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A New Arithmetic Optimization Algorithm for Solving Real-World Multiobjective CEC-2021 Constrained Optimization Problems: Diversity Analysis and Validations

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ABSTRACT In this paper, a new Multi-Objective Arithmetic Optimization Algorithm (MOAOA) is proposed for solving Real-World constrained Multi-objective Optimization Problems (RWMOPs). Such problems can be found in different areas, including mechanical engineering, chemical engineering, process and synthesis, and power electronics systems. MOAOA is inspired by the distribution behavior of the main arithmetic operators in mathematics. The proposed multi-objective version is formulated and developed from the recently introduced single-objective Arithmetic Optimization Algorithm (AOA) through an elitist non-dominance sorting and crowding distance-based mechanism. For the performance evaluation of MOAOA, a set of 35 constrained RWMOPs and five ZDT unconstrained problems are considered. For the fitness and efficiency evaluation of the proposed MOAOA, the results obtained from the MOAOA are compared with four other state-of-the-art multi-objective algorithms. In addition, five performance indicators, such as Hyper-Volume (HV), Spread (SP), Inverse Generalized Distance (IGD), Runtime (RT), and Generalized Distance (GD), are calculated for the rigorous evaluation of the performance and feasibility study of the MOAOA. The findings demonstrate the superiority of the MOAOA over other algorithms with high accuracy and coverage across all objectives. This paper also considers the Wilcoxon signed-rank test (WSRT) for the statistical investigation of the experimental study. The coverage, diversity, computational cost, and convergence behavior achieved by MOAOA show its high efficiency in solving ZDT and RWMOPs problems.

INDEX TERMS Arithmetic Optimization Algorithm (AOA); CEC-2021 real-world problems; Constrained optimization; Multi-Objective Arithmetic Optimization Algorithm (MOAOA).

I. INTRODUCTION

Recently, computer technology advancements have increased the quality of addressing complex problems and decreased the time and cost of producing the optimal solution. However, human input is yet needed to determine the best of different solutions. Significant efforts can be seen in the literature to produce a system that optimally solves the given problem without any human effort [1]. One of the most reliable methods to accomplish this depends on optimization techniques. In many instances, most engineering problems, such as city programming, program management, investment decision, control system design, engineering design, and university timetable, the objectives conflict by nature. Thus, one objective cannot be developed without the depravity of another objective. This kind of problem is called multi-objective optimization problems (MOPs), producing various optimal solutions identified as Pareto optimal solutions [2]. Hence, the multi-objective problem also varies from the single-objective.

In multi-objective problems, different tasks are considered to solve the problem: a searching task whose aim is to obtain Pareto optimal solutions and the decision-making task, most of the selected solutions are taken from Pareto optimal solutions. In other words, the two main tasks in multi-objective optimization are to get a set of non-dominated solutions as similarly as possible to the true Pareto optimal Front (PF) and keep a set of well-categorized solutions along with the Pareto optimal front [3]. Therefore, multi-objective methods intend to discover a set of reasonable trade-off solutions, and a decision-maker is required to choose one of them. There are several targets for multi-objective optimization problems, often in dispute, since they are difficult problems to solve because of their complex structure [4]. A selection of candidate solutions used progressively by the optimization technique to solve the given problem is the standard key to such optimization issues. It is called optimum solutions from Pareto. Due to MOPs, arithmetic operators do not apply to multiple optimized solutions [5]-[6]. The optimal dominance theory of Pareto helps to compare two solutions in a multi-objective space. The Pareto optimal solutions demonstrate the best state of equilibrium relating to the given objectives [7]. With generality in mind, the MOPs can be expressed as a minimization concept and expressed as follows.

$$\left.\begin{array}{c}\text{Minimize: } F(\vec{x}) = \left[f_1(\vec{x}), f_2(\vec{x}), \ldots, f_j(\vec{x}), \ldots, f_q(\vec{x})\right] \\ \text{Subjected to:} \\ h_i(\vec{x}) = 0, \;\; i = 1, 2, \ldots, p \\ g(\vec{x}) \geq 0, \;\; i = 1, 2, \ldots, m \\ LB_i \leq x_i \leq UB_i, \;\; i = 1, 2, \ldots, n \end{array}\right\} \quad (1)$$

where *q* denotes a total number of objectives, *m* and *p* denote the number of inequality and equality constraints, respectively, LB_i is the lower bound of the i^{th} variable, and UB_i is the upper bound of the i^{th} variable.

Recently, optimization algorithms have been successfully applied to solve MOPs [8]-[9]. Thanks to their ability to determine a Pareto optimal solution in a specific run, these optimization algorithms tend to be more beneficial than the traditional algorithms. Although optimization algorithms use a set of candidate solutions, they can be expanded to retain varied solutions in a given run. Many optimization algorithms solve MOPs using non-dominated ranking and Pareto strategy to provide different Pareto optimal solutions [10]. This paper explores the non-dominated approach to rank the solutions and crowding distance mechanism to maintain diversified *Pareto* optimal solutions.

The literature indicates that many multi-objective evolutionary algorithms (MOEAs), such as the Non-dominated Sorting Genetic Algorithm (NSGA-II) [9], Decomposition-based Multi-Objective Evolutionary Algorithm (MOEA/D) [11], multi-objective swarm algorithms, such as Multi-Objective Ant Lion Optimization (MOALO) [12], Multi-Objective Grey Wolf Optimizer (MOGWO) [13], and Multi-Objective Particle Swarm Optimization (MOPSO) [14] have been proposed that can successfully approximate the true Pareto-optimal solutions for many MOPs. Nevertheless, the baseline optimization techniques for such MOPs, such as the particle swarm optimization (PSO) for MOPSO, the genetic algorithm (GA) for NSGA-II, grey wolf optimizer (GWO) for MOGWO, and ant lion optimizer (ALO) for MOALO, are not considered to be sufficiently advanced and efficient.

There are a variety of metaheuristics suggested in recent decades. Examples of the very newly enacted approaches of nature-inspired techniques include new techniques focused on grey wolf optimizer (GWO) [15], tunicate swarm optimizer [16], heap optimizer [17], gradient-based optimizer [18], jellyfish optimizer [19], Jaya algorithm [20], and red deer algorithm [21], among others. The readers should go through scientific studies for more details on many other optimization methods [22]. In general, such algorithms' regulating parameters are found to operate with the initial constant value. Such algorithms are indeed not versatile enough to turn their attention to either exploration or exploitation as required.

The No-Free Lunch theorem [23] for the development of optimization allows researchers to improve or refine new optimization algorithms because it logically proves that no single algorithm can solve all optimization problems. This theory provides guidelines for researchers to implement new algorithms or improve existing algorithms to achieve enhanced efficiency. These are indeed the reasons behind the new research described in this paper, in which a Multi-objective Arithmetic Optimization (MOAOA) focused on the newly published Arithmetic Optimization Algorithm (AOA) proposed by Abualigah et al. [24] in 2021 that employs the distribution of leading arithmetic operator's behavior in mathematics.

When working with MOAOA, one of the most important questions is why this algorithm needs to be applied for a constraint optimization problem. The NFL theorem can answer this question, which indicates that no metaheuristic exists to solve all types of practical applications. Due to many optimizers' insufficient accuracy in providing solutions for constraint optimization problems, MOAOA is achieved the best solutions in this work. This motivates us to propose a new metaheuristic multi-objective algorithm to handle the constrained multi-objective problems released by the CEC community. In this paper, MOAOA is proposed to solve various challenging Real-World constrained Multi-Objective Problems (RWMOPs). The proposed MOAOA is formulated as similar to single-objective AOA, and it has been converted as a multi-objective algorithm by utilizing the elitist non-dominance-sorting mechanism. Comprehensive experiments have been conducted on 35 CEC-2021 real-world constrained optimization problems. The results reported that the proposed MOAOA provides a promising performance compared with other multi-objective algorithms reported in the literature. Moreover, MOAOA results in an equilibrium between the exploration and exploitation search approach efficiently. Consequently, the contributions of this paper are as follows.

- A new MOAOA is formulated by employing an elitist non-dominance sorting mechanism to maintain Pareto optimal dominance and a crowding distance mechanism to improve convergence and solution diversity.
- A thorough and informative examination is provided on the performance of the MOAOA on various unconstrained ZDT benchmark test problems, and the performance of the MOAOA is compared with the other algorithms in terms of the Generational Distance (GD), Spread (SD), Hyper-Volume (HV), Runtime (RT), and Inverted Generational Distance (IGD).
- The proposed MOAOA is provided with an updated epsilon constraint-handling mechanism and experimented with CEC-2021 35 challenging real-world constrained MOPs, and the results are compared with other state-of-the-art algorithms.

The rest of this paper is organized as follows. Section 2 discusses the related works. Section 3 briefly explains the basic arithmetic optimization algorithm and explains the procedure to convert AOA into MOAOA. Section 4 provides the experimental results on all five ZDT test suites and 35 RWMOPs. Also, the performance comparison with other state-of-the-art algorithms is discussed in Section 4. Section 5 concludes the paper.

II. LITERATURE REVIEW

This section first introduces the preliminary definitions of multiple-objective optimization, such as Pareto optimal front, Pareto optimal set, Pareto optimal dominance, and Pareto optimality. The definitions are as follows.

Def. 1 Pareto optimal front (POF) [25]:
A set that includes the value of objective functions for the *Pareto* solutions set.

$$P_f \coloneqq \{F(\vec{x}) | \vec{x} \in P_s\} \quad (2)$$

Def. 2 Pareto optimal set (POS) [25]:
The set all *Pareto*-optimal solutions are called *Pareto* set as follows:

$$P_s \coloneqq \{x, y \in X \mid \exists F(\vec{y}) \succ F(\vec{x})\} \quad (3)$$

Def. 3 Pareto Optimality [25]:
A solution $\vec{x} \in X$ is called *Pareto*-optimum if and only if:

$$\nexists\, \vec{y} \in X \mid F(\vec{y}) \prec F(\vec{x}) \quad (4)$$

Def. 4 Pareto Dominance [25]:
Assume two vectors such as: $\vec{x} = (x_1, x_2, \dots, x_k)$ and $\vec{y} = (y_1, y_2, \dots, y_k)$. Vector *x* is said to dominate vector *y* (denote as $\vec{x} \prec \vec{y}$) if and only if:

$$\forall i \in \{1,2,\dots,k\}: f_i(\vec{x}) \le f_i(\vec{y}) \quad \wedge \quad \exists i \in \{1,2,\dots,k\}: f_i(\vec{x}) < f_i(\vec{y}) \quad (5)$$

As shown in Fig. 1, the objective space represents a set of non-dominated solutions called Pareto optimum solutions for maximization or a minimization problem, and the parametric space represents a set of dominated solutions. A relation between parametric spaces to the objective space is called optimum Pareto front (PF).

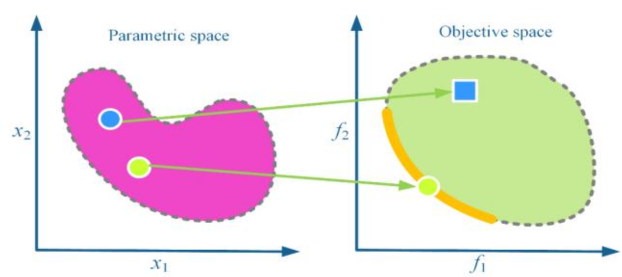

FIGURE 1. Objective space and Parameter space in multi-objective optimization.

The techniques for solving MOPs are primarily divided into a priori and posteriori methods [26]. Priori techniques usually focus on solving MOPs by transforming them to a single objective. Linear programming and weighted-sum methods, introduced in the 1950s, are in this category [3]. Surveys have shown that the priori method is faced with various problems, such as local optimization, large processing time, etc., when handling the MOPs [27]. Posteriori techniques have been developed to establish multiple strategies, and it has significant benefits, such as low computational complexity and generating good results independently of the problem structure. While solving MOPs with multi-objective metaheuristic optimizers (MOMOs), each solution has a quality score dependent on its similarity to PF and spread (diversity). These metrics are used in the identification of parents and in the evaluation of solutions that survives. There are three main steps used to rank the solution [9].

(i) Pareto-based approach
(ii) Indicator-based approach
(iii) Decomposition-based approach

Pareto-based approach - Goldberg [28] initially realized in 1989 that the *Pareto*-dominance principle could be used to evaluate the optimal solution. Oriented by this theory, many MOMOs have proposed a variety of frameworks that use the *Pareto*-dominance rule to calculate the similarity of the optimal solution to PF. For instance, Deb et al. rank the optimum solutions using the non-dominated-ranking approach in the NSGA-II [9], and Zitzler et al. rank the optimum solutions in Strength *Pareto* Evolutionary Algorithm (SPEA2) [29]. In general, multiple methods, such as fitness-sharing, clustering [29], and crowding-distance [9], are used to calculate the Spread of optimal solution in PF. Most of the *Pareto*-based MOMOs are: MOPSO [14], Multi-Objective Multi-Verse Optimizer (MOMVO) [30], Multi-Objective Heat Transfer Search (MOHTS) [31], Non-dominated Sorting MFO (NSMFO) [32], Non-dominated Sorting GWO (NSGWO) [33], Multi-Objective Slime Mould Optimizer (MOSMA) [34], MOALO [12], Non-dominated Sorting WOA (NSWOA) [35], and Multi-Objective Passing Vehicle Search (MOPVS) [31].

Indicator-based approach - Many performance indicators have been suggested in the literature to quantify the level to which the PF obtained by the MOMOs for a problem displays the complete PF in terms of diversity, coverage, and spread. The limited indicators only assess the convergence output (Epsilon [36], GD [37], etc.) or diversity (Spread [36], Spacing [38], etc.) of the PF collected, whereas others assess both diversity and convergence (HV [39], IGD [40], RT [41], etc.). Nowadays, researchers have been using these metrics as indicators to direct the discovery process in solving MOPs. Zitzler and Künzli suggested an indicator-based evolutionary algorithm (IBEA) that measures optimal solution output with quantitative performance indicators [42]. Performance metrics are used in indicator-based algorithms' environmental selection process. There are many metrics for checking the effectiveness of algorithms, which measure diversity and convergence, or both simultaneously. In specific, using the HV indicator [39] or performance metrics based on reference sets such as R2 [43], IGD [40], or Δ_p [44], a reasonably good PF depiction of a MOP can be accomplished. As previously stated, IBEAs based on reference sets rely on the reference set, which is often hard to determine before starting the quest. Nevertheless, numerous studies have discovered new strategies for constructing the reference set, as evidenced by the studies published for IGD/IGD$^+$ [45], R2 [43], and the Δ_p indicator [44]. HV-based IBEAs, on the other hand, only need a single reference vector to calculate the hypervolume indicator. Nevertheless, such methods are restricted by the HV indicator's high computational cost, which rises as the number of objectives rises.

Decomposition-based approach - The POS can be the ideal choice of the scalar function achieved by integrating all the fitness functions of the MOPs. The POF can therefore be decomposed into a variety of scalar optimization problems [46]. Decomposition-based strategies use this core principle to optimize the decomposition of the cost function produced by a certain weight vector. A variety of decomposition-based methods have been discussed and recommendations by researchers. Zhang and Li first introduce the MOEA/D algorithm in [11]. Some of these optimizers are MOEA/D with Uniform Design (MOEA/D-UD) [47], MOEA based on Hierarchical Decomposition (MOEA/HD) [48], MOEA/D with Adaptive Weight Vector Adjustment (MOEA/D-AWA [49], MOGWO/D [50], and MOPSO/D [51].

Works such as [9] and [11] on multi-objective optimization algorithms are suggested for further reading by interested readers. As per the No-Free-Lunch theory, it may now be likely to create a new algorithm that can solve an unsolved problem described in the literature or solve an existing solved problem with improved results. Furthermore, the basic AOA version is claimed to be an easy and straightforward algorithm based on the mathematics operator with very fewer tuning parameters. The AOA was shown to perform very well on constrained and unconstrained benchmark test suites and real-world problems. The convergence and diversity of the solutions are balanced efficiently in AOA. Consequently, compared to several other traditional algorithms, it is very likely that the multi-objective variant of the basic AOA has the maximum performance.

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III. MULTI-OBJECTIVE ARITHMETIC OPTIMIZATION ALGORITHM (MOAOA)

In this section, the single-objective version of AOA is first presented. Then, the multi-objective version is proposed. The computational complexity of MOAOA is discussed in the end.

A. BASIC VERSION OF ARITHMETIC OPTIMIZATION ALGORITHM (AOA)

The principle of the basic model of the AOA is briefly discussed in this section. This algorithm was proposed in [24], which is motivated by the use of arithmetic operators to solve mathematical problems. The arithmetic operators, such as multiplication, division, subtraction, and addition, are utilized in scientific optimization to find the best solution subjected to specific criteria from some set of candidate solutions. The performance of the above-said operators and their impact on the algorithm are discussed in this section. The initialization of the AOA begins with 'n' quantities of initial random solutions where the solution has 'm' control variables. The solution group is then upgraded in each generation 'g' (g=1, 2, 3, ..., g_{max}; g_{max} is the maximum number of generations) to support the four-phase arithmetic operator search process. The better functional value of the modified solution was found to result in greedy selection within the AOA. The best solutions replace the worst solution in the population, and the duplicate solution is replaced by randomly generated solutions following a greedy selection process. For further information on four-phase arithmetic operators, please refer to [24]. The pseudocode of the AOA can be detailed in Fig. 1, and the flowchart of AOA is illustrated in Fig. 2. Fig. 1 and Fig. 2 explain the complete procedure of the AOA in detail.

```
START
Define objective function F(X), population size (n), set number of design variables (m), limits on design variables
(LB,UB), and set termination criterion ('FEmax', or 'gmax'); where, F(X) is the objective function and 'X' is the
design vector. The set of population size (i=1,2,,n), α=5, μ=0.4999.                      /* Initialization /*
Initialize the random generated population within its (LB,UB) bounds and evaluate it.   /* Initialize population /*
FE = 0;
Identify the best solution (Xbest) of the population.
for g = 1 to gmax do                                                      /* Initialize the optimization loop /*
        MOP=1- (g^((1/α) ))/(gmax^((1/α) ) )                                       /* MOP is AOA parameter /*
        MOA=0.2+g × (0.8/gmax)                                                    /* MOA is AOA parameter /*
        for i = 1 to n do                                                          /* Update the population /*
                for j = 1 to m do                                                  /* Update design variable /*
                        r1=rand(0,1)                                   /* r1, r2, and r3 are random numbers /*
                        r2=rand(0,1)
                        r3=rand(0,1)
                        if r1>MOA
                           if r2>0.5 (Exploration Phase)
                             Xi,j'=Xbest÷(MOP+ϵ) ×((UBj-LBj )×μ + LBj )   /* Divide operator (D,"÷") /*
                           else
                             Xi,j'=Xbest×(MOP) ×((UBj-LBj )×μ + LBj )     /* Multiply operator (M,"×")/*
                           end if
                        else
                           if r3>0.5 (Exploitation Phase)
                             Xi,j'=Xbest-(MOP) ×((UBj-LBj )×μ + LBj )  /* Subtraction operator (S,"-") /*
                           else
                             Xi,j'=Xbest+(MOP) ×((UBj-LBj )×μ + LBj )     /* Addition operator (A,"+") /*
                           end if
                        end if
                end for
                FE = FE + 1                                                    /* Count Function evaluation /*
                if F(Xi)<F(Xbest) then                    /* Update the best solution /* /* Greedy selection /*
                        Xbest=Xi
                end if
                if FE ≥ FEmax then                                                  /* Termination criterion /*
                        break optimization loop
                end if
        end for                                                                   /* Population loop ends /*
end for                                                                         /* Optimization loop ends /*
STOP
```

FIGURE 2. Pseudocode of the AOA

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B. MULTI-OBJECTIVE ARITHMETIC OPTIMIZATION ALGORITHM (MOAOA)

The proposed MOAOA utilizes an elitist non-dominated sorting (NDS) approach and diversity maintenance by the crowding distance (CD) framework [9]. The NDS comprises the subsequent phases.

- First, determining the non-dominated solution
- Second, the application of the NDS approach
- For all non-dominated solutions, non-dominated ranking (NDR) is calculated

The ranking procedure occurs between two fronts. The first front solutions assign a ‘0’ index because the solutions are not dominated; simultaneously, the second front solutions are dominated by a minimum of one solution in the first front. The NDR process is illustrated in Fig. 4. Such a non-dominated ranking of the solutions is equal to the solutions that dominate others. The crowding-distance framework is illustrated in Fig. 5, and it is utilized to maintain diversity between the generated solutions.

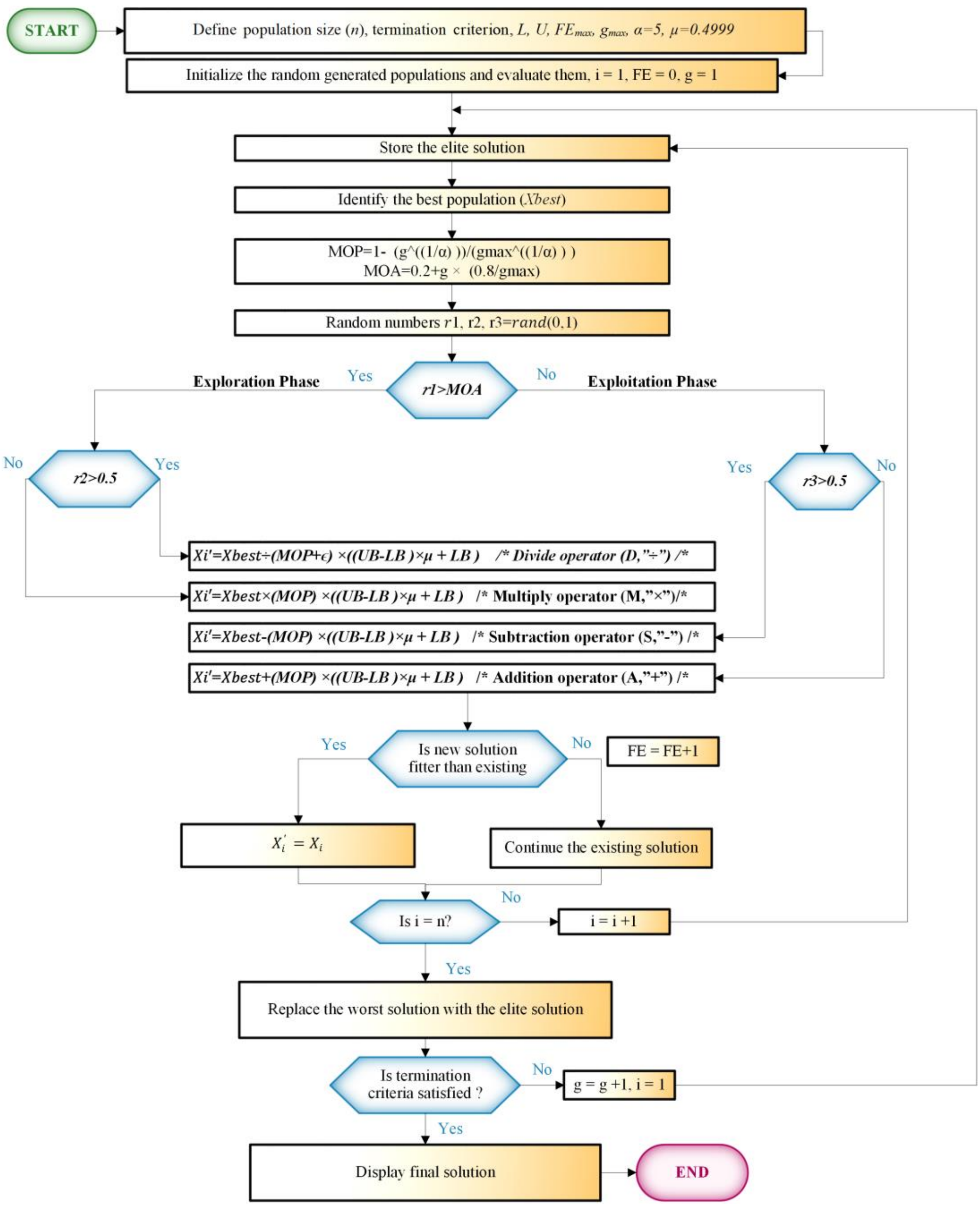

FIGURE 3. Flowchart of the basic version of AOA

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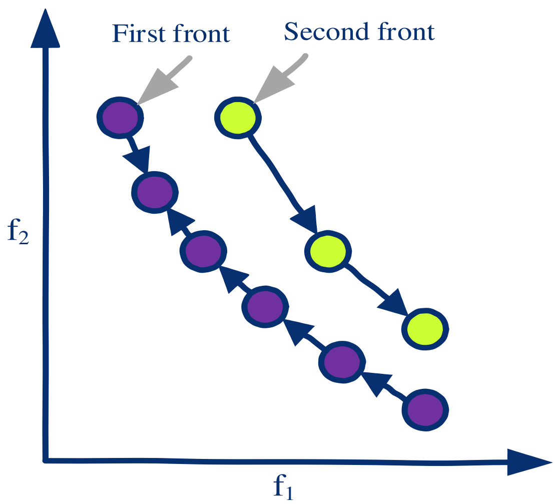

FIGURE 4. Schematic representation of NDS

The crowding-distance framework is well-defined as follows.

$$CD_j^i = \frac{fobj_j^{i+1} - fobj_j^{i-1}}{fobj_j^{max} - fobj_j^{min}} \quad (6)$$

where $fobj_j^{max}$ and $fobj_j^{min}$ are the maximum and minimum values of j^{th}objective function. The diagrammatic illustration of an NDS-based approach is illustrated in Fig. 6.

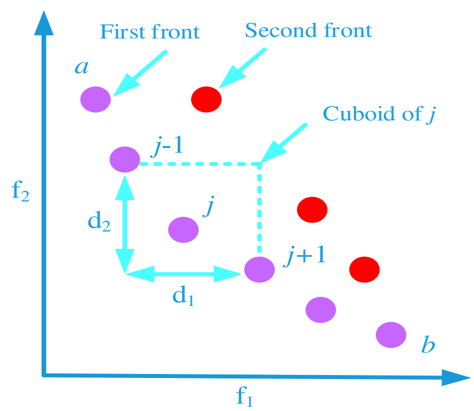

FIGURE 5. Schematic representation of CD mechanism

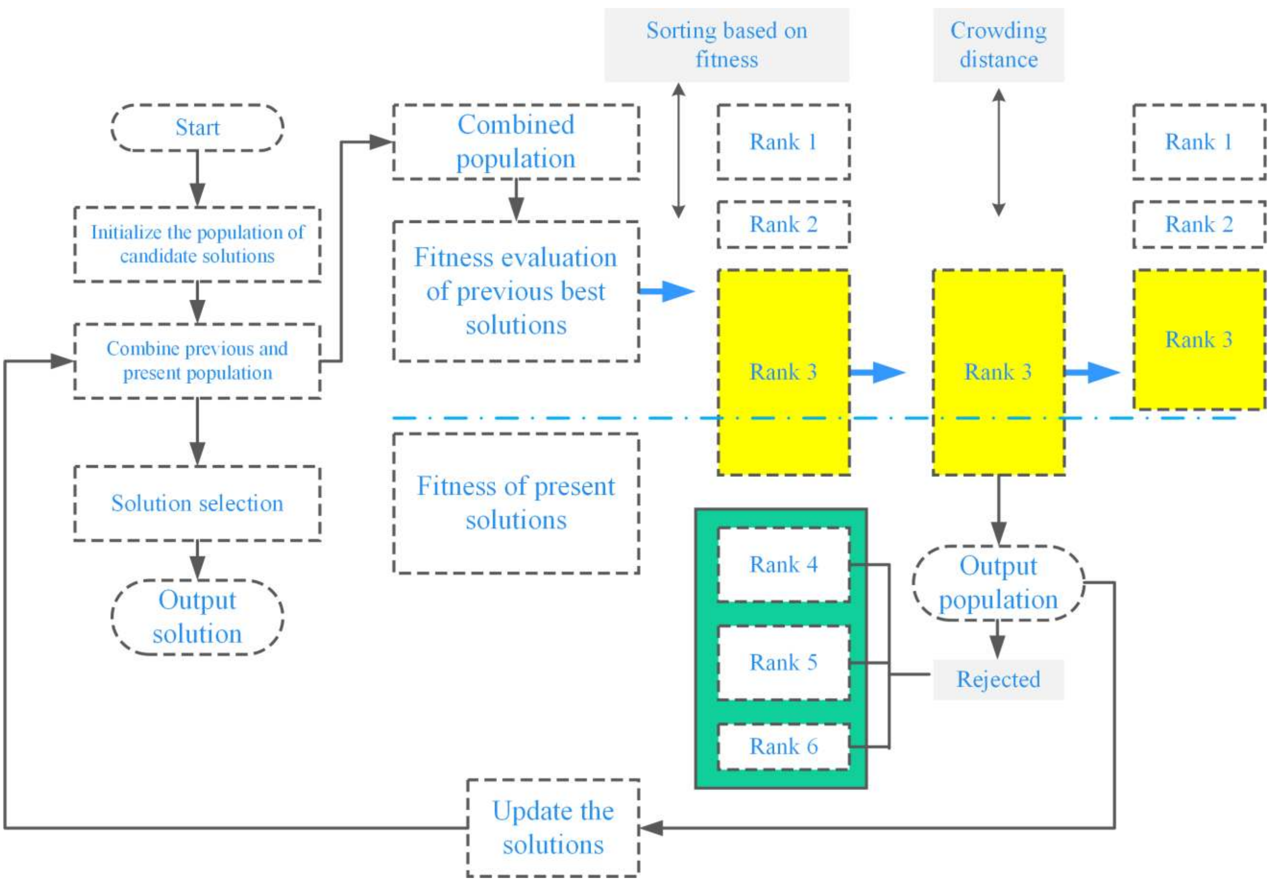

FIGURE 6. Graphical illustration of NDS based algorithm

The pseudocode of the MOAOA is illustrated in *Algorithm 1*. The initial phase of the algorithm is to define the required parameters, such as the maximum number of iteration (IT_{max})/maximum number of generations, population size

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(N_p), and termination criteria. Then, parent population P_o is randomly generated in the region of feasible search space, and each fitness function in the objective vector space F for P_o is assessed. Apply the CD and NDS based on the elitist framework to P_o. The new population of P_j is generated and combined with P_o to obtain population P_i. Then, P_i is arranged based on the elitist non-dominated sorting approach and obtained the values of CD and NDR. The best N_p solutions are reviewed to make an updated parent population. Lastly, this procedure is repetitive till the termination criteria. Fig. 7 shows the flowchart of MOAOA.

***Algorithm 1:* MOAOA-Pseudocode**

Step 1:	Primarily generate random population (P_o) in solution space (S)
Step 2:	Assess objective vector space (F) for the generated P_o
Step 3:	**B**ased on elitist NDS method, sort the solutions and calculate the NDR and fronts
Step 4:	Calculate CD for each front
Step 5:	Update solutions (P_j) using ***Fig. 2***
Step 6:	Merge P_o and P_j to create P_i=P_o U P_j
Step 7:	For P_i perform **Step 2**
Step 8:	Based on NDR and CD sort P_i
Step 9:	Replace P_o with P_i for N_p first members of P_i

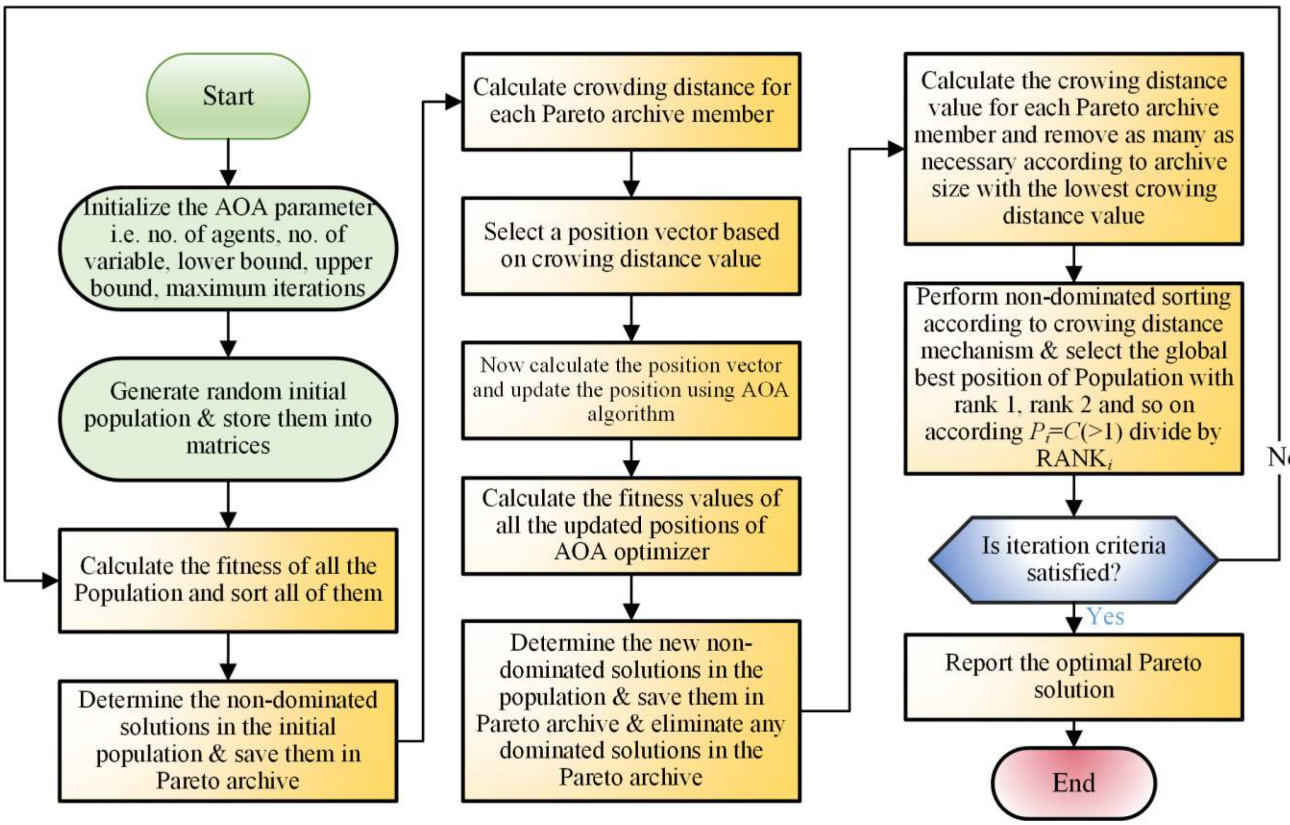

FIGURE 7. Flowchart of the proposed MOAOA

C. COMPUTATION COMPLEXITY OF MOAOA

The computational complexity of the MOAOA algorithm is represented in terms of time and space complexity. As per the previous discussion, the suggested MOAOA utilizes the NSGA-II operators [9]. Subsequently, the concept of CD and NDS mechanisms is taken from NSGA-II. Therefore, the computational space complexity of MOAOA is similar to

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MOMVO, NSGWO, MOALO, and MOSMA optimizers are $O(MN_p)^2$, where N_p is the population size, and M is the total number of objective functions.

IV. SIMULATION RESULTS AND DISCUSSIONS

In order to assess whether the suggested MOAOA is efficient in solving multi-objective optimization problems, several experiments are conducted on the unconstrained ZDT1-4, ZDT6 multi-objective problems with two objectives [52], and the CEC-2021 test problems with two, three, and five objectives [53] with different performance metrics. The proposed MOAOA results are compared with four state-of-the-art optimizers, namely NSGWO [33], MOMVO [30], MOALO [12], and MOSMA [34]. In the following subsection, the test problems and performance metrics adopted are briefly introduced. Afterward, the parameter settings of all algorithms, constraint handling approach, and best-compromised solutions (BCS) approach are introduced. Finally, the experimental results, together with the analysis and the comparative results, are discussed comprehensively.

A. MULTI-OBJECTIVE TEST SUITES

Firstly, in experimentation, the suggested MOAOA is compared with ZDT1-4, ZDT6 from the ZDT test suite, and secondly, challenging Real-world constrained multi-objective problems from the CEC-2021 test suite are selected as the test instances for empirical comparisons in this study for testing the efficiency of the proposed MOAOA on MOPs. The number of objectives $M \subseteq \{2, 3, 5\}$. These test suites are composed of optimization problems with linear, mixed, partially separable, concave, and disconnected *Pareto* optimal fronts characteristics of ZDT and CEC-2021 problems, as shown in Fig. 5.

B. PERFORMANCE METRICS

The generational distance (GD), Spread (SD), hypervolume (HV), runtime (RT), and inverted generational distance (IGD) [54] metrics are selected to evaluate the performance of the proposed MOAOA. HV and IGD deliver joint statistics of the diversity of the obtained set of solutions and convergence. Simultaneously, spread (SD) and GD metrics are the diversity and convergence measure, and RT metric provides average CPU time called computational complexity of each algorithm, respectively. The usage and formulas to calculate all performance metrics are presented in Fig. 9.

C. PARAMETER SETTINGS

For statistical comparisons, all selected algorithms are run 30 times independently on each test instance with the maximum number of function evaluations (MAX_{FES}), for each problem is established [50] as follows.

$$MAX_{FES} = \begin{cases} 2\times10^4, & \text{if } D \le 10, M=2 \\ 8\times10^4, & \text{else if } D > 10, M=2 \\ 2.6250\times10^4, & \text{else if } D \le 10, M=3 \\ 1.05\times10^4, & \text{else if } D > 10, M=3 \\ 5.3*10^4, & \text{else} \end{cases} \quad (7)$$

7 is applicable for all selected algorithms, such as MOAOA, NSGWO, MOMVO, MOALO, and MOSMA. Other specific parameter settings of each algorithm are the same as suggested in the references.

D. CONSTRAINT HANDLING APPROACH

An updated epsilon constraint-handling [55] to handle the constraints is applied to the proposed MOAOA. The formula to handle the constraint is given as follows.

$$\varepsilon(k) = \begin{cases} (1-\tau)\varepsilon(k-1), \text{if } rf_k < \alpha \\ \varepsilon(0)\left(1-\frac{k}{T_c}\right)^{cp}, \text{if } rf_k \ge \alpha \end{cases} \quad (8)$$

where $\tau \in [0, 1]$, τ denotes control parameter to reduce the constraints relaxation in the case of $rf_k < \alpha$, rf_k is the ratio of feasible to infeasible solutions in the k^{th} generation, $\alpha \in [0, 1]$, α controls the searching priority between the infeasible and the feasible regions, *cp* control parameter to reduce the constraints relaxation in the case of $rf_k \ge \alpha$, and $\varepsilon(k)$ is updated till the generation k achieves the control generation T_c.

E. BEST COMPROMISE SOLUTION (BCS) BASED ON FUZZY DECISION

After obtaining the Pareto-optimal package, a fuzzy membership strategy [56] is introduced in this paper to achieve a suitable and BCS over the compromise curve.

$$\mu_i^j = \begin{cases} 1, \ f_i^j \leqslant f_{\min}^j \\ \frac{f_{\max}^j - f_i^j}{f_{\max}^j - f_{\min}^j}, f_{\min}^j \leqslant f_i^j \leqslant f_{\max}^j \\ 0, \ f_i^j \geqslant f_{\max}^j \end{cases} \quad (9)$$

The normalized membership function can be constructed at each non-dominated solution as follows.

$$\mu_i = \frac{\sum_{j=1}^{N_{obj}} \mu_{ij}}{\sum_{i=1}^{M} \sum_{j=1}^{N_{obj}} \mu_{ij}} \quad (10)$$

where M is the number of non-dominated solutions, N_{obj} is the number of the objective functions, and $f_{\max}^{j}$ and $f_{\min}^{j}$ are the maximum and minimum values of the respective objective function. The BCS is the one with a high value of μ_i.

Problems	Name	M	D	ng	nh
Mechanical Design Problems					
RWMOP01	Pressure Vessel Design	2	2	2	2
RWMOP02	Vibrating Platform Design	2	5	5	0
RWMOP03	Two Bar Truss Design	2	3	3	0
RWMOP04	Welded Beam Design	2	4	4	0
RWMOP05	Disc Brake Design	2	4	4	0
RWMOP06	Speed Reducer Design	2	7	11	0
RWMOP07	Gear Train Design	2	4	1	0
RWMOP08	Car Side Impact Design	3	7	9	0
RWMOP09	Four Bar Plane Truss	2	4	1	0
RWMOP10	Two Bar Plane Truss	2	2	2	0
RWMOP11	Water Resources Management	5	3	7	0
RWMOP12	Simply Supported I-beam Design	2	4	1	0
RWMOP13	Gear Box Design	3	7	11	0
RWMOP14	Multiple Disk Clutch Brake Design	2	5	8	0
RWMOP15	Spring Design	2	3	8	0
RWMOP16	Cantilever Beam Design	2	2	2	0
RWMOP17	Bulk Carrier Design	3	6	9	0
RWMOP18	Front Rail Design	2	3	3	0
RWMOP19	Multi-product Batch Plant	3	10	10	0
RWMOP20	Hydro-static Thrust Bearing Design	2	4	7	0
RWMOP21	Crash Energy Management for High-speed Train	2	6	4	0
Chemical Engineering Problems					
RWMOP22	Haverly's Pooling Problem	2	9	2	4
RWMOP23	Reactor Network Design	2	6	1	4
RWMOP24	Heat Exchanger Network Design	3	9	0	6
Process, Design and Synthesis Problems					
RWMOP25	Process Synthesis Problem	2	2	2	0
RWMOP26	Process Synthesis and Design Problem	2	3	1	1
RWMOP27	Process Flow Sheeting Problem	2	3	3	0
RWMOP28	Two Reactor Problem	2	7	4	4
RWMOP29	Process Synthesis Problem	2	7	9	0
Power Electronics Problems					
RWMOP30	Synchronous Optimal Pulse-width Modulation of 3-level Inverters	2	25	24	0
RWMOP31	Synchronous Optimal Pulse-width Modulation of 5-level Inverters	2	25	24	0
RWMOP32	Synchronous Optimal Pulse-width Modulation of 7-level Inverters	2	25	24	0
RWMOP33	Synchronous Optimal Pulse-width Modulation of 9-level Inverters	2	30	29	0
RWMOP34	Synchronous Optimal Pulse-width Modulation of 11-level Inverters	2	30	29	0
RWMOP35	Synchronous Optimal Pulse-width Modulation of 13-level Inverters	2	30	29	0

FIGURE 8. Characteristics of CEC-2021 Real-world constrained multi-objective problems [57]

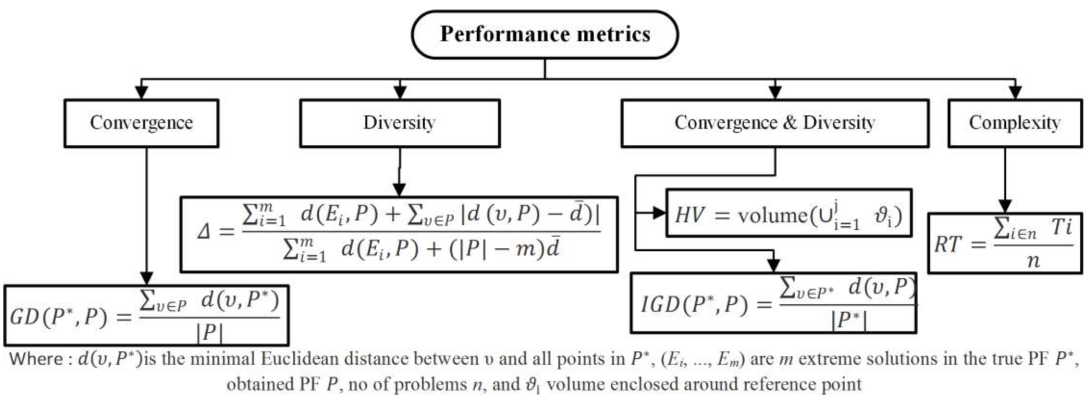

FIGURE 9. Performance metrics of MOPs

F. RESULTS ON ZDT TEST PROBLEMS

Before discussing the performance of the various optimizers for CEC-2021 Real-World constrained optimization, it is interesting to compare them using the standard test suites, which are constrained multi-objective optimization problems, including ZDT1-4 and ZDT6 benchmark problems [52]. The comprehensive experiments are conducted to measure the performance when handling general-purpose multi-objective optimization. The MOAOA, MOMVO, MOALO, MOSMA, and the NSGWO algorithms are chosen to solve the test problems for 30 runs where the comparative results are based on GD, Spread, IGD, HV, and RT indicators are presented in Table 1.

In this table, each cell on the table presents the mean (standard deviation) and the results of the Wilcoxon rank-sum test (WSRT) values obtained from various optimizers. The bold font represents the best performance of all algorithms on the respective problem. From the results, the best GD, SD, IGD, HV, and RT mean values for MOAOA, i.e., 3/5, 4/5, 4/5, 3/5, and 4/5, NSGWO, i.e., 0/5, 0/5, 0/5, 0/5, and 0/5, MOMVO, i.e., 0/5, 1/5, 1/5, 0/5, and 1/5, MOALO, i.e., 0/5, 0/5, 0/5, 1/5, and 0/5, and MOSMA, i.e., 2/5, 0/5, 0/5, 1/5, and 0/5 best results for ZDT1-4, ZDT6 problems. Overall, it was noticed that the proposed MOAOA could expose the best convergence, coverage, diversity, and computational complexity as compared to MOMVO, MOALO, MOSMA, and NSGWO algorithms for the standard multi-objective optimization problem. In the WSRT test, each cell in the last row with $+/-/\approx$ of Table 1 presents the numbers of test instances for which the compared algorithms perform significantly better than, significantly worse than, and statistically similar to the proposed MOAOA, respectively. It can be seen in Table 1 that the MOAOA significantly outperforms the other four algorithms in terms of the GD, IGD, Spread, RT, and HV metrics. Additionally, in Fig. 10 for ZDT problems, evaluating the IGD values versus function evolutions (FEs). Fig. 10 shows that the MOAOA has shown successful convergence ability on ZDT problems. Nevertheless, as stated in the No-Free-Lunch theory, it cannot be guaranteed that a meta-heuristic with good performance when solving a particular problem will be efficient for another one. Thus, the study of applied metaheuristics is always a challenging issue. For the studied CEC-2021 RWMOP, the GD, spread, IGD, HV, and RT comparison outcomes for all considered design problems are discussed in subsequent sections.

TABLE I.
GD/SPREAD/IGD/HV/RT-METRICS (MEAN AND STD VALUES) OF ALL ALGORITHMS ON THE ZDT BENCHMARK TEST SUITE

Problem	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
			GD		
ZDT1	1.1164e-1 (2.17e-2) -	6.5495e-3 (8.86e-4) -	5.2746e-3 (2.07e-3) -	**1.7064e-4 (1.32e-5) =**	1.8394e-4 (2.00e-5)
ZDT2	1.235e+0 (3.04e-1) -	8.9903e-3 (1.93e-3) -	4.8162e-3 (1.52e-3) -	1.9038e-4 (2.49e-5) =	**1.5869e-4 (6.08e-5)**
ZDT3	1.1900e-1 (3.69e-2) -	6.4039e-3 (2.45e-3) -	1.0794e-2 (3.36e-3) -	1.3712e-4 (3.15e-5) =	**1.1025e-4 (1.88e-5)**
ZDT4	8.564e+0 (7.56e+0) -	6.0155e-3 (1.69e-3) -	2.5584e-1 (2.41e-1) -	7.1331e-4 (1.17e-4) =	**4.4424e-4 (8.40e-5)**
ZDT6	1.2526e-1 (1.96e-1) -	6.9421e-3 (2.00e-3) -	9.4918e-2 (2.03e-2) -	**2.4914e-4 (9.49e-5) =**	4.4362e-4 (3.90e-4)
WSRT (+/-/=)	0/5/0	0/5/0	0/5/0	0/0/5	
			SPREAD		
ZDT1	8.7416e-1 (2.75e-2) -	1.4169e-1 (8.32e-3) =	4.9308e-1 (6.24e-2) -	6.3166e-1 (1.14e-1) -	**1.2752e-1 (1.38e-2)**
ZDT2	4.0675e-1 (7.78e-2) -	1.3692e-1 (9.73e-3) =	5.0675e-1 (9.78e-2) -	6.4653e-1 (1.16e-1) -	**1.3684e-1 (1.54e-2)**
ZDT3	8.8486e-1 (2.94e-2) -	1.6715e-1 (3.81e-2) =	4.9998e-1 (1.08e-1) -	8.6684e-1 (1.16e-1) -	**1.6115e-1 (5.48e-3)**
ZDT4	9.828e-1 (2.19e-2) =	**1.464e-1 (1.99e-2) =**	7.9300e-1 (1.73e-1) -	1.0673e+0 (3.60e-2) -	1.6418e-1 (2.41e-2)
ZDT6	1.164e+0 (2.63e-1) -	1.4807e-1 (5.20e-2) =	4.3637e-1 (4.90e-2) -	5.6692e-1 (6.20e-2) -	**1.4390e-1 (2.08e-2)**
+/-/=	0/3/1	0/0/5	0/5/0	0/5/0	
			IGD		
ZDT1	1.154e+0 (1.56e-1) -	4.3795e-3 (4.47e-3) =	5.3472e-2 (2.15e-2) -	7.3136e-2 (1.28e-4) -	**4.1437e-3 (8.17e-5)**
ZDT2	1.317e+0 (2.88e-1) -	**4.4717e-3 (1.19e-2) =**	5.7696e-2 (1.49e-2) -	1.7108e-2 (1.68e-4) -	4.6360e-3 (2.68e-4)
ZDT3	7.7030e-1 (3.15e-1) -	5.2142e-3 (1.31e-2) =	1.0487e-1 (3.42e-2) -	5.1988e-2 (2.37e-4) -	**5.0147e-3 (1.89e-4)**
ZDT4	2.796e+1 (2.58e+0) -	8.8661e-3 (1.71e-1) =	2.5180e-1 (5.22e-1) -	8.0786e-1 (1.46e-3) -	**6.2557e-3 (7.17e-4)**
ZDT6	5.854e-3 (2.21e-2) =	5.2947e-3 (7.67e-3) =	3.4416e-2 (5.50e-4) -	4.7419e-3 (5.87e-4) -	**3.2705e-3 (2.90e-3)**
+/-/=	0/4/1	0/0/5	0/5/0	0/5/0	
			HV		
ZDT1	6.3320e-3 (1.27e-2) -	6.6097e-1 (6.48e-3) -	6.5377e-1 (2.61e-2) -	**7.1875e-1 (2.52e-4) =**	7.1863e-1 (1.85e-4)
ZDT2	0.000e+0 (0(e-0)) -	3.7337e-1 (1.63e-2) -	3.8052e-1 (2.07e-2) -	4.4288e-1 (4.31e-4) =	**4.4331e-1 (7.25e-4)**
ZDT3	4.9129e-2 (5.57e-2) -	5.5975e-1 (8.04e-3) -	5.3850e-1 (2.70e-2) -	5.9858e-1 (2.91e-4) =	**5.9870e-1 (2.63e-4)**
ZDT4	0.000e+0 (0(e-0)) -	5.9421e-1 (1.10e-1) -	1.3824e-1 (1.67e-1) -	7.1127e-1 (2.24e-3) =	**7.1489e-1 (1.30e-3)**
ZDT6	2.886e-1 (1.92e-1) =	3.3594e-1 (9.51e-3) -	**3.8688e-1 (6.09e-4) =**	3.8610e-1 (1.12e-3) =	3.8396e-1 (4.80e-3)
+/-/=	0/4/1	0/5/0	0/4/1	0/0/5	

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RUNTIME					
ZDT1	4.73E+00	**1.08E+00**	5.70E+00	9.30E+00	1.09E+00
ZDT2	4.07E+00	9.88E-01	5.69E+00	7.70E+00	**8.18E-01**
ZDT3	4.39E+00	1.02E+00	5.48E+00	8.01E+00	**1.00E+00**
ZDT4	3.09E+00	8.81E-01	6.06E+00	6.23E+00	**8.30E-01**
ZDT6	3.28E+00	9.94E-01	5.78E+00	6.51E+00	**8.47E-01**
Time Complexity	3.91E+00	9.93E-01	5.74E+00	7.55E+00	**9.18E-01**

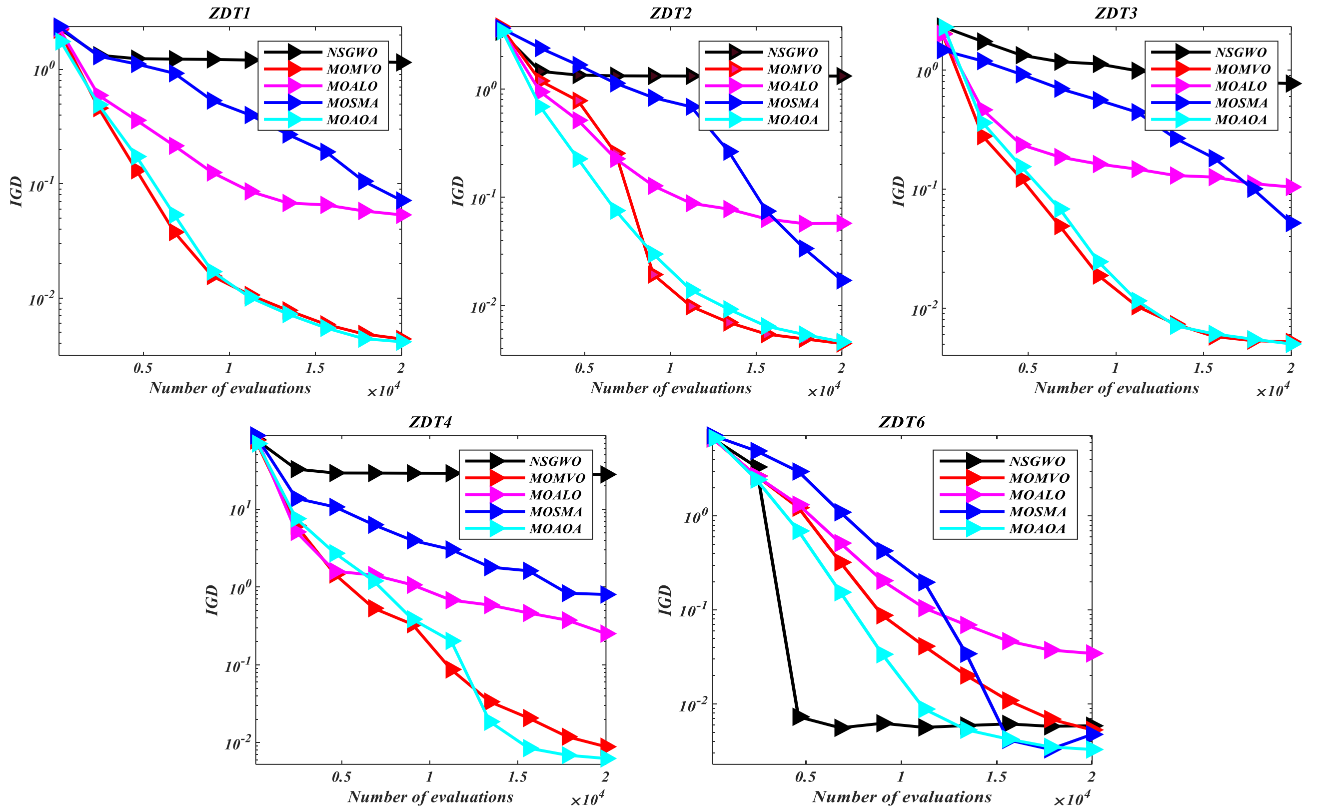


FIGURE 10. Curves of the mean IGD values versus FEs on the ZDT1-4, ZDT6 test benchmarks

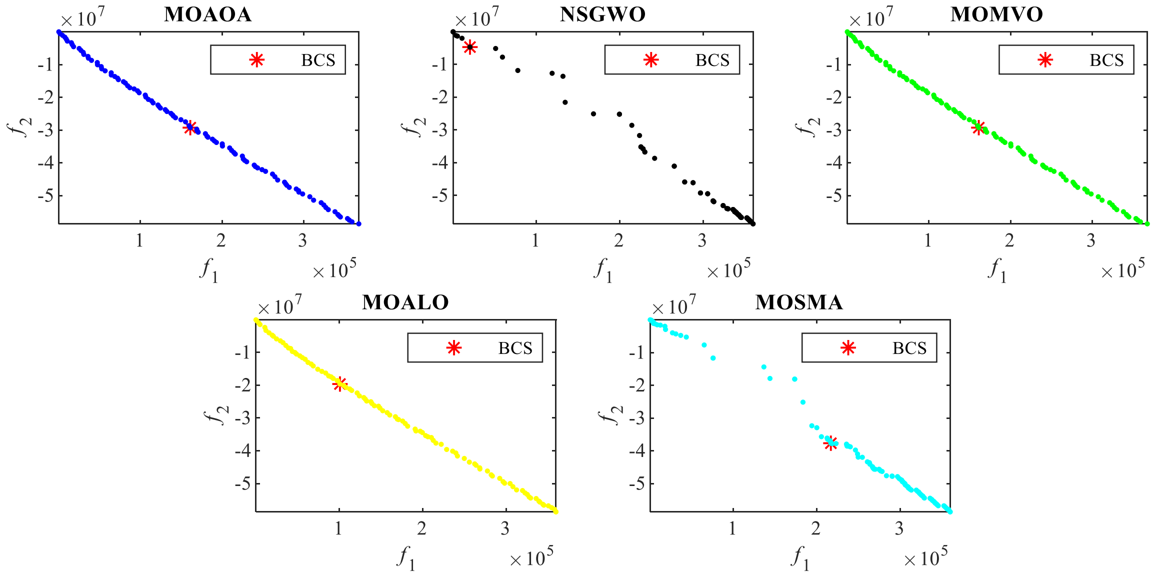
G. RESULTS ON CEC-2021 REAL-WORLD CONSTRAINED OPTIMIZATION PROBLEMS

Recently, 35 CEC-2021 real-world constrained optimization problems are released by the optimization community to make a challenging test suite for evaluating the efficiency of various algorithms [57]. CEC-2021 RWMOPs are combinations of mechanical design (RWMOP1-RWMOP21) problems, chemical engineering (RWMOP22-RWMOP24) problems, process, synthesis, and design (RWMOP25-RWMOP29) problems, and power electronics (RWMOP30-RWMOP35) problems [57]. Basic descriptions of these problems, such as the number of objective functions (M), number of decision variables (D), number of equality constraints (n_h), and inequality constraints (n_g), are reported in Fig. 8. As illustrated in Fig. 8, M varies from 2 to 5, D varies from 2 to 34, n_g varies from 0 to 29, and n_h vary from 0 to 26. Here, two algorithms, such as self-adaptive spherical search optimizer [58] and modified covariance matrix adaptation evolution strategy [59], are used to calculate the ideal and nadir points of all objectives of all problems of the test suite as these algorithms are the top-ranked algorithms of special session & competition on real-world constrained optimization organized at WCCI 2020 and GECCO 2020. The proposed MOAOA successfully solved a variety of ZDT1-4 and ZDT6 test suites. Therefore, it is appropriate to apply and evaluate its performance over challenging real-world CEC-2021 problems. In all the selected problems, constraints are handled using the penalty function approach [55] and the Fuzzy-based [56] approach to locate the best compromise solution (BCS) in obtained PF for each problem. To further verify the effectiveness of the MOAOA in solving CEC-2021, the above cases are optimized using NSGWO, MOMVO, MOALO, and MOSMA, and the comparisons of the optimized results are discussed. In each case, all the five algorithms are run independently 30 times, and the obtained results are shown and discussed in this section.

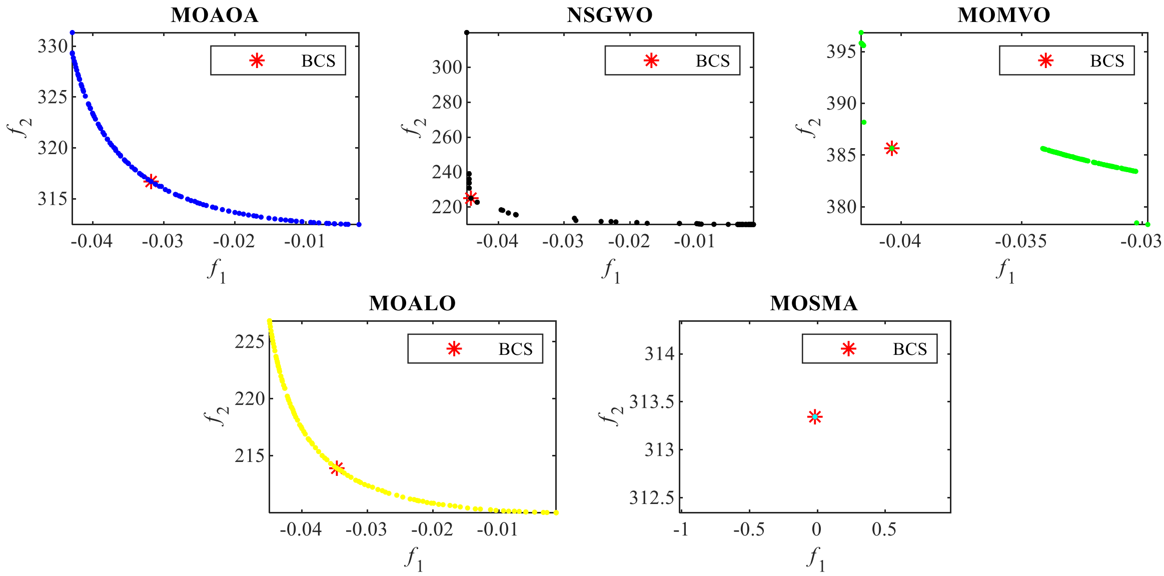
G.1. RESULTS ON CEC-2021 MECHANICAL DESIGN PROBLEMS (RWMOP1-RWMOP21)

The qualitative and quantitative results obtained by MOAOA, NSGWO, MOMVO, MOALO, and MOSMA while solving mechanical design problems are collectively described in Table 2-6. Fig. 11 shows the best PF and BCS of all the problems for visualizing the performance of the MOAOA.

RWMOP1



RWMOP2



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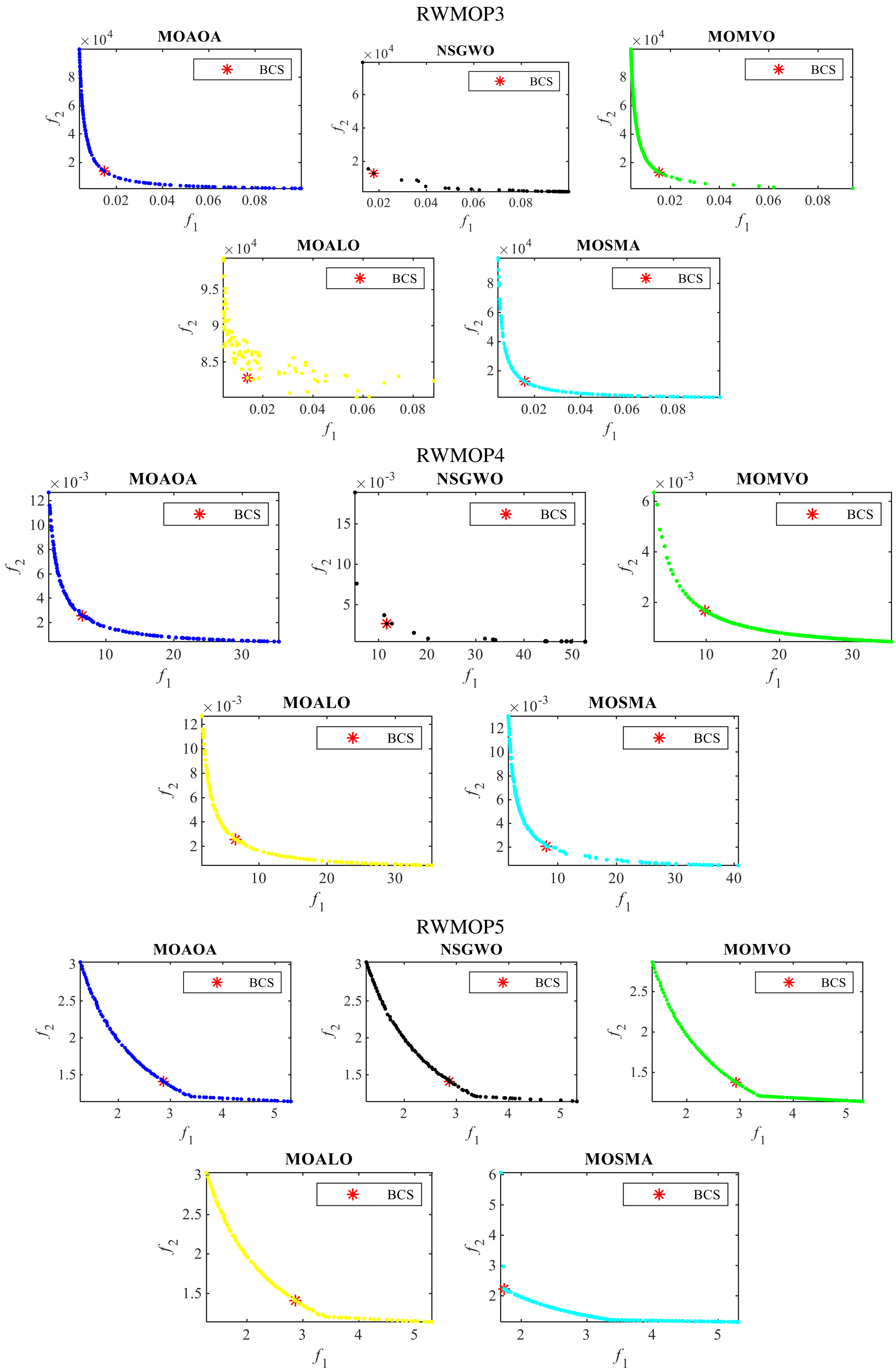

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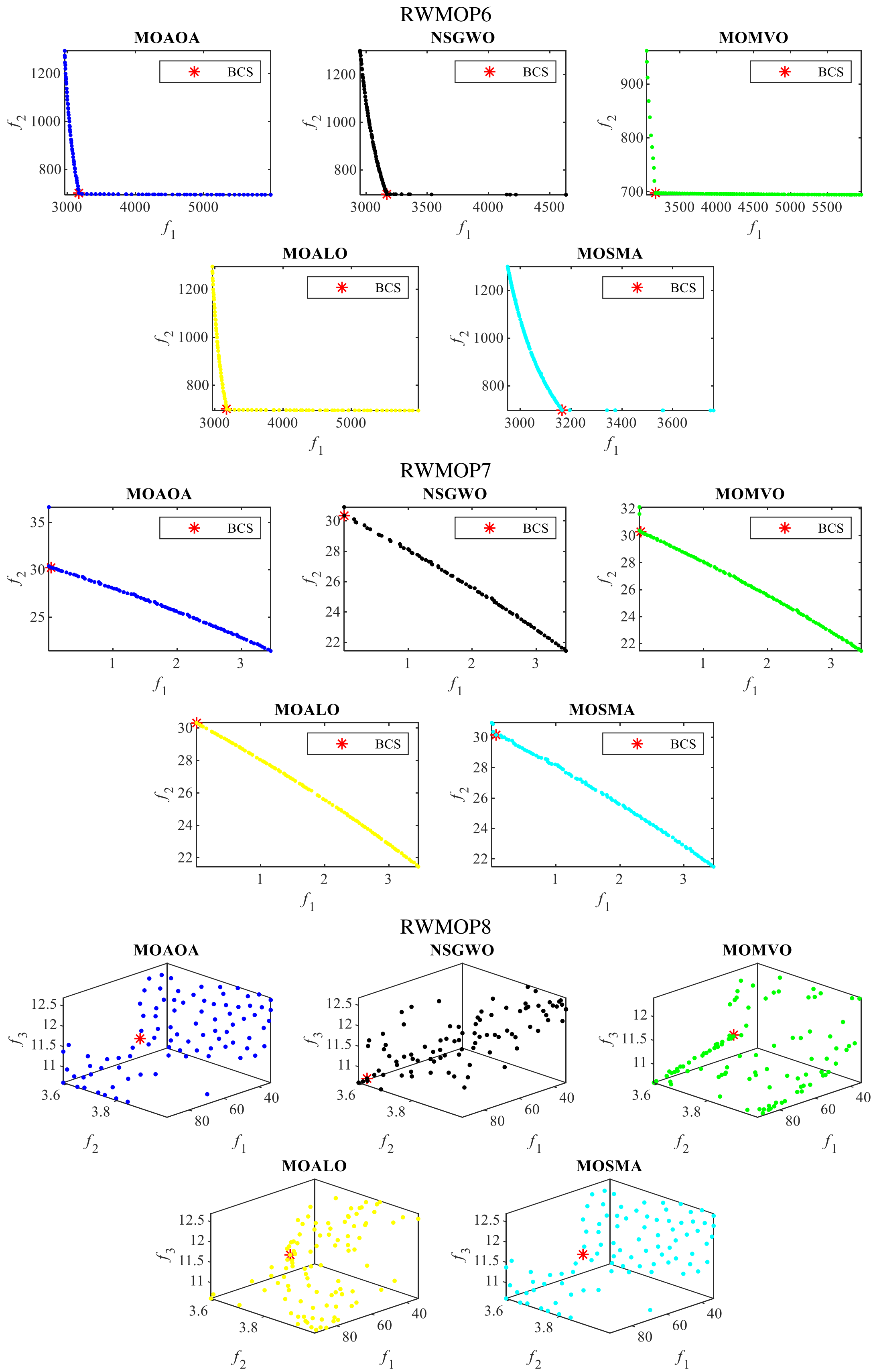

FIGURE 11. PFs of all the algorithms on mechanical design (RWMOP1-RWMOP21) problems (the rest of the FIGUREs can be found in the appendix)

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TABLE II.
GD METRIC RESULTS OF VARIOUS OPTIMIZERS ON MECHANICAL DESIGN PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP1	20000	5.9671e+6 (1.26e+6)	3.5345e+6 (5.28e+4)	4.6138e+6 (2.67e+5)	3.4959e+6 (2.20e+4)	**3.4094e+6 (5.18e+4)**
RWMOP2	20000	1.0535e+1 (8.16e-2)	2.6332e+1 (3.21e+1)	2.9037e+2 (4.23e+2)	**6.4018e+0 (6.98e+0)**	9.1620e+0 (1.57e+0)
RWMOP3	20000	9.6759e+3 (5.27e+1)	1.0913e+4 (5.48e+3)	8.3295e+3 (1.25e+2)	7.7001e+3 (1.38e+2)	**5.7943e+3 (8.79e+1)**
RWMOP4	20000	2.4710e+0 (8.65e-1)	2.6080e+0 (4.82e-2)	2.8529e+0 (5.98e-2)	2.6073e+0 (8.41e-2)	**2.0188e+0 (4.44e-2)**
RWMOP5	20000	3.2927e-1 (4.79e-3)	3.1728e-1 (4.25e-4)	2.9704e-1 (1.69e-2)	3.1200e-1 (3.53e-3)	**2.9310e-1 (1.62e-3)**
RWMOP6	20000	2.9101e+2 (2.77e+0)	2.4293e+2 (1.52e+0)	2.9408e+2 (1.10e+0)	2.5373e+2 (1.67e+1)	**2.4138e+2 (5.22e+1)**
RWMOP7	20000	1.9792e+0 (1.12e-2)	1.9617e+0 (9.54e-4)	1.9584e+0 (1.07e-2)	**1.9569e+0 (1.57e-2)**	1.9695e+0 (1.32e-2)
RWMOP8	26250	4.2462e+0 (2.20e-1)	4.1297e+0 (7.43e-2)	4.7562e+0 (7.09e-2)	4.1225e+0 (1.61e-1)	**3.6860e+0 (7.12e-2)**
RWMOP9	20000	1.3101e+2 (5.43e+0)	1.0805e+2 (2.86e+0)	1.2820e+2 (2.66e+0)	1.1021e+2 (7.29e-1)	**9.4923e+1 (3.23e-1)**
RWMOP10	20000	1.4653e+1 (1.49e-1)	1.2743e+1 (1.94e-1)	1.4562e+1 (1.21e-1)	1.2790e+1 (3.99e-2)	**9.1378e+0 (6.52e-2)**
RWMOP11	53000	5.3046e+5 (9.23e+3)	4.7339e+5 (8.66e+3)	5.4132e+5 (8.10e+3)	4.8700e+5 (1.22e+4)	**4.2443e+5 (2.51e+3)**
RWMOP12	20000	2.7771e+1 (1.60e+0)	2.4589e+1 (2.88e-1)	2.3257e+1 (1.64e+0)	2.4145e+1 (6.09e-1)	**2.1495e+1 (2.19e-1)**
RWMOP13	26250	2.7968e+2 (8.05e-1)	2.5387e+2 (4.35e+0)	2.6692e+2 (4.60e+0)	2.5099e+2 (7.44e+0)	**2.0187e+2 (3.35e+0)**
RWMOP14	20000	1.0775e-1 (3.45e-3)	8.5385e-2 (1.10e-3)	9.4933e-2 (4.39e-3)	8.4988e-2 (1.11e-3)	**6.5349e-2 (9.46e-4)**
RWMOP15	20000	1.4954e+4 (8.39e+3)	8.9207e+3 (1.91e+2)	3.5864e+4 (1.44e+4)	9.0699e+3 (1.52e+2)	**8.4701e+3 (3.34e+2)**
RWMOP16	20000	2.1899e-1 (3.03e-3)	1.9667e-1 (2.44e-3)	2.0713e-1 (4.99e-3)	1.9647e-1 (1.47e-3)	**1.5060e-1 (1.83e-3)**
RWMOP17	26250	7.5967e+7 (1.52e+8)	**1.7744e+7 (3.31e+7)**	7.6574e+8 (8.65e+8)	3.6860e+7 (3.73e+7)	1.0370e+8 (1.82e+8)
RWMOP18	20000	**1.3804e-2 (2.47e-4)**	1.4640e-2 (1.05e-4)	1.4293e-2 (3.00e-4)	1.4711e-2 (1.79e-4)	1.4175e-2 (1.17e-4)
RWMOP19	26250	1.6336e+4 (5.83e+2)	1.5250e+4 (5.57e+2)	2.7503e+4 (1.43e+4)	1.4003e+4 (8.33e+2)	**1.3903e+4 (7.37e+2)**
RWMOP20	20000	3.3715e+3 (1.83e+3)	1.3829e+3 (7.93e+2)	4.2546e+2 (1.30e+2)	**3.9614e+2 (8.53e+1)**	7.0704e+2 (2.53e+2)
RWMOP21	20000	4.6959e-1 (3.12e-2)	4.1156e-1 (5.20e-3)	4.5520e-1 (2.32e-2)	4.0392e-1 (4.45e-3)	**3.9555e-1 (6.27e-3)**

TABLE III.
SD METRIC RESULTS OF VARIOUS OPTIMIZERS ON MECHANICAL DESIGN PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP1	20000	3.6042e+5 (1.07e+3)	3.6191e+5 (2.11e+3)	3.6259e+5 (5.48e+3)	**3.5996e+5 (7.50e+2)**	3.9525e+5 (4.39e+4)
RWMOP2	20000	8.3127e+1 (1.54e+1)	9.7874e+1 (2.00e+1)	2.9956e+2 (4.18e+2)	6.5762e+1 (4.31e+1)	**3.2343e+1 (2.50e+1)**
RWMOP3	20000	5.7138e+2 (4.54e+2)	4.0598e+3 (4.85e+3)	2.1311e+3 (1.69e+3)	**1.1680e+2 (1.59e+2)**	6.4856e+4 (3.09e+4)
RWMOP4	20000	1.3117e+0 (4.12e-2)	1.3393e+0 (3.12e-2)	**2.6754e-1 (1.71e-1)**	1.2656e+0 (9.05e-2)	1.1704e+0 (1.10e+0)
RWMOP5	20000	1.8881e+0 (1.59e-4)	1.8873e+0 (1.43e-3)	1.8884e+0 (4.77e-4)	1.8884e+0 (2.02e-4)	**1.8830e+0 (5.26e-3)**
RWMOP6	20000	1.4156e+3 (9.47e+2)	**6.0545e+2 (1.77e-1)**	2.6232e+3 (2.83e+2)	8.6452e+2 (5.15e+2)	1.9221e+3 (7.10e+2)
RWMOP7	20000	1.4717e+1 (7.68e-1)	**1.2137e+1 (5.81e+0)**	1.4068e+1 (6.02e-1)	1.3857e+1 (2.94e+0)	1.5326e+1 (2.81e-1)
RWMOP8	26250	2.1298e+0 (9.20e-5)	2.1299e+0 (0(e-0))	**1.9357e+0 (3.66e-1)**	2.1288e+0 (1.98e-3)	2.1142e+0 (3.12e-2)
RWMOP9	20000	**3.7239e-2 (0(e-0))**	3.7239e-2 (0(e-0))	2.3151e+2 (5.16e+1)	3.723e-2 (1.91e-10)	1.7305e+1 (2.64e+1)
RWMOP10	20000	4.0912e-3 (4.80e-3)	2.2411e-2 (2.70e-2)	1.1286e+1 (6.57e+0)	**3.9048e-3 (3.14e-3)**	1.2710e+1 (8.25e+0)
RWMOP11	53000	**2.3913e+6 (5.84e+4)**	2.3982e+6 (3.09e+4)	2.4934e+6 (2.32e+4)	2.4669e+6 (1.90e+4)	2.5334e+6 (3.77e+4)
RWMOP12	20000	2.0780e+0 (2.29e+0)	**1.6201e+0 (1.17e+0)**	2.1023e+1 (3.33e+1)	1.9430e+0 (2.54e+0)	1.3371e+1 (3.84e+0)
RWMOP13	26250	**3.4776e+2 (8.41e+0)**	4.7863e+2 (6.75e+1)	6.2192e+2 (3.33e+2)	4.2371e+2 (1.16e+2)	6.4315e+2 (1.42e+1)
RWMOP14	20000	**1.2137e-2 (0(e-0))**	1.2137e-2 (0(e-0))	2.6082e-1 (2.92e-1)	1.2137e-2 (6.91e-9)	1.2710e-1 (1.69e-1)
RWMOP15	20000	3.0086e+3 (3.98e+3)	**5.7037e+2 (3.53e+2)**	2.1326e+4 (1.91e+4)	2.7765e+3 (4.04e+3)	1.8136e+4 (8.79e+3)
RWMOP16	20000	1.9989e-3 (0(e-0))	1.9989e-3 (0(e-0))	1.9989e-3 (0(e-0))	1.9989e-3 (0(e-0))	**1.9989e-3 (0(e-0))**
RWMOP17	26250	4.9943e+3 (7.64e+2)	6.2638e+3 (3.24e+3)	1.1592e+4 (8.90e+3)	**4.8889e+3 (3.42e+2)**	1.5132e+4 (2.38e+4)
RWMOP18	20000	9.4534e-2 (4.53e-5)	9.4473e-2 (7.32e-5)	**9.4196e-2 (1.73e-4)**	9.4312e-2 (1.54e-4)	9.4426e-2 (2.49e-4)
RWMOP19	26250	1.1433e+5 (1.15e+4)	6.7975e+4 (2.47e+4)	1.4073e+5 (2.10e+4)	8.1388e+4 (2.23e+4)	**5.5351e+4 (2.42e+4)**

RWMOP20	20000	3.1697e+3 (1.24e+3)	2.6368e+3 (1.04e+3)	2.4816e+3 (1.07e+2)	**2.0068e+3 (2.64e+2)**	4.2110e+3 (1.26e+3)
RWMOP21	20000	1.8529e-1 (2.57e-1)	1.6050e-2 (0(e-0))	7.1704e-1 (3.59e-1)	1.6050e-2 (6.78e-7)	**1.6050e-2 (0(e-0))**

TABLE IV.
IGD METRIC RESULTS OF VARIOUS OPTIMIZERS ON MECHANICAL DESIGN PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP1	20000	3.6042e+5 (1.07e+3)	3.6191e+5 (2.11e+3)	3.6259e+5 (5.48e+3)	**3.5996e+5 (7.50e+2)**	3.9525e+5 (4.39e+4)
RWMOP2	20000	8.3127e+1 (1.54e+1)	9.7874e+1 (2.00e+1)	2.9956e+2 (4.18e+2)	**3.2343e+1 (2.50e+1)**	6.5762e+1 (4.31e+1)
RWMOP3	20000	5.7138e+2 (4.54e+2)	4.0598e+3 (4.85e+3)	2.1311e+3 (1.69e+3)	**1.1680e+2 (1.59e+2)**	6.4856e+4 (3.09e+4)
RWMOP4	20000	1.3117e+0 (4.12e-2)	1.3393e+0 (3.12e-2)	**2.6754e-1 (1.71e-1)**	1.2656e+0 (9.05e-2)	1.1704e+0 (1.10e+0)
RWMOP5	20000	1.8881e+0 (1.59e-4)	1.8873e+0 (1.43e-3)	1.8884e+0 (4.77e-4)	1.8884e+0 (2.02e-4)	**1.8830e+0 (5.26e-3)**
RWMOP6	20000	1.4156e+3 (9.47e+2)	**6.0545e+2 (1.77e-1)**	2.6232e+3 (2.83e+2)	8.6452e+2 (5.15e+2)	1.9221e+3 (7.10e+2)
RWMOP7	20000	1.4717e+1 (7.68e-1)	**1.2137e+1 (5.81e+0)**	1.4068e+1 (6.02e-1)	1.3857e+1 (2.94e+0)	1.5326e+1 (2.81e-1)
RWMOP8	26250	2.1298e+0 (9.20e-5)	2.1299e+0 (0(e-0))	**1.9357e+0 (3.66e-1)**	2.1288e+0 (1.98e-3)	2.1142e+0 (3.12e-2)
RWMOP9	20000	**3.7239e-2 (0(e-0))**	3.7239e-2 (0(e-0))	2.3151e+2 (5.16e+1)	3.7239e-2 (1.91e-10)	1.7305e+1 (2.64e+1)
RWMOP10	20000	4.0912e-3 (4.80e-3)	2.2411e-2 (2.70e-2)	1.1286e+1 (6.57e+0)	1.2710e+1 (8.25e+0)	**3.9048e-3 (3.14e-3)**
RWMOP11	53000	**2.3913e+6 (5.84e+4)**	2.3982e+6 (3.09e+4)	2.4934e+6 (2.32e+4)	2.4669e+6 (1.90e+4)	2.5334e+6 (3.77e+4)
RWMOP12	20000	2.0780e+0 (2.29e+0)	**1.6201e+0 (1.17e+0)**	2.1023e+1 (3.33e+1)	1.9430e+0 (2.54e+0)	1.3371e+1 (3.84e+0)
RWMOP13	26250	**3.4776e+2 (8.41e+0)**	4.7863e+2 (6.75e+1)	6.2192e+2 (3.33e+2)	4.2371e+2 (1.16e+2)	6.4315e+2 (1.42e+1)
RWMOP14	20000	**1.2137e-2 (0(e-0))**	1.2137e-2 (0(e-0))	2.6082e-1 (2.92e-1)	1.2137e-2 (6.91e-9)	1.2710e-1 (1.69e-1)
RWMOP15	20000	3.0086e+3 (3.98e+3)	**5.7037e+2 (3.53e+2)**	2.1326e+4 (1.91e+4)	2.7765e+3 (4.04e+3)	1.8136e+4 (8.79e+3)
RWMOP16	20000	1.9989e-3 (0(e-0))	1.9989e-3 (0(e-0))	1.9989e-3 (0(e-0))	1.9989e-3 (0(e-0))	**1.9989e-3 (0(e-0))**
RWMOP17	26250	4.9943e+3 (7.64e+2)	6.2638e+3 (3.24e+3)	1.1592e+4 (8.90e+3)	**4.8889e+3 (3.42e+2)**	1.5132e+4 (2.38e+4)
RWMOP18	20000	9.4534e-2 (4.53e-5)	9.4473e-2 (7.32e-5)	**9.4196e-2 (1.73e-4)**	9.4312e-2 (1.54e-4)	9.4426e-2 (2.49e-4)
RWMOP19	26250	1.1433e+5 (1.15e+4)	6.7975e+4 (2.47e+4)	1.4073e+5 (2.10e+4)	8.1388e+4 (2.23e+4)	**5.5351e+4 (2.42e+4)**
RWMOP20	20000	3.1697e+3 (1.24e+3)	2.6368e+3 (1.04e+3)	2.4816e+3 (1.07e+2)	**2.0068e+3 (2.64e+2)**	4.2110e+3 (1.26e+3)
RWMOP21	20000	1.8529e-1 (2.57e-1)	1.6050e-2 (0(e-0))	7.1704e-1 (3.59e-1)	1.6050e-2 (6.78e-7)	**1.6050e-2 (0(e-0))**

TABLE V.
HV METRIC RESULTS OF VARIOUS OPTIMIZERS ON MECHANICAL DESIGN PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP1	20000	6.0546e-1 (9.45e-4)	4.6638e-1 (5.87e-2)	5.5263e-1 (1.66e-2)	6.0516e-1 (4.14e-4)	**6.0787e-1 (4.61e-4)**
RWMOP2	20000	2.9329e-1 (1.96e-1)	**3.9056e-1 (1.53e-3)**	2.6234e-2 (5.25e-2)	2.2138e-1 (9.08e-2)	2.9469e-1 (1.96e-1)
RWMOP3	20000	8.9868e-1 (6.09e-4)	8.2941e-1 (1.46e-2)	9.0146e-1 (9.63e-5)	**9.0200e-1 (1.69e-4)**	6.3605e-1 (3.02e-1)
RWMOP4	20000	8.5550e-1 (3.43e-3)	8.1688e-1 (5.99e-2)	8.5623e-1 (1.30e-3)	8.5929e-1 (3.45e-3)	**8.6183e-1 (4.32e-4)**
RWMOP5	20000	4.3378e-1 (4.67e-4)	4.3313e-1 (2.98e-4)	4.2431e-1 (7.17e-3)	4.3304e-1 (1.09e-3)	**4.3447e-1 (1.93e-4)**
RWMOP6	20000	2.7594e-1 (1.65e-3)	2.7647e-1 (3.38e-4)	2.7668e-1 (2.30e-4)	**2.7716e-1 (2.89e-5)**	2.7677e-1 (1.16e-4)
RWMOP7	20000	4.8433e-1 (6.68e-5)	4.8337e-1 (1.26e-4)	4.8354e-1 (1.74e-4)	4.8396e-1 (6.66e-5)	**4.8436e-1 (1.04e-4)**
RWMOP8	26250	**2.5946e-2 (6.91e-5)**	2.3449e-2 (4.86e-4)	2.5837e-2 (1.40e-4)	2.5862e-2 (1.20e-4)	2.5704e-2 (1.06e-4)
RWMOP9	20000	4.0937e-1 (2.36e-4)	3.9108e-1 (5.50e-3)	3.8809e-1 (1.03e-3)	4.0909e-1 (7.43e-5)	**4.0942e-1 (4.47e-5)**
RWMOP10	20000	8.4151e-1 (1.46e-3)	8.4696e-1 (4.57e-5)	8.4709e-1 (1.85e-4)	8.4721e-1 (2.83e-4)	**8.4741e-1 (3.94e-5)**
RWMOP11	53000	9.4649e-2 (9.37e-4)	9.7566e-2 (4.33e-4)	**9.7961e-2 (6.05e-4)**	9.4178e-2 (1.45e-3)	8.7709e-2 (3.21e-4)
RWMOP12	20000	5.5357e-1 (5.69e-3)	5.3356e-1 (1.20e-2)	5.4476e-1 (1.12e-2)	5.5980e-1 (1.80e-4)	**5.6046e-1 (5.98e-5)**
RWMOP13	26250	8.8826e-2 (2.32e-4)	8.9488e-2 (9.01e-5)	**9.0187e-2 (1.08e-4)**	8.9462e-2 (2.01e-4)	8.9300e-2 (2.79e-4)
RWMOP14	20000	6.1465e-1 (2.85e-3)	5.7878e-1 (1.94e-2)	6.1188e-1 (3.88e-3)	**6.1782e-1 (1.23e-3)**	6.1763e-1 (1.81e-4)
RWMOP15	20000	5.3807e-1 (2.17e-3)	5.0063e-1 (5.91e-2)	3.9907e-1 (6.87e-2)	5.4222e-1 (2.17e-4)	**5.4310e-1 (5.94e-5)**
RWMOP16	20000	7.6242e-1 (2.95e-4)	7.6134e-1 (1.12e-3)	7.5290e-1 (7.44e-3)	7.6380e-1 (1.03e-4)	**7.6381e-1 (6.18e-5)**
RWMOP17	26250	**3.2139e-1 (6.20e-2)**	2.4320e-1 (4.33e-2)	2.5307e-1 (5.03e-2)	2.5987e-1 (1.26e-2)	2.2196e-1 (8.71e-2)
RWMOP18	20000	**4.0515e-2 (2.62e-6)**	4.0468e-2 (2.16e-5)	4.0481e-2 (2.45e-5)	4.0490e-2 (6.77e-6)	4.0493e-2 (6.36e-6)

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RWMOP19	26250	3.1132e-1 (1.40e-2)	3.1610e-1 (1.54e-2)	2.6711e-1 (3.67e-2)	3.3280e-1 (4.48e-3)	**3.5393e-1 (7.74e-3)**
RWMOP20	20000	00(e-0) (0(e-0))	**00(e-0) (0(e-0))**	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))
RWMOP21	20000	3.1742e-2 (2.25e-5)	3.1639e-2 (3.03e-5)	3.1594e-2 (5.20e-5)	3.1753e-2 (1.63e-6)	**3.1756e-2 (6.43e-7)**

TABLE VI.
RT METRIC RESULTS OF VARIOUS OPTIMIZERS ON MECHANICAL DESIGN (RWMOP1-RWMOP21) PROBLEMS

Problem	M	D	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP1	2	4	20000	6.28E+00	1.34E+01	**1.62E+00**	2.72E+01	1.62E+00
RWMOP2	2	5	20000	6.07E+00	1.07E+01	2.67E+00	2.77E+01	**1.22E+00**
RWMOP3	2	3	20000	5.71E+00	1.53E+01	4.55E+00	2.63E+01	**1.19E+00**
RWMOP4	2	4	20000	5.93E+00	1.47E+01	1.33E+00	2.51E+01	**1.11E+00**
RWMOP5	2	4	20000	6.05E+00	1.36E+01	1.42E+00	2.56E+01	**1.08E+00**
RWMOP6	2	7	20000	6.01E+00	1.04E+01	1.68E+00	2.32E+01	**1.15E+00**
RWMOP7	2	4	20000	6.08E+00	1.05E+01	1.49E+00	3.12E+01	**1.03E+00**
RWMOP8	3	7	26250	8.25E+00	2.83E+01	1.94E+00	3.27E+01	**1.47E+00**
RWMOP9	2	4	20000	6.20E+00	1.35E+01	9.09E-01	3.21E+01	**6.55E-01**
RWMOP10	2	2	20000	6.06E+00	2.02E+01	8.50E-01	2.88E+01	**6.49E-01**
RWMOP11	5	3	53000	1.64E+01	6.72E+01	3.95E+00	8.36E+01	**3.03E+00**
RWMOP12	2	4	20000	6.23E+00	2.13E+01	1.55E+00	2.19E+01	**1.08E+00**
RWMOP13	3	7	26250	8.16E+00	1.96E+01	2.47E+00	3.91E+01	**1.62E+00**
RWMOP14	2	5	20000	7.28E+00	1.40E+01	1.41E+00	2.57E+01	**1.10E+00**
RWMOP15	2	3	20000	6.76E+00	1.17E+01	1.56E+00	3.20E+01	**1.12E+00**
RWMOP16	2	2	20000	6.21E+00	2.68E+01	1.44E+00	2.68E+01	**1.06E+00**
RWMOP17	3	6	26250	7.88E+00	1.42E+01	3.40E+00	3.21E+01	**1.65E+00**
RWMOP18	2	3	20000	6.01E+00	2.10E+01	1.36E+00	2.93E+01	**1.05E+00**
RWMOP19	3	10	26250	8.39E+00	9.28E+00	1.56E+00	3.13E+01	**1.06E+00**
RWMOP20	2	4	20000	6.20E+00	5.77E+00	3.95E+00	2.30E+01	**8.88E-01**
RWMOP21	2	6	20000	6.24E+00	2.16E+01	1.31E+00	3.14E+01	**1.05E+00**

CEC-20221 mechanical design problems are discrete and continuous problems, and it is more complicated than the ZDT benchmark suite. Test problems from RWMOP8, RWMOP13, RWMOP19, and RWMOP20 are multimodal in design and offer difficulty for convergence to true PF. However, the MOAOA has provided greater convergence and divergence of the solutions than other optimizers. RWMOP1-RWMOP7, RWMOP13-RWMOP18 have degenerate PF, making it simpler to converge than SD, the NDS, along with the whole PF. NSGWO and MOSMA could not search the lower part of the true PF on RWMOP15, RWMOP17, and RWMOP20 problems. However, MOAOA has covered the entire PF along with the end solutions. In other words, MOAOA is successful in achieving convergence and diversity on RWMOP9, RWMOP10, and RWMOP 21. The test problem, RWMOP20, has disconnected PF, which is a combination of the convex and concave types of PFs. It also has a disconnected search space. For this problem, NSGWO, MOSMA, and MOMVO performed poorly; however, MOAOA performed exceedingly better by solving RWMOP11 with five objective functions. Therefore, it is claimed that the complexity level of these cases is considerably low, as other optimizers, except NSGWO, quickly access the feasible solutions of the constrained PF of mechanical design problems.

G.2. RESULTS ON CEC-2021 CHEMICAL ENGINEERING PROBLEMS (RWMOP22-RWMOP24)

The qualitative and quantitative results obtained by MOAOA, NSGWO, MOMVO, MOALO, and MOSMA optimizers while solving chemical engineering problems are described in Table 7, Table 8, Table 9, Table 10, and Table 11, collectively. Fig. 12 shows the best PF and BCS of all the chemical engineering problems for visualizing the performance of the proposed MOAOA.

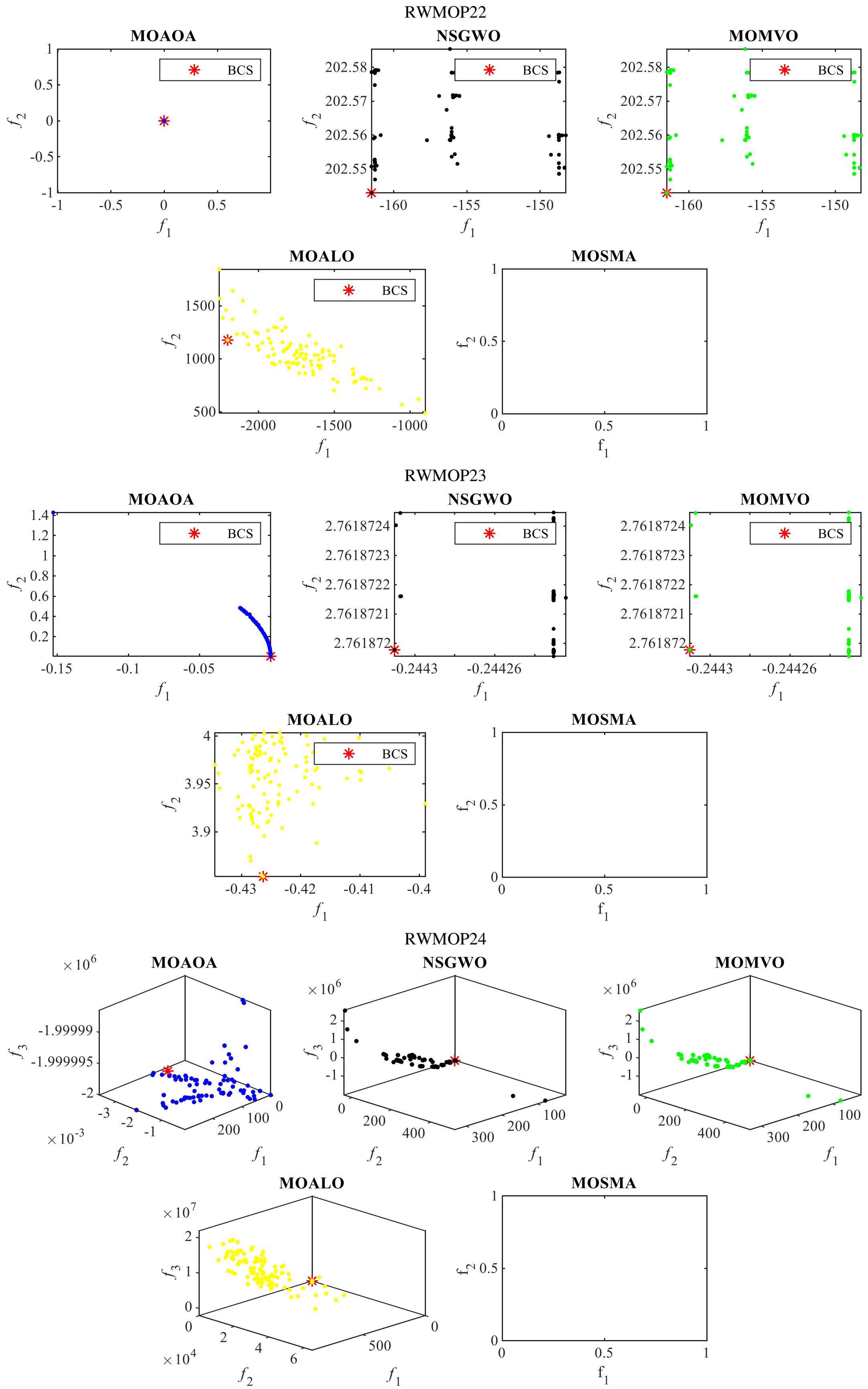

FIGURE 12. PFs of all the algorithms on chemical engineering (RWMOP22-RWMOP24) problems

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TABLE 7.
GD METRIC RESULTS OF VARIOUS OPTIMIZERS ON CHEMICAL ENGINEERING PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP22	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.1169e+3 (4.23e+2)**
RWMOP23	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	8.5366e-1 (0(e-0))	**7.1196e-1 (3.79e-1)**
RWMOP24	26250	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.4959e+3 (2.01e+3)**

TABLE VIII.
SPREAD METRIC RESULTS OF VARIOUS OPTIMIZERS ON CHEMICAL ENGINEERING PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP22	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.0028e+3 (8.97e-5)**
RWMOP23	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.2586e+0 (0(e-0))**	3.6062e+0 (6.87e-1)
RWMOP24	26250	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**3.3652e+0 (2.32e+0)**

TABLE IX.
IGD METRIC RESULTS OF VARIOUS OPTIMIZERS ON CHEMICAL ENGINEERING PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP22	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.0028e+3 (8.97e-5)**
RWMOP23	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.2586e+0 (0(e-0))**	3.6062e+0 (6.87e-1)
RWMOP24	26250	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**3.3652e+0 (2.32e+0)**

TABLE X.
HV METRIC RESULTS OF VARIOUS OPTIMIZERS ON CHEMICAL ENGINEERING PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP22	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.0000e+0 (0(e-0))**
RWMOP23	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	6.0228e-1 (0(e-0))	**9.9108e-1 (1.97e-1)**
RWMOP24	26250	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**5.9444e-1 (3.82e-1)**

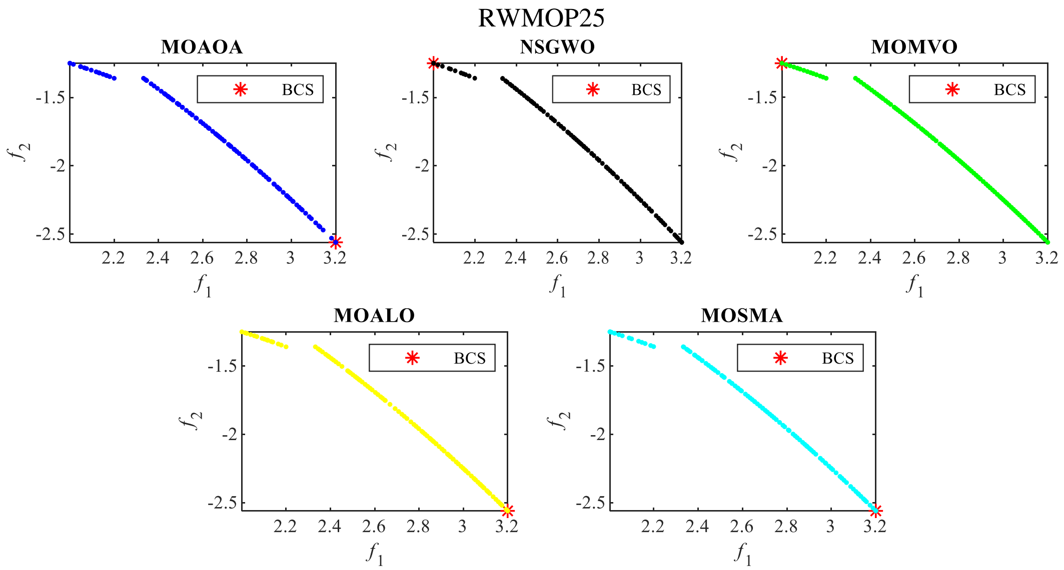
TABLE XI.
RT METRIC RESULTS OF VARIOUS OPTIMIZERS ON CHEMICAL ENGINEERING PROBLEMS

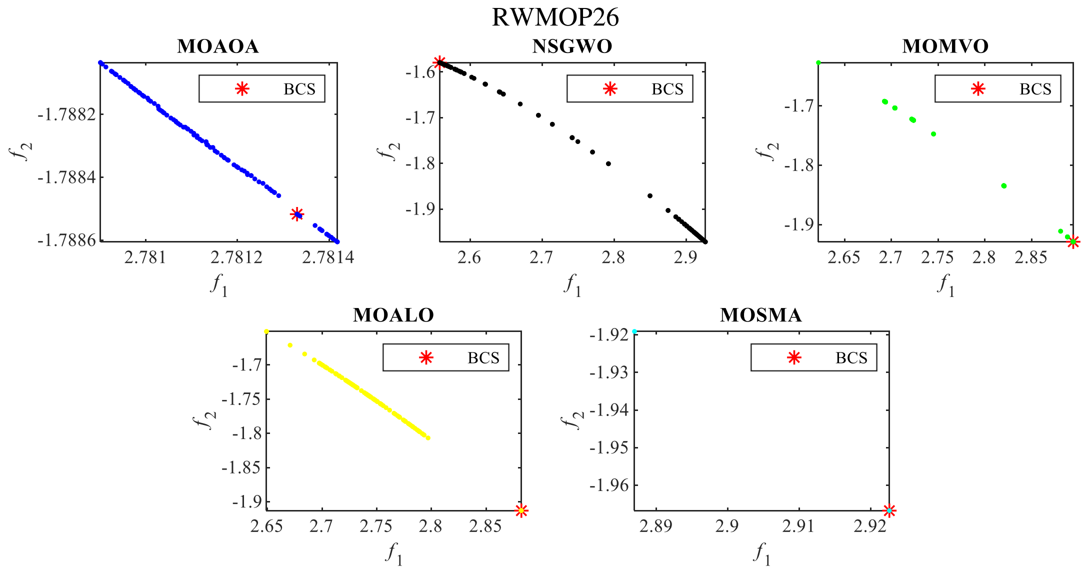
Problem	M	D	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP22	2	9	20000	5.70E+00	4.89E+00	4.32E+00	2.07E+01	**2.28E+00**
RWMOP23	2	6	20000	5.64E+00	7.07E+00	4.15E+00	2.64E+01	**2.21E+00**
RWMOP24	3	9	26250	7.73E+00	9.93E+00	5.69E+00	2.67E+01	**3.14E+00**

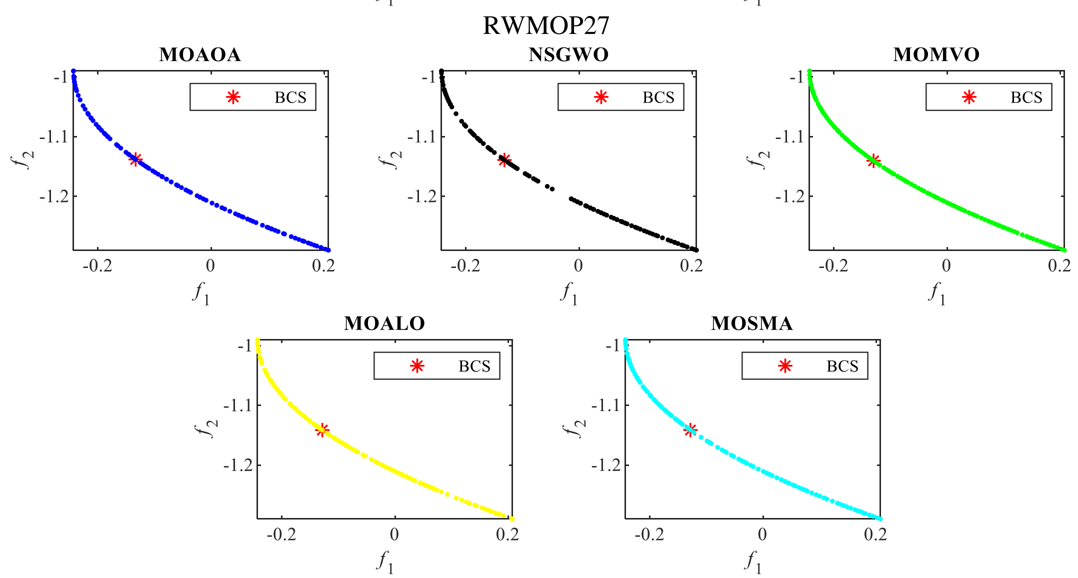
Multimodality of CEC-2021 chemical engineering benchmark functions is a concern for the convergence of solutions. RWMOP22 has provided a convergence difficulty as it includes a variety of local optima. Even so, the MOAOA method is not trapped at the local PF for any problems with the CEC-2021 chemical engineering problems. This performance is due to its explorative potential. Equally, RWMOP23 and RWMOP24 provided the convergence task and the distribution of solutions for NSGWO, MOMVO, and MOALO. RWMOP24 has found it hard to maintain final solutions for all optimizers except the MOAOA. RWMOP23 is structured to have a solution distribution challenge. MOSMA has not been capable of achieving a whole distribution of solutions across the entire PF. In addition, the search for accurate end solutions on RWMOP22 proved to be difficult for competitive optimizers. It can be stated that the level of complexity of such issues is significantly higher compared to ZDT and CEC-2021 mechanical design problems, as the state-of-the-art optimizers cannot find a single, realistic solution to two out of three cases. In RWMOP23, MOSMA optimizers identify feasible solutions in several runs, but these possible solutions are not restricted to PF.

G.3. RESULTS ON CEC-2021 PROCESS, SYNTHESIS, AND DESIGN PROBLEMS (RWMOP25-RWMOP29)

The qualitative and quantitative results obtained by MOAOA, NSGWO, MOMVO, MOALO, and MOSMA optimizers while solving process, synthesis, and design problems are described in Table 12, Table 13, Table 14, Table 15, and Table 16, collectively. Fig. 13 shows the best PF and BCS of all the process, synthesis, and design problems for visualizing the performance of the proposed MOAOA.


RWMOP25
MOAOA
NSGWO
MOMVO
MOALO
MOSMA
BCS
f_1
f_2


RWMOP26
MOAOA
NSGWO
MOMVO
MOALO
MOSMA
BCS
f_1
f_2


RWMOP27
MOAOA
NSGWO
MOMVO
MOALO
MOSMA
BCS
f_1
f_2

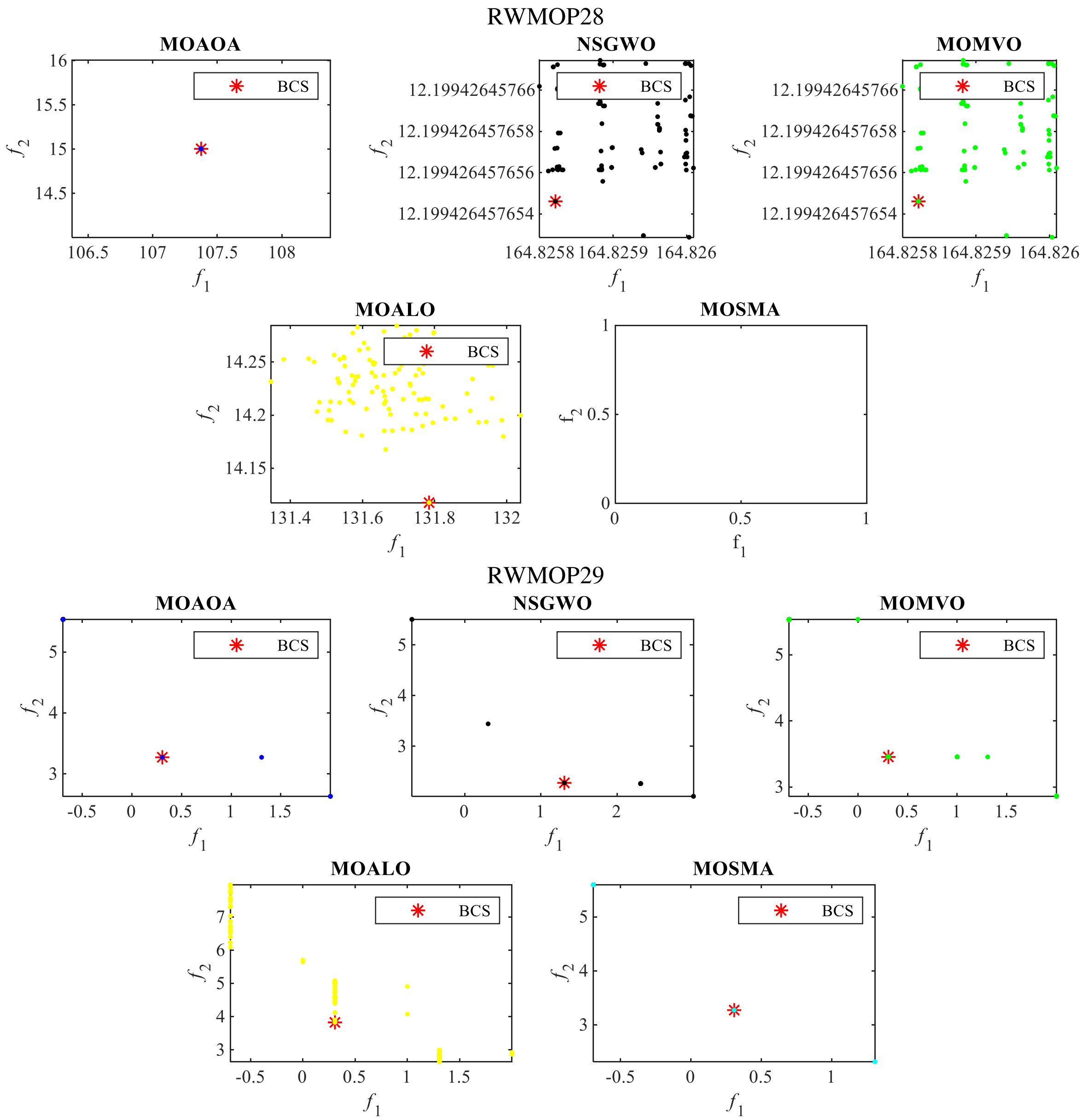


FIGURE 13. PFs of all the algorithms on process, design, and synthesis (RWMOP25-RWMOP29) problems

TABLE XII.
GD METRIC RESULTS OF VARIOUS OPTIMIZERS ON PROCESS, DESIGN, AND SYNTHESIS PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP25	20000	9.5236e-2 (8.39e-4)	9.5953e-2 (8.99e-5)	9.5490e-2 (7.23e-4)	9.5258e-2 (1.48e-3)	**9.5086e-2 (5.09e-4)**
RWMOP26	20000	3.4427e-2 (4.57e-4)	**2.9059e-2 (2.97e-3)**	3.2134e-1 (6.49e-2)	3.0594e-2 (3.77e-3)	3.0752e-2 (2.74e-3)
RWMOP27	20000	1.2013e-1 (3.78e-4)	1.1951e-1 (6.72e-4)	**1.1929e-1 (3.68e-4)**	1.1972e-1 (2.84e-4)	1.1963e-1 (3.02e-4)
RWMOP28	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.0243e+1 (3.33e+0)**
RWMOP29	20000	**3.0103e+0 (2.50e-1)**	6.1811e+0 (8.09e-1)	5.5464e+0 (3.43e+0)	5.3355e+0 (3.09e+0)	5.2707e+0 (6.35e-1)

TABLE XIII.
SPREAD METRIC RESULTS OF VARIOUS OPTIMIZERS ON THE PROCESS, DESIGN, AND SYNTHESIS PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP25	20000	7.4391e-1 (1.38e-5)	7.4389e-1 (1.97e-5)	7.4393e-1 (5.56e-5)	**7.4387e-1 (1.11e-5)**	7.4392e-1 (4.25e-5)
RWMOP26	20000	2.8162e-1 (2.38e-2)	2.5844e-1 (2.05e-2)	3.6909e-1 (1.95e-2)	2.7214e-1 (3.96e-2)	**2.4813e-1 (1.55e-3)**

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3085529, IEEE Access

RWMOP27	20000	9.8999e-1 (9.80e-6)	9.9001e-1 (8.00e-5)	9.9010e-1 (2.87e-4)	9.8999e-1 (1.01e-5)	**9.8993e-1 (1.33e-4)**
RWMOP28	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.8807e+1 (4.66e+0)**
RWMOP29	20000	9.1974e+0 (6.28e-2)	9.6236e+0 (9.39e-1)	**8.6623e+0 (6.27e-1)**	1.0519e+1 (8.59e-1)	9.2363e+0 (2.53e-2)

TABLE XIV.
IGD METRIC RESULTS OF VARIOUS OPTIMIZERS ON PROCESS, DESIGN, AND SYNTHESIS PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP25	20000	7.4391e-1 (1.38e-5)	7.4389e-1 (1.97e-5)	7.4393e-1 (5.56e-5)	**7.4387e-1 (1.11e-5)**	7.4392e-1 (4.25e-5)
RWMOP26	20000	2.8162e-1 (2.38e-2)	2.5844e-1 (2.05e-2)	3.6909e-1 (1.95e-2)	2.7214e-1 (3.96e-2)	**2.4813e-1 (1.55e-3)**
RWMOP27	20000	9.8999e-1 (9.80e-6)	9.9001e-1 (8.00e-5)	9.9010e-1 (2.87e-4)	9.8999e-1 (1.01e-5)	**9.8993e-1 (1.33e-4)**
RWMOP28	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.8807e+1 (4.66e+0)**
RWMOP29	20000	9.1974e+0 (6.28e-2)	9.6236e+0 (9.39e-1)	**8.6623e+0 (6.27e-1)**	1.0519e+1 (8.59e-1)	9.2363e+0 (2.53e-2)

TABLE XV.
HV METRIC RESULTS OF VARIOUS OPTIMIZERS ON PROCESS, DESIGN, AND SYNTHESIS PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP25	20000	**2.4154e-1 (4.70e-6)**	2.4121e-1 (7.90e-5)	2.4129e-1 (7.99e-5)	2.4102e-1 (2.62e-5)	2.4120e-1 (4.24e-5)
RWMOP26	20000	**1.6145e-1 (3.33e-2)**	1.5338e-1 (2.60e-2)	9.6129e-2 (4.22e-3)	1.5821e-1 (2.70e-2)	1.4316e-1 (1.33e-3)
RWMOP27	20000	1.01e+10 (1.63e+10)	5.8879e+7 (6.48e+7)	3.1787e+7 (3.80e+7)	**9.47e+10 (9.08e+10)**	2.5892e+8 (5.07e+8)
RWMOP28	20000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**7.1843e-3 (1.24e-2)**
RWMOP29	20000	7.6489e-1 (1.14e-2)	6.8829e-1 (9.19e-2)	6.7184e-1 (1.28e-1)	7.5545e-1 (6.68e-3)	**7.7859e-1 (1.12e-2)**

TABLE XVI.
RT METRIC RESULTS OF VARIOUS OPTIMIZERS ON PROCESS, DESIGN, AND SYNTHESIS PROBLEMS

Problem	M	D	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP25	2	2	20000	5.88E+00	2.22E+01	8.24E-01	2.89E+01	**5.84E-01**
RWMOP26	2	3	20000	5.74E+00	7.61E+00	4.13E+00	2.12E+01	**7.02E-01**
RWMOP27	2	3	20000	5.92E+00	1.96E+01	9.51E-01	2.87E+01	**6.39E-01**
RWMOP28	2	7	20000	5.93E+00	6.22E+00	4.30E+00	2.74E+01	**1.55E+00**
RWMOP29	2	7	20000	5.79E+00	3.92E+00	4.18E+00	2.22E+01	**1.06E+00**

The challenges presented by the process, design, and synthesis test suites in terms of different features, such as non-separability, multimodality, bias, deceptiveness, many-to-one mappings, a combination of PF shapes, specific search domains, etc. makes the optimization process complex. RWMOP28-RWMOP29 gives a greater stiffness to the convergence of solutions on the true PF. All competitive optimizers were trapped at the local PF except MOAOA. RWMOP27 is relatively simple, and the MOAOA has obtained well-distributed solutions and final solutions compared to most optimizers. For RWMOP26, all optimizers (except MOAOA) could not obtain well-converged solutions until the stopping criterion was met. Even so, it is clear from Table 12 – Table 16 that the MOAOA has obtained greater convergence and diversity compared to other approaches.

G.4. RESULTS ON CEC-2021 POWER ELECTRONICS PROBLEMS (RWMOP30-RWMOP35)

The qualitative and quantitative results obtained by MOAOA, NSGWO, MOMVO, MOALO, and MOSMA optimizers while solving power electronics problems are described in Table 17, Table 18, Table 19, Table 20, and Table 21, collectively. Fig. 14 shows the best PF and BCS of all the power electronics problems for visualizing the performance of the proposed MOAOA.

TABLE XVII.
GD METRIC RESULTS OF VARIOUS OPTIMIZERS ON POWER ELECTRONICS PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP30	80000	1.8856e-2 (2.59e-3)	0(e-0) (0(e-0))	**1.3111e-2 (0(e-0))**	1.9560e-2 (0(e-0))	3.0041e-2 (0(e-0))
RWMOP31	80000	8.9802e-2 (7.68e-2)	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**4.4475e-2 (2.91e-3)**	7.4820e-2 (7.28e-2)
RWMOP32	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**3.4253e-2 (0(e-0))**	1.3685e-1 (1.38e-1)	3.9861e-2 (3.28e-3)
RWMOP33	80000	3.5776e-1 (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	2.0379e-1 (0(e-0))	**5.3870e-2 (1.80e-2)**
RWMOP34	80000	3.4590e-1 (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.0385e-1 (0(e-0))**
RWMOP35	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**6.5411e-1 (0(e-0))**	1.3837e+0 (0(e-0))

TABLE XVIII.
SPREAD METRIC RESULTS OF VARIOUS OPTIMIZERS ON POWER ELECTRONICS PROBLEMS

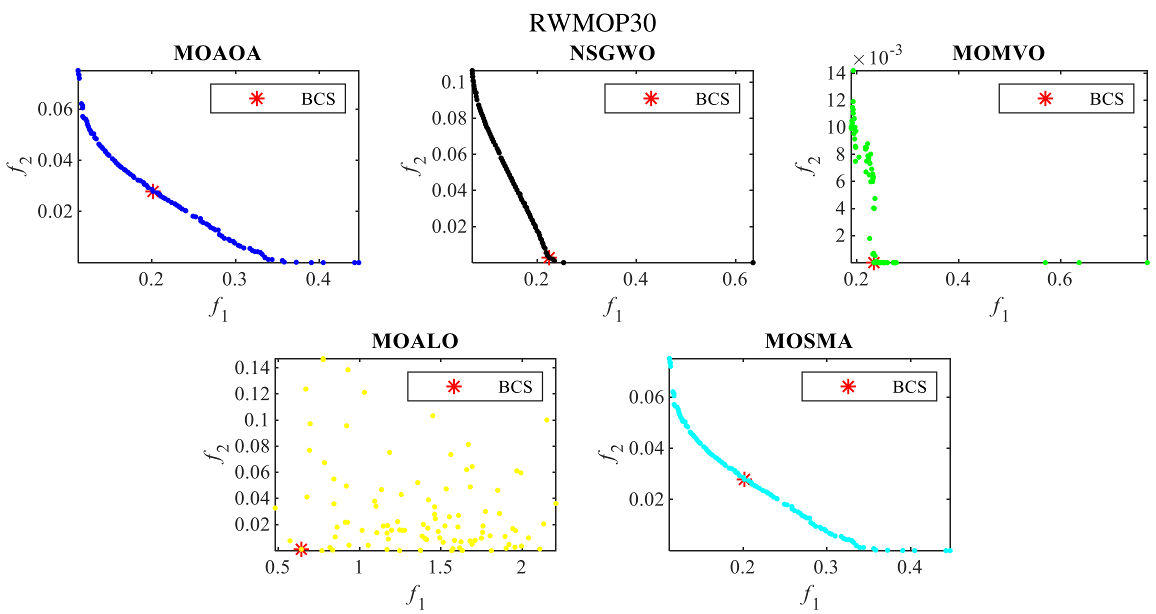
Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP30	80000	1.0332e-1 (0(e-0))	0(e-0) (0(e-0))	**8.9486e-2 (0(e-0))**	1.9533e-1 (0(e-0))	1.0566e-1 (2.40e-3)
RWMOP31	80000	1.3456e-1 (1.15e-1)	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.3277e-1 (1.51e-1)**	3.2832e-1 (4.08e-1)
RWMOP32	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	1.6855e-1 (0(e-0))	2.9756e-1 (1.36e-1)	**1.4294e-1 (1.39e-2)**
RWMOP33	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.3364e-1 (9.95e-2)**	1.7185e+0 (0(e-0))	1.9971e+0 (0(e-0))
RWMOP34	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.0121e+0 (0(e-0))**	2.0836e+0 (0(e-0))
RWMOP35	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	5.8240e+0 (0(e-0))	**5.6247e+0 (0(e-0))**

TABLE XIX.
IGD METRIC RESULTS OF VARIOUS OPTIMIZERS ON POWER ELECTRONICS PROBLEMS.

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP30	80000	1.0332e-1 (0(e-0))	0(e-0) (0(e-0))	**8.9486e-2 (0(e-0))**	1.9533e-1 (0(e-0))	1.0566e-1 (2.40e-3)
RWMOP31	80000	1.3456e-1 (1.15e-1)	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.3277e-1 (1.51e-1)**	3.2832e-1 (4.08e-1)
RWMOP32	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	1.6855e-1 (0(e-0))	2.9756e-1 (1.36e-1)	**1.4294e-1 (1.39e-2)**
RWMOP33	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**2.3364e-1 (9.95e-2)**	1.7185e+0 (0(e-0))	1.9971e+0 (0(e-0))
RWMOP34	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**1.0121e+0 (0(e-0))**	2.0836e+0 (0(e-0))
RWMOP35	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	5.8240e+0 (0(e-0))	**5.6247e+0 (0(e-0))**

TABLE XX.
HV METRIC RESULTS OF VARIOUS OPTIMIZERS ON POWER ELECTRONICS PROBLEMS

Problem	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP30	80000	3.9123e-1 (0(e-0))	0(e-0) (0(e-0))	4.8422e-1 (0(e-0))	6.2937e-1 (0(e-0))	**6.6027e-1 (1.37e-1)**
RWMOP31	80000	1.5577e-1 (1.35e-1)	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**3.5104e-1 (4.96e-1)**	1.6044e-1 (2.78e-1)
RWMOP32	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	7.1079e-1 (0(e-0))	3.2454e-1 (4.59e-1)	**7.2921e-1 (7.84e-2)**
RWMOP33	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**0(e-0) (0(e-0))**
RWMOP34	80000	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**0(e-0) (0(e-0))**
RWMOP35	80000	5.4220e-1 (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	0(e-0) (0(e-0))	**5.8022e-1 (0(e-0))**



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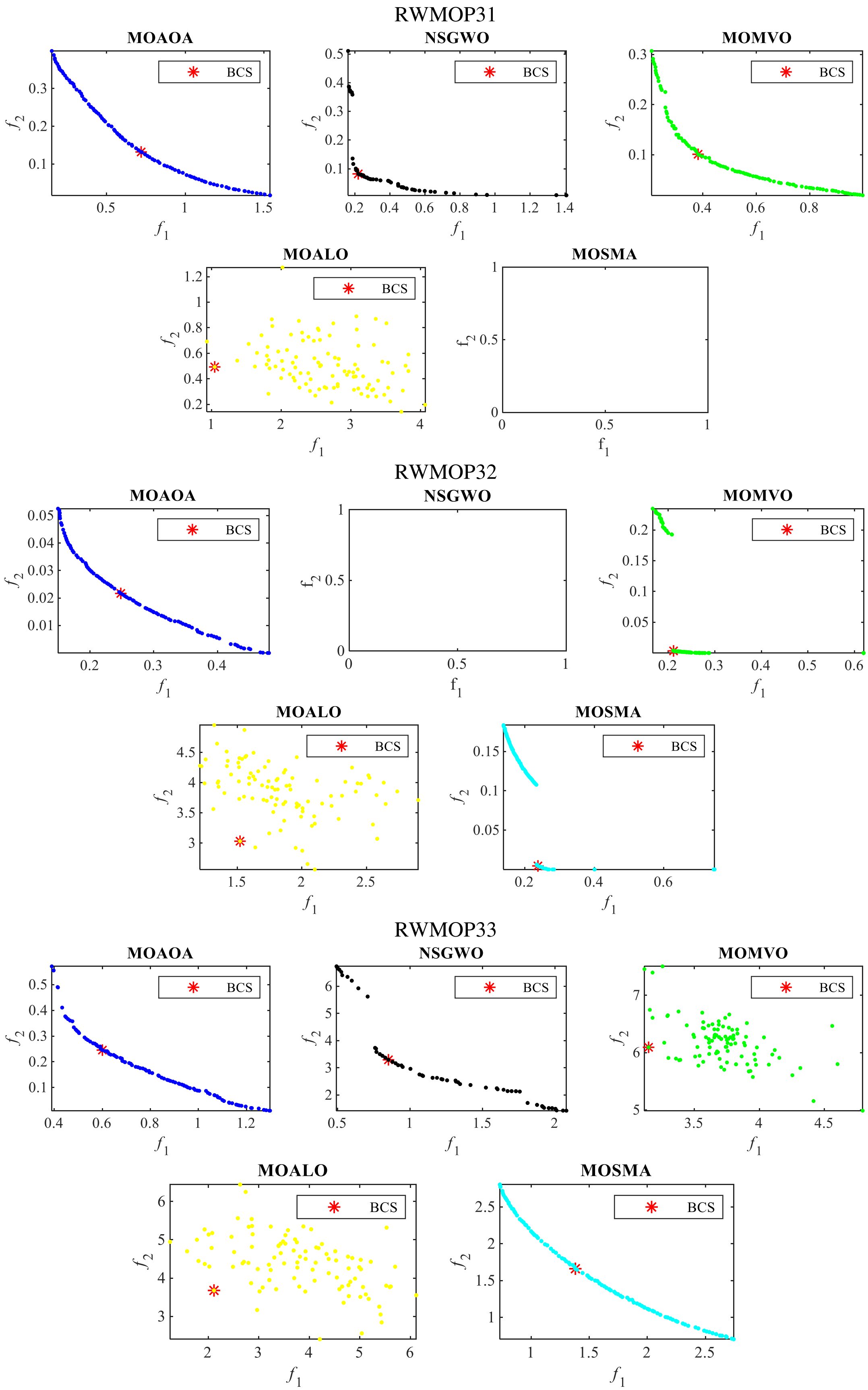

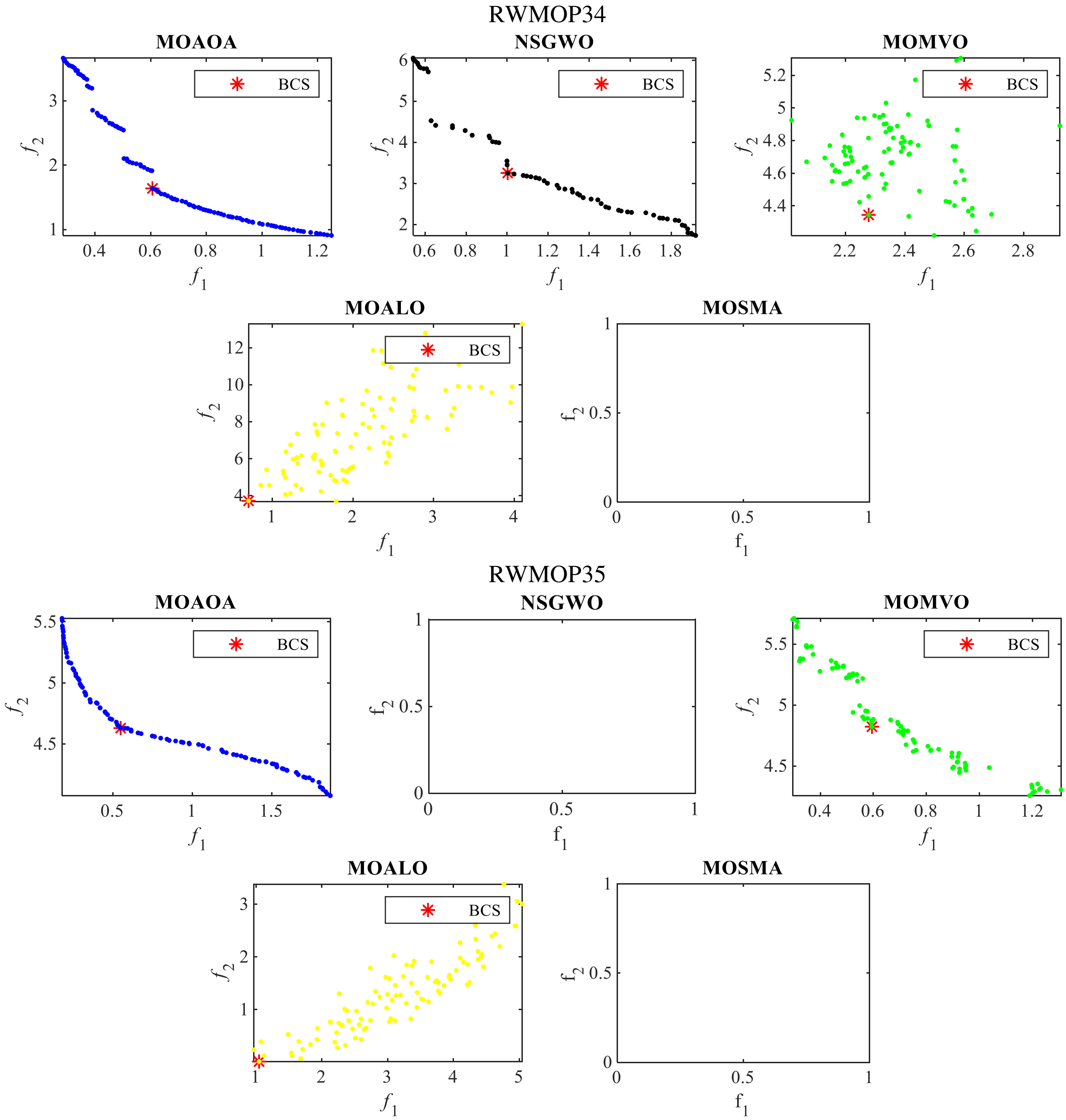

FIGURE 14. PFs of all the algorithms on power electronics (RWMOP30-RWMOP35) problems

TABLE XXI.
RT METRIC RESULTS OF VARIOUS OPTIMIZERS ON POWER ELECTRONICS PROBLEMS

Problem	M	D	FEs	NSGWO	MOMVO	MOALO	MOSMA	MOAOA
RWMOP30	2	25	80000	3.45E+01	2.42E+01	2.83E+01	1.10E+02	**1.65E+01**
RWMOP31	2	25	80000	3.38E+01	2.61E+01	2.70E+01	1.08E+02	**1.57E+01**
RWMOP32	2	25	80000	3.34E+01	3.28E+01	2.75E+01	1.09E+02	**1.82E+01**
RWMOP33	2	30	80000	3.48E+01	3.18E+01	2.92E+01	1.08E+02	**2.02E+01**
RWMOP34	2	30	80000	3.51E+01	2.55E+01	2.94E+01	1.05E+02	**1.80E+01**
RWMOP35	2	30	80000	3.65E+01	2.61E+01	2.92E+01	1.04E+02	**1.70E+01**

CEC-2021 power electronics is one of the most challenging test suites on a global scale. RWMOP30-RWMOP35 problems are distinguished by a non-linear solution space (multimodal). All problems are a challenge to the convergence, distribution, and diversity of the NDS across the entire PF. Popular optimizers NSGWO, MOMVO, MOALO, MOSMA, etc., are explicitly developed to solve large-scale problems. Even so, the solution could not be identified within a small number of FEs, and the NDS could not be obtained. Most of the algorithms show premature convergence in these case studies. MOAOA has performed higher than other competitive algorithms on all efficiency metrics.

G.5. CONVERGENCE TOWARDS PF ANALYSIS

Table 2, Table 7, Table 12, and Table 17 show the mean (STD) GD metric, which evaluates the similarity of the solutions to the actual PF; the obtained PF using MOAOA optimizer for different cases are discussed. These statistical results demonstrate that MOAOA shows a promising efficiency in handling unconstrained and CEC-2021 problems, as it is best on 26 out of 40 cases on the GD metric. By contrast, NSGWO, MOMVO, MOALO, and MOSMA are respectively best on 2, 2, 3, and 7 cases for GD. All the tables mentioned above show that the MOAOA's efficiency outperforms NSGWO, MOMVO, MOALO, and MOSMA; it leads to better convergence toward PF than NSGWO, MOMVO, MOALO, and MOSMA.

G.6. COVERAGE/DIVERSITY ANALYSIS

It is observed from Table 3, Table 8, Table 13, and Table 18 the mean (STD) Spread metric, which evaluates the distribution of solutions in the search space, the obtained PF using MOAOA for different cases are discussed. These statistical results present that MOAOA has a promising efficiency in handling unconstrained and CEC-2021 problems, as it is best on 16 out of 40 cases on the Spread metric. By contrast, NSGWO, MOMVO, MOALO, and MOSMA are respectively best on 4, 5, 6, and 9 cases for Spread. All the tables mentioned above show that the MOAOA's efficiency outperforms NSGWO, MOMVO, MOALO, and MOSMA; it leads to a better ND solution distribution PF than NSGWO, MOMVO, MOALO, and MOSMA.

G.7. BALANCE ANALYSIS BETWEEN CONVERGENCE AND DIVERSITY

It is seen from Table 4, Table 5, Table 9, Table 10, Table 14, Table 15, Table 19, and Table 20 the mean (STD) IGD and HV metrics, which evaluate the closer and diverse the corresponding results approach the PF, the obtained PF using MOAOA for different cases are discussed. These statistical results present that MOAOA has a promising efficiency in handling unconstrained and CEC-2021 problems, as it is best on 16 out of 40 cases on IGD and best on 24 out of 40 cases on HV metrics. By contrast, NSGWO, MOMVO, MOALO, and MOSMA are respectively best on 4, 5, 6, and 9 cases for IGD and best on 5, 2, 3, and 6 cases for HV metrics. All the tables mentioned above show that the MOAOA's efficiency outperforms NSGWO, MOMVO, MOALO, and MOSMA, leading to better convergence and diverse solutions toward PF than NSGWO, MOMVO, MOALO, and MOSMA.

G.8. RUNTIME ANALYSIS

Finally, it addresses five optimizers' computation time measured by the average running time of 30 separate trials. The CPU's average time is summarized in Table 6, Table 11, Table 16, and Table 21. These statical results present that MOAOA has a promising efficiency in handling unconstrained and CEC-2021 problems, as it is best on 38 out of 40 cases on RT metric. These statistical results present that MOAOA has a promising efficiency in handling unconstrained and CEC-2021 problems, as it is best on 38 out of 40 cases on RT metric. By contrast, NSGWO, MOMVO, MOALO, and MOSMA are respectively best on 0, 1, 1, and 0 cases for RT. The above-mentioned tables show that the MOAOA's efficiency outperforms NSGWO, MOMVO, MOALO, and MOSMA; it leads to a better CPU time than NSGWO, MOMVO, MOALO, and MOSMA algorithms. Since our implemented optimizer could perform better than all the selected optimizers with better CPU time, MOAOA would help the decision-makers find better alternatives to solve their problems.

Why does the proposed MOAOA perform best? Here’s a brief analysis of the reasons. Based on the proposed MOAOA, the models' CD, NDS, adds the historical information of individuals in previous iterations to the generation of offspring. The individuals selected in this model are chosen randomly or fixedly rather than the optimal individuals in the population, which leads to the individuals selected may be bad or good. To a certain level, it restricts the optimizer's convergence rate and prevents local optimization due to rapid convergence. Besides, this way of randomly choosing entities also increases the diversity of the optimizer, resulting in smaller HV, IGD values. As classical convergence-diversity metrics, HV, IGD is closely related to the diversity and convergence of algorithms. The better the diversity and convergence, the smaller the IGD, HV values. In this paper, the proposed MOAOA contributes to improving the other state-of-art algorithms' diversity and convergence. From the above experimental results, it can be seen that the HV, IGD values of the MOAOA using the NDS and CD are better than that of the other selected algorithms.

V. CONCLUSION

The proposed MOAOA is formulated with AOA, non-dominance sorting, and crowding distance-based mechanisms. The MOAOA outperformed comparative optimizers, such as NSGWO, MOMVO, MOALO, and MOSMA, in multiple benchmark test suites, including ZDT and CEC-2021 RWMOP test suites. Various performance indicators, such as GD, Spread, IGD, HV, and RT, are used for quantitative performance evaluation. Even then, an exploratory analysis of performance indicators showed a clear statistical association between some metrics. The WSRT is a non-parametric test for the rating of all optimizers for each metric. In other terms, it

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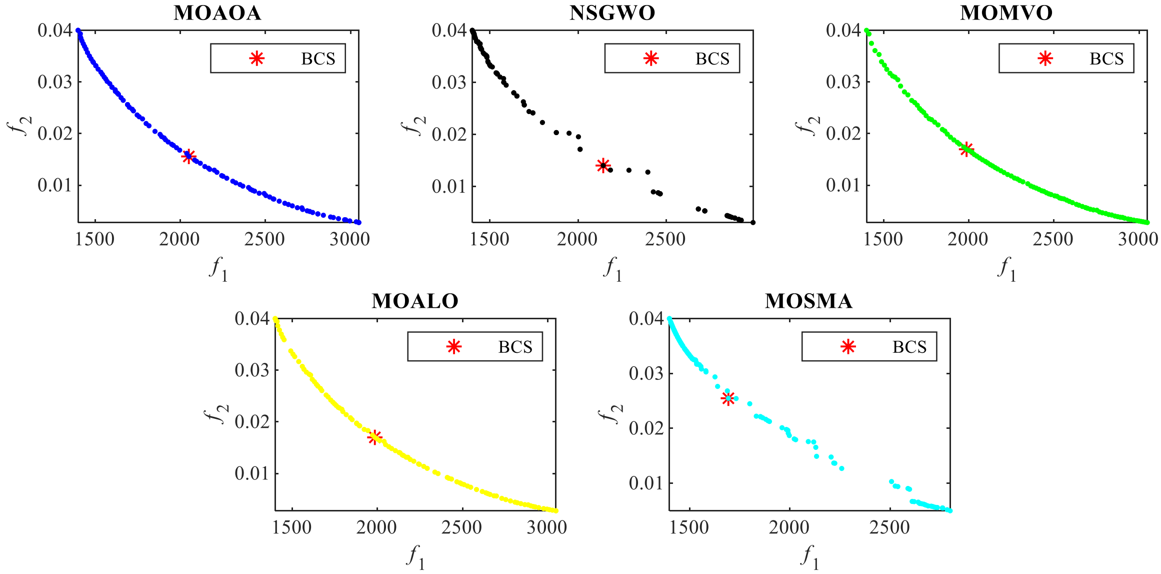
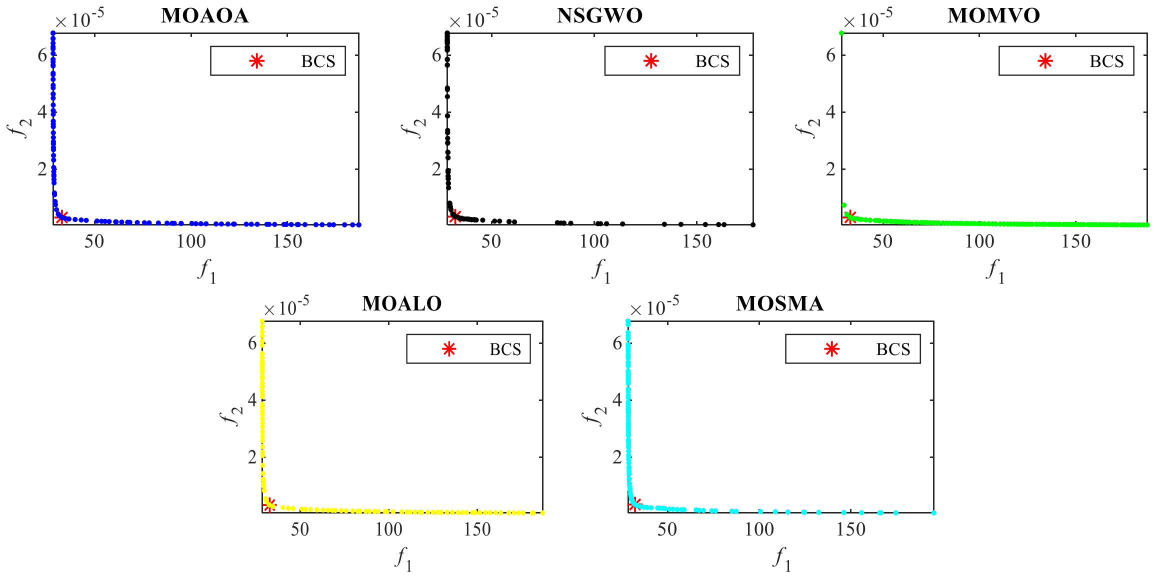
clearly showed the variations in the performance of the optimizers, which required more exploration and verification of the differences. In this way, the efficiency of the MOAOA is numerically examined and tested for coverage, convergence, diversity, and computational cost metrics.
The effectiveness of the MOAOA in finding a significant number of NDS in several FEs is due to the different conceptual features implemented. They're a CD and an NDS process. These features enabled MOAOA to optimize acceptable balance among exploration and exploitation so as to address the crises and escape saturation. Such functions also help to stabilize exploration and exploitation at the FEs stage and navigate the search for a promising optimal solution. The excellent results of the proposed MOAOA over the ZDT and CEC-2021 constrained RWMOPs test suites led to its application to the real-world MOPs problems of CEC-2021. CEC-2021 Real-world constrained RWMOPS problems are overcome using MOAOA. The PF achieved by the MOAOA is much superior to the competitive NSGWO, MOMVO, MOALO, and MOSMA optimizers. The development of the CD criteria showed the reliability, efficiency, and effectiveness of the MOAOA, while its deployment across different test suites demonstrated its robustness in the achievement of non-dominated solutions. In conclusion, MOAOA is one of Pareto's robust non-dominant optimizers to achieving better convergence, coverage, diversity, and computational cost.

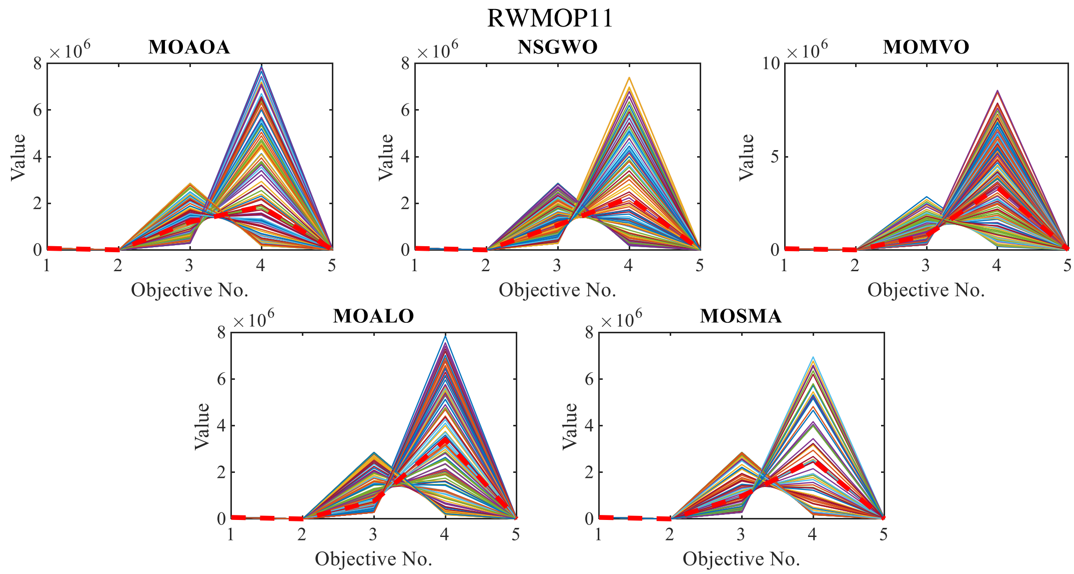
APPENDIX

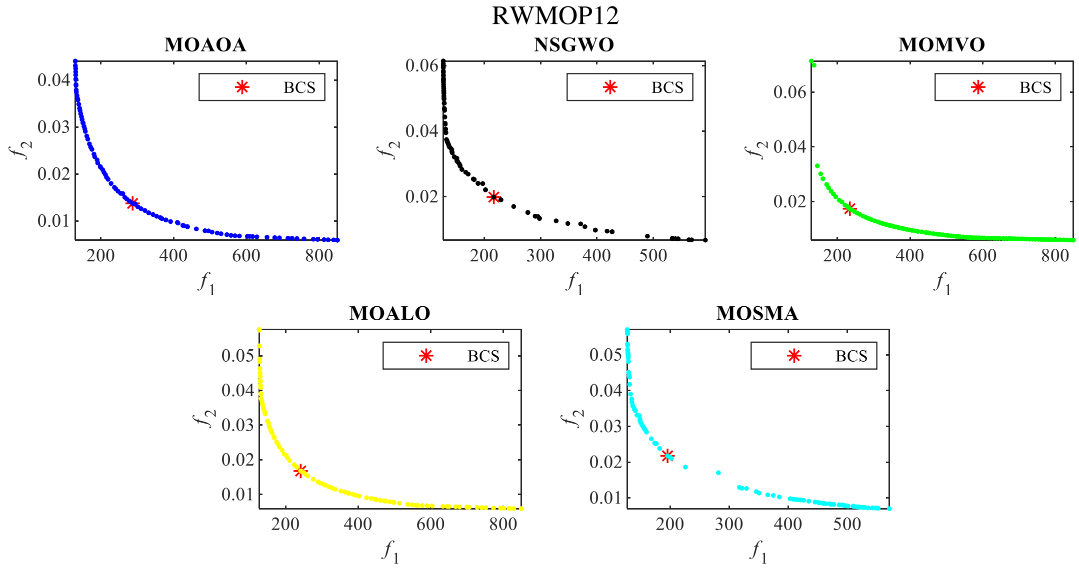
RWMOP9

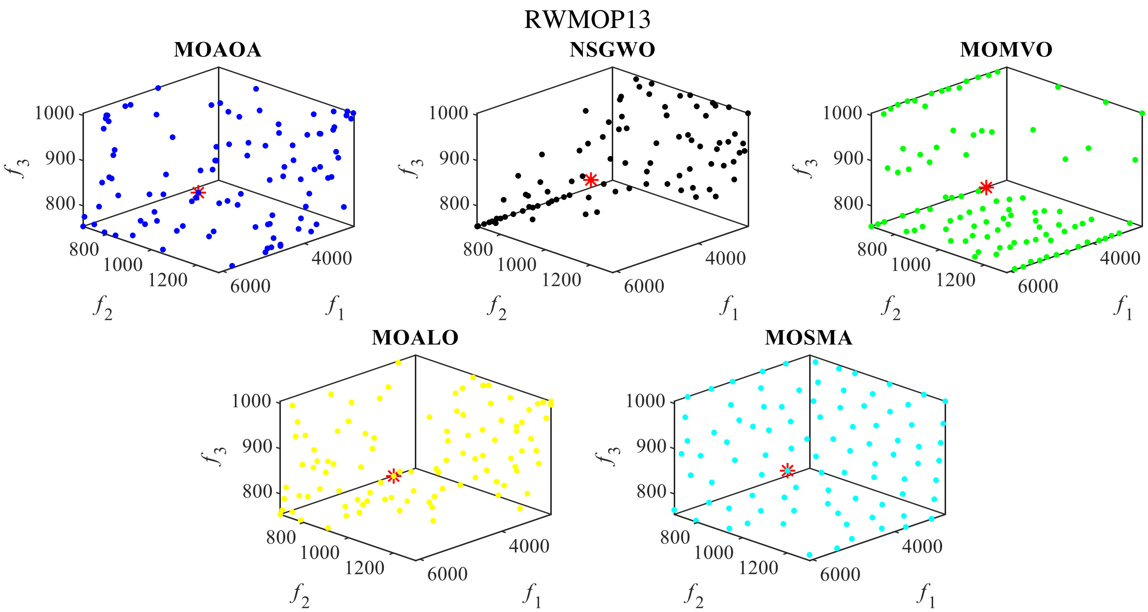


RWMOP10




RWMOP11
MOAOA
NSGWO
MOMVO
MOALO
MOSMA
Value
Objective No.
$\times 10^6$


RWMOP12
MOAOA
NSGWO
MOMVO
MOALO
MOSMA
BCS
f_2
f_1


RWMOP13
MOAOA
NSGWO
MOMVO
MOALO
MOSMA
f_3
f_2
f_1

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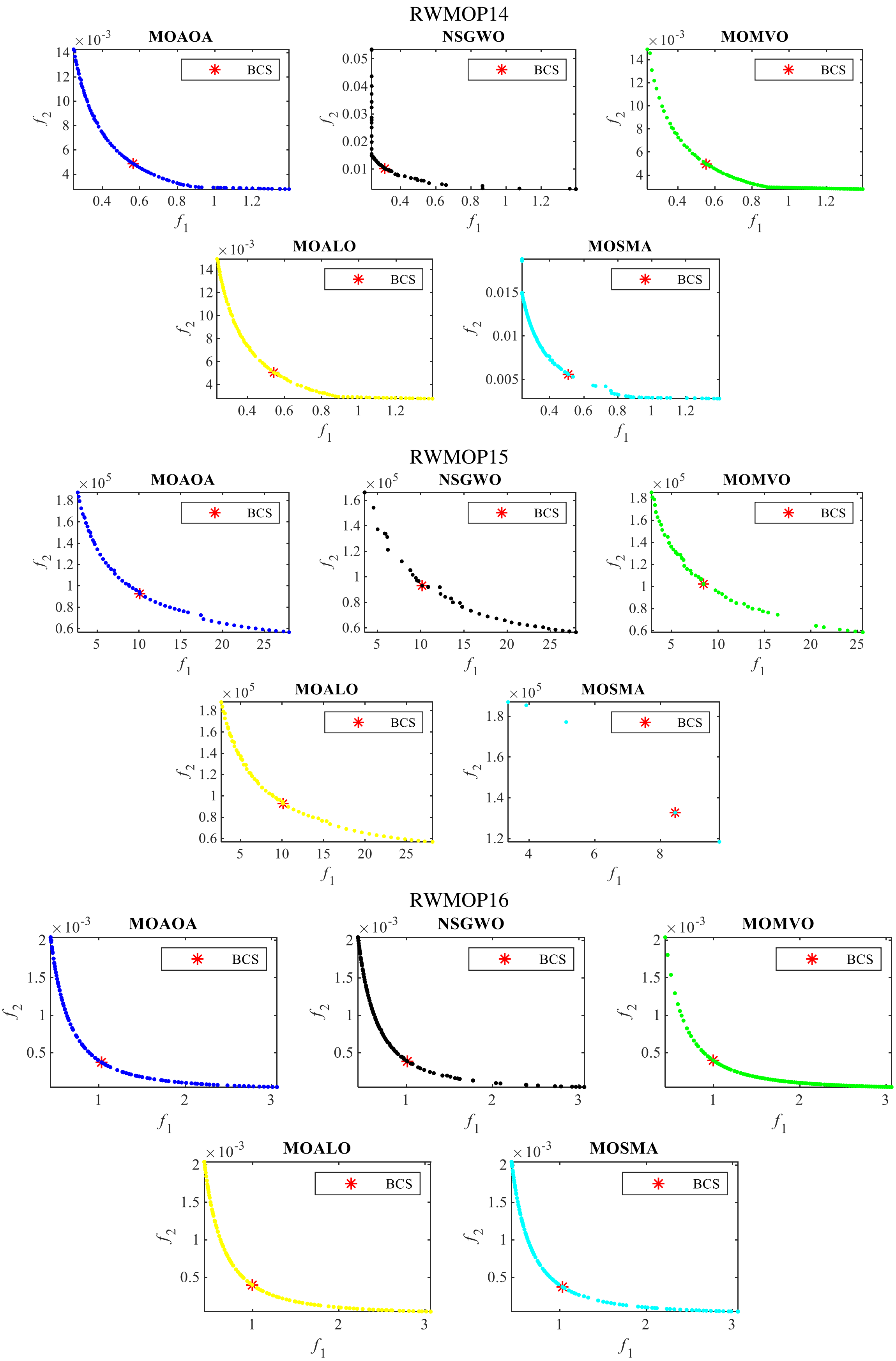

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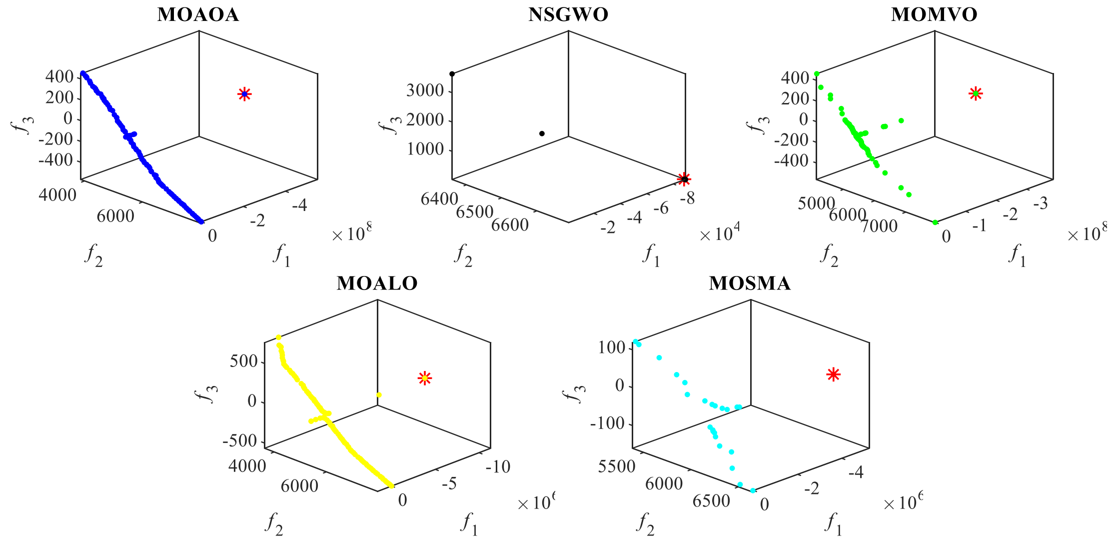
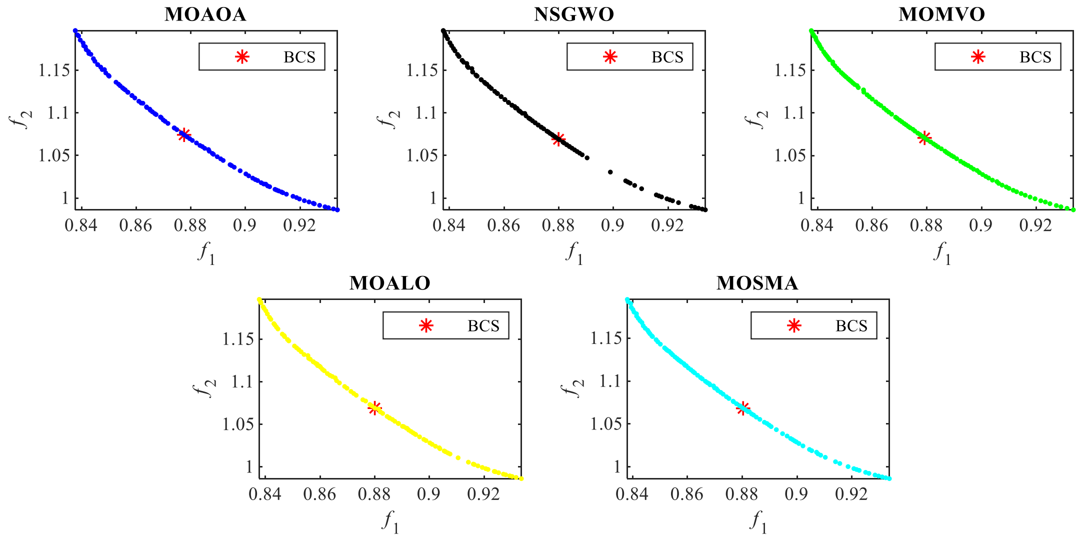
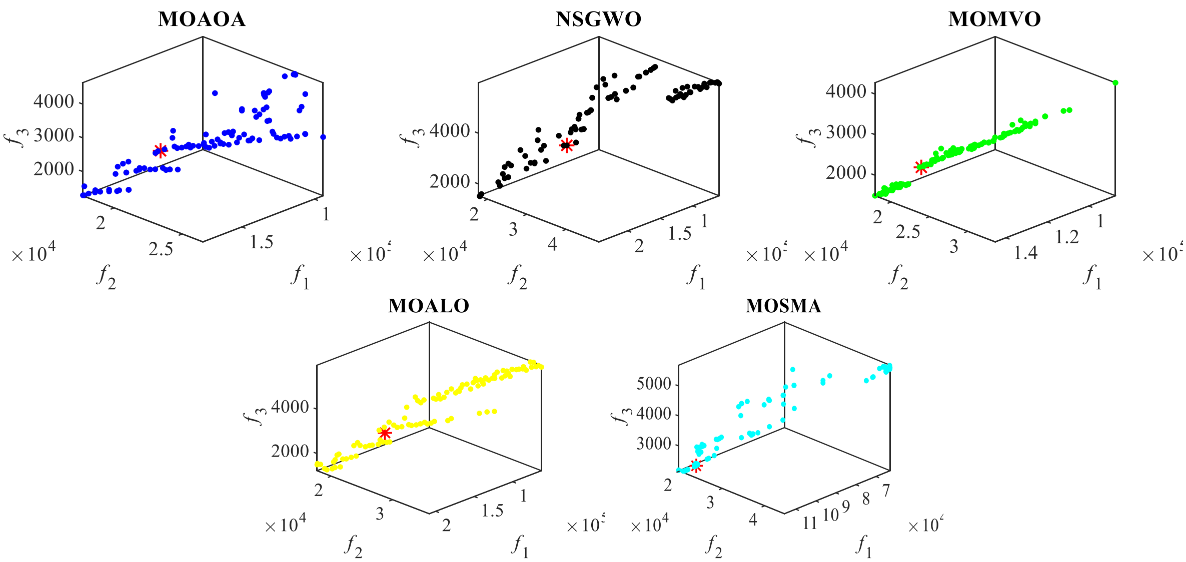
RWMOP17



RWMOP18



RWMOP19



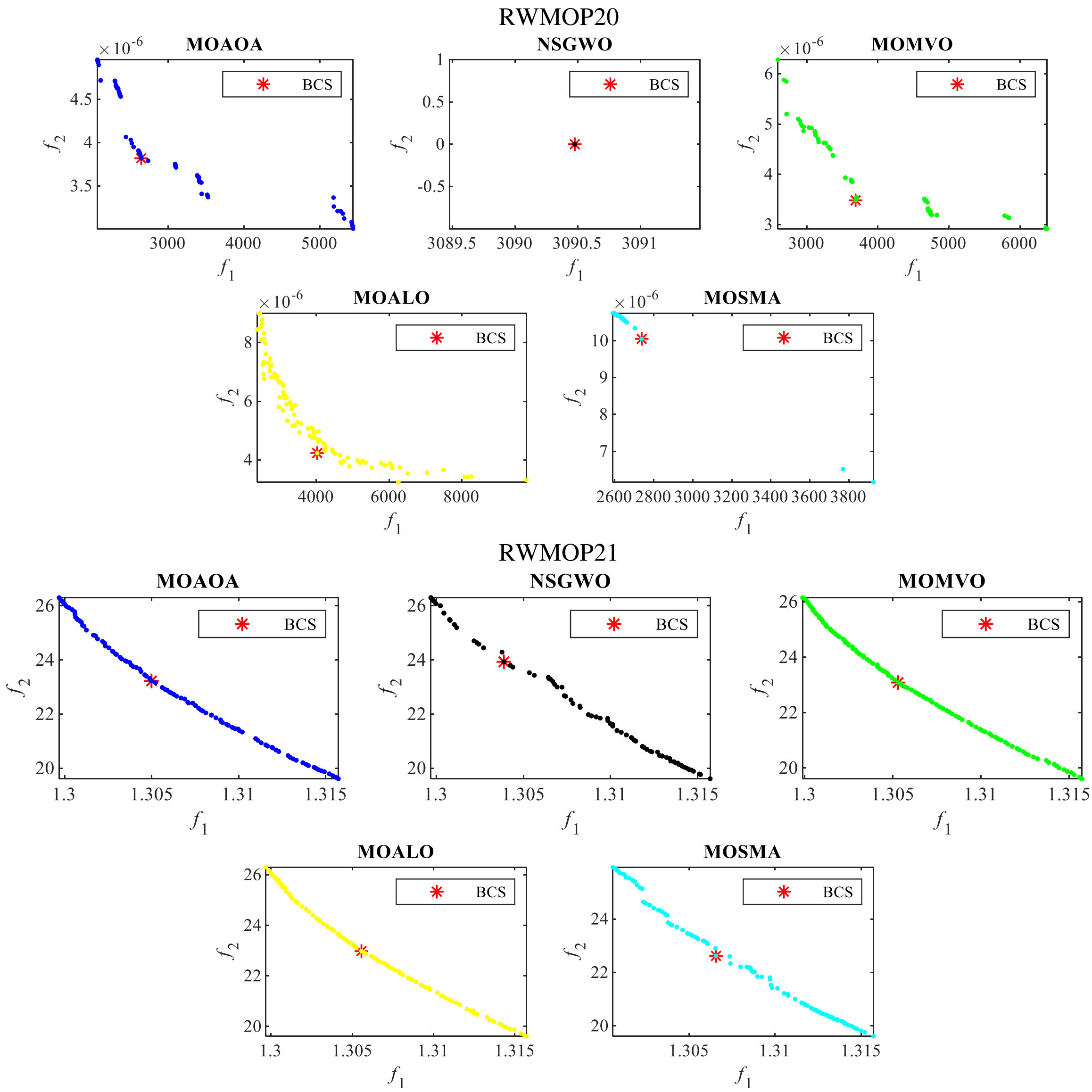


FIGURE 11. Contd.

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