistics of the logarithmic returns (see Table 3), from which we can see a large skewness and kurtosis. To better describe the time series, we also plot the integrated profiles $I(t) = \sum_{i=1}^{t} (|r(i)| - \langle |r| \rangle)$ in Fig. 3, from which one can find that they are highly correlated.

Table 3 Mean, standard deviation, skewness and kurtosis of crude oil returns

	Mean	Std. dev.	Skewness	Kurtosis
WTI	0.0177	0.0186	4.0157	46.274
Brent	0.0168	0.0172	3.9382	43.757

In Table 3 the asymmetry and large kurtosis can be clearly seen in crude oil returns, which imply existence of power-law tails in probability density function (PDF). Podobnik *et al.* analyze the asymmetry in presence of long-range correlations by a new process [40]. We investigate the tails in PDF, following the method suggested by Podobnik *et al.* [25], which is also proposed by Ren and Zhou [41] independently. Several studies investigate the returns intervals τ between consecutive price fluctuations above a volatility threshold q, especially in Chinese stock markets [41-45]. The PDF of returns intervals $P_q(\tau)$ scales with the mean of τ as in Ref. [46]:

$$P_q(\tau) = \frac{1}{\bar{\tau}} f(\frac{\tau}{\bar{\tau}}) \tag{11}$$

where $f(x)$ is a stretched exponential function. Then Podobnik *et al.* [25] propose a new estimate for the power-law exponent by

$$\bar{\tau}_q \propto q^{\alpha} \tag{12}$$

By means of Eqs. (11) and (12), we obtain the relationships between average return interval τ and threshold q (in units of standard deviation) (see Fig. 4). From this log-log plot, the linear relationships can be clearly identified, which implies that there exist power law relationships between the intervals and thresholds for both

WTI and Brent prices; and the estimated exponents, that is, $\widetilde{\alpha} = 2.7655 \pm 0.0775$ (WTI) and $\widetilde{\alpha} = 2.6864 \pm 0.0419$ (Brent), are slightly less than the inverse cubic law reported in Ref. [47].

To provide a practical example, we further applied DMCA and DCCA methods to quantify the cross-correlation relationships between WTI and Brent crude oil spot markets respectively (see Fig. 5 and Table 4). From Fig. 5 and Table 4, the power-law cross-correlation relationships can be found for both markets. The results measured by DMCA and DCCA (see Table 4) tell us that the exponents are around 0.5 for original returns, but approximate to 1 for the absolute returns. It suggests that price fluctuations can propagate and then transmit to the other; and that one large volatility of price change in one market is likely to cause another large volatility in the correlated market.

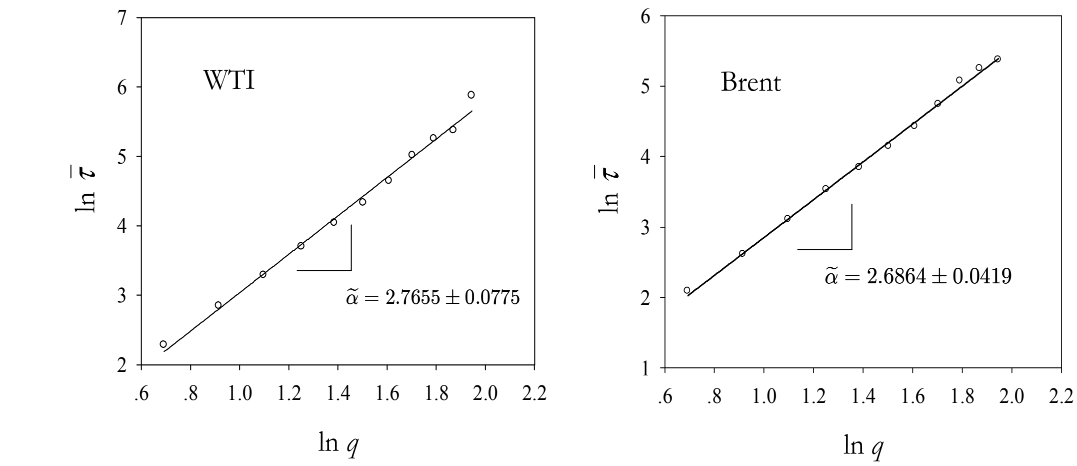

Fig. 4 The log-log plot for average return interval τ *vs.* threshold q (in units of standard deviation). One can find distinct power law relationships, whose exponents are 2.7655 ± 0.0775 (WTI) and 2.6864 ± 0.0419 (Brent), which are slightly less than the inverse cubic law.

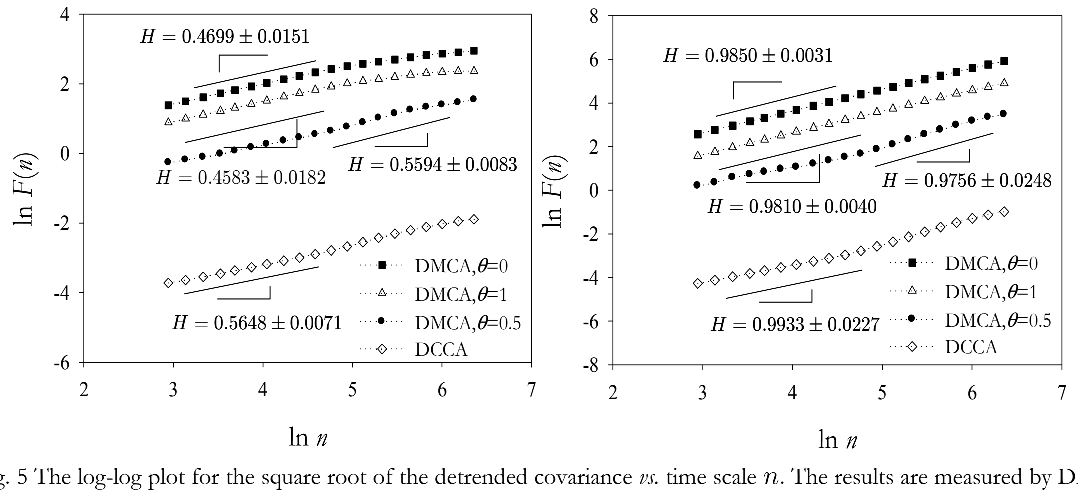

Fig. 5 The log-log plot for the square root of the detrended covariance *vs.* time scale n. The results are measured by DMCA and DCCA, using (a) the original returns and (b) absolute returns series. In order to make clearer contrast among the different curves, some constants are subtracted from the original results. One can find that there is power-law cross-correlation between WTI and Brent crude oil spot markets, where the exponents are around 0.5 (for original returns) and approximate to 1 (for absolute returns or volatility).

Table 4 The cross-correlation exponents between WTI and Brent crude oil spot markets

	Original Returns	Absolute Returns
DCCA	0.5648±0.0071	0.9933±0.0227
DMCA($\theta = 0$)	0.4699±0.0151	0.9850±0.0031
DMCA($\theta = 0.5$)	0.5594±0.0083	0.9756±0.0248
DMCA($\theta = 1$)	0.4583±0.0182	0.9810±0.0040

5. Conclusions

We proposed a new method Detrended Moving-average Cross-correlation Analysis (DMCA) by combining DCCA and DMA. Many comparisons were made between this new method and DCCA. To make a practical example, we also applied DMCA to investigate empirically the cross-correlation between real world data, i.e. WTI and Brent crude oil spot markets in this paper. Our conclusions can be summarized as follows:

First, we proposed a new approach—DMCA, which can efficiently quantify the power-law correlation between two non-stationary time series.

Second, for long time series with trend, DCCA and DMCA (for centered moving average case) outperform DMCA (for background and forward moving average case); but if time series are short, the latter method seems to be a better choice. Since the real world data are often short in length, DMCA (for background and forward moving average cases) may be a more practical choice compared with DCCA.

Third, this new approach can outperform the present method by significantly reducing the amount of calculations and the effect of trend, although it can not completely eliminate the effect.

Fourth, the empirical study also shows that by means of this method, some commodity markets (in this paper, WTI and Brent crude oil spot markets) are found to be power-law cross-correlated. We thereby present an example that our method has potential application to real world problems.

In all, our method provides another practical choice that dedicates to the identification of the cross-correlation between two non-stationary time series, especially of short periods.

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