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A New Chaotic System with a Pear-Shaped Equilibrium and Its Circuit Simulation

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ABSTRACT

This paper reports the finding a new chaotic system with a pear-shaped equilibrium curve and makes a valuable addition to existing chaotic systems with infinite equilibrium points in the literature. The new chaotic system has a total of five nonlinearities. Lyapunov exponents of the new chaotic system are studied for verifying chaos properties and phase portraits of the new system are unveiled. An electronic circuit simulation of the new chaotic system with pear-shaped equilibrium curve is shown using Multisim to check the model feasibility

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1. INTRODUCTION

In chaos theory, the key important topics are modeling and applications of nonlinear dynamical systems exhibiting chaotic dynamical behavior. Chaotic systems have generated good interest via various science and engineering applications [1]-[7]. Chaos theory has been also applied for special applications such as voice encryption [8], image encryption [9], robotics [10], secure communication [11]-[13] etc.

Many scientists have studied the modelling of chaotic systems having special types of equilibrium curves such as line equilibrium [14, 15], circle [16], hyperbola [17], parabola [18], cloud-shaped curve [19], rectangle [20], ellipse [20], square [21], hyperbolic sine curve [22], hyperbolic tangent curve [23], exponential curve [24], heart-shaped curve [25], conic-shaped curve [26], axe-shaped curve [27], conch-shaped curve [28] etc.

In this work, we derive a new chaotic system with a pear-shaped equilibrium curve. Our new system exhibits hidden attractors [29]- [30] as it possesses an infinite number of equilibrium points on a pear-shaped curve. This work makes a new valuable addition to the chaotic systems with closed curves of equilibrium points. We also unveil an electronic circuit simulation of the new chaotic system with a pear-shaped curve of equilibrium points. It is known that checking the feasibility of a chaotic system with electronic circuit realization has practical applications [31]-[35].

2. A NEW CHAOTIC SYSTEM WITH A PEAR-SHAPED EQUILIBRIUM CURVE

Motivated by the method and structure proposed in [26], we report a new three-dimensional dynamical system given by

$$\left.\begin{aligned} \dot{x} &= z \\ \dot{y} &= z(-ay - bxz) \\ \dot{z} &= cy^2 - dx^3 + x^4 \end{aligned}\right\} \tag{1}$$

Which has a total of five nonlinearities in the dynamics. We show that the system (1) is chaotic for the parameter values $(a, b, c, d) = (15, 0.02, 6, 6)$.

For numerical simulations of phase portraits and for the calculation of Lyapunov chaos exponents, we take the initial values as $X(0) = (0.2, 0.2, 0.2)$ and parameter set as $(a, b, c, d) = (15, 0.02, 6, 6)$.

The Lyapunov chaos exponents are determined as $(L_1, L_2, L_3) = (0.0073, 0, -0.0084)$. Since $L_1 > 0$, the new system (1) is chaotic. By adding L_1, L_2 and L_3, we get the sum as -0.0011, which is negative. This shows that the new system (1) is dissipative. The Kaplan-Yorke dimension is determined as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.8690 \tag{2}$$

which is a high value showing the complexity of the new system.

The equilibrium points of the new system (1) are tracked by solving the following system:

$$\left.\begin{aligned} 0 &= z \\ 0 &= z(-ay - bxz) \\ 0 &= cy^2 - dx^3 + x^4 \end{aligned}\right\} \tag{3}$$

Simplifying (3), we see that the equilibrium points of the system (1) are characterized by the two equations

$$z = 0 \text{ and } cy^2 = x^3(x - d) \tag{4}$$

which is a pear-shaped curve in the (x, y) plane as shown in Figure 1.

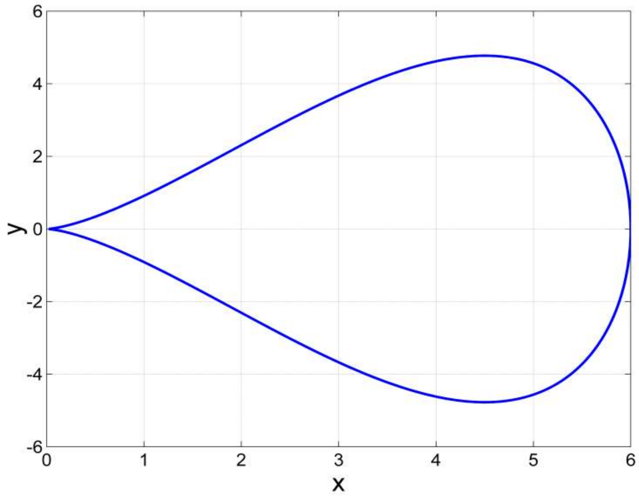

Figure 1. Pear-shaped curve of equilibrium points of the new system (1)

The phase portraits of the new chaotic system (1) with pear-shaped equilibrium curve are displayed in Figure 2. The Lyapunov chaos exponents of the new chaotic system (1) are displayed in Figure 2 (d).

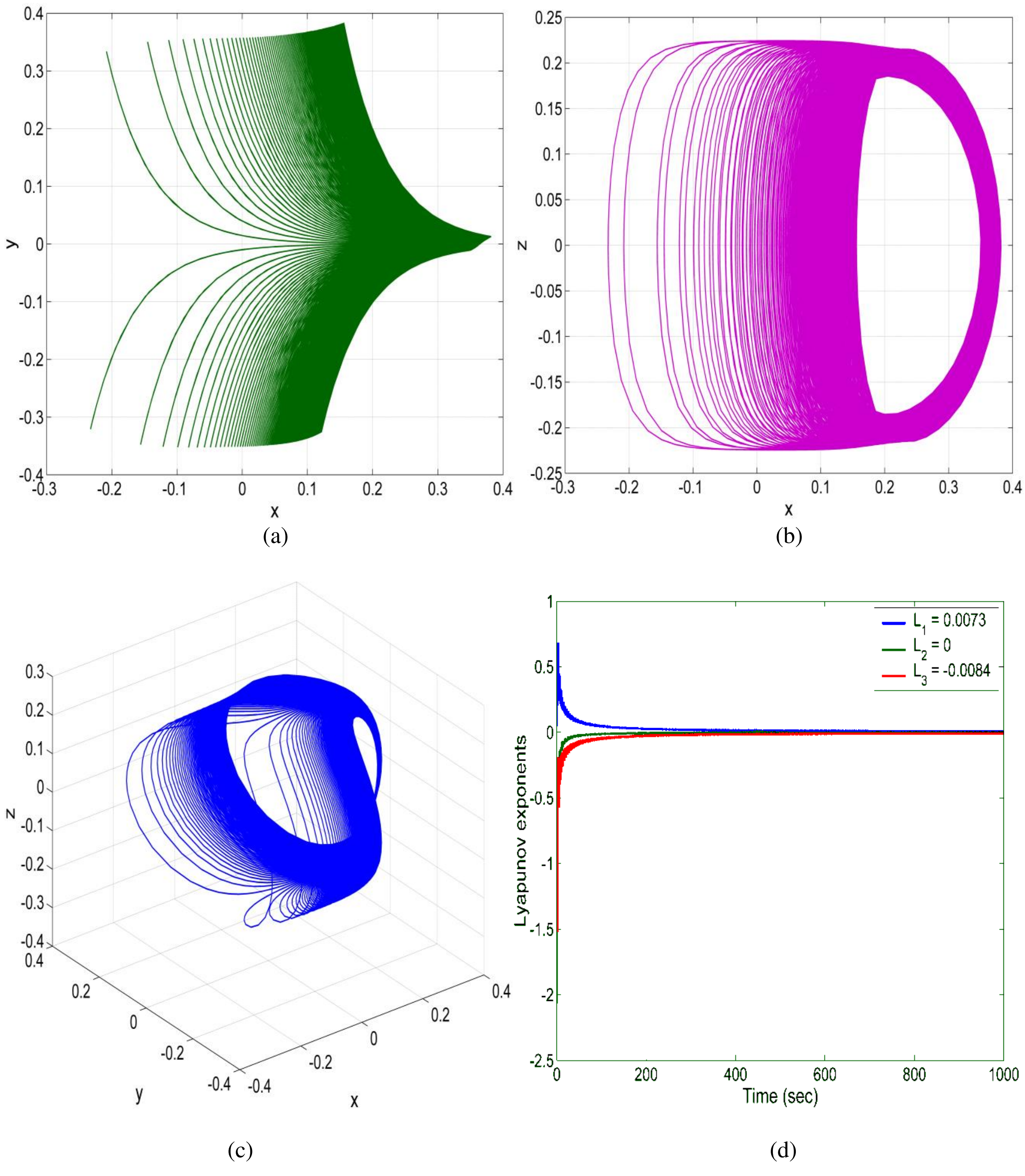

Figure 2. Numerical simulations of phase portraits of the new chaotic system (1) for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b, c, d) = (15, 0.02, 6, 6)$: (a) *x*-*y* plane, (b) *x*-*z* plane, (c) R^3 and (d) Lyapunov exponents

3. CIRCUIT IMPLEMENTATION OF THE NEW CHAOTIC SYSTEM

In order to prove the chaotic behaviors of system (1), a simulation circuit is constructed in this study, which is shown in Figure 3. The new chaotic system (1) can be implemented by the resistance, capacitance, operational amplifier and analog multiplier. Here the variables *x*, *y*, *z* of new chaotic system (1) are the voltages across the capacitor C_1, C_2 and C_3, respectively.

In this section, three state variables of new chaotic system (1) x; y; z are rescaled with amplitude control methods. Therefore the system will be changed int to:

$$\left.\begin{aligned} \dot{x} &= z \\ \dot{y} &= z\left(\frac{-ay}{2} - \frac{bxz}{4}\right) \\ \dot{z} &= \frac{cy^2}{2} - \frac{dx^3}{4} + \frac{x^4}{8} \end{aligned}\right\} \tag{5}$$

To determine the dynamical equations of the circuit, we apply Kirchhoff's circuit laws into the circuit in Figure 3 so that we have the following circuital equations:

$$\left.\begin{aligned} \dot{x} &= \frac{1}{C_1R_1}z \\ \dot{y} &= -\frac{1}{C_2R_2}yz - \frac{1}{C_2R_3}xz^2 \\ \dot{z} &= \frac{1}{C_3R_4}y^2 - \frac{1}{C_3R_5}x^3 + \frac{1}{C_3R_6}x^4 \end{aligned}\right\} \tag{6}$$

In (6), x, y, z are the voltages on capacitors (C_1, C_2, C_3), respectively. We choose R_1 = 400 kΩ, R_2 = 53.33 kΩ, R_3 = 80 MΩ, R_4 = 133.33 kΩ, R_5 = 266.67 kΩ, R_6 = 3.2 MΩ, $R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12}$ = 100 kΩ, $C_1 = C_2 = C_3$ = 1 nF.

The Multisim simulation oscilloscope outputs (phase portraits) of circuitry of the re-scaled new chaotic system, for parameters (a, b, c, d) = (15, 0.02, 6, 6) are seen in Figure 4. A good agreement between the outputs of the theoretical model and chaotic attractors shown in Figures 2 and Figures 4 confirms the feasibility of the new chaotic system (1).

4. CONCLUSION

Discovering chaotic systems with infinite number of equilibrium points such as curve equilibrium is an active topic of research in the chaos literature. In this work, we reported a new chaotic system with a pear-shaped equilibrium curve. We also showed an electronic circuit simulation of the new chaotic system to check the feasibility of the chaotic system model.

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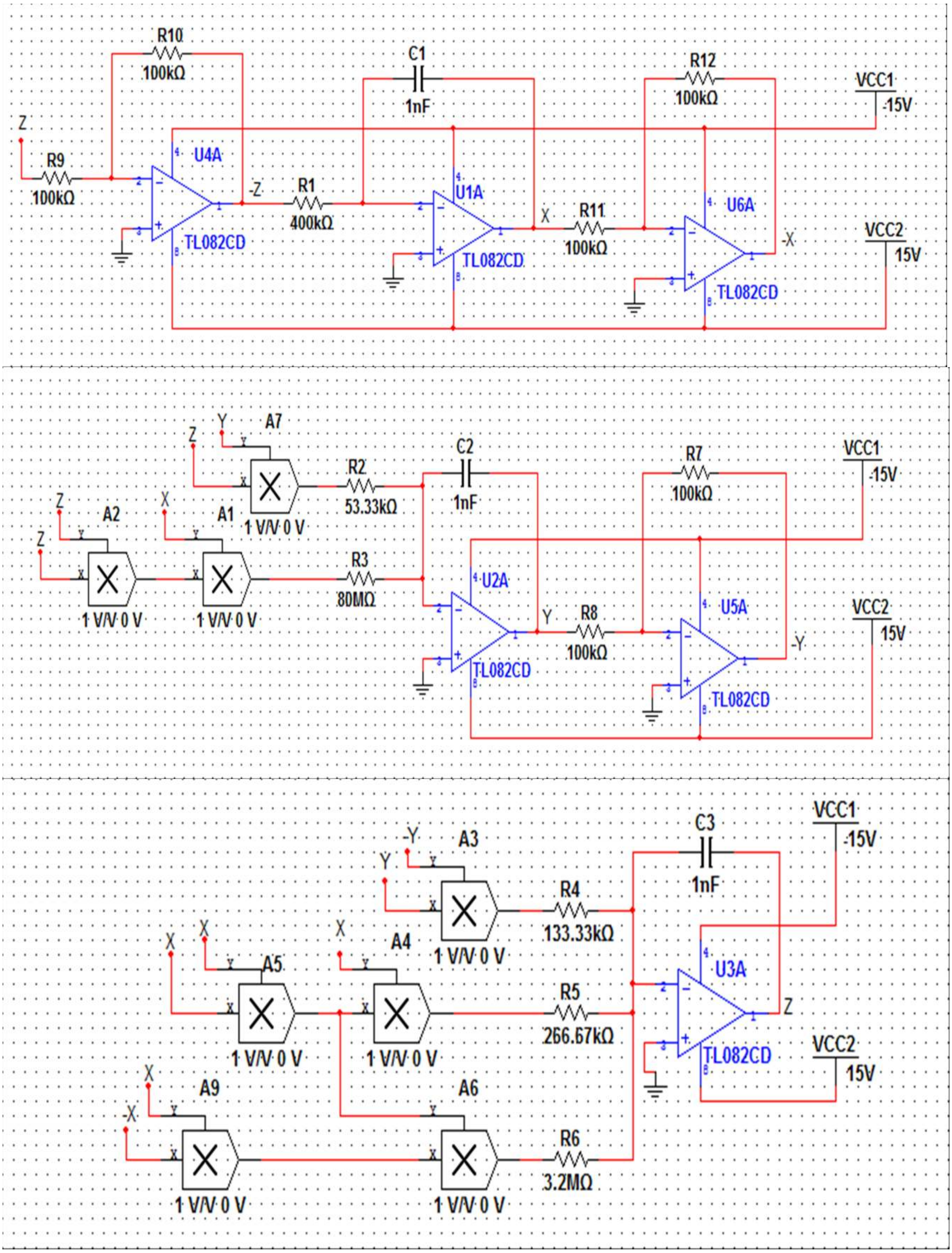
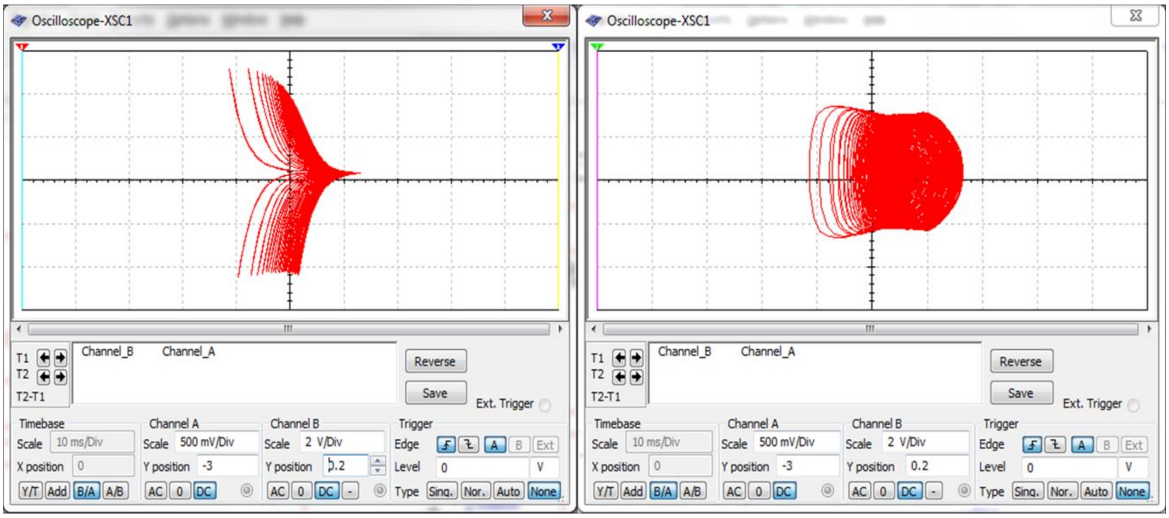


Figure 3. Circuit design of new chaotic system (1)



(a) (b)

Figure 4. Chaotic attractors of system (1) using Multisim circuit simulation:
(a) *x-y* plane, and (b) *x-z* plane

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