

## A NEW CLASS OF GENERALIZED NONLINEAR MULTI-VALUED QUASI-VARIATIONAL-LIKE INCLUSIONS WITH $H$ -MONOTONE MAPPINGS

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*Abstract.* In this paper, we introduce and study a new class of generalized nonlinear multi-valued quasi-variational-like inclusions with  $H$ -monotone operators in Hilbert spaces. By using the resolvent operator method associated with  $H$ -monotone operator due to Fang and Huang, we construct a new iterative algorithm for solving this kind of nonlinear multi-valued variational inclusions. We also prove the existence of solutions for the nonlinear multi-valued variational inclusions and the convergence of iterative sequences generated by the algorithm. Our results improve and generalize many known corresponding results.

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*Key words and phrases:* generalized nonlinear multi-valued variational inclusion with  $H$ -monotone operator, resolvent operator technique, relaxed cocoercive mapping, iterative algorithm with errors, existence and convergence.

### REFERENCES

- [1] S. ADLY, *Perturbed algorithm and sensitivity analysis for a general class of variational inclusions*, J. Math. Anal. Appl., **201**, (1996), 609–630.
- [2] R. AHMAD, Q. H. ANSARI, *An iterative algorithm for generalized nonlinear variational inclusions*, Appl. Math. Lett., **13**, (5) (2000), 23–26.
- [3] Y. J. CHO, J. H. KIM, N. J. HUANG AND S. M. KANG, *Ishikawa and Mann iterative processes with errors for generalized strongly nonlinear implicit quasi-variational inequalities*, Publ. Math. Debrecen, **58**, (2001), 635–649.
- [4] X. P. DING, *Perturbed proximal point algorithm for generalized quasivariational inclusions*, J. Math. Anal. Appl., **210**, (1) (1997), 88–101.
- [5] X. P. DING, *Generalized quasi-variational-like inclusions with nonconvex functionals*, Appl. Math. & Comput., **122**, (3) (2001), 267–282.
- [6] X. P. DING, C. L. LUO, *Perturbed proximal point algorithms for general quasi-variational-like inclusions*, J. Comput. Appl. Math., **113**, (2000), 153–165.
- [7] Y. P. FANG, N. J. HUANG,  *$H$ -monotone operator and resolvent operator technique for variational inclusions*, Applied Mathematics and Computation, **145**, (2003), 795–803.
- [8] Y. P. FANG, N. J. HUANG,  *$H$ -monotone operators and system of variational inclusions*, Communications on Applied Nonlinear Analysis, **11**, (2004), 93–101.
- [9] A. HASSOUNI, A. MOUDAFI, *perturbed algorithm for variational inclusions*, J. Math. Anal. Appl., **185**, (3) (1994), 706–712.
- [10] N. J. HUANG, *Mann and Ishikawa type perturbed iterative algorithms for generalized nonlinear implicit quasi-variational inclusions*, Computers Math. Appl., **35**, (10) (1998), 1–7.
- [11] N. J. HUANG, *Generalized nonlinear variational inclusions with noncompact valued mappings*, Appl. Math. Lett., **9**, (3) (1996), 25–29.
- [12] N. J. HUANG, *Generalized nonlinear implicit quasivariational inclusion and an application to implicit variational inequalities*, Z. Angew. Math. Mech., **79**, (8) (1999), 569–575.

- [13] M. M. JIN, *Perturbed algorithm and stability for strongly nonlinear quasi-variational inclusion involving  $H$ -monotone operators*, *Math. Inequal. Appl.*, **9**, (4) (2006), 771–779.
- [14] J. S. JUNG, C. H. MORALES, *The Mann process for perturbed  $m$ -accretive operators in Banach spaces*, *Nonlinear Anal.* **46**, (2) (2001), 231–243.
- [15] H. Y. LAN, J. K. KIM AND N. J. HUANG, *On the generalized nonlinear quasi-variational inclusions involving non-monotone set-valued mappings*, *Nonlinear Funct. Anal. & Appl.*, **9**, (3) (2004), 451–465.
- [16] C. H. LEE, Q. H. ANSARI AND J. C. YAO, *A perturbed algorithm for strongly nonlinear variational-like inclusions*, *Bull. Austral. Math. Soc.*, **62**, (2000), 417–426.
- [17] L. W. LIU, Y. Q. LI, *On generalized set-valued variational inclusions*, *J. Math. Anal. Appl.*, **261**, (1) (2001), 231–240.
- [18] S. B. NALDER, *Multi-valued contraction mappings*, *Pacific J. Math.*, **30**, (1969), 475–488.
- [19] Z. NANIEWICZ, P. D. PANAGIOTOPOULOS, *Mathematical Theory of Hemi-variational Inequalities and Applications*, Marcel Dekker, New York, 1995.
- [20] E. ZEIDLER, *Nonlinear Functional Analysis and its Applications II: Monotone Operators*, Springer-Verlag, Berlin, 1985.
- [21] M. A. NOOR, K. I. NOOR AND T. M. RASSIAS, *Invitation to variational inequalities*, in: *Analysis, Geometry and Groups: A Riemann Legacy Volume*, Hadronic, FL, (1993), 373–448.
- [22] SALAHUDDIN, A. RAIS, *Generalized multivalued nonlinear quasi-variational like inclusions*, *Nonlinear Anal. Forum*, **6**, (2) (2001), 409–416.
- [23] S. H. SHIM, S. M. KANG, N. J. HUANG AND Y. J. CHO, *Perturbed iterative algorithms with errors for completely generalized strongly nonlinear implicit quasivariational inclusions*, *J. Inequal. Appl.*, **5**, (4) (2000), 381–395.
- [24] R. U. VERMA, *Nonlinear  $H$ -monotone variational inclusions and resolvent operator technique*, *Int. J. Pure & Appl. Math. Sci.*, **2**, (1) (2005), 53–57.
- [25] R. U. VERMA, *Generalized system for relaxed cocoercive variational inequalities and projection methods*, *Journal of Optimization Theory and Applications*, **121**, (2004), 203–210.
- [26] R. U. VERMA, *Partially relaxed cocoercive variational inequalities and auxiliary problem principle*, *Journal of Applied Mathematics and Stochastic Analysis*, **17**, (2) (2004), 143–148.
- [27] GEORGE X. Z. YUAN, *KKM Theory and Applications in Nonlinear Analysis*, Marcel Dekker, New York, 1999.