

A New Class of Binary Zero Correlation Zone Sequence Sets

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Abstract: This paper proposes a new class of binary zero correlation zone (ZCZ) sequence sets, in which the periodic correlation functions of the proposed sequence set is zero for the phase shifts within the zero-correlation zone. It is shown that the proposed zero correlation zone sequence set can reach the upper bound on the ZCZ codes.

Keywords- Sequence design, theoretical upper bound, zero correlation zone (ZCZ) sequences.

I. Introduction

Zero correlation zone (ZCZ) sequences can be used in spread spectrum systems and CDMA systems to reduce the multiple access interference and co-channel interference [1]. There are several intensive studies on the constructing of ZCZ sequences set [1-8].

The ZCZ sequences set construction is limited by the correlation property and theory bounds [1-7]. Generally, sets of ZCZ sequences are characterized by the period of sequences L , the family size, namely the number of sequences M , and the length of the zero-correlation zone Z_{CZ} . A ZCZ (L, M, Z_{CZ}) sequence set that satisfies the theoretical bound defined by the ratio $M(Z_{CZ} + 1)/L = 1$ is called an optimal zero-correlation zone sequence set [4]. Compared with earlier works on binary ZCZ sequence sets [1, 6], our proposed zero-correlation zone sequence set is, in all cases, an optimal or an approach optimal ZCZ sequence set.

The paper is organized as follows. Section 2 introduces the notations required for the subsequent sections, the proposed scheme for sequence construction is explained in section 3. Examples of new ZCZ sequence sets are presented in Section 4. The properties of the proposed sequence sets are shown in Section 5. Finally, we draw the concluding remarks.

II. Notations

2.1 Definition 1: Suppose $X_j = (x_{j,0}, x_{j,1}, \dots, x_{j,L-1})$ and, $X_v = (x_{v,0}, x_{v,1}, \dots, x_{v,L-1})$ are two sequences of period L . Sequence pair (X_j, X_v) is called a binary sequence pair if

$$x_{j,i}, x_{v,i} \in \{-1, +1\}, i = 0, 1, 2, \dots, L-1 \quad [7].$$

The Periodic Correlation Function (PCF) between X_j and X_v at a shift τ is defined by [4]:

$$\forall \tau \geq 0, \theta_{(X_j, X_v)}(\tau) = \sum_{i=0}^{L-1-\tau} x_{j,i} x_{v,(i+\tau) \bmod L} \quad (1)$$

2.2 Definition 2: A set of M sequences $\{X_0, X_1, X_2, \dots, X_{M-1}\}$ is denoted by $\{X_j\}_{j=0}^{M-1}$.

A set of sequences $\{X_j\}_{j=0}^{M-1}$ is called zero correlation zone sequence set, denoted by $Z(L, M, Z_{CZ})$ if the periodic correlation functions satisfy [4]:

$$\forall j, 0 < |\tau| \leq Z_{CZ}, \theta_{(X_j, X_j)}(\tau) = 0 \quad (2)$$

$$\forall j, (j \neq v), |\tau| \leq Z_{CZ}, \theta_{(X_j, X_v)}(\tau) = 0 \quad (3)$$

III. Proposed Sequence Construction

In this section, a new method for constructing sets of binary ZCZ sequences is proposed. The construction is accomplished through three steps.

3.1 Step 1: The j^{th} row of the Hadamard matrix H of order n is denoted by

$$h_j = [h_{j,0}, h_{j,1}, \dots, h_{j,n-1}].$$

A set of $2n$ sequences d_j , each of length $2n$, is constructed as follows:

For $0 \leq j < n$,

$$d_{j+0} = [-h_j, h_j] \tag{4}$$

$$d_{j+1} = [h_j, h_j] \tag{5}$$

3.2 Step 2: For a fixed integer value n , and for the first stage, $p = 0$, we can generate, based on the scheme for sequence construction in [6], a series of sets $\{B_j\}_{j=0}^{2n-1}$ of $2n$ sequences as follows:

A sequence set $\{B_j\}_{j=0}^{2n-1}$ is constructed from the sequences set $\{d_j\}_{j=0}^{2n-1}$. A pair of sequences B_{j+0} and B_{j+1} of length $(2^{p+2}n)$ are constructed by the process of interleaving a sequence pair d_{j+0} and d_{j+1} as follows:

For $0 \leq j < n$,

$$B_{j+0} = [d_{j+0,0}, d_{j+1,0}, d_{j+0,1}, d_{j+1,1}, \dots, \dots, d_{j+0,2n-1}, d_{j+1,2n-1}] \tag{6}$$

and,

$$B_{j+1} = [d_{j+0,0}, -d_{j+1,0}, d_{j+0,1}, -d_{j+1,1}, \dots, \dots, d_{j+0,2n-1}, -d_{j+1,2n-1}] \tag{7}$$

The member size of the sequence set $\{B_j\}$ is $2n$.

3.3 Step 3: For $p > 0$, we can recursively construct a new series of set, $\{B_j\}_{j=0}^{2n-1}$, by interleaving of actual $\{B_j\}_{j=0}^{2n-1}$.

The $\{B_j\}_{j=0}^{2n-1}$ is generated as follows:

For $0 \leq j < n$,

$$B_{j+0} = [B_{j+0,0}, B_{j+1,0}, B_{j+0,1}, B_{j+1,1}, \dots, \dots, B_{j+0,4n-1}, B_{j+1,4n-1}] \tag{8}$$

and,

$$B_{j+1} = [B_{j+0,0}, -B_{j+1,0}, B_{j+0,1}, -B_{j+1,1}, \dots, \dots, B_{j+0,4n-1}, -B_{j+1,4n-1}] \tag{9}$$

The length of both sequences B_{j+0} and B_{j+1} is equal to $(2^{p+2}n)$.

IV. Example of Construction

4.1 Step 1: Let H be a Hadamard matrix of order $n=2$, given by:

$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} h_0 \\ h_1 \end{bmatrix}$$

A set of $2n = 4$ sequences d_j , each of length $2n = 4$, is constructed as follows:

For $0 \leq j < 2$,

$$d_{0+0} = [-h_0, h_0] = [-1, -1, 1, 1]$$

$$d_{1+0} = [-h_1, h_1] = [-1, 1, 1, -1]$$

$$d_{0+1} = [h_0, h_0] = [1, 1, 1, 1]$$

$$d_{1+1} = [h_1, h_1] = [1, -1, 1, -1]$$

4.2 Step 2: For the first iteration, $p = 0$.

A pair of sequences B_{j+0} and B_{j+1} of length $(2^{p+2}n) = 8$ are constructed by interleaving a sequence pair d_{j+0} and d_{j+1} as follows:

For $0 \leq j < 2$,

$$B_{j+0} = [d_{j+0,0}, d_{j+1,0}, d_{j+0,1}, d_{j+1,1}, \dots, \dots, d_{j+0,3}, d_{j+1,3}]$$

$$B_{0+0} = [-1, 1, -1, 1, 1, 1, 1, 1]$$

$$B_{1+0} = [-1, 1, 1, -1, 1, 1, -1, -1]$$

and,

$$B_{j+1} = [d_{j+0,0}, -d_{j+1,0}, d_{j+0,1}, -d_{j+1,1}, \dots, \dots, d_{j+0,3}, -d_{j+1,3}]$$

$$B_{0+1} = [-1, -1, -1, -1, 1, 1, -1, -1]$$

$$B_{1+1} = [-1, -1, 1, 1, 1, -1, -1, 1]$$

4.3 Step 3: For the next iteration $p = 1$, The $\{B_j\}_{j=0}^3$ is generated as follows:

$$B_{j+0} = [B_{j+0,0}, B_{j+1,0}, B_{j+0,1}, B_{j+1,1}, \dots, \dots, B_{j+0,7}, B_{j+1,7}]$$

$$B_{0+0} = [-1, -1, 1, -1, -1, -1, 1, -1, 1, 1, 1, -1, 1, 1, 1, -1],$$

$$B_{1+0} = [-1, -1, 1, -1, 1, 1, -1, 1, 1, 1, -1, -1, -1, -1, 1],$$

and,

$$B_{j+1} = [B_{j+0,0}, -B_{j+1,0}, B_{j+0,1}, -B_{j+1,1}, \dots, \dots, B_{j+0,7}, -B_{j+1,7}]$$

$$B_{0+1} = [-1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1],$$

$$B_{1+1} = [-1, 1, 1, 1, -1, -1, -1, 1, -1, 1, 1, -1, 1, -1, -1].$$

The length of both sequences B_{j+0} and B_{j+1} is equal to $(2^{p+2}n) = 16$.

The member size of the sequence set $\{B_j\}$ is $2n = 4$.

Iteratively for $p = 2$, The $\{B_j\}_{j=0}^3$ is generated as follows:

$$\begin{aligned}
 B_{0+0} &= [-1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1, -1, 1, -1, 1] \\
 B_{1+0} &= [-1, -1, -1, 1, 1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1, 1, 1, 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, -1] \\
 B_{0+1} &= [-1, 1, -1, -1, 1, -1, -1, -1, -1, 1, -1, -1, 1, -1, -1, -1, 1, -1, 1, 1, 1, -1, -1, -1, 1, -1, 1, 1, -1, -1, -1] \\
 B_{1+1} &= [-1, 1, -1, -1, 1, -1, -1, -1, 1, -1, 1, 1, -1, 1, 1, 1, -1, 1, 1, 1, -1, -1, -1, -1, 1, -1, -1, -1, 1, 1, 1]
 \end{aligned}$$

Also, we can find out that the zero correlation zone length of $\{B_j\}_{j=0}^3$ is 4. For example, for $\tau = 0, 1, \dots, \dots, 31$, the periodic auto-correlation functions (PACF) $(\forall j, j = v)$ given in (1) of $B_{0+0}, B_{1+0}, B_{0+1}$ and B_{1+1} are calculated as follows:

$$\begin{aligned}
 &\theta_{(B_{0+0}, B_{0+0})}(\tau) \\
 &= \{32, 0, 0, 0, 0, 4, 0, -4, 16, -4, 0, 4, 0, 8, 0, -8, 0, -8, 0, 8, 0, 4, 0, -4, 16, -4, 0, 4, 0, 0, 0\} \\
 &\theta_{(B_{1+0}, B_{1+0})}(\tau) = \\
 &\{32, 0, 0, 0, 0, -4, 0, 4, -16, 4, 0, -4, 0, 8, 0, -8, 0, -8, 0, 8, 0, -4, 0, 4, -16, 4, 0, -4, 0, 0, 0, 0\} \\
 &\theta_{(B_{0+1}, B_{0+1})}(\tau) \\
 &= \{32, 0, 0, 0, 0, -4, 0, 4, 16, 4, 0, -4, 0, -8, 0, 8, 0, 8, 0, -8, 0, -4, 0, 4, 16, 4, 0, -4, 0, 0, 0, 0\} \\
 &\theta_{(B_{1+1}, B_{1+1})}(\tau) \\
 &= \{32, 0, 0, 0, 0, 4, 0, -4, -16, -4, 0, 4, 0, -8, 0, 8, 0, 8, 0, -8, 0, 4, 0, -4, -16, -4, 0, 4, 0, 0, 0, 0\}
 \end{aligned}$$

The periodic cross-correlation functions (PCCF) $(\forall j, j \neq v)$ given in (1) are calculated as follows:

$$\begin{aligned}
 &\theta_{(B_{0+0}, B_{1+0})}(\tau) \\
 &= \{0, 0, 0, 0, 0, -4, 0, 4, 16, 4, 0, -4, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, -4, -16, -4, 0, 4, 0, 0, 0, 0\} \\
 &\theta_{(B_{0+0}, B_{0+1})}(\tau) \\
 &= \{0, 0, 0, 0, 0, -4, -8, -4, 0, 4, -8, 4, 0, -8, -16, -8, 0, 8, -16, 8, 0, -4, -8, -4, 0, 4, -8, 4, 0, 0, 0, 0\} \\
 &\theta_{(B_{0+0}, B_{1+1})}(\tau) \\
 &= \{0, 0, 0, 0, 0, 4, 8, 4, 0, -4, 8, -4, 0, 0, 0, 0, 0, 0, 0, 0, -4, -8, -4, 0, 4, -8, 4, 0, 0, 0, 0\} \\
 &\theta_{(B_{1+0}, B_{0+1})}(\tau) \\
 &= \{0, 0, 0, 0, 0, -4, -8, -4, 0, 4, -8, 4, 0, 0, 0, 0, 0, 0, 0, 0, 4, 8, 4, 0, -4, 8, -4, 0, 0, 0, 0\} \\
 &\theta_{(B_{1+0}, B_{1+1})}(\tau) \\
 &= \{0, 0, 0, 0, 0, 4, 8, 4, 0, -4, 8, -4, 0, -8, -16, -8, 0, 8, -16, 8, 0, 4, 8, 4, 0, -4, 8, -4, 0, 0, 0, 0\} \\
 &\theta_{(B_{0+1}, B_{1+1})}(\tau) = \{0, 0, 0, 0, 0, 4, 0, -4, 16, -4, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, -4, 0, 4, -16, 4, 0, -4, 0, 0, 0, 0\}
 \end{aligned}$$

We notice that the zero correlation zone length of $\{B_j\}_{j=0}^3$ is 4.

The PACF and PCCF confirm that $\{B_j\}$ is a ZCZ $(32, 4, 4)$ sequence set.

V. The Properties of the Proposed Sequence

The proposed zero-correlation zone sequence set can be generated from an arbitrary Hadamard matrix of order n .

The length of B_j in equations (6) and (7), equal to $(2^{p+2}n)$, is twice that of B_j in equations (8) and (9).

The proposed ZCZ sequence set can satisfy the ideal autocorrelation and cross-correlation properties in the zero-correlation zone.

The generated sequence set satisfies the following properties:

$$\forall j, \forall \tau \neq 0, |\tau| \leq 2^p$$

$$\theta_{(B_j, B_j)}(\tau) = 0$$

(10)

and,

$$\forall j \neq v, \forall \tau, |\tau| \leq 2^p$$

$$\theta_{(B_j, B_v)}(\tau) = 0.$$

(11)

The $\{B_j\}$ is a ZCZ sequence set having parameters $(L, M, Z_{CZ}) = (2^{p+2}n, 2n, 2^p)$, its parameters perfectly satisfy the theory bound of ZCZ sequences set [4, 7]:

$$M(Z_{CZ} + 1) \leq L \tag{12}$$

Let $\mu = \frac{M(Z_{CZ}+1)}{L}$, if $\mu = 1$, it indicates that the ZCZ sequence set is optimal [1, 6].

For the proposed ZCZ $(2^{p+2}n, 2n, 2^p)$ sequence set, $\mu = \frac{M(Z_{CZ}+1)}{L} = \frac{2n(2^p+1)}{2^{p+2}n} = \frac{(2^p+1)}{2^{p+1}}$

- 1) For $p = 0$, $\mu = 1$, the proposed ZCZ sequence set is optimal.
- 2) For $p > 0$, $1/2 \leq \mu < 1$ and $\lim_{p \rightarrow \infty}(\mu) = 1/2$.

For a given family size M, we can construct different sets of sequences with different lengths L and zero correlation zone Z_{CZ} . As an example, assuming that M=16, we can construct sets of ZCZ sequences with different lengths L=32, 64, 128, 256, 512, 1024, 2048, 4096....., and different ZCZs, $Z_{CZ}=1, 2, 4, 8, 16, 32, 64, 128, \dots$ and different parameters $\mu=1, 0.75, 0.625, 0.5625, 0.53125, 0.515625, 0.5078125, 0.5039063, \dots$

In step 1, other set of $2n$ sequences d_j , each of length $2n$, can be constructed as follows:

1- For $0 \leq j < n$,

$$d_{j+0} = [h_j, -h_j]$$

(13)

$$d_{j+1} = [h_j, h_j]$$

(14)

In this case, the set of $\{B_j\}_{j=0}^3$ is generated as follows:

$$B_{0+0} = [1,1,1, -1, 1, 1, -1, 1,1,1,1, -1, 1, 1, -1, 1, -1, -1, -1, -1,1,1, 1, -1, 1, -1, -1, -1, -1,1,1, 1, -1, 1, -1, -1, 1]$$

$$B_{1+0} = [1,1,1, -1, 1, 1, -1, 1, -1, -1, -1, 1, -1, -1, 1, -1, -1, -1, -1, 1,1, 1, 1, -1, 1,1,1,1, -1, -1, -1,1, -1]$$

$$B_{0+1} = [1, -1,1,1, 1, -1, -1, -1,1, -1,1,1, 1, -1, -1, -1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, -1, 1, -1, -1, 1, -1, -1, -1]$$

$$B_{1+1} = [1, -1,1,1, 1, -1, -1, -1, 1, -1, -1, -1, 1,1, 1, -1, 1, -1, -1, 1, -1, -1, -1, 1, -1, 1, -1, 1,1, -1,1,1,1]$$

2- For $0 \leq j < n$,

$$d_{j+0} = [h_j, h_j]$$

(15)

$$d_{j+1} = [-h_j, h_j]$$

(16)

The set of $\{B_j\}_{j=0}^3$ is generated as follows:

$$B_{0+0} = [1,1,1, -1, -1, -1, 1, -1,1,1,1, -1, -1, -1, 1, -1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1, -1, 1]$$

$$B_{1+0} = [1,1,1, -1, -1, -1, 1, -1, -1, -1, 1, 1, 1, -1, 1, 1, 1, 1, -1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, 1, -1]$$

$$B_{0+1} = [1, -1,1,1, -1,1,1,1,1, -1,1,1, -1,1,1, 1, -1, 1, 1, 1, -1, -1, -1, 1, -1, 1, 1, 1, -1, -1, -1]$$

$$B_{1+1} = [1, -1,1,1, -1,1,1,1, -1, 1, -1, -1, 1, -1, -1, 1, -1, 1, 1, 1, -1, -1, -1, -1, -1, -1, -1, -1, 1, -1, -1, 1, 1, 1]$$

3- For $0 \leq j < n$,

$$d_{j+0} = [h_j, h_j]$$

(17)

$$d_{j+1} = [h_j, -h_j]$$

(18)

It should be noted that in [9], the limits of the correlation function of binary zero-correlation zone sequences $\{B_j\}_{j=0}^3$ obtained from equation (17) and (18) are evaluated.

VI. Conclusion

In this paper, we have proposed a new method for constructing sets of binary ZCZ sequences. The structure of the proposed ZCZ sequences set is simple and thus it is easy to be generated. The PACF side lobes and PCCF of the proposed sequence set is zero for the phase shifts within the zero-correlation zone. The proposed ZCZ sequence set with $(2^{p+2}n, 2n, 2^p)$ is optimal or asymptotically optimal ZCZ sequence set. This method is useful for designing spreading sequences for multi-user CDMA system.

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