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K. SHIN and H. HIRAYAMA

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A NEW APPROXIMATING MODEL FOR GAMMA-RAY BUILDUP FACTORS OF STRATIFIED SHIELDS

Kazuo Shin

Department of Nuclear Engineering, Kyoto University,
Yoshida, Sakyo-ku, Kyoto 606-01, Japan

And

Hideo Hirayama

National Laboratory for Hight Energy Physics,
1-1 Oh-ho, tsukuba, Ibaraki 305, Japan

ABSTRACT

A new approximate expression for gamma-ray buildup factors of multilayered shields is proposed. The expression is formulated based on the vector form and considers the gamma-ray energy spectrum directly. It treats the gamma-ray transmission by a transmission matrix and the backscattering by an albedo matrix. Its capability of reproducing the buildup factors for multilayered shields was demonstrated by using double layered shields composed of two materials of water, iron and lead at 1 and 10 MeV. The data of three layered shields of these materials are also very well reproduced.

The mechanism of arising the density effect which appeared in the buildup factor for a point isotropic source was clearly interpreted by the present method to be a geometrical effect. A correction factor for incorporating the density effect into the present expression was derived. The modified expression was successfully applied to buildup factors for a 0.5-MeV point isotropic source for two layered shields of water and iron.

1 Introduction

The point kernel method has been widely used for a quick estimation of the gamma-ray attenuation in shields. Buildup factors are key parameters for this calculation method. Since systematic data of gamma-ray buildup factors were published by Goldstein and Wilkins[1] in 1954, many efforts[2-11] have been made to improve the accuracy of the calculation of buildup factors. Experiments[12-19] for gamma-ray attenuation and buildup factors have stimulated the improvement of the calculations. A number of studies[21-29] to develop empirical formulae, which enabled the generated buildup factors to be incorporated in point kernel codes, have been made. Through these works, a systematic compilation of buildup factor data and a very good empirical expression of them are now available[29] for single layered shields.

Shielding configurations appearing in nuclear facilities are almost always stratified ones. Correspondingly, many works[30-37] were carried out to develop analysis method of gamma-ray attenuation in the stratified shields and empirical formulae simply to describe the buildup factors for these shield configurations. However, the behavior of gamma rays in the stratified shields is complicated, reflecting changes in the energy and angular distributions of gamma rays near the material boundary in the shields.

Moreover, as was showed by Engholm[38], the "density effect" was observed in the gamma-ray buildup factors for doubly stratified shields of the point isotropic source geometry. The density distribution had a pronounced effect on the calculated buildup factor, whereas in the plane case no effect was observed. For example, he showed concrete followed iron resulted in excess buildup as compared with concrete alone, and the reverse happened when iron followed concrete. When the density was adjusted, the behavior of the buildup factor became similar to those in the plane case. Recently, Harima and Hirayama[11] pointed out that the excess buildup arose corresponding to a small deviation in the forward angular flux of gamma rays in stratified shields from that in the single layered shield. However, the mechanism of arising the effect is still not well understood.

Because of these reasons, the empirical expressions of buildup factors proposed so far did not succeed to reproduce the data of stratified shields for generic combinations of materials. They only fit the data for specific combinations of shield materials in a not wide energy range.

Since too many combinations of materials are required, including the thickness of each layer, in generating systematic data of the buildup factors for stratified configurations, it is not practical to give all the data directly. It is inevitable to develop a generic expression for the buildup factor data, that has a clear physical base. And the generation of the buildup factors for the stratified shields should be made just to settle the expression. Once the expression is settled, one can handle the buildup factors by the expression.

The present paper proposes a new approximate expression of the gamma-ray buildup factors for multilayered shields. The proposed expression treats quantities in vectors and uses transmission and albedo matrices by which changes in the gamma-ray energy spectrum in the shield are automatically taken into consideration. Therefore complicated changes in the buildup factor between different materials are well reproduced. An usage of the transmission matrix in the gamma-ray attenuation calculation was first made by Yarmush et al.[39] as one of rigorous calculation methods of gamma-ray transmission in shields. Here is used a similar method, which is modified and simplified to be an easily tractable formula of the buildup factor. Therefore, the present formula posseses clear physical bases and shows more generic reproducibility of the data than other formulae proposed so far.

The physical background of the density effect is directly understood by the present method. That is a geometrical effect. Through the analysis of this effect, an approximate correction factor to the transmission matrix for describing the density effect is given.

The expression and calculational method of the transmission and albedo matrices are outlined in Section 2. The application of the method to the buildup factors for laminated shields in the plane geometry is described in Section 3. The analysis of the density effect and the application of the modified formula to the buildup factor of the point isotropic source geometry will be written in Section 4. The last section 5 is for conclusions.

2 Approximate Expression of Buildup Factors

We use the vector form to express the energy spectrum of gamma rays, dividing the energy range from 0 to the source energy into G discrete groups. We define a transmission matrix T by a 1-mfp thick material layer, where a gamma ray with the energy E_0 is normally injected. As the albedo of gamma rays is usually not large, we ignore backscattering effects of gamma rays.

Then, the gamma-ray dose D behind a n mean-free-path(mfp) thick layer of this material is written as,

$$D = \vec{C} T^n \vec{S} \quad , \tag{1}$$

where $\vec{S}$ is a source intensity vector whose explicit expression is $\vec{S} = (1, 0, \cdots 0)$, and $\vec{C}$ converts the gamma-ray energy flux to the dose.

When the shield is composed of multilayers of N materials, i.e. n_1-mfp thick material 1, n_2-mfp thick material 2, and so on, Eq.(1) becomes as,

$$D = \vec{C} \left(\prod_{l=1}^{N} T_l^{n_l} \right) \vec{S} \quad , \tag{2}$$

where T_l is the transmission matrix of the material l.

We use a small number of groups, i.e. $G = 3$ or 4, to make the expression easily tractable. For the gamma-ray source energy E_0, the group structures assumed are showed in Table 1, where the 1st group is for the direct (uncollided) component and the remainder for the scattered ones.

Under these group structures, the fluence to dose conversion factor η_g at the g-th

group is defined as,

$$\eta_g = \frac{\int_{E_g} \eta(E)\, dE}{\Delta E_g} \quad , \tag{3}$$

where E_g is the energy range of the g-th group, ΔE_g its width, and $\eta(E)$ is the conversion factor at the energy E. To make the dose D in Eqs.(1) and (2) be the buildup factor that is the ratio of the total gamma-ray dose to the corresponding one of uncollided gamma rays, the vector $\vec{C}$ and the matrix T are defined as follows: $\vec{C}$ is given by Eq.(4) as

$$C_g = \frac{\eta_g}{\eta_1} \quad , \tag{4}$$

where C_g is the g-th element of the vector $\vec{C}$.

The transmission matrix T is defined as the follows:

$$T_{ij} = K_{ij} \frac{\phi_{ij}}{\phi_{11}} \quad , \tag{5}$$

where T_{ij} is the ij-element of the matrix T. Here ϕ_{ij} is defined by the equation (6),

$$\phi_{ij} = \frac{\int_{E_i} \phi_j(E) \eta(E)\, dE}{\eta_i} \quad , \tag{6}$$

using $\phi_j(E)$ which is transmitted gamma-ray flux behind the 1-mfp thick layer(1 mfp at the energy E_0) with assuming the j-th group source (stepwise spectrum assumed) of unity intensity normally incident to the layer. The transmitted flux ϕ_{ij} in Eq.(6) is the flux weighted by η such that the dose may be conserved. The calculation of ϕ_{ij} was made by the Monte Carlo code EGS4.[40]

The factors K_{ij} in Eq.(5) are empirical parameters which make corrections for not considering angular distributions of gamma rays and for using the small number of groups. In the present work, the parameters K_{ij} are assumed as the follow (shown for the case of $G = 4$);

$$K_{ij} = \begin{cases} 1 & (j = 1) \\ \alpha & (j = 2) \\ \beta & (j = 3) \\ \gamma & (j = 4) \end{cases} \quad , \tag{7}$$

and α, β, γ are determined by the least square method so that the expression (2) gives the best fit to the data of buildup factors. It is noted that the parameters α, β and γ are independent of the index i.

We assumed so far that the albedo was negligible. However, actually the albedo value may not always be very small for low Z materials. Since the buildup factor is defined in an infinite medium, backscattering effects may affect the buildup factors. For this case, we modify Eq.(2) by taking the backscattering effects into account by an approximate manner; we add a 1-mfp thick layer of the material N behind the considered N-layered shield. The reflection of gamma rays from this added layer after the penetration of the original shield is added to Eq.(2) to yield Eq.(8),

$$B_D = \vec{C}(I + B_N)(\prod_{l=1}^{N} T_l^{n_l})\vec{S} \quad , \tag{8}$$

where B_N is the albedo matrix defined for the added layer, and I the unit matrix.

The ij-element of the albedo matrix B_N is given by the equation (6) for the ϕ_{ij}, replacing the transmitted flux $\phi(E)$ to the reflected flux $\phi_B(E)$ from the 1-mfp thick layer of the material N with assuming the normal incidence of the j-th group gamma rays. Since the energy of backscattered gamma rays drops drastically, all the reflected gamma rays are gathered in the last group: $(B_N)_{ij} = 0$, unless $i = G$.

The equation (8) is the final form of the present approximate expression of the buildup factor for the multilayered plane shield.

3 Application of Expression to Buildup Factors for Multilayered Plane Geometry

3.1 Method for Determining Empirical Parameters

The empirical parameters α, β, γ are considered to be energy and material dependent, and are determined by the least square method using reference buildup factors for

single- and double-layered shields of two materials at each energy. Consider two materials 1 and 2, and reference buildup factors are calculated for four shield configurations composed of the two materials, i.e. the single layered shields of material 1 and 2, and the double layered shield of n-mfp material 1 followed by material 2, and the same thickness shield in mfp where the order of the materials is reversed.

We define the square of the relative difference in buildup factors W by the equation (9);

$$W = \sum_{m=1}^{4} \sum_{n'=n+1}^{N} \left\{ \frac{B_m(n') - B_{0m}(n')}{B_{0m}(n')} \right\}^2 \quad , \tag{9}$$

where $B_m(n')$ is the buildup factor given by Eq.(8) at the depth of n' mfp, $B_{0m}(n')$ the corresponding reference data, and $m = 1$ to 4 representing the above four material configurations. The value W is the function of 6 parameters, $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ for materials 1 and 2, respectively.

These 6 parameters were changed gradually in a direction where the value W most quickly approached the minimum one untill W became less than a predetermined small value.

3.2 Application of the Expression to Multilayered Plane Geometry

3.2.1 Three Group Approximation

The expression (8) is applied to fit buildup factors for double layered shields in the plane geometry. Water and lead were assumed for shield materials, and reference data were calculated by the EGS4 code assuming plane normal sources of 1, 3, 6, and 10 MeV. The 3 group expression of Table 1 was used in the fit but the albedo was neglected, i.e. $B_N = 0$.

Figures 1 through 4 show comparisons between the reference data and Eq.(8) calculated with the 6 empirical parameters determined by the data fitting, for the 10-, 6-, 3- and 1-MeV sources, respectively. Value of the fitting parameters α and β are listed in Table 2 for 10-, 6- and 3-MeV cases. It is noted that the 2 fitting parameters were used in the 3 group approximation. Figures show that the expression (8) reproduced the buildup factors well for all cases. In more detail, there is an underestimation by Eq.(8) seen in the 1-MeV case of Fig.4(shown by broken lines) for the water shield at shallow locations and for the double layered shield of lead followed by water in the water region. The data are already underestimated at the 1-mfp depth for the water shield, since the backscattering from the backward material is not included.

When the transmission matrix of water was slightly modified so that the buildup factor at 1 mfp was reproduced by Eq.(8), the fitting was improved for the monolayer water shield, but for the double layered shield the similar underestimation was still seen.(See dotted line in Fig. 4.)

So the transmission matrix of water was reset to the original one, and the albedo was included for both of water and lead. The calculation of the albedo matrix was made by the EGS4 code. The inclusion of the albedo improved the fitting very much as is shown by the solid line in Fig. 4.

The amount of backscattered gamma rays very much differed between water and lead. Corresponding to this, the buildup factors of double layered shields calculated by EGS4 at the material boundary differed from corresponding ones of the single layered shields. This behavior of the buildup factor was very well reproduced by Eq.(8) by taking albedos into account.

The 3-group expression by Eq.(8) was applied to the buildup factors in the case of 0.5-MeV plane normal source of the same geometries. However, the fitting was not made well even with the albedo. The reason of this was considered that the absorption cross sections of lead became larger quickly with decreasing the gamma-ray energy and the change in the energy spectrum was very steep in the energy range corresponding to the 2nd group. This means one more group is needed to express the spectrum change in

this group.

Therefore, we adopted the 4 groups, of which energy structure is shown in Table 1. Using these new groups, the buildup factors of the 0.5-MeV source were fitted by Eq.(8) with the albedo. The results are shown in Fig. 5 in comparison with the reference data. The data are now reproduced very well.

We will use Eq.(8) of the 4-group expression with the albedo in all the cases below in this section.

3.2.2 Consistency of Determination of Fitting Parameters

It was shown that the 4-group expression reproduced the reference data very well by fitting parameters α, β, γ for each material and energy. However, it is important to test whether or not the thus obtained parameters strongly depend on the combination of materials used in determining the parameters. The geometry assumed for that test is two and three layered shields in the plane geometry with the plane normal source. The materials used are water, iron, and lead. The energies of the source gamma rays are 10 and 1 MeV. All the reference data were calculated by the EGS4 code.

The procedures of the test is as the follows:

1. Obtain the parameters for lead and water by the fit of Eq.(8) to the reference data for single and double layered shields of these materials.
2. Obtain the parameter for iron using the data for the iron shield and double layered shields of iron and lead, together with parameters of lead given by the step (1).
3. Calculate buildup factors of double layered shields of iron and water by Eq.(8) with the above obtained parameters for these materials, and compare them with the reference data.

4. Repeat the step (2) and determine the parameters for iron, replacing lead in the double layered shields with water.

5. Same as the step (3) except that water is replaced with lead.

6. Compare the buildup factor for three layered shields of the three materials between the reference data and those reproduced by the parameters of the steps (1) and (2).

The fitting of the steps (1), (2) and (4) were made very well and the reference data used in the fitting were quite well reproduced by these processes. The obtained fitting parameters for 10 MeV are tabulated in Table 3. Figure 6 shows the comparison of the step (3) at 10 MeV. It is seen that Eq.(8) with pre-obtained parameters very well reproduced the buildup factors of the double layered shield of the different combination of materials.

For the 1-MeV source energy, the comparison is shown in Fig. 7, which exhibits a fairly well agreement between Eq.(8) and the reference data. In more detail, slight discrepancies are seen for the data of water followed by iron, where the reference data approach quickly to the buildup factor of the iron shield while the value by Eq.(8) approaches to it more moderately. This is not due to the misfitting, but probably coming from too wide energy range of the 4th group. A very low energy part in the gamma-ray spectrum which buildups greatly in water diminishes quickly as gamma rays cross the boundary from water to iron. However, this spectrum change is moderately reproduced by the 4-group structure of Table 1.

The comparisons of the step (5) of the buildup factors for the combination of lead and iron shields are showed in Figs. 8 and 9, for the 10- and 1-MeV sources, respectively. Both figures show a good agreement between the reference data and Eq.(8). The reproduced values by the step (5) were almost the same as those when Eq.(8) was directly fitted to the reference data in the step (2), although the obtained fitting parameter values in Table 3 were slightly affected by changing the material combination.

Results of the test of the step (6) by using the three-layered shields of 4-mfp iron+3-

mfp water+lead and 4-mfp water+3-mfp iron+lead are showed in Figs. 10 and 11 for the 10- and 1-MeV sources, respectively. Again, the overall behavior of the buildup factors in the shields was very well reproduced by Eq.(8), except for small deviations which are seen in the 3rd layer for the 10-MeV source and the 2nd layer for the 1-MeV source. The latter one is the same one already pointed out regarding Fig. 7. The former one also arised from the similar reason, that is the too wide energy region of the 4th group. At higher energies, as Harima and Hirayama[11] pointed out, the low energy part in the gamma-ray spectrum builtup quickly in lead through the emission of the blemsstrahlung. But this spectrum change is moderately followed by the present method owing to the wide energy structure of the 4th group.

4 Application of Expression to Point Isotropic Geometry

4.1 Description of Density Effect

Consider the transmission probability of gamma rays through a 1-mfp thick slab. The gamma rays are directed to an angle θ with respect to the normal direction to the slab surface. The probability is always the same regardless of the slab location in the shield, when the plane geometry is considered.

In a spherical geometry, the slab is replaced by a spherical shell of 1-mfp thickness. In this case, the transmission probability of gamma rays depends on the radius (i.e. the location) of the shell. The shell of smaller radius gives shorter penetration length, resulting larger transmission probability.

To estimate the dependence of the probability on the shell radius, we carried out Monte Carlo caluclations by EGS4 assuming a shell of 1-mfp thickness for 0.5-MeV gamma rays with a source at the inner surface of the shell which was directed to the angle θ from the radial direction. The calculation simulated scattered gamma rays for the source energy of 0.5 MeV. Therefore, in the calculation, gamma rays having a stepwise spectrum either in the 2nd, 3rd or the 4th group of the 4 group structure in Table 1 were assumed, and transmitted gamma-ray fluxes of the 3rd and 4th groups were estimated. The angle θ was $cos\theta = 1$ for the 2nd group source, $cos\theta = 0.25$ for the 3rd and 4th group sources. Materials assumed in the calculations were water, alminum, iron and lead.

Typical examples of the calculated results are shown in Fig. 12. The vertical axis of the figure is,

$$\Delta R_j(r_{out}) = \frac{\phi_j(r_{out}) - \phi_j(r_\infty)}{\phi_j(r_\infty)} \quad , \tag{10}$$

where r_{out} is the outer radius of the shell, $\phi_j(r_{out})$ is the transmitted j-th group flux, and $\phi_j(r_\infty)$ is the transmitted flux for the shell with very large outer radius($\sim 200\ cm$). The horizontal axis is the ratio of r_{out} to r_0(1-mfp thickness).

From these results, we may assume that the relative change in an element of the transmission matrix, which corresponds to scattered gamma rays, is approximted by a straight line in the log-log plot:

$$\ln \Delta R = -a \ln \left(\frac{r_{out}}{r_0}\right) + b' \quad , \tag{11}$$

where

$$\Delta R = \frac{P(r_{out}) - P_\infty}{P_\infty} \quad . \tag{12}$$

Here $P(r_{out})$ and P_∞ are the transmission matrix element considered and the element for the very large radius, respectively, and a, b' are constants.

We will define a geometrical factor R for the transmission matrix element as,

$$R = \frac{P(r_{out})}{P_\infty} \quad . \tag{13}$$

From Eqs.(11) and (12), R is written by Eq.(14),

$$R = 1 + \Delta R = 1 + \frac{b}{\left(\frac{r_{out}}{r_0}\right)^a} \quad , \tag{14}$$

where b is a new constant.

Since the transmission matrix is calculated correctly for $r_{out} = r_0$ (1 mfp thickness), the value R in Eq.(14) is renormalized so that $R = 1$ at $r_{out} = r_0$;

$$R = \left\{1 + \frac{b}{\left(\frac{r_{out}}{r_0}\right)^a}\right\} / (1 + b) \quad . \tag{15}$$

Using the geometrical factor R in Eq.(15), the transmission matrix for a point isotropic source is given as,

$$\overline{T}_{ij} = f T_{ij} \quad , \tag{16}$$

and

$$f = \begin{cases} 1 & (j = 1) \\ R & (j \geq 2), \end{cases} \tag{17}$$

where T_{ij} is defined by Eq.(5) and calculated assuming a 1-mfp radius sphere. The parameters a, b are material-dependent empirical parameters which are determined together with α, β, γ values by the least square method so that the expression (8) with $\overline{T}_{ij}$ may best fit the reference data of buildup factors for doubly stratified spheres.

4.2 Application of Expression with Geometrical Factor to Buildup Factors of Point Istotropic Source

Engholm[38] calculated buildup factors of a 0.5-MeV point isotropic source for iron and concrete doublly stratified shields. The density effect was observed in his calculated results.

We consider the mechanism by which the density effect arises based on the geometrical factor R in Eq.(15). Consider the shield of concrete followed by iron and compare buildup factors in iron of this shield with ones in the iron monolayered shield. Since the 1-mfp thickness r_0 is larger for concrete than for iron, the r_{out} in the doubly stratified shield is larger than the corresponding one at the same mfp depth in the iron monolayered shield. In iron of the both shields, r_0 which is the 1-mfp thickness of iron, is the same. Consequently, R in iron of the stratified shield is less than the corresponding one at the same mean free path depth in the monolayered shield of iron, which means the transmission matrix elements of iron modified by Eq.(16) become smaller for the stratified shield even at the same mfp depth. This results in the smaller buildup factors in iron of the stratified shield as compared with the corresponding ones of the iron monolayered shield at the same depth in mfp. This is "undershooting" of the buildup factor. When the order of materials in the stratified shield is reversed, the similar discussion leads to "overshooting" of the buildup factor for the stratified shield to the corresponding one for the concrete monolayered shield.

When the density of concrete is increased for the doubly layered shield of concrete followed by iron, the radius of the concrete layer decreases and the radius of the iron layer also decreases. Then in the second layer, the geometrical factor R varies to the direction of resolving the above undershooting. It is noted that the geometrical factor of the first layer is not affected by this density change.

The buildup factors for the 0.5-MeV point isotropic source were given by Engholm for concrete and iron up to 20-mfp thickness, and stratified shields of the same thickness composed of 5-mfp, 10-mfp and 15-mfp iron followed by concrete and the same thickness shields where the order of the materials was reversed. Using the buildup factors for these 8 shields as reference data, empirical parameters $\alpha, \beta, \gamma, a, b$ for iron and concrete were simultaneously fitted by the least square method of Sec. 3.1. The albedo was not considered in this data fitting, i.e. $B_N = 0$.

Reproducibility of the buildup factors by this parameter fitting is shown in Fig. 13. It is known that the expression (8) together with the geometrical correction R reproduced the density effect in the buildup factor of the stratified shields for the point isotropic source very well.

5 Conclusions

The new approximate expression of gamma ray buildup factors for multilayered shields was proposed. The expression utilized the transmission and reflection probabilities which were formulated in the matrix form to describe changes in gamma-ray energy spectrum in the shields. The empirical parameters α, β, γ were adopted for correcting effects owing to the angular distribiton of gamma rays and the few group approximation.

It was showed that these parameters were determined consistently by the least square fitting for three materials of water, iron and lead. Using the obtained parameters, the buildup factors for three layered shields composed of these materials were well reproduced by the proposed expression.

The mechanism of arising the density effect in the buildup factor of the point isotropic source was analyzed. It was showed that the density effect was the geometrical effect that allowed more scattered gamma rays transmit through the same thickness shell at smaller radius than at larger radius. The geometrical effect was approximately described and incorporated in the present expression. The usefulness of the obtained expression for the point isotropic source was demonstrated by applying it to the data for the stratified shields of iron and water given by Engholm, showing good reproducibility of the data in all the cases.

The matrices utilized in this work were not showed. Also, the parameter values of α, β and γ were in most cases not showed, because these were tentative values in the process of the feasibility test of the formula. Final parameter values and the matrices will be published later, when the parametrization study of buildup factors is accomplished. The following works are needed before the parametrization study: (a) feasibility tests for the point isotropic source problem, (b) tests of applicability of the formula to much thicker shields, (c) optimization of the group structure, (d) tests for the interpolation of the parameters and the matrices with the gamma-ray energy.

It is planned to incorporate the present formula in point kernel codes in a near future.

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Table 1: Energy Group Structures

Group	*Energy*	*Group*	*Energy*
1	$0.99E_0$-E_0	1	$0.99E_0$-E_0
2	$0.5E_0$-$0.99E_0$	2	$0.8E_0$-$0.99E_0$
3	0-$0.5E_0$	3	$0.45E_0$-$0.8E_0$
		4	0-$0.45E_0$

Table 2: Fitting Parameters in Three Group Approximation

Source	*Water*		*Lead*	
	α	β	α	β
10 MeV	1.05	0.598	0.923	0.772
6 MeV	1.09	0.812	1.05	0.844
3 MeV	0.924	0.589	0.894	0.393

Table 3: Fitting Parameters for 10 MeV in Four Group Approximation

Step	*Material*	*Parameters*		
		α	β	γ
(1)	Lead	0.905	1.08	0.581
	Water	1.11	0.879	0.588
(2)	Iron	1.01	0.962	0.521
(4)	Iron	1.02	0.904	0.569

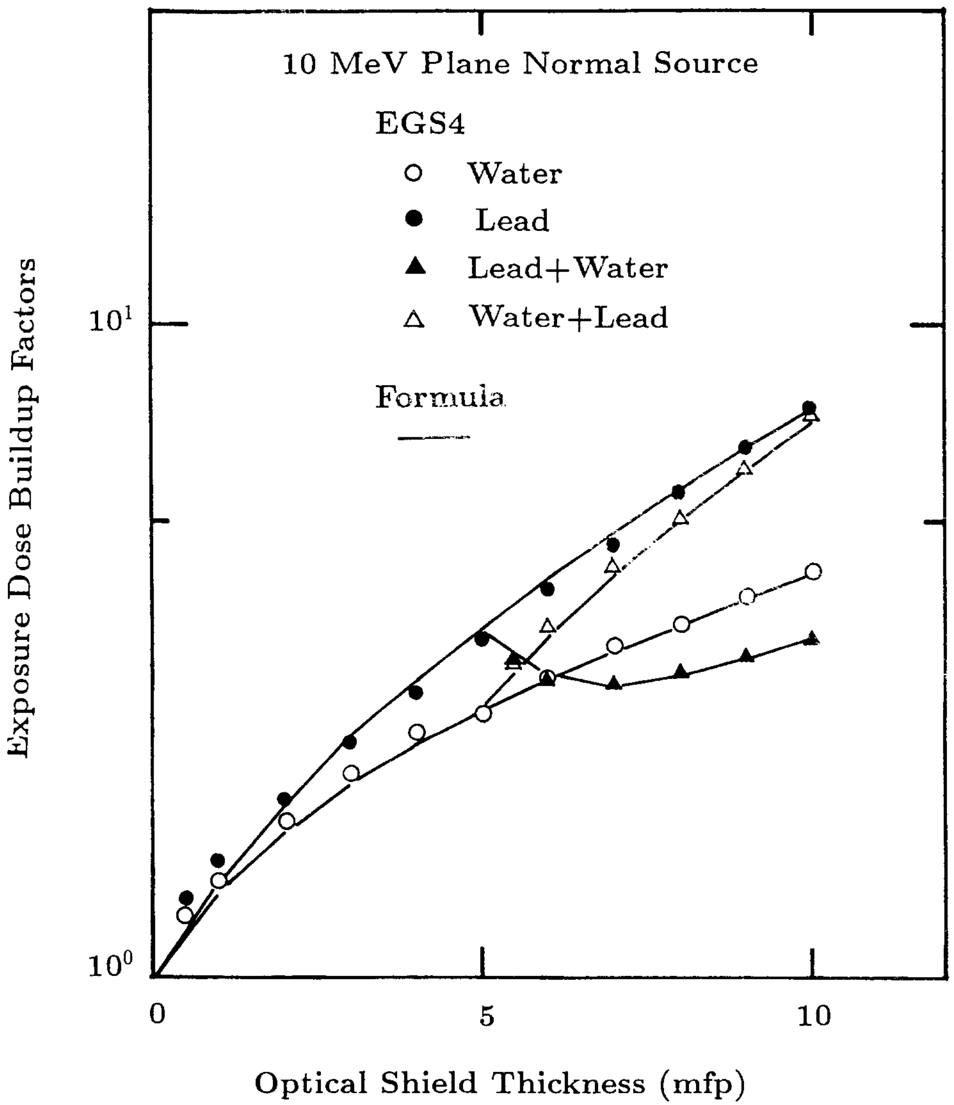

Figure 1 Buildup factors for a 10-MeV plane normal source of heterogeneous shields composed of water and lead.

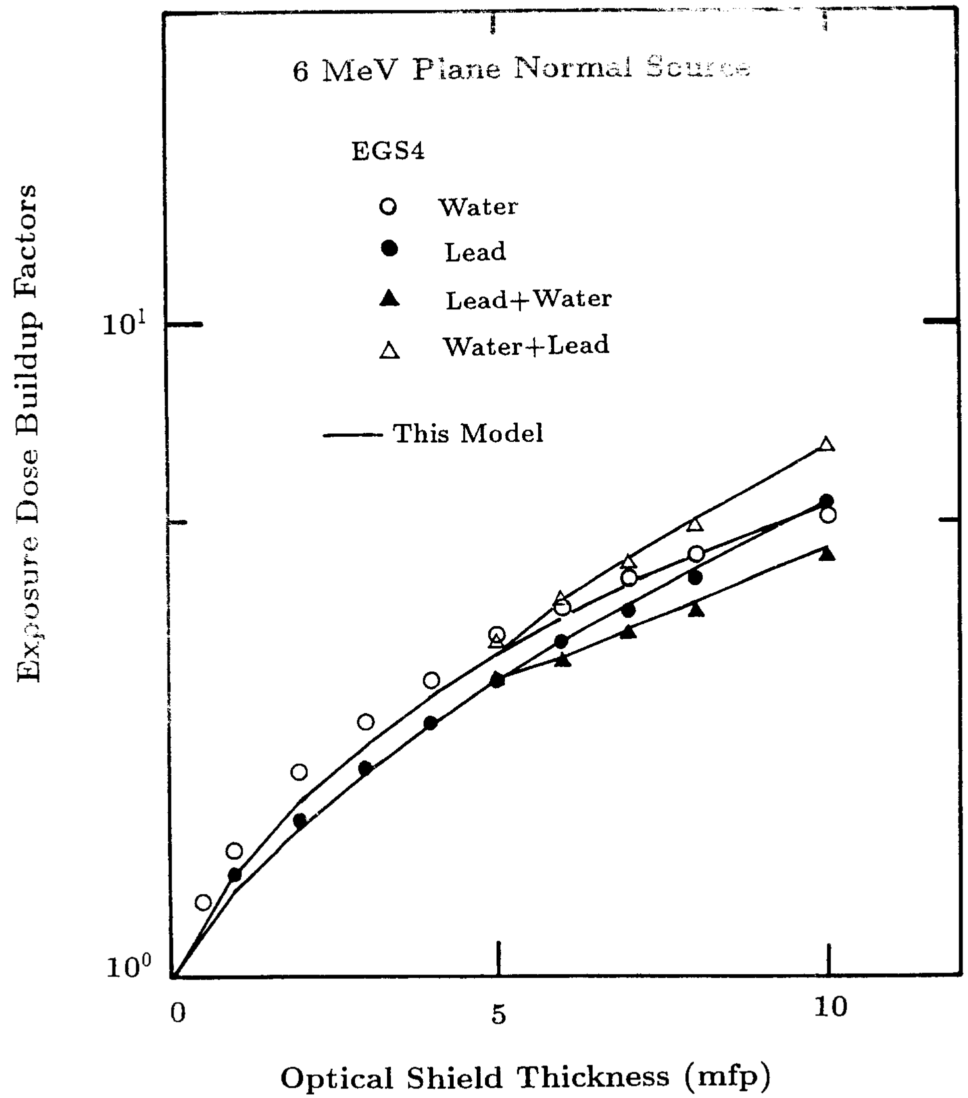

Figure 2 Buildup factors for a 6-MeV plane normal source of heterogeneous shields composed of water and lead.

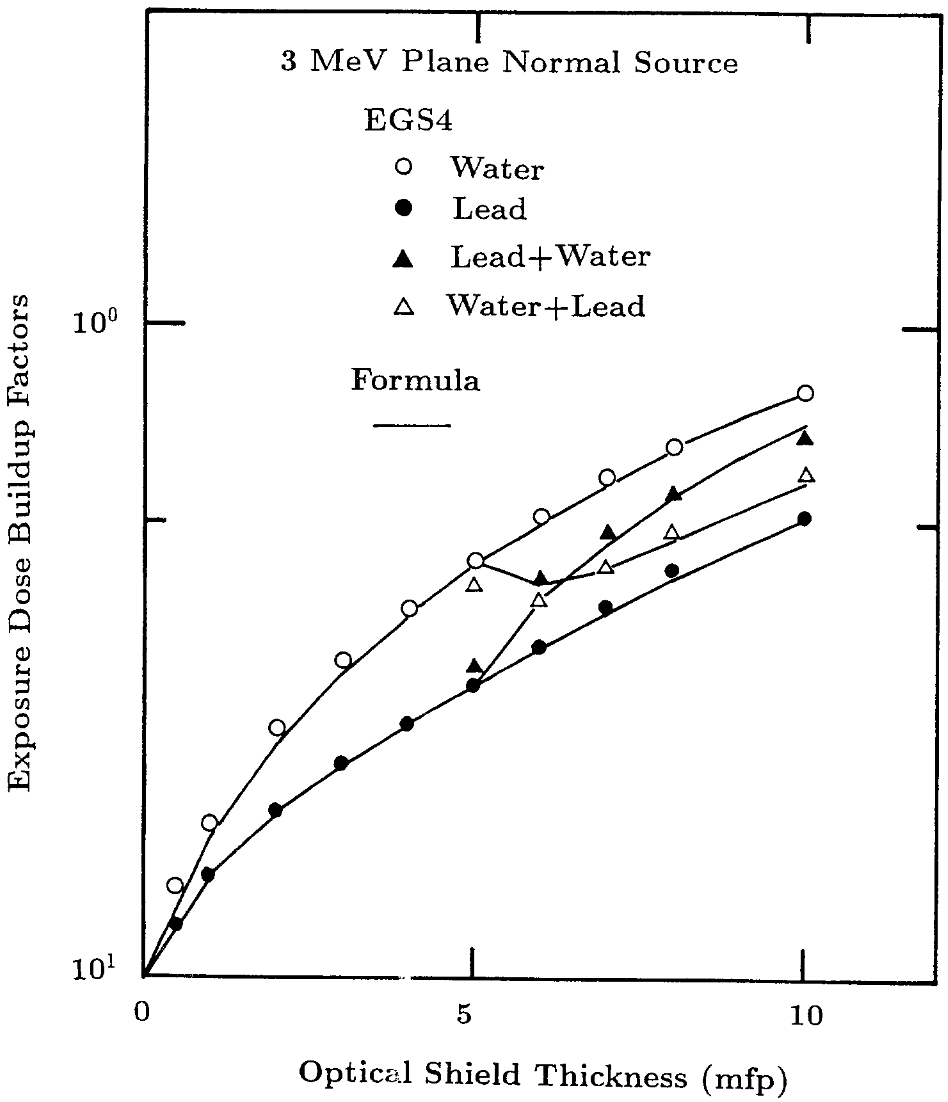

Figure 3 Buildup factors for a 3-MeV plane normal source of heterogeneous shields composed of water and lead.

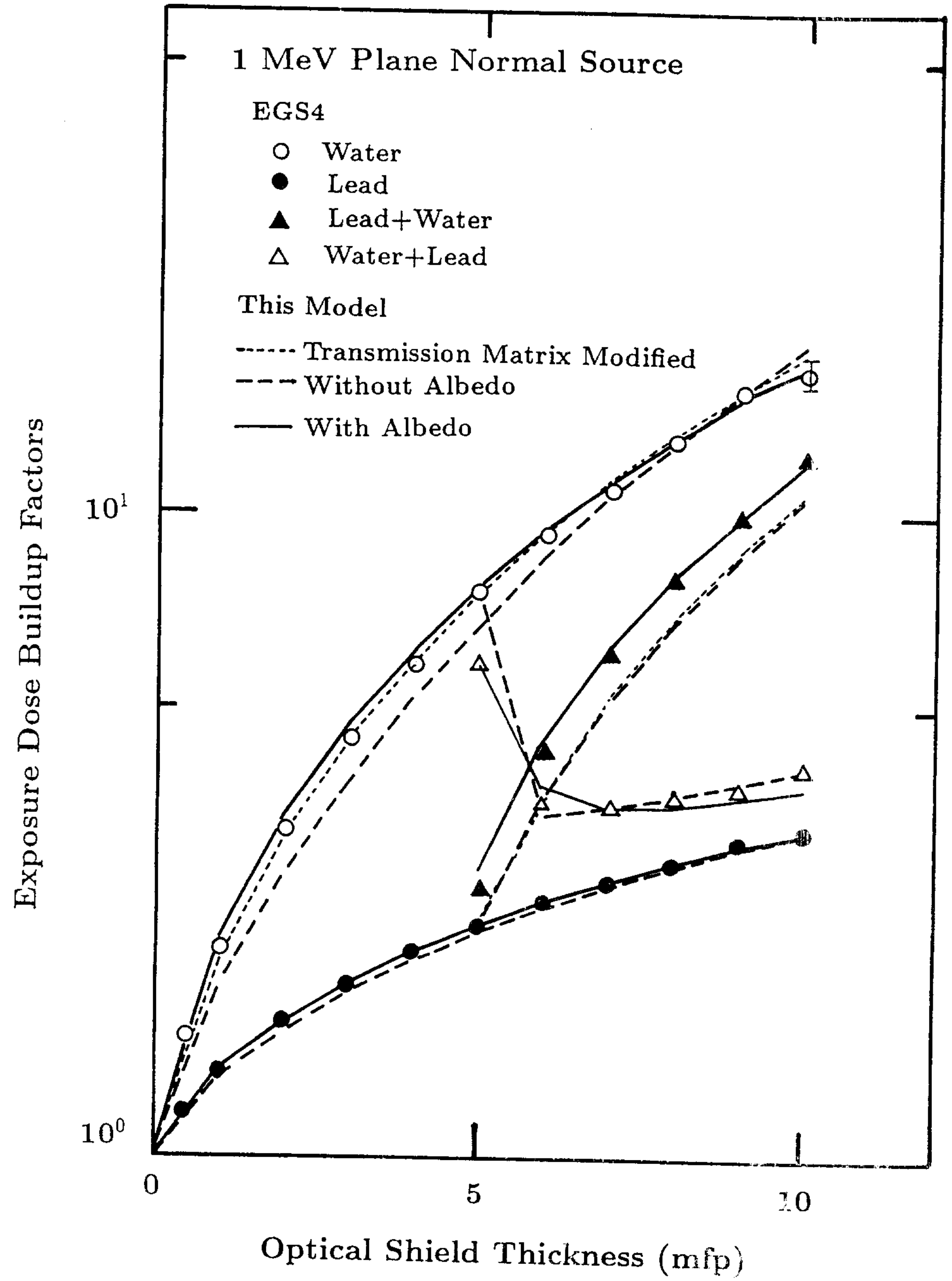

Figure 4 Buildup factors for a 1-MeV plane normal source of heterogeneous shields composed of water and lead.

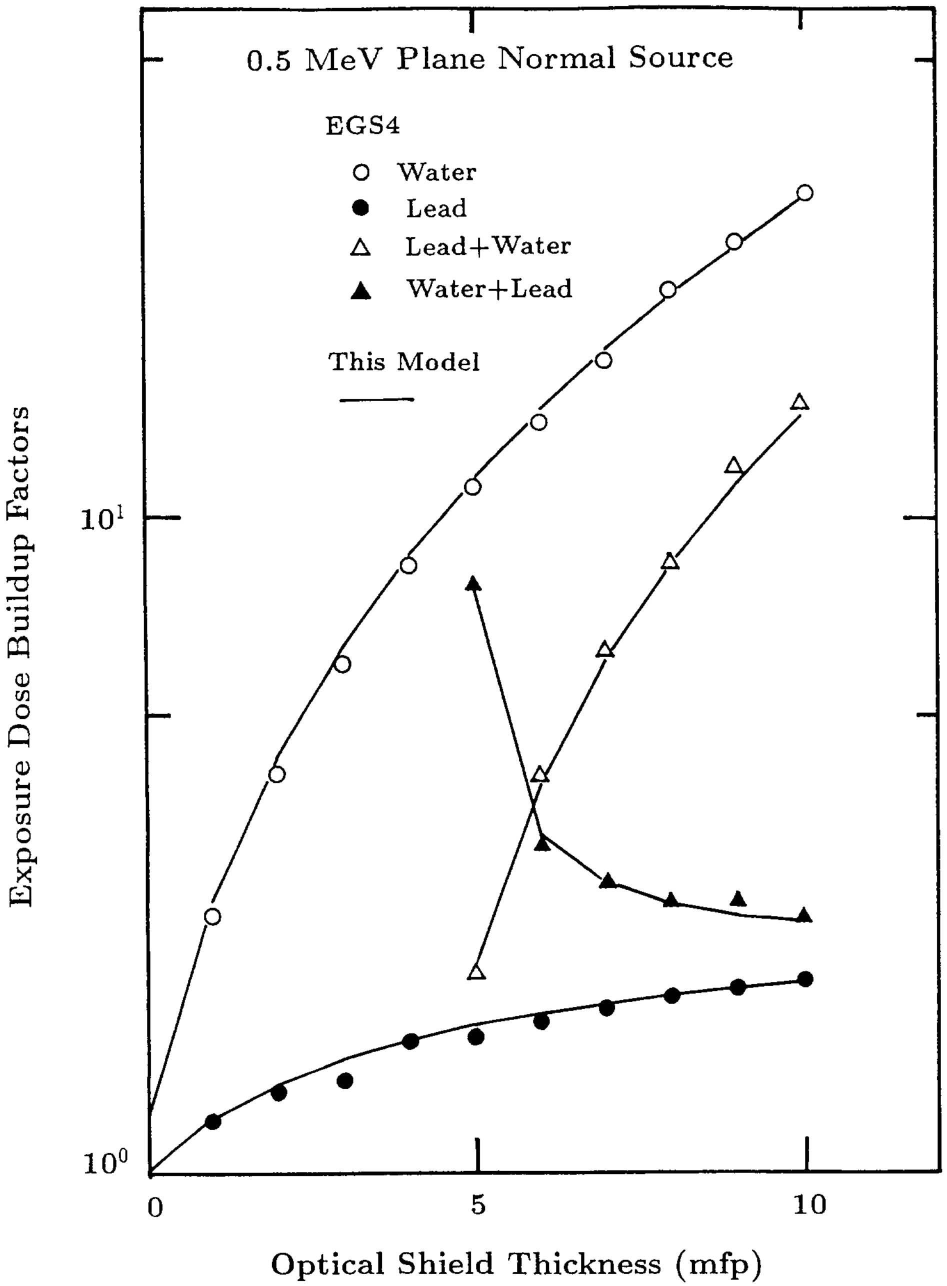

Figure 5 Buildup factors for a 0.5-MeV plane normal source of heterogeneous shields composed of water and lead.

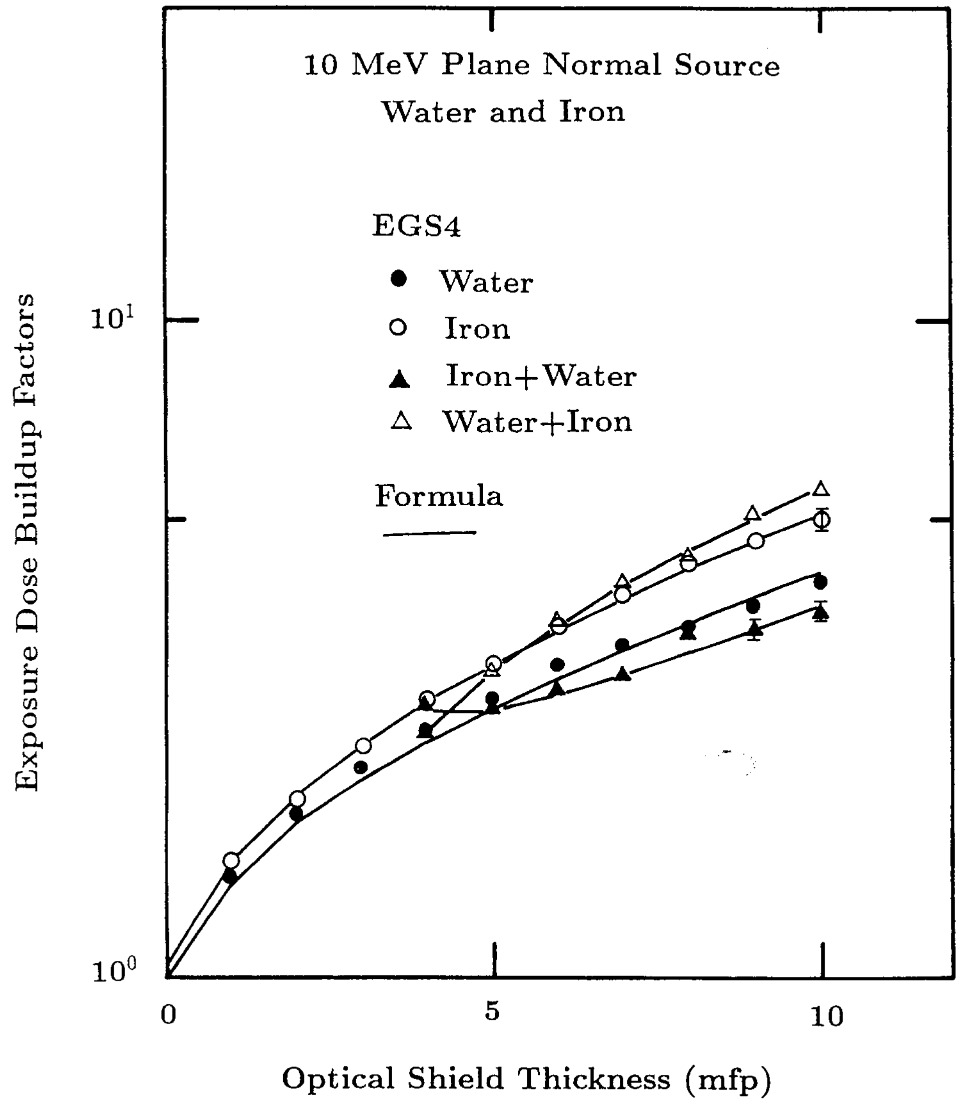

Figure 6 Test of reproducibility in step(3) of buildup factors at 10-MeV by present formula with parameters adjusted by different combinations of materials.

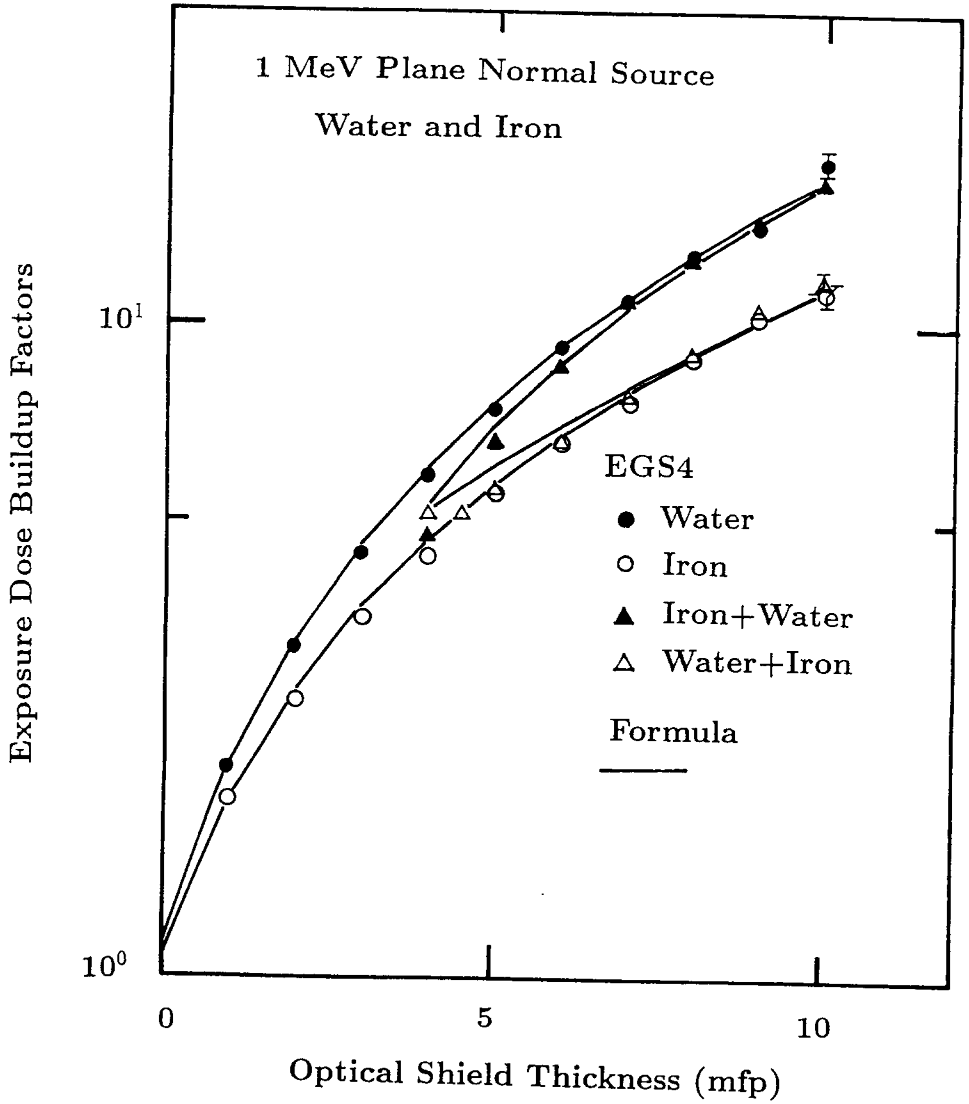

Figure 7 Test of reproducibility in step(3) of buildup factors at 1-MeV by present formula with parameters adjusted by different combinations of materials.

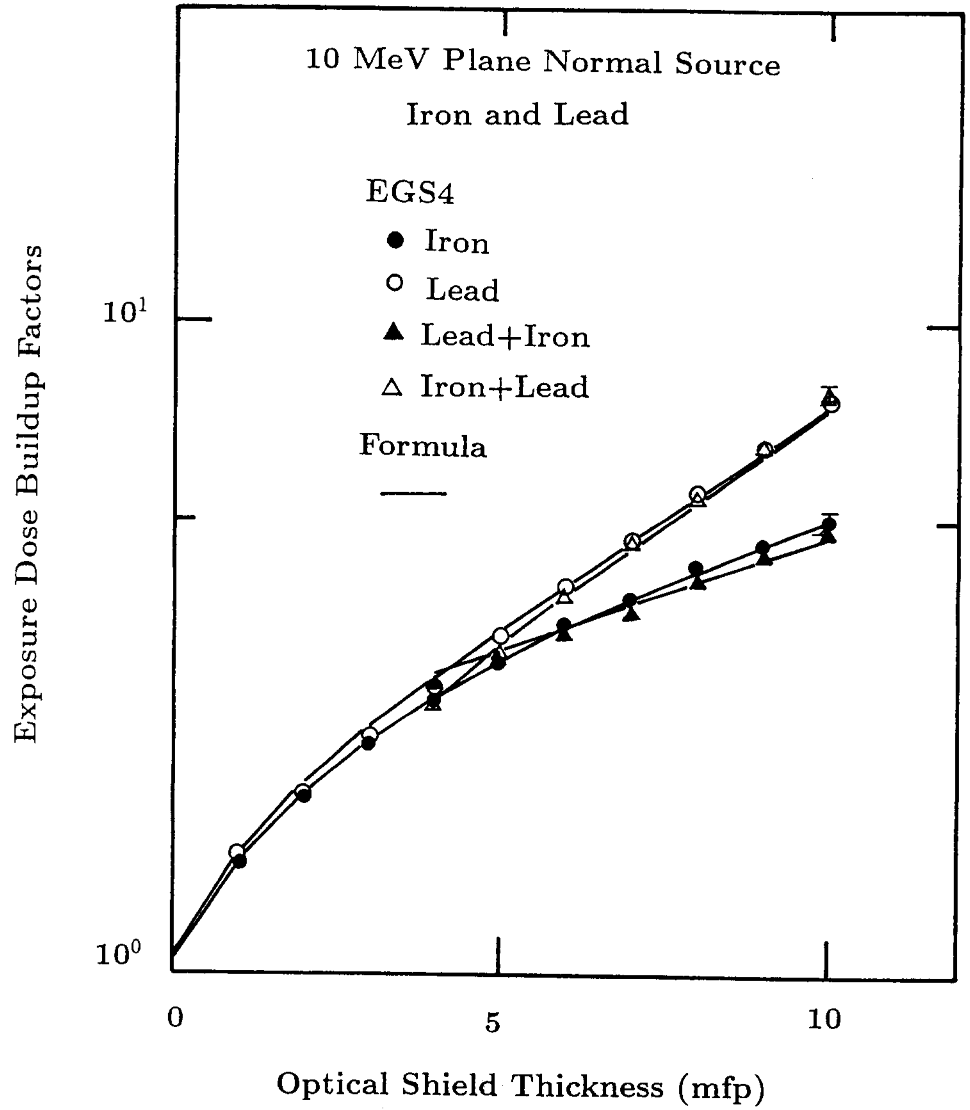

Figure 8 Test of reproducibility in step(5) of buildup factors at 10-MeV by present formula with parameters adjusted by different combinations of materials.

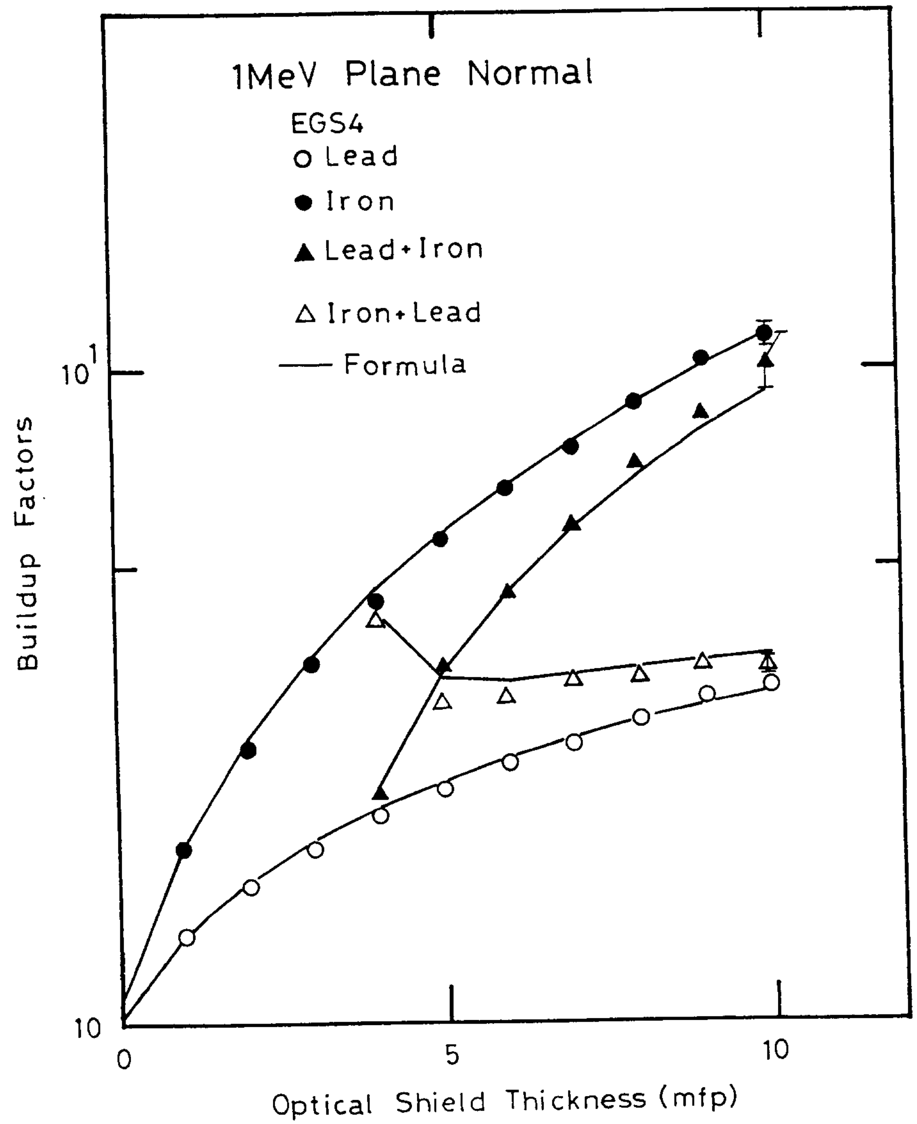

Figure 9 Test of reproducibility in step(5) of buildup factors at 1-MeV by present formula with parameters adjusted by different combinations of materials.

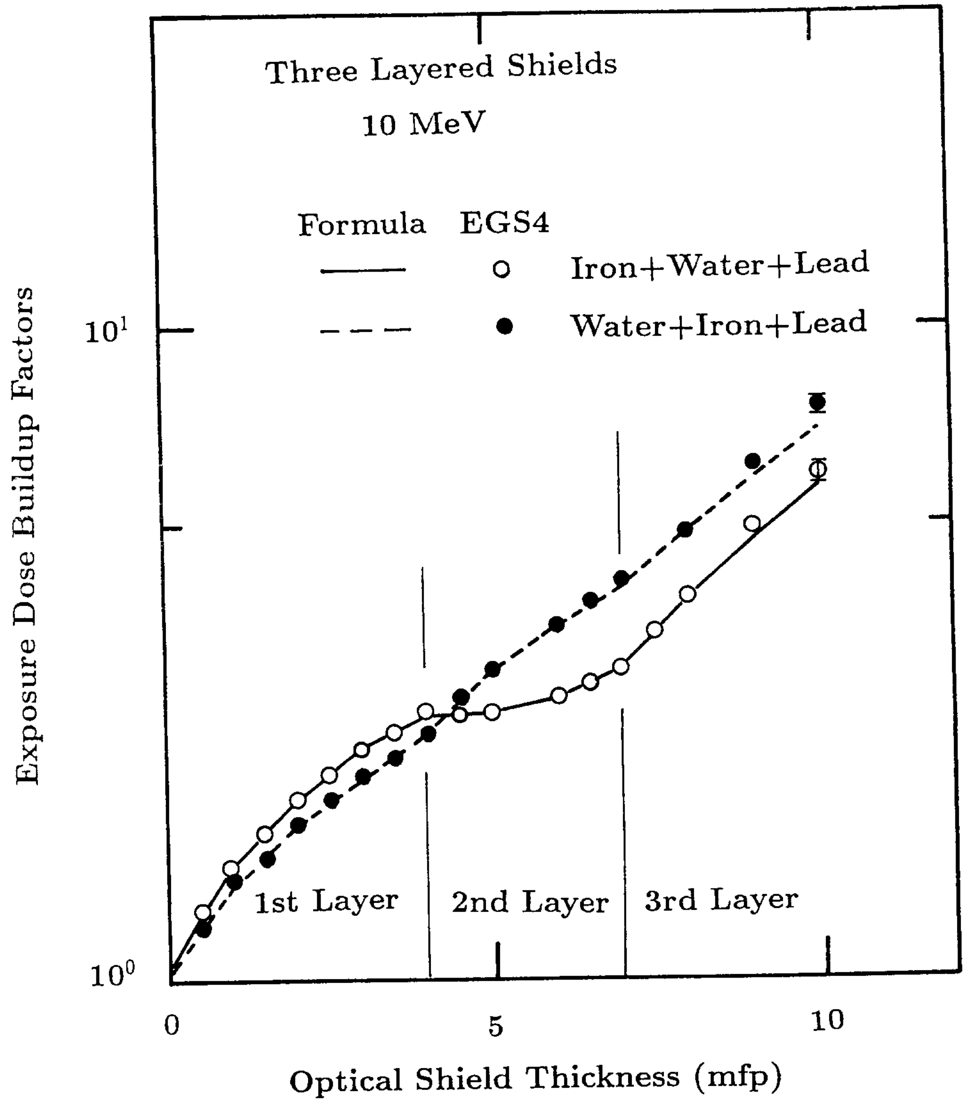

Figure 10 Buildup factors at 10 MeV for three layered shields composed of water, iron and lead.

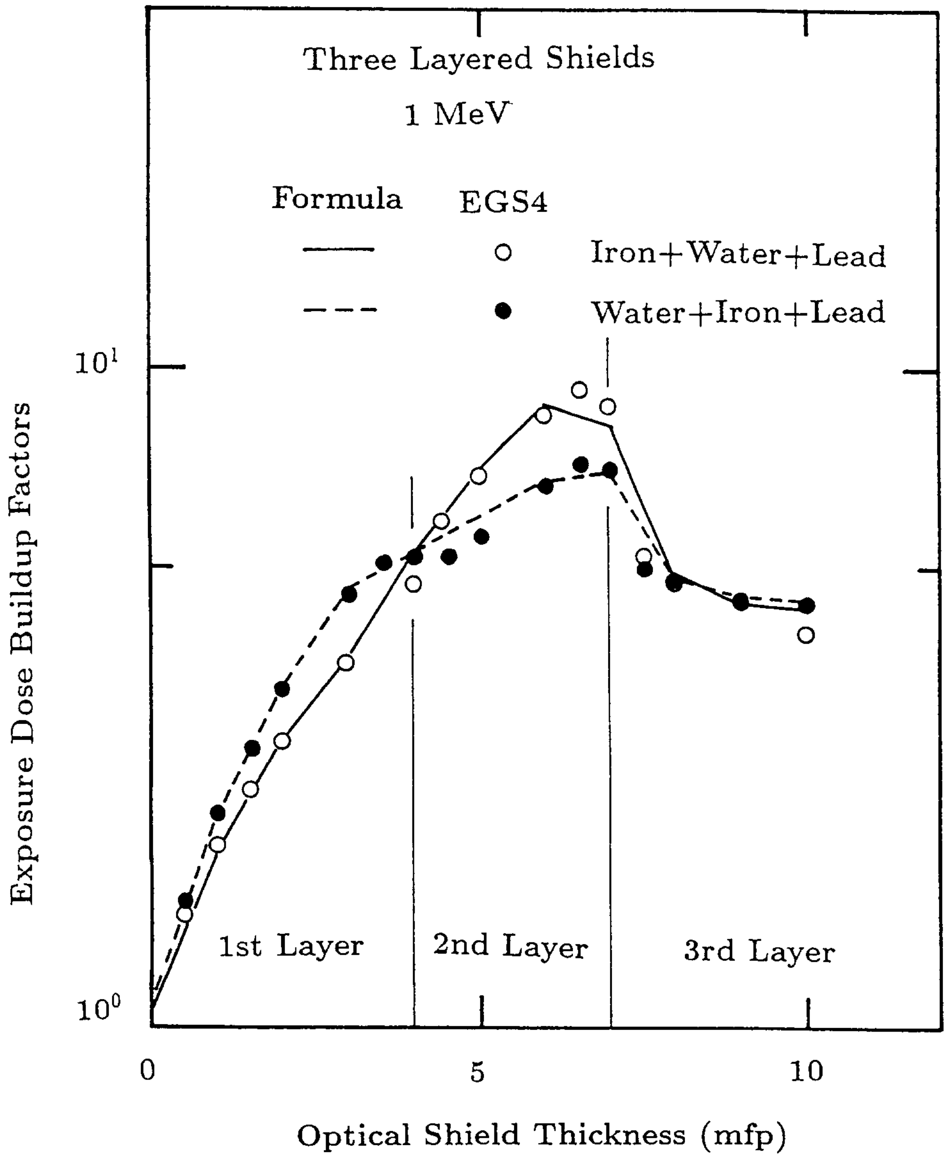

Figure 11 Buildup factors at 1 MeV for three layered shields composed of water, iron and lead.

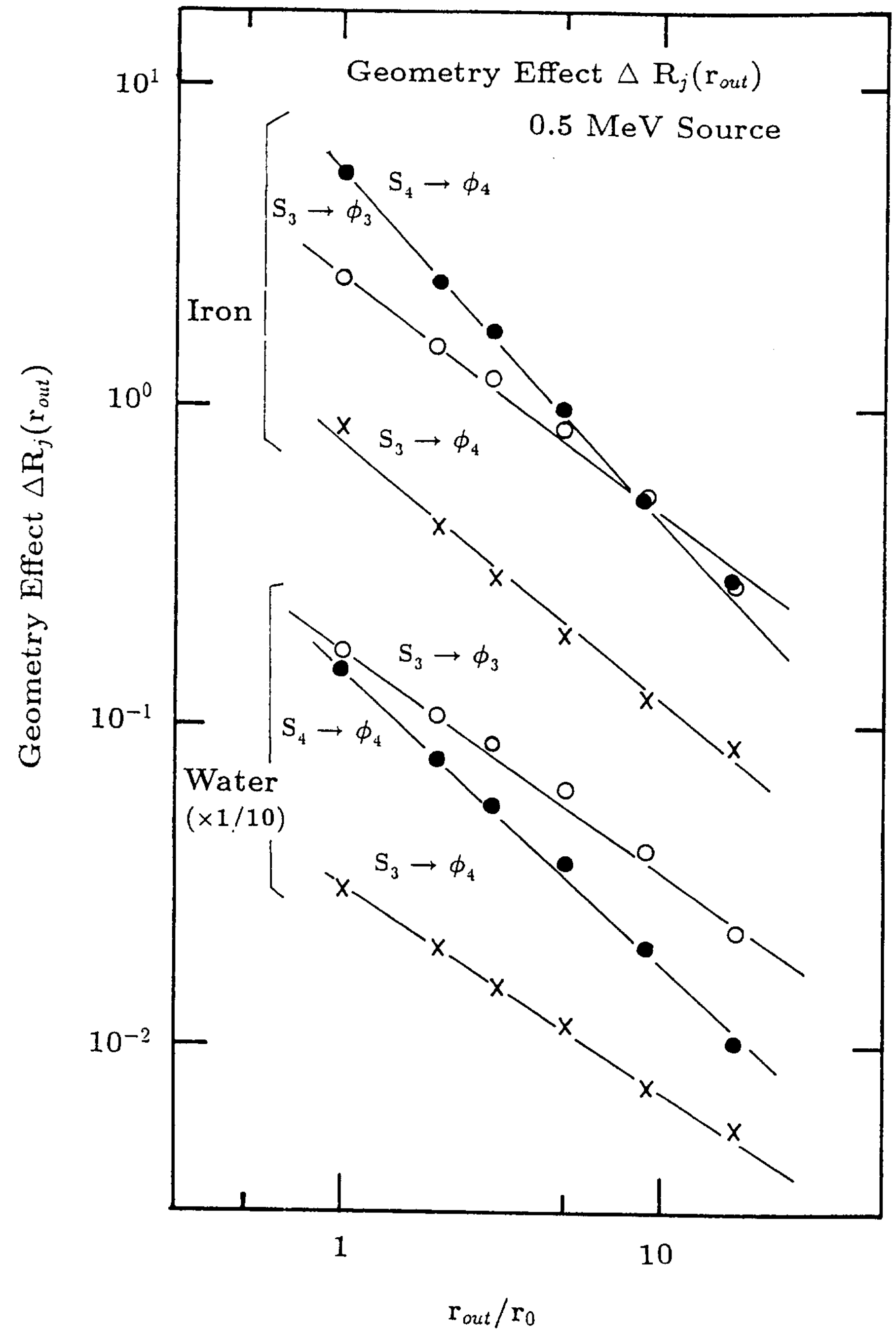

Figure 12 Relative increase in tansmitted gamma-ray fluxes with shield outer radius.

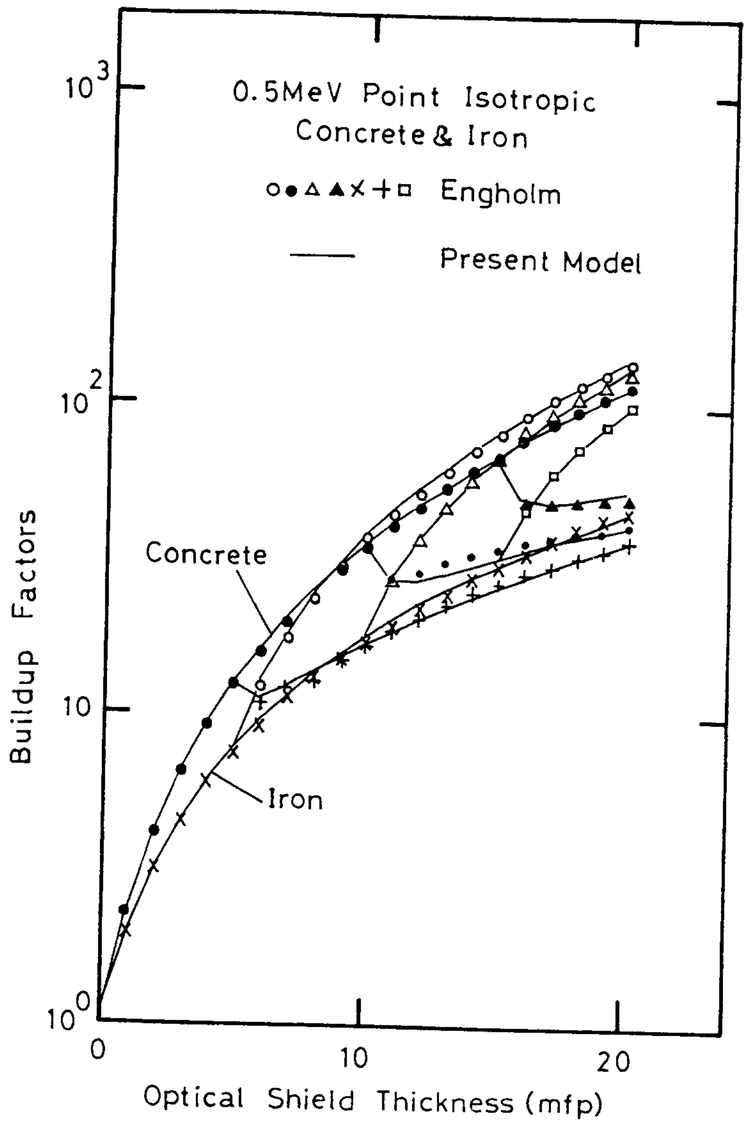

Figure 13 Buildup factors of 0.5-MeV poinst isotropic source for double layered shields composed of concrete and iron.