Hindawi Publishing Corporation
ISRN Mathematical Analysis
Volume 2013, Article ID 695891, 18 pages
http://dx.doi.org/10.1155/2013/695891

Research Article

A New Existence Theory for Positive Periodic Solutions to a Class of Neutral Delay Model with Feedback Control and Impulse

Zhenguo Luo,[1,2] Jianhua Huang,[1] and Binxiang Dai[3]

[1] *Department of Mathematics, National University of Defense Technology, Changsha 410073, China*
[2] *Department of Mathematics, Hengyang Normal University, Hengyang 421008, China*
[3] *School of Mathematical Sciences and Statistics, Central South University, Changsha 410075, China*

Correspondence should be addressed to Zhenguo Luo; robert186@163.com

Received 10 May 2013; Accepted 9 July 2013

Academic Editors: M. Ehrnström, S. U. Islam, G. Ólafsson, G. Schimperna, and L. Wang

We acquire some sufficient and realistic conditions for the existence of positive periodic solution of a general neutral impulsive n-species competitive model with feedback control by applying some analysis techniques and a new existence theorem, which is different from Gaines and Mawhin's continuation theorem and abstract continuation theory for k-set contraction. As applications, we also examine some special cases, which have been studied extensively in the literature, some known results are improved and generalized.

1. Introduction

In this paper, we consider the existence of the positive periodic solution of the following impulsive n-species competition system with multiple delays and feedback control:

$$\begin{aligned}
N_i'(t) = N_i(t)\Bigg[& r_i(t) - \sum_{j=1}^{n} a_{ij}(t) N_j(t) \\
& - \sum_{j=1}^{n} b_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) N_j(s)\,ds \\
& - \sum_{j=1}^{n} c_{ij}(t) N_j\left(t-\gamma_{ij}(t)\right) \\
& - \sum_{j=1}^{n} d_{ij}(t) N_j'\left(t-\tau_{ij}(t)\right) - e_i(t) u_i(t) \\
& - f_i(t) u_i\left(t-\delta_i(t)\right)\Bigg], \\
& i = 1,2,\ldots,n,\ t \neq t_k, \\
u_i'(t) = & -\alpha_i(t) u_i(t) + \beta_i(t) N_i(t) \\
& + \theta_i(t) N_i\left(t-\sigma_i(t)\right), \quad t \geq 0, \\
\Delta N_i(t_k) = & \left(p_{ik} + q_{ik}\right) N_i(t_k), \\
& i = 1,2,\ldots,n,\ k = 1,2,\ldots,
\end{aligned} \tag{1}$$

with the following initial conditions:

$$\begin{aligned}
& N_i(\xi) = \phi_i(\xi), \qquad N_i'(\xi) = \phi_i'(\xi), \\
& \xi \in [-\tau, 0], \quad \phi_i(0) > 0, \\
& \phi_i \in C\left([-\tau, 0), R^+\right) \bigcap C^1\left([-\tau, 0), R^+\right) \\
& u_i(\xi) = \varphi_i(\xi), \quad \xi \in [-\tau, 0], \ \varphi_i(0) > 0, \\
& \varphi_i \in C\left([-\tau, 0), R^+\right), \quad i = 1,2,3,\ldots,n,
\end{aligned} \tag{2}$$

where $a_{ij}, b_{ij}, c_{ij}, e_i, f_i, \alpha_i, \beta_i, \theta_i \in C(R,[0,+\infty))$, $d_{ij} \in C^1(R,[0,+\infty))$, $\gamma_{ij}, \delta_i \in C^1(R,R)$, and $\tau_{ij} \in C^2(R,R)$ are continuous ω-periodic functions; $r_i \in C(R,R)$ are continuous ω-periodic functions with $\int_0^{\omega} r_i(t)dt > 0$. The growth functions r_i are not necessarily positive; since the environment fluctuates randomly, in some conditions, r_i may be negative. Consider the following: $\tau = \max_{t\in[0,\omega]}\{\gamma_{ij}(t), \tau_{ij}(t), \delta_i(t), \sigma_i(t), 1 \le i, j \le n\}$; and $\int_0^{\infty} k_{ij}(s)ds = 1$, $\int_0^{+\infty} sk_{ij}(s)ds < +\infty$, and $i, j = 1, 2, \ldots, n$. And p_{ik} and q_{ik} represent the birth rate and the harvesting (or stocking) rate of N_i at time t_k, respectively. When $q_{ik} > 0$, it stands for harvesting, while $q_{ik} < 0$ means stocking. For the ecological justification of (1) and the similar types, refer to [1–14].

In 1991, in [1], Gopalsamy et al. have established the existence of a positive periodic solution for a periodic neutral delay logistic equation

$$\frac{dN}{dt} = r(t) N(t) \left[1 - \frac{N(t-mT) - c(t) N'(t-mT)}{K(t)}\right], \tag{3}$$

where $K(t), r(t)$, and $c(t)$ are positive continuous T-periodic functions with $T > 0$ and m is a positive integer. In 1993, in [2], Kuang proposed an open problem (Open problem 9.2) to obtain sufficient conditions for the existence of a positive periodic solution of the following equation:

$$\begin{aligned}\frac{dN}{dt} = N(t) \big[&a(t) - \beta(t) N(t) - b(t) N(t-\tau(t)) \\ &-c(t) N'(t-\tau(t))\big].\end{aligned} \tag{4}$$

In [3], Li tried to give an affirmative answer to the previous open problem; however, there is a mistake in the proof of Theorem 2 in [3]. With the aim of giving a right answer to the previous open problem, [4–6] also have investigated the previous question. However, it is more complex to check the sufficient conditions of the system [5, 6]. Moreover, in [7], Li studied the existence of positive periodic solution of the neutral Lotka-Volterra equation with several delays

$$\begin{aligned}N'(t) = N(t) \Bigg[&a(t) - \sum_{i=1}^{n} b_i(t) N(t-\tau_i) \\ &-\sum_{i=1}^{n} c_i(t) N'(t-\gamma_i)\Bigg],\end{aligned} \tag{5}$$

where $a(t), b_i(t)$, and $c_i(t)$ are positive continuous T-periodic functions and τ_i, γ_i $(i = 1, \ldots, n)$ are nonnegative constants. Recently, in [8], Lu and Ge investigated a neutral delay population model with multiple delays:

$$\begin{aligned}\frac{dN}{dt} = N(t) \Bigg[&a(t) - \beta(t) N(t) - \sum_{j=1}^{n} b_j(t) N\left(t-\sigma_j(t)\right) \\ &-\sum_{i=1}^{m} c_i(t) N'(t-\tau_i(t))\Bigg].\end{aligned} \tag{6}$$

They applied the theory of abstract continuous theorem of k-set contractive operator and some analysis techniques to obtain some sufficient conditions for the existence of positive periodic solutions of the model (6).

It is of course very interesting to study the neutral delay population model for higher dimensional systems. In fact, in [9], Li has studied the neutral Lotka-Volterra system with constant delays

$$\begin{aligned}N_i'(t) = N_i(t) \Bigg[&a_i(t) - \sum_{j=1}^{n} b_{ij}(t) N_j\left(t-\tau_{ij}\right) \\ &-\sum_{j=1}^{n} c_{ij}(t) N_j'\left(t-\gamma_{ij}\right)\Bigg],\end{aligned} \tag{7}$$

where $i = 1, \ldots, n$, and $a_i(t)$, $b_{ij}(t)$, and $c_{ij}(t)$ are positive continuous T-periodic functions, and τ_{ij}, γ_{ij} are nonnegative constants. He obtained sufficient conditions that guarantee the existence of positive periodic solution of the system (7), by applying a continuation theorem based on Gaines and Mawhin's coincidence degree. Noticing that delays arise frequently in practical applications, it is difficult to measure them precisely. In population dynamics, it is clear that a constant delay is only a special case. In most situations, delays are variable, and so in [10], Liu and Chen investigated the following general neutral Lotka-Volterra system with unbounded delays:

$$\begin{aligned}N_i'(t) = N_i(t) \Bigg[&a_i(t) - \sum_{j=1}^{n} \beta_{ij}(t) N_j(t) \\ &- \sum_{j=1}^{n} b_{ij}(t) N_j\left(t-\tau_{ij}(t)\right) \\ &-\sum_{j=1}^{n} c_{ij}(t) N_j'\left(t-\gamma_{ij}(t)\right)\Bigg], \quad i = 1, 2, \ldots, n.\end{aligned} \tag{8}$$

They introduced a new existence theorem to obtain a set of sufficient conditions for the existence of positive periodic solutions for the system (8), and their results improved and generalized some known results.

Moreover, in some situations, people may wish to change the position of the existing periodic solution but keep its stability. This is of significance in the control of ecology balance. One of the methods for its realization is to alter the system structurally by introducing some feedback control variables so as to get a population stabilizing at another periodic solution. The realization of the feedback control mechanism might be implemented by means of some biological control schemes or by harvesting procedure. In fact, during the last decade, the qualitative behaviors of the population dynamics with feedback control have been studied extensively; see [11, 15–23]. Recently, in [11], Chen considered the following neutral Lotka-Volterra competition model with feedback control of the form:

$$\begin{aligned} y_i'(t) = y_i(t)\Bigg[& r_i(t) - \sum_{j=1}^{n} a_{ij}(t)\, y_j(t) \\ & - \sum_{j=1}^{n} b_{ij}(t)\, y_j\left(t - \tau_{ij}(t)\right) \\ & - \sum_{j=1}^{n} c_{ij}(t)\, y_j'\left(t - \gamma_{ij}(t)\right) \\ & - f_i(t)\, u_i(t) - e_i(t)\, u_i\left(t - \sigma_i(t)\right) \Bigg], \\ u_i'(t) = {} & -\alpha_i(t)\, u_i(t) + \beta_i(t)\, y_i(t) \\ & + \gamma_i(t)\, y_i\left(t - \delta_i(t)\right), \quad i = 1, 2, \ldots, n. \end{aligned} \tag{9}$$

With the help of a continuation theorem based on Gaines and Mawhin's coincidence degree, he established easily verifiable criteria for the global existence of positive periodic solutions of the system (9), and his results extended and improved existing results.

On the other hand, there are some other perturbations in the real world, such as fires and floods, that are not suitable to be considered continually. These perturbations bring sudden changes to the system. Systems with such sudden perturbations involving impulsive differential equations have attracted the interest of many researchers in the past twenty years, see [12–14, 24–30], since they provide a natural description of several real processes subject to certain perturbations whose duration is negligible in comparison with the duration of the process. Such processes are often investigated in various fields of science and technology such as physics, population dynamics, ecology, biological systems, and optimal control. For details, see [31–33].

In [12], Huo studied the following neutral impulsive delay Lotka-Volterra system:

$$\begin{aligned} N_i'(t) = N_i(t)\Bigg[& \alpha_i(t) - \sum_{j=1}^{n} \beta_{ij}(t)\, N_j\left(t - \tau_{ij}(t)\right) \\ & - \sum_{j=1}^{n} c_{ij}(t)\, N_j'\left(t - \gamma_{ij}(t)\right) \Bigg], \\ & i = 1, 2, \ldots, n, \; t \neq t_k, \\ \Delta N_i(t) = {} & N_i(t^+) - N_i(t_k) = b_{ik} N_i(t_k), \\ & i = 1, 2, \ldots, n, \; k = 1, 2, \ldots. \end{aligned} \tag{10}$$

By using some techniques of Mawhin's coincidence degree theory, he obtained sufficient conditions for the existence of periodic positive solutions of the system (10).

In [13], Wang and Dai investigated the following periodic neutral population model with delays and impulse:

$$\begin{aligned} N'(t) = N(t)\Bigg[& a(t) - e(t)\, N(t) - \sum_{j=1}^{n} b_j(t)\, N\left(t - \sigma_j(t)\right) \\ & - \sum_{i=1}^{m} c_i(t)\, N'\left(t - \tau_i(t)\right) \Bigg], \quad t \neq t_k, \\ N(t^+) = {} & \left(1 + \theta_k\right) N(t_k), \quad k = 1, 2, \ldots. \end{aligned} \tag{11}$$

They obtained some sufficient conditions for the existence of positive periodic solutions of the model (11) by using the theory of abstract continuous theorem of k-set contractive operator and some analysis techniques.

Recently, in [14], Luo et al. studied the following n-species competition system with general periodic neutral delay and impulse:

$$\begin{aligned} N_i'(t) = N_i(t)\Bigg[& r_i(t) - \sum_{j=1}^{n} a_{ij}(t)\, N_j(t) - \sum_{j=1}^{n} b_{ij}(t) \\ & \times \int_{-\infty}^{t} k_{ij}(t-s)\, N_j(s)\, ds \\ & - \sum_{j=1}^{n} c_{ij}(t)\, N_j\left(t - \tau_{ij}(t)\right) \\ & - \sum_{j=1}^{n} d_{ij}(t)\, N_j'\left(t - \gamma_{ij}(t)\right) \Bigg], \\ & i = 1, 2, \ldots, n, \; t \neq t_k, \\ \Delta N_i(t_k) = {} & N_i(t_k^+) - N_i(t_k) = \theta_{ik} N_i(t_k), \\ & i = 1, 2, \ldots, n, \; k = 1, 2, \ldots. \end{aligned} \tag{12}$$

They obtained some sufficient and realistic conditions for the existence of positive periodic solutions of the system (12), by using a new existence theorem, which is different from Gaines and Mawhin's continuation theorem and abstract continuation theory for k-set contraction.

However, to this day, no scholars had done works on the existence of positive periodic solution of the system (1). One could easily see that systems (3)–(12) are all special cases of the system (1). Therefore, we propose and study the system (1) in this paper.

For the sake of generality and convenience, we make the following notations and assumptions: let $\omega > 0$ be a constant and

$C_\omega = \{x \mid x \in C(R, R), x(t + \omega) = x(t)\}$, with the norm defined by $|x|_0 = \max_{t\in[0,\omega]} |x(t)|$;

$C_\omega^1 = \{x \mid x \in C^1(R, R), x(t + \omega) = x(t)\}$, with the norm defined by $\|x\| = \max_{t\in[0,\omega]} \{|x|_0, |x'|_0\}$;

$PC = \{x \mid x : R \to R^+, \lim_{s\to t} x(s) = x(t)$, if $t \neq t_k, \lim_{t\to t_k^-} x(t) = x(t_k), \lim_{t\to t_k^+} x(t)$ exists, $k \in Z^+\}$;

$PC^1 = \{x \mid x : R \to R^+, x' \in PC\}$;

$PC_\omega = \{x \mid x \in PC, x(t+\omega) = x(t)\}$, with the norm defined by $|x|_0 = \max_{t\in[0,\omega]} |x(t)|$;

$PC_\omega^1 = \{x \mid x \in PC^1, x(t+\omega) = x(t)\}$, with the norm defined by $\|x\| = \max_{t\in[0,\omega]}\{|x|_0, |x'|_0\}$.

Then, the previous spaces are all Banach spaces. We also denote that

$$\overline{g} = \frac{1}{\omega}\int_0^\omega g(t)\,dt,$$
$$g^L = \min_{t\in[0,\omega]} g(t), \quad \text{for any } g \in PC_\omega, \tag{13}$$
$$\Delta_{ik} = 1 + p_{ik} + q_{ik}, \quad i = 1,2,\ldots,n,\ k = 1,2,\ldots,$$

and make the following assumptions:

(A) a_{ij}, b_{ij}, c_{ij}, e_i, f_i, α_i, β_i, $\theta_i \in C(R,[0,+\infty))$, $d_{ij} \in C^1(R,[0,+\infty))$, $\gamma_{ij}, \delta_i \in C^1(R,R)$, and $\tau_{ij} \in C^2(R,R)$ are continuous ω-periodic functions, and $\int_0^\infty k_{ij}(s)ds = 1$, $\int_0^{+\infty} s k_{ij}(s)ds < +\infty$, and $i,j = 1,2,\ldots,n$;

(B) $[t_k]_{k\in N}$ satisfies $0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$;

(C) $\{\Delta_{ik}\}$ is a real sequence such that $\Delta_{ik} > 0$, $\prod_{0<t_k<t} \Delta_{ik}$ $(i = 1,2,\ldots,n,\ k = 1,2,\ldots)$ are ω-periodic functions.

The organization of this paper is as follows. In the following section, we introduce some lemmas and an important existence theorem developed in [34, 35]. In the third section, we derive some sufficient conditions, which ensure the existence of positive periodic solution of system (1) by applying this theorem and some other techniques. Finally, we study some special cases of system (1), which have been studied extensively in the literature. These examples show that our sufficient conditions are new, and some known results can be improved and generalized.

2. Preliminaries

In this section, in order to obtain the existence of a periodic solution for system (1) and (2), we will give some concepts and results from [35], and we will state an existence theorem and some lemmas.

For a fixed $\tau > 0$, let $C =: C([-\tau,0];R^n)$, if $x \in C([-\tau,0];R^n)$ for some $\delta > 0$ and $\eta \in R$, then $x_t \in C$ for $t \in [\eta,\eta+\delta]$ is defined by $x_t(\theta) = x(t+\theta)$ for $\theta \in [-\tau,0]$. The supremum norm in C is denoted by $\|\cdot\|$, that is, $\|\phi\| = \max_{t\in[-\tau,0]}|\phi(\theta)|$ for $\phi \in C$, where $|\cdot|$ denotes the norm in R^n and $|u| = \sum_{j=1}^n |u_i|$ for $u = (u_1,\ldots,u_n) \in R^n$. Consider the following neutral functional differential equation:

$$\frac{d}{dt}\left[x(t) - b(t,x_t)\right] = f(t,x_t), \tag{14}$$

where $f : R\times C \to R^n$ is completely continuous, and $b : R\times C \to R^n$ is continuous. Moreover, we assume the following:

(1) there exists $\omega > 0$ such that for every $(t,\phi) \in R\times C$, we have $b(t+\omega,\phi) = b(t,\phi)$ and $f(t+\omega,\phi) = f(t,\phi)$;

(2) there exists a constant $k < 1$ such that $|b(t,\phi) - b(t,\varphi)| \le k\|\phi-\varphi\|$, for $t \in R$ and $\phi,\varphi \in C$.

By using the continuation theorem for composite coincidence degree, in [34], Erbe et al. proved the following existence theorem (see also Theorem 4.7.1 in [35]).

Lemma 1. *Assume that there exists a constant $M > 0$ such that*

(i) *for any $\lambda \in (0,1)$ and any ω-periodic solution x of the system*

$$\frac{d}{dt}\left[x(t) - \lambda b(t,x_t)\right] = f(t,x_t). \tag{15}$$

One has that $|x(t)| < M$ for $t \in R$;

(ii) *$h(u) =: \int_0^\omega f(s,\hat{u})ds \neq 0$ for $u \in \partial B_M(R^n)$, where $B_M(R^n) = \{u \in R^n : |u| < M\}$, and $\hat{u}$ denotes the constant mapping from $[-\tau,0]$ to R^n with the value $u \in R^n$;*

(iii) *$\deg(h, B_M(R^n)) \neq 0$. Then, there exists at least one ω-periodic solution of the system (14) that satisfies $\sup_{t\in R}|x(t)| < M$.*

The following remark is introduced by Fang (see Remark 1 in [36]).

Remark 2. Lemma 1 remains valid if the assumption (ii) is replaced by the following:

(ii*) there exists a constant $k < 1$ such that $|b(t,\phi) - b(t,\varphi)| \le k\|\phi-\varphi\|$ for $t \in R$ and $\phi,\varphi \in \{\phi \in C : \|\phi\| < M\}$ with M as given in condition (i) of Lemma 1.

We will also need the following lemmas.

Lemma 3 (see [8, 13]). *Suppose that $\sigma \in C_\omega^1$ and $\sigma'(t) < 1$, $t \in [0,\omega]$. Then, the function $t - \sigma(t)$ has a unique inverse $\mu(t)$ satisfying $\mu \in C(R,R)$ with $\mu(a+\omega) = \mu(a)+\omega$, $\forall a \in R$, and if $h \in PC_\omega$, $\sigma'(t) < 1$, and $t \in [0,\omega]$, then $h(\mu(t)) \in PC_\omega$.*

Lemma 4 (see [27]). *Suppose that $x(t)$ is a differently continuous ω-periodic function on R with $(\omega > 0)$. Then, to any $t_* \in R$, $\max_{t_*\le t\le t_*+\omega}|x(t)| \le |x(t_*)| + (1/2)\int_0^\omega |x'(t)|dt$.*

Lemma 5. *Consider that $R_+^{2n} = \{(N_i(t),u_i(t)) : N_i(0) > 0, u_i(0) > 0, i = 1,2,\ldots,n\}$ is the positive invariable region of the system* (1) *and* (2).

Proof. In view of biological population, we obtain $N_i(0) > 0$, $u_i(0) > 0$. By the system (1) and (2), we have

$$\begin{aligned} N_i(t) &= N_i(0)\exp\left\{\int_0^t \left[r_i(\xi) - \sum_{j=1}^n a_{ij}(\xi) N_j(\xi) - \sum_{j=1}^n b_{ij}(\xi) \times \int_{-\infty}^t K_{ij}(\xi - s) N_j(s)\, ds - \sum_{j=1}^n c_{ij}(\xi) N_j(\xi - \gamma_{ij}(\xi)) - \sum_{j=1}^n d_{ij}(\xi) N_j'(\xi - \tau_{ij}(\xi)) - e_i(\xi) u_i(\xi) - f_i(\xi) u_i \times (\xi - \sigma_i(\xi)) \right] d\xi \right\}, \\ & \qquad t \in [0, t_1],\ i = 1, 2, \dots, n, \\ N_i(t) &= N_i(t_k)\exp\left\{\int_{t_k}^t \left[r_i(\xi) - \sum_{j=1}^n a_{ij}(\xi) N_j(\xi) - \sum_{j=1}^n b_{ij}(\xi) \times \int_{-\infty}^t K_{ij}(\xi - s) N_j(s)\, ds - \sum_{j=1}^n c_{ij}(\xi) N_j(\xi - \gamma_{ij}(\xi)) - \sum_{j=1}^n d_{ij}(\xi) N_j'(\xi - \tau_{ij}(\xi)) - e_i(\xi) u_i(\xi) - f_i(\xi) u_i \times (\xi - \sigma_i(\xi)) \right] d\xi \right\}, \\ & \qquad t \in (t_k, t_{k+1}],\ i = 1, 2, \dots, n,\ k \ge 1, \\ N_i(t_k^+) &= e^{(p_{ik} + q_{ik})} N_i(t_k) > 0, \\ & \qquad k \in N,\ i = 1, 2, \dots, n, \\ u_i(t) &= \int_t^{t+\omega} G_i(t,s) \left[\beta_i(s) N_i(s) + \vartheta_i(s) N_i \times (s - \gamma_i(s))\right] ds := (\phi_i N_i)(t), \end{aligned} \tag{16}$$

where

$$G_i(t,s) = \frac{\exp\left\{\int_t^s \alpha_i(\xi)\, d\xi\right\}}{\exp\left\{\int_t^s \alpha_i(\xi)\, d\xi\right\} - 1}. \tag{17}$$

Then, the solution of the systems (1) and (2) is positive. □

Definition 6. A function $N_i : [-\tau, 0] \to [0, +\infty)$ $(i = 1, 2, \dots, n)$ is said to be a positive solution of (1) and (2) on $[-\tau, \infty]$, if the following conditions are satisfied:

(a) $N_i(t)$ is absolutely continuous on each (t_k, t_{k+1});

(b) for each $k \in Z_+$, $N_i(t_k^+)$ and $N_i(t_k^-)$ exist, and $N_i(t_k^-) = N_i(t_k)$;

(c) $N_i(t)$ satisfies the first equation of (1) and (2) for almost everywhere (for short a.e.) in $[0, \infty] \setminus \{t_k\}$ and satisfies $N_i(t_k^+) = \Delta_{ik} N_i(t_k)$ for $t = t_k, k \in Z_+ = \{1, 2, \dots\}$.

Consider the following nonimpulsive delay differential equation:

$$\begin{aligned} y_i'(t) &= y_i(t) \left[r_i(t) - \sum_{j=1}^n A_{ij}(t) y_j(t) - \sum_{j=1}^n B_{ij}(t) \times \int_{-\infty}^t k_{ij}(t - s) y_j(s)\, ds - \sum_{j=1}^n C_{ij}(t) y_j(t - \gamma_{ij}(t)) - \sum_{j=1}^n D_{ij}(t) y_j'(t - \tau_{ij}(t)) - e_i(t) u_i(t) - f_i(t) u_i(t - \delta_i(t)) \right], \\ \frac{du_i(t)}{dt} &= -\alpha_i(t) u_i(t) + \beta_i^*(t) y_i(t) + \theta_i^*(t) y_i(t - \sigma_i(t)), \end{aligned} \tag{18}$$

with the following initial conditions:

$$\begin{gathered} y_i(\xi) = \phi_i(\xi), \qquad y_i'(\xi) = \phi_i'(\xi), \\ \xi \in [-\tau, 0], \qquad \phi_i(0) > 0, \\ \phi_i \in C([-\tau, 0], R^+) \bigcap C^1([-\tau, 0], R^+), \\ u_i(\xi) = \varphi_i(\xi), \quad \xi \in [-\tau, 0], \quad \varphi_i(0) > 0, \\ \varphi_i \in C([-\tau, 0), R^+), \quad i = 1, 2, 3, \dots, n, \end{gathered} \tag{19}$$

where

$$
\begin{aligned}
A_{ij}(t) &= a_{ij}(t)\prod_{0<t_k<t}\Delta_{ik}, \qquad B_{ij}(t) = b_{ij}(t)\prod_{0<t_k<t}\Delta_{ik},\\
C_{ij}(t) &= c_{ij}(t)\prod_{0<t_k<t-\gamma_{ij}(t)}\Delta_{ik},\\
D_{ij}(t) &= d_{ij}(t)\prod_{0<t_k<t-\tau_{ij}(t)}\Delta_{ik},\\
\tau &= \max_{t\in[0,\omega]}\left\{\gamma_{ij}(t),\tau_{ij}(t),\delta_i(t),\sigma_i(t),1\le i,j\le n\right\},\\
\beta_i^*(t) &= \beta_i(t)\prod_{0<t_k<t}\Delta_{ik},\\
\theta_i^*(t) &= \theta_i(t)\prod_{0<t_k<t-\sigma_i(t)}\Delta_{ik}, \quad i,j=1,2,\ldots,n.
\end{aligned}
\tag{20}
$$

The following lemmas will be used in the proofs of our results. The proof of Lemma 7 is similar to that of Theorem 1 in [24].

Lemma 7. *Suppose that (A)–(C) hold, then*

(i) *if $(y_i(t),u_i(t))^T$ $(i=1,2,\ldots,n)$ is a solution of* (18) *and* (19) *on $[-\tau,+\infty)$, then $(N_i(t),u_i(t))^T$ $(i=1,2,\ldots,n)$ is a solution of* (1) *and* (2) *on $[-\tau,+\infty)$, where $N_i(t)=\prod_{0<t_k<t}\Delta_{ik}y_i(t)$;*

(ii) *if $(N_i(t),u_i(t))^T$ $(i=1,2,\ldots,n)$ is a solution of* (1) *and* (2) *on $[-\tau,+\infty)$, then $(y_i(t),u_i(t))^T$ $(i=1,2,\ldots,n)$ is a solution of* (18) *and* (19) *on $[-\tau,+\infty)$, where $y_i(t)=\prod_{0<t_k<t}\Delta_{ik}^{-1}N_i(t)$.*

Proof. (i) It is easy to see that $N_i(t)=\prod_{0<t_k<t}\Delta_{ik}y_i(t)$ $(i=1,2,\ldots,n)$ is absolutely continuous on every interval $(t_k,t_{k+1}]$, $t\neq t_k$, $k=1,2,\ldots$,

$$
\begin{aligned}
N_i'(t) &- N_i(t)\Bigg[r_i(t)-\sum_{j=1}^n a_{ij}(t)N_j(t)\\
&-\sum_{j=1}^n b_{ij}(t)\int_{-\infty}^t k_{ij}(t-s)N_j(s)\,ds\\
&-\sum_{j=1}^n c_{ij}(t)N_j\left(t-\gamma_{ij}(t)\right)\\
&-\sum_{j=1}^n d_{ij}(t)N_j'\left(t-\tau_{ij}(t)\right)\\
&-e_i(t)u_i(t)-f_i(t)u_i\left(t-\delta_i(t)\right)\Bigg]\\
&=\prod_{0<t_k<t}\Delta_{ik}y_i'(t)\\
&-\prod_{0<t_k<t}\Delta_{ik}y_i(t)\\
&\times\Bigg[r_i(t)-\sum_{j=1}^n a_{ij}(t)\prod_{0<t_k<t}\Delta_{ik}y_j(t)\\
&-\sum_{j=1}^n b_{ij}(t)\prod_{0<t_k<t}\Delta_{ik}\int_{-\infty}^t k_{ij}(t-s)\,y_j(s)ds\\
&-\sum_{j=1}^n c_{ij}(t)\prod_{0<t_k<t-\gamma_{ij}(t)}\Delta_{ik}y_j\left(t-\tau_{ij}(t)\right)\\
&-\sum_{j=1}^n d_{ij}(t)\prod_{0<t_k<t-\gamma_{ij}(t)}\Delta_{ik}y_j'\left(t-\tau_{ij}(t)\right)\\
&-e_i(t)u_i(t)-f_i(t)\\
&\times u_i\left(t-\delta_i(t)\right)\Bigg]\\
&=\prod_{0<t_k<t}\Delta_{ik}\Bigg\{y_i'(t)-y_i(t)\\
&\times\Bigg[r_i(t)-\sum_{j=1}^n A_{ij}(t)y_j(t)\\
&-\sum_{j=1}^n B_{ij}(t)\int_{-\infty}^t k_{ij}(t-s)\,y_j(s)\,ds\\
&-\sum_{j=1}^n C_{ij}(t)y_j\left(t-\gamma_{ij}(t)\right)\\
&-\sum_{j=1}^n D_{ij}(t)y_j'\left(t-\tau_{ij}(t)\right)\\
&-e_i(t)u_i(t)-f_i(t)u_i\\
&\times\left(t-\delta_i(t)\right)\Bigg]\Bigg\}=0,
\end{aligned}
$$

$$
\begin{aligned}
u_i'(t) &+ \alpha_i(t)u_i(t)-\beta_i(t)N_i(t)-\theta_i(t)N_i\left(t-\sigma_i(t)\right)\\
&=u_i'(t)+\alpha_i(t)u_i(t)-\beta_i^*(t)\,y_i(t)\\
&\quad-\theta_i^*(t)\,y_i\left(t-\gamma_i(t)\right)=0.
\end{aligned}
\tag{21}
$$

On the other hand, for any $t=t_k$, $k=1,2,\ldots$,

$$
\begin{aligned}
N_i\left(t_k^+\right) &= \lim_{t\to t_k^+}\prod_{0<t_j<t}\Delta_{ik}y_i(t)=\prod_{0<t_j\le t_k}\Delta_{ik}y_i\left(t_k\right),\\
N_i\left(t_k\right) &= \prod_{0<t_j<t_k}\Delta_{ik}y_i\left(t_k\right),
\end{aligned}
\tag{22}
$$

thus,

$$\Delta N_i(t_k^+) = \Delta_{ik} y_i(t_k), \tag{23}$$

which implies that $(N_i(t), u_i(t))^T$ $(i = 1, 2, \ldots, n)$ is a solution of the system (1) and (2). Therefore, if $(y_i(t), u_i(t))^T$ $(i = 1, 2, \ldots, n)$ is a solution of the system (18) and (19) on $[-\tau, +\infty)$, we can prove that $(N_i(t), u_i(t))^T$ $(i = 1, 2, \ldots, n)$ are solutions of the system (1) and (2) on $[-\tau, +\infty)$.

(ii) Since $N_i(t) = \prod_{0<t_k<t} \Delta_{ik} y_i(t)$ $(i = 1, 2, \ldots, n)$ is absolutely continuous on every interval $(t_k, t_{k+1}], t \neq t_k$, $k = 1, 2, \ldots$, and in view of (23), it follows that for any $k = 1, 2, \ldots$,

$$\begin{aligned} y_i(t_k^+) &= \prod_{0<t_j\le t_k} \Delta_{ik}^{-1} N_i(t_k^+) \\ &= \prod_{0<t_j<t_k} \Delta_{ik}^{-1} N_i(t_k) = y_i(t_k), \\ y_i(t_k^-) &= \prod_{0<t_j<t_k} \Delta_{ik}^{-1} N_i(t_k^-) \\ &= \prod_{0<t_j\le t_k^-} \Delta_{ik}^{-1} N_i(t_k^-) = y_i(t_k), \\ &\qquad i = 1, 2, \ldots, n, \end{aligned} \tag{24}$$

which implies that $(y_i(t), u_i(t))^T$ $(i = 1, 2, \ldots, n)$ is continuous on $[-\tau, +\infty)$. It is easy to prove that $(y_i(t), u_i(t))^T$ $(i = 1, 2, \ldots, n)$ is absolutely continuous on $[-\tau, +\infty)$. Similar to the proof of (i), we can check that $(y_i(t), u_i(t))^T$ $(i = 1, 2, \ldots, n)$ is a solution of the system (18) and (19) on $[-\tau, +\infty)$. The proof of Lemma 7 is completed. □

Lemma 8. *Consider that $(y_i(t), u_i(t))$ is a ω-periodic solution of* (18) *and* (19) *if and only if $y_i(t)$ is a ω-periodic solution of the following system:*

$$\begin{aligned} \frac{dy_i(t)}{dt} = y_i(t) \Bigg[& r_i(t) - \sum_{j=1}^n A_{ij}(t) y_j(t) \\ & - \sum_{j=1}^n B_{ij}(t) \int_{-\infty}^t k_{ij}(t-s) y_j(s)\, ds \\ & - \sum_{j=1}^n C_{ij}(t) y_j(t - \gamma_{ij}(t)) \\ & - \sum_{j=1}^n D_{ij}(t) y_j'(t - \tau_{ij}(t)) \\ & - e_i(t)(\psi_i y_i)(t) - f_i(t)(\psi_i y_i) \\ & \times (t - \delta_i(t)) \Bigg], \end{aligned} \tag{25}$$

where

$$\begin{aligned} (\psi_i y_i)(t) := \int_t^{t+\omega} & G_i(t,s) [\beta_i^*(s) y_i(s) + \theta_i^*(s) y_i \\ & \times (s - \sigma_i(s))]\, ds, \end{aligned} \tag{26}$$

and $G_i(t,s)$ is defined by (17).

Proof. The proof of Lemma 8 is similar to that of Lemma 2.2 in [11], and we omit the details here. □

From Lemmas 7 and 8, if we want to discuss the existence of positive periodic solutions of systems (1) and (2), we only discuss the existence of positive periodic solutions of systems (25) and (26).

3. The Main Result

Since $\gamma_{ij}'(t) < 1, \tau_{ij}'(t) < 1, t \in [0, \omega]$, we see that $\gamma_{ij}(t), \tau_{ij}(t)$ all have their inverse function. Throughout the following part, we set to $\vartheta_{ij}(t), \nu_{ij}(t)$ that represent the inverse function of $t - \gamma_{ij}(t), t - \tau_{ij}(t)$, respectively. We denote that

$$\begin{aligned} \Gamma_{ij}(t) &= A_{ij}(t) + B_{ij}(t) + \frac{C_{ij}(\vartheta_{ij}(t))}{1 - \gamma_{ij}'(\vartheta_{ij}(t))} - \frac{D_{ij}'(\nu_{ij}(t))}{1 - \tau_{ij}'(\nu_{ij}(t))}, \\ \Gamma_i^1(t) &= e_i(t)(\psi_i 1)(t), \qquad \Gamma_i^2(t) = f_i(t)(\psi_i 1)(t - \sigma_i(t)). \end{aligned} \tag{27}$$

Remark 9. From Lemma 3, we get that $\vartheta_{ij}(\omega) = \vartheta_{ij}(0) + \omega$, $\nu_{ij}(\omega) = \nu_{ij}(0) + \omega$, $i, j = 1, 2, \ldots, n$, then

$$\begin{aligned} \int_0^\omega \frac{C_{ij}(\vartheta_{ij}(s))}{1 - \gamma_{ij}'(\vartheta_{ij}(s))} ds &= \int_{\vartheta_{ij}(0)}^{\vartheta_{ij}(\omega)} \frac{C_{ij}(t)(1 - \gamma_{ij}'(t))}{1 - \gamma_{ij}'(t)} dt \\ &= \int_{\vartheta_{ij}(0)}^{\vartheta_{ij}(0)+\omega} C_{ij}(t)\, dt = \overline{C_{ij}}\omega, \\ &\qquad i, j = 1, 2, \ldots, n. \end{aligned} \tag{28}$$

Similarly,

$$\begin{aligned} \int_0^\omega \frac{D_{ij}'(\nu_{ij}(s))}{1 - \tau_{ij}'(\nu_{ij}(s))} ds &= \int_{\nu_{ij}(0)}^{\nu_{ij}(\omega)} \frac{D_{ij}'(t)(1 - \tau_{ij}'(t))}{1 - \tau_{ij}'(t)} dt \\ &= \int_{\nu_{ij}(0)}^{\nu_{ij}(0)+\omega} D_{ij}'(t)\, dt = 0, \\ &\qquad i, j = 1, 2, \ldots, n. \end{aligned} \tag{29}$$

Thus,

$$\overline{\Gamma_{ij}}\omega = \int_0^\omega \Gamma_{ij}(t)\, dt = (\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}})\,\omega. \tag{30}$$

Here, we have the following notations:

$$\rho_{ij} = \frac{\Gamma_{ij}^{L}\left(1-\gamma_{ij}'\right)^{L}}{\left(1-\gamma_{ij}'\right)^{L} + \left|D_{ij}\right|_0}, \qquad \overline{R_i} = \frac{1}{\omega}\int_0^{\omega} \left|r_i(t)\right| dt,$$

$$L_0 = \max\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|D_{ij}\right|_0, \sum_{i=1}^{n}\sum_{j=1}^{n}\left|D_{0,ij}\right|_0\right\},$$

$$M_0 = \max\left\{\sum_{i=1}^{n}\left|\ln\mu_i^*\right|_0, H, \frac{1}{2}\omega\Lambda^* + \sum_{i=1}^{n}\Lambda_i\right\},$$

$$H = \max_{i\in[1,n]}\{H_i\}, \qquad H_i = \ln\frac{\overline{r_i}}{\rho_{ii}} + \sum_{j=1}^{n}\frac{\overline{r_i}}{\rho_{ij}} + \left(\overline{R_i} + \overline{r_i}\right)\omega,$$

$$\Lambda^* = \left(\sum_{i=1}^{n}\left|r_i\right|_0 + \sum_{i=1}^{n}\sum_{j=1}^{n}\left|A_{ij}\right|_0 e^{H_j} + \sum_{i=1}^{n}\sum_{j=1}^{n}\left|B_{ij}\right|_0 e^{H_j} + \sum_{i=1}^{n}\sum_{j=1}^{n}\left|C_{ij}\right|_0 e^{H_j} + \left(\left|e_i\right|_0 + \left|f_i\right|_0\right)\frac{\left|\beta_i^*\right|_0 + \left|\theta_i^*\right|_0}{\alpha_i^L}e^{H_i}\right) \times \left(1 - \sum_{i=1}^{n}\sum_{j=1}^{n}\left|D_{0,ij}\right|_0 e^{H_j}\right)^{-1},$$

$$\Lambda_i = \max\left\{\left|\ln\frac{r_i}{\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}}}\right|, \left|\ln\frac{\overline{r_i} - \sum_{j=1,j\neq i}^{n}\left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}}\right)e^{H_j}}{\overline{A_{ii}}+\overline{B_{ii}}+\overline{C_{ii}}+\left(\overline{e_i}+\overline{f_i}\right)\left(\left|\beta_i^*\right|_0+\left|\theta_i^*\right|_0/\alpha_i^L\right)}\right|\right\}, \tag{31}$$

where $\Gamma_{ij}(t)$, $\Gamma_i^1(t)$, and $\Gamma_i^2(t)$ are defined by (27), and $D_{0,ij}(t) = D_{ij}(t)(1-\gamma_{ij}'(t))$.

Theorem 10. *Suppose that the following conditions hold:*

(1) *the system of algebraic equations*

$$f^*(\mu) = \left(\overline{r_i} - \sum_{j=1}^{n}\left(\left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}}\right)\mu_j + \left(\overline{\Gamma_i^1} + \overline{\Gamma_i^2}\right)\mu_i\right)\right)_{n\times 1} = 0 \tag{32}$$

has a unique positive solution $\mu^* = (\mu_1^*, \ldots, \mu_n^*)$;

(2) $\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}} > 0$, $\overline{r_i} > \sum_{j=1,j\neq i}^{n}(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}})e^{H_j}$, $\gamma_{ij}'(t) < 1$, $\tau_{ij}'(t) < 1$ *and* $\Gamma_{ij}(t) > 0$;

(3) $K_0 =: L_0 e^{M_0} < 1$.

Then the system (1) *and* (2) *has at least one positive* ω*-periodic solution.*

To prove the previous theorem, we make the change of variables

$$y_i(t) = e^{x_i(t)}, \quad i = 1, 2, \ldots, n. \tag{33}$$

Then, the system (25) can be rewritten in the following form:

$$\begin{aligned} x_i'(t) = r_i(t) &- \sum_{j=1}^{n}A_{ij}(t)e^{x_j(t)} \\ &- \sum_{j=1}^{n}B_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)e^{x_j(s)}ds \\ &- \sum_{j=1}^{n}C_{ij}(t)e^{x_j(t-\gamma_{ij}(t))} \\ &- \sum_{j=1}^{n}D_{0ij}(t)x_j'\left(t-\tau_{ij}(t)\right)e^{x_j(t-\tau_{ij}(t))} \\ &- e_i(t)\left(\psi_i e^{x_i}\right)(t) \\ &- f_i(t)\left(\psi_i e^{x_i}\right)(t)\left(t-\delta_i(t)\right). \end{aligned} \tag{34}$$

Let X denote the linear space of real value continuous ω-periodic functions on R. The linear space X is a Banach space with the usual norm $\|x\|_0 = \max_{t\in R}|x(t)| = \max_{t\in R}\sum_{j=1}^{n}|x_i(t)|$ for a given $x = (x_1, \ldots, x_n) \in X$.

We define the following maps:

$$b : R\times C \longrightarrow R^n,$$

$$b(t,\phi) = \left(b_1(t,\phi), b_2(t,\phi), \ldots, b_n(t,\phi)\right),$$

$$b_i(t,\phi) = -\sum_{j=1}^{n}D_{ij}(t)e^{\phi_j(-\gamma_{ij}(t))},$$

$$f : R\times C \longrightarrow R^n,$$

$$f(t,\phi) = \left(f_1(t,\phi), f_2(t,\phi), \ldots, f_n(t,\phi)\right),$$

$$f_i(t,\phi) = r_i(t) - \sum_{j=1}^{n} A_{ij}(t) e^{\phi_j(0)} - \sum_{j=1}^{n} B_{ij}(t) \int_{-\infty}^{0} k_{ij}(t-s) e^{\phi_j(s)} ds - \sum_{j=1}^{n} C_{ij}(t) e^{\phi_j(-\tau_{ij}(t))} + \sum_{j=1}^{n} D'_{ij}(t) e^{\phi_j(-\gamma_{ij}(t))} - e_i(t)\left(\psi_i e^{\phi_i}\right)(t) - f_i(t)\left(\psi_i e^{\phi_i}\right)(t-\delta_i(t)),$$
$$i = 1,2,\ldots,n, \quad \phi = (\phi_1,\phi_2,\ldots,\phi_n) \in C, \quad t \in R. \tag{35}$$

Clearly, $b : R \times C \to R^n$ and $f : R \times C \to R^n$ are complete continuation functions, and system (34) takes the form

$$\frac{d}{dt}\left[x(t) - b(t,x_t)\right] = f(t,x_t). \tag{36}$$

In the proof of our main result below, we will use the following two important lemmas.

Lemma 11. *If the assumptions of Theorem 10 are satisfied and if $\Omega = \{\phi \in C : \|\phi\| < M\}$, where $M > M_0$ such that $k = L_0 e^M < 1$, then $|b(t,\phi) - b(t,\varphi)| \le k\|\phi - \varphi\|$, for $t \in R$ and $\phi, \varphi \in \Omega$.*

Proof. For $t \in R$ and $\phi, \varphi \in \Omega$, we have

$$\begin{aligned} \left|b_i(t,\phi) - b_i(t,\varphi)\right| &\le \sum_{j=1}^{n} D_{ij}(t) \left|e^{\phi_j(-\gamma_{ij}(t))} - e^{\varphi_j(-\gamma_{ij}(t))}\right| \\ &\le \sum_{j=1}^{n} D_{ij}(t) e^{\sigma_{ij}\phi_j(-\gamma_{ij}(t)) + (1-\sigma_{ij})\varphi_j(-\gamma_{ij}(t))} \\ &\quad \times \left|\phi_j(-\gamma_{ij}(t)) - \varphi_j(-\gamma_{ij}(t))\right|, \end{aligned} \tag{37}$$

for some $\sigma_{ij} \in (0,1)$. Then, we get

$$\left|b_i(t,\phi) - b_i(t,\varphi)\right| \le \sum_{j=1}^{n} \left|D_{ij}\right|_0 e^M \|\phi - \varphi\|. \tag{38}$$

Hence,

$$\begin{aligned} \left|b(t,\phi) - b(t,\varphi)\right| &\le \sum_{i=1}^{n}\sum_{j=1}^{n} \left|D_{ij}\right|_0 e^M \|\phi - \varphi\| \\ &\le L_0 e^M \|\phi - \varphi\| = k\|\phi - \varphi\|. \end{aligned} \tag{39}$$

The proof of Lemma 11 is thus completed. □

Lemma 12. *If the assumptions of Theorem 10 are satisfied, then every solution $x \in X$ of the system*

$$\frac{d}{dt}\left[x(t) - \lambda b(t,x_t)\right] = f(t,x_t), \quad \lambda \in (0,1) \tag{40}$$

satisfies $\|x\|_0 \le M_0$.

Proof. Let $(d/dt)[x(t) - \lambda b(t,x_t)] = f(t,x_t)$, for $x \in X$, that is,

$$\begin{aligned} \left[x_i(t) + \lambda \sum_{j=1}^{n} D_{ij}(t) e^{x_j(t-\tau_{ij}(t))}\right]' &= \lambda \Bigg[r_i(t) - \sum_{j=1}^{n} A_{ij}(t) e^{x_j(t)} - \sum_{j=1}^{n} B_{ij}(t) \\ &\quad \times \int_{-\infty}^{t} k_{ij}(t-s) e^{x_j(s)} ds \\ &\quad - \sum_{j=1}^{n} C_{ij}(t) e^{x_j(t-\gamma_{ij}(t))} \\ &\quad + \sum_{j=1}^{n} D'_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \\ &\quad - e_i(t)\left(\psi_i e^{x_i}\right)(t) - f_i(t)\left(\psi_i e^{x_i}\right) \\ &\quad \times (t)\left(t - \delta_i(t)\right) \Bigg], \\ & i = 1,2,\ldots,n; \ \lambda \in (0,1), \end{aligned} \tag{41}$$

which yields, after integrating from 0 to ω, that

$$\begin{aligned} &\int_0^{\omega} \sum_{j=1}^{n} \Big[A_{ij}(t) e^{x_j(t)} + B_{ij}(t) \\ &\quad \times \int_{-\infty}^{t} k_{ij}(t-s) e^{x_j(s)} ds \\ &\quad + C_{ij}(t) e^{x_j(t-\gamma_{ij}(t))} - D'_{ij}(t) e^{x_j(t-\tau_{ij}(t))} + e_i(t) \\ &\quad \times \left(\psi_i e^{x_i}\right)(t) + f_i(t)\left(\psi_i e^{x_i}\right)(t)\left(t-\delta_i(t)\right)\Big] dt \\ &= \int_0^{\omega} \sum_{j=1}^{n} \Gamma_{ij}(t) e^{x_j(t)} dt + \int_0^{\omega} \Big[e_i(t)\left(\psi_i e^{x_i}\right)(t) + f_i(t) \\ &\qquad \times \left(\psi_i e^{x_i}\right)(t)\left(t - \delta_i(t)\right)\Big] dt \\ &= \int_0^{\omega} r_i(t) dt = \overline{r_i}\omega, \quad i = 1,2,\ldots,n, \end{aligned} \tag{42}$$

where $\Gamma_{ij}(t)$ is defined by (27). From (41), we derive

$$\begin{aligned}
&\int_0^{\omega}\left|\left[x_i(t)+\lambda\sum_{j=1}^{n}D_{ij}(t)e^{x_j(-\tau_{ij}(t))}\right]'\right|dt \\
&= \lambda\int_0^{\omega}\left|\left[r_i(t)-\sum_{j=1}^{n}A_{ij}(t)e^{x_j(t)}-\sum_{j=1}^{n}B_{ij}(t)\right.\right. \\
&\qquad \times\int_{-\infty}^{t}k_{ij}(t-s)e^{x_j(s)}ds \\
&\qquad -\sum_{j=1}^{n}C_{ij}(t)e^{x_j(t-\gamma_{ij}(t))} \\
&\qquad +\sum_{j=1}^{n}D'_{ij}(t)e^{x_j(t-\tau_{ij}(t))} \\
&\qquad -e_i(t)\left(\psi_i e^{x_i}\right)(t)-f_i(t)\left(\psi_i e^{x_i}\right) \\
&\qquad \left.\left.\times(t)\left(t-\delta_i(t)\right)\right]\right|dt \\
&\le \int_0^{\omega}\left|r_i(t)\right|dt+\int_0^{\omega}\left|\sum_{j=1}^{n}\left[A_{ij}(t)e^{x_j(t)}+B_{ij}(t)\right.\right. \\
&\qquad \times\int_{-\infty}^{t}k_{ij}(t-s)e^{x_j(s)}ds \\
&\qquad +C_{ij}(t)e^{x_j(t-\gamma_{ij}(t))} \\
&\qquad -D'_{ij}(t)e^{x_j(t-\tau_{ij}(t))} \\
&\qquad +e_i(t)\left(\psi_i e^{x_i}\right)(t) \\
&\qquad +f_i(t)\left(\psi_i e^{x_i}\right)(t) \\
&\qquad \left.\left.\times\left(t-\delta_i(t)\right)\right]\right|dt.
\end{aligned} \tag{43}$$

It follows from (41)–(43) that

$$\int_0^{\omega}\left|\left[x_i(t)+\lambda\sum_{j=1}^{n}D_{ij}(t)e^{x_j(-\tau_{ij}(t))}\right]'\right|dt \le \left(\overline{R_i}+\overline{r_i}\right)\omega, \quad i=1,2,\ldots,n. \tag{44}$$

By amplification, it follows from (42) that

$$\begin{aligned}
\overline{r_i}\omega &\ge \sum_{j=1}^{n}\int_0^{\omega}\Gamma_{ij}(t)e^{x_j(t)}dt \\
&= \sum_{j=1}^{n}\int_0^{\omega}\left[\Gamma_{ij}(t)e^{x_j(t)}-\left(\rho_{ij}e^{x_j(t)}+\rho_{ij}D_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\right)\right. \\
&\qquad \left.+\left(\rho_{ij}e^{x_j(t)}+\rho_{ij}D_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\right)\right]dt \\
&= \sum_{j=1}^{n}\int_0^{\omega}\left[\Gamma_{ij}(t)e^{x_j(t)}-\left(\rho_{ij}e^{x_j(t)}+\rho_{ij}D_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\right)\right]dt \\
&\quad +\sum_{j=1}^{n}\int_0^{\omega}\left[\rho_{ij}e^{x_j(t)}+\rho_{ij}D_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\right]dt.
\end{aligned} \tag{45}$$

In view of Remark 9 and by a similar analysis, we obtain

$$\begin{aligned}
&\sum_{j=1}^{n}\int_0^{\omega}\left[\Gamma_{ij}(t)e^{x_j(t)}-\left(\rho_{ij}e^{x_j(t)}+\rho_{ij}D_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\right)\right]dt \\
&\quad = \sum_{j=1}^{n}\int_0^{\omega}\left[\Gamma_{ij}(s)-\rho_{ij}-\rho_{ij}\frac{D_{ij}\left(\nu_{ij}(s)\right)}{1-\tau'_{ij}\left(\nu_{ij}(s)\right)}\right]e^{x_j(s)}dt.
\end{aligned} \tag{46}$$

As $\rho_{ij}=\Gamma_{ij}^{L}(1-\tau'_{ij})^{L}/((1-\tau'_{ij})^{L}+|D_{ij}|_0)$, it follows that $\Gamma_{ij}(s)-\rho_{ij}-\rho_{ij}(D_{ij}(\nu_{ij}(s))/(1-\tau'_{ij}(\nu_{ij}(s))))\ge 0$. So we find from (45) that

$$\overline{r_i}\omega \ge \sum_{j=1}^{n}\int_0^{\omega}\left[\rho_{ij}e^{x_j(t)}+\rho_{ij}D_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\right]dt. \tag{47}$$

That is,

$$\overline{r_i}\omega \ge \int_0^{\omega}\sum_{j=1}^{n}\left[\rho_{ij}e^{x_j(t)}+\rho_{ij}D_{ij}(t)e^{x_j(t-\tau_{ij}(t))}\right]dt. \tag{48}$$

By the mean value theorem, we see that there exist points $\zeta_i\in[0,\omega]$, $(i=1,\ldots,n)$ such that

$$\overline{r_i}\ge\sum_{j=1}^{n}\rho_{ij}e^{x_j(\zeta_i)}+\sum_{j=1}^{n}\rho_{ij}D_{ij}\left(\zeta_i\right)e^{x_j(\zeta_i-\tau_{ij}(\zeta_i))}, \quad i=1,\ldots,n, \tag{49}$$

which implies that

$$\begin{aligned}
x_i\left(\zeta_i\right) &\le \ln\frac{\overline{r_i}}{\rho_{ii}}, \\
D_{ij}\left(\zeta_i\right)e^{x_j(\zeta_i-\tau_{ij}(\zeta_i))} &\le \frac{\overline{r_i}}{\rho_{ij}}, \\
i &= 1,\ldots,n.
\end{aligned} \tag{50}$$

By (44) and (50), we can see that

$$\begin{aligned} x_i(t) + \lambda \sum_{j=1}^{n} D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} &\le x_i(\zeta_i) + \lambda \sum_{j=1}^{n} D_{ij}(\zeta_i) e^{x_j(\zeta_i - \tau_{ij}(\zeta_i))} \\ &\quad + \int_0^{\omega} \left| \left[x_i(t) + \lambda \sum_{j=1}^{n} D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \right]' \right| dt \\ &\le \ln \frac{\overline{r_i}}{\rho_{ii}} + \sum_{j=1}^{n} \frac{\overline{r_i}}{\rho_{ij}} + \left(\overline{R_i} + \overline{r_i}\right)\omega =: H_i, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{51}$$

For $\lambda \sum_{j=1}^{n} D_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \ge 0$, one can find that

$$x_i(t) \le H_i, \quad i = 1, \ldots, n. \tag{52}$$

Besides, from (41), we have

$$\begin{aligned} x_i'(t) = \lambda \Bigg[& r_i(t) - \sum_{j=1}^{n} A_{ij}(t) e^{x_j(t)} - \sum_{j=1}^{n} B_{ij}(t) \\ & \times \int_{-\infty}^{t} k_{ij}(t-s) e^{x_j(s)} ds - \sum_{j=1}^{n} C_{ij}(t) e^{x_j(t-\tau_{ij}(t))} \\ & - \sum_{j=1}^{n} D_{0,ij}(t) x_j'\left(t - \gamma_{ij}(t)\right) e^{x_j(t-\gamma_{ij}(t))} \\ & - e_i(t) \left(\psi_i e^{x_i}\right)(t) - f_i(t) \left(\psi_i e^{x_i}\right)(t) \\ & \times \left(t - \delta_i(t)\right) \Bigg], \quad i = 1, 2, \ldots, n. \end{aligned} \tag{53}$$

Notice that for all $t \in R$, one has

$$\begin{aligned} \left(\psi_i\right)(t) &= \int_t^{t+\omega} G_i(t,s) \left[\beta_i^*(s) + \theta_i^*(s)\right] ds \\ &= \int_t^{t+\omega} G_i(t,s) \alpha_i(s) \frac{\beta_i^*(s) + \theta_i^*(s)}{\alpha_i(s)} ds \\ &\le \frac{|\beta_i^*|_0 + |\theta_i^*|_0}{\alpha_i^L} \int_t^{t+\omega} G_i(t,s) \alpha_i(s) ds \\ &= \frac{|\beta_i^*|_0 + |\theta_i^*|_0}{\alpha_i^L}, \quad i = 1, \ldots, n. \end{aligned} \tag{54}$$

Then, by (52) and (54), we get

$$\begin{aligned} \left|x_i'\right|_0 \le \Bigg| & r_i(t) + \sum_{j=1}^{n} A_{ij}(t) e^{x_j(t)} + \sum_{j=1}^{n} B_{ij}(t) \\ & \times \int_{-\infty}^{t} k_{ij}(t-s) e^{x_j(s)} ds + \sum_{j=1}^{n} C_{ij}(t) e^{x_j(t-\gamma_{ij}(t))} \\ & + \sum_{j=1}^{n} D_{0,ij}(t) x_j'\left(t - \tau_{ij}(t)\right) e^{x_j(t-\tau_{ij}(t))} + e_i(t) \\ & \times \left(\psi_i e^{x_i}\right)(t) + f_i(t) \left(\psi_i e^{x_i}\right)(t) \left(t - \delta_i(t)\right) \Bigg| \\ \le & \left|r_i\right|_0 + \sum_{j=1}^{n} \left|A_{ij}\right|_0 e^{H_j} + \sum_{j=1}^{n} \left|B_{ij}\right|_0 e^{H_j} \\ & + \sum_{j=1}^{n} \left|C_{ij}\right|_0 e^{H_j} + \sum_{j=1}^{n} \left|D_{0,ij}\right|_0 \left|x_j'\right|_0 e^{H_j} + \left(|e_i|_0 + |f_i|_0\right) \\ & \times \frac{|\beta_i^*|_0 + |\theta_i^*|_0}{\alpha_i^L} e^{H_i}, \\ & i = 1, 2, \ldots, n. \end{aligned} \tag{55}$$

Furthermore, we have

$$\begin{aligned} \left\|x'\right\|_0 &= \sum_{i=1}^{n} \left|x_i'\right|_0 \\ &\le \sum_{i=1}^{n} \left|r_i\right|_0 + \sum_{i=1}^{n} \sum_{j=1}^{n} \left|A_{ij}\right|_0 e^{H_j} \\ &\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} \left|B_{ij}\right|_0 e^{H_j} \\ &\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} \left|C_{ij}\right|_0 e^{H_j} + \left(|e_i|_0 + |f_i|_0\right) \frac{|\beta_i^*|_0 + |\theta_i^*|_0}{\alpha_i^L} e^{H_i} \\ &\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} \left|D_{0,ij}\right|_0 \left\|x'\right\|_0 e^{H_j}. \end{aligned} \tag{56}$$

By the assumption (3) of Theorem 10, we see that

$$\begin{aligned} \sum_{i=1}^{n} \sum_{j=1}^{n} \left|D_{ij}\right|_0 e^{H_j} &\le \sum_{i=1}^{n} \sum_{j=1}^{n} \left|D_{0,ij}\right|_0 e^{H} \\ &\le \sum_{i=1}^{n} \sum_{j=1}^{n} \left|D_{0,ij}\right|_0 e^{M_0} < 1. \end{aligned} \tag{57}$$

Then, we have

$$\begin{aligned}\left\|x'\right\|_0 \le \Bigg(& \sum_{i=1}^{n}\left|r_i\right|_0 + \sum_{i=1}^{n}\sum_{j=1}^{n}\left|A_{ij}\right|_0 e^{H_j} \\ & + \sum_{i=1}^{n}\sum_{j=1}^{n}\left|B_{ij}\right|_0 e^{H_j} \\ & + \sum_{i=1}^{n}\sum_{j=1}^{n}\left|C_{ij}\right|_0 e^{H_j} \\ & + \left(\left|e_i\right|_0 + \left|f_i\right|_0\right) \frac{\left|\beta_i^*\right|_0 + \left|\theta_i^*\right|_0}{\alpha_i^L} e^{H_i} \Bigg) \\ & \times \left(1 - \sum_{i=1}^{n}\sum_{j=1}^{n}\left|D_{0,ij}\right|_0 e^{H_j}\right)^{-1} =: \Lambda^*. \end{aligned} \tag{58}$$

Since $x(t) = (x_1(t), \dots, x_n(t)) \in X$, there exists a $\xi_i \in [0, \omega]$ such that

$$x_i\left(\xi_i\right) = \inf_{t \in [0,\omega]} x_i(t), \quad i = 1, \dots, n. \tag{59}$$

It follows from (42) that

$$\begin{aligned} \overline{r_i}\omega &\ge \sum_{j=1}^{n} \int_0^{\omega} \Gamma_{ij}(t) e^{x_j(t)} dt \\ &= \sum_{j=1}^{n} e^{x_j(\xi_j)} \int_0^{\omega} \Gamma_{ij}(t)\, dt, \quad i = 1, \dots, n. \end{aligned} \tag{60}$$

It follows from (30) and (60) that

$$\overline{r_i} \ge \sum_{j=1}^{n} e^{x_j(\xi_j)} \left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}}\right), \quad i = 1, \dots, n. \tag{61}$$

From (61), we obtain

$$e^{x_i(\xi_i)}\left(\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}}\right) \le \overline{r_i}, \quad i = 1, \dots, n. \tag{62}$$

As $\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}} > 0$, it follows from the previous formula that

$$x_i\left(\xi_i\right) \le \ln \frac{\overline{r_i}}{\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}}}, \quad i = 1, \dots, n. \tag{63}$$

On the other hand, there also exists a $\eta_i \in [0, \omega]$ such that

$$x_i\left(\eta_i\right) = \sup_{t \in [0,\omega]} x_i(t), \quad i = 1, \dots, n. \tag{64}$$

It follows from (42), (54), and (64) that

$$\begin{aligned} \overline{r_i} &\le \sum_{j=1}^{n} \overline{\Gamma_{ij}} e^{x_j(\eta_i)} + \left(\overline{e_i} + \overline{f_i}\right) \frac{\left|\beta_i^*\right|_0 + \left|\theta_i^*\right|_0}{\alpha_i^L} e^{x_i(\eta_i)} \\ &= \sum_{j=1, j\ne i}^{n} \overline{\Gamma_{ij}} e^{x_j(\eta_i)} \\ &\quad + \left[\overline{\Gamma_{ii}} + \left(\overline{e_i} + \overline{f_i}\right) \frac{\left|\beta_i^*\right|_0 + \left|\theta_i^*\right|_0}{\alpha_i^L}\right] e^{x_i(\eta_i)}, \\ &\qquad i = 1, \dots, n. \end{aligned} \tag{65}$$

From (41), (54), and (64), we can have

$$\begin{aligned} &\left[\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}} + \left(\overline{e_i} + \overline{f_i}\right) \frac{\left|\beta_i^*\right|_0 + \left|\theta_i^*\right|_0}{\alpha_i^L}\right] e^{x_i(\eta_i)} \\ &\quad \ge \overline{r_i} - \sum_{j=1, j\ne i}^{n} \left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}}\right) e^{x_j(\eta_i)} \\ &\quad \ge \overline{r_i} - \sum_{j=1, j\ne i}^{n} \left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}}\right) e^{H_j}, \quad i = 1, \dots, n. \end{aligned} \tag{66}$$

That is,

$$x_i\left(\eta_i\right) \ge \ln \frac{\overline{r_i} - \sum_{j=1, j\ne i}^{n} \left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}}\right) e^{H_j}}{\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}} + \left(\overline{e_i} + \overline{f_i}\right)\left(\left|\beta_i^*\right|_0 + \left|\theta_i^*\right|_0 / \alpha_i^L\right)}, \quad i = 1, \dots, n. \tag{67}$$

Now, from (60) and (63) we know that there exist $\varsigma_i \in [0, \omega]$ $(i = 1, \dots, n)$ such that

$$\begin{aligned} &\left|x_i\left(\varsigma_i\right)\right| \\ &\le \max\left\{\left|\ln \frac{r_i}{\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}}}\right|, \right. \\ &\qquad \left. \left|\ln \frac{\overline{r_i} - \sum_{j=1, j\ne i}^{n} \left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}}\right) e^{H_j}}{\overline{A_{ii}} + \overline{B_{ii}} + \overline{C_{ii}} + \left(\overline{e_i} + \overline{f_i}\right)\left(\left|\beta_i^*\right|_0 + \left|\theta_i^*\right|_0 / \alpha_i^L\right)}\right|\right\} \\ &=: \Lambda_i, \quad i = 1, \dots, n. \end{aligned} \tag{68}$$

From (54), (64), and Lemma 3, we have

$$\begin{aligned} \left|x_i\right| &\le \left|x_i\left(\varsigma_i\right)\right| + \frac{1}{2}\int_0^{\omega} \left|x_i'(t)\right| dt \\ &\le \Lambda_i + \frac{1}{2}\int_0^{\omega} \left|x_i'\right| dt, \quad i = 1, \dots, n. \end{aligned} \tag{69}$$

Then,

$$\begin{aligned} \|x\|_0 &\le \sum_{i=1}^{n} \left|x_i\right| \le \sum_{i=1}^{n} \Lambda_i + \frac{1}{2}\int_0^{\omega} \left\|x'\right\|_0 dt \\ &< \sum_{i=1}^{n} \Lambda_i + \frac{1}{2}\Lambda^* \omega \le M_0. \end{aligned} \tag{70}$$

Obviously, M_0 is independent of λ; the proof of Lemma 12 is completed. □

Based on the previous results, we can now apply Lemma 1 and Remark 2 to (34) and obtain a proof of Theorem 10.

Proof. Obviously, for M as given in Lemma 11, condition (i) in Lemma 1 is satisfied. Let $h(\mu) = (h_1(\mu), \ldots, h_n(\mu))$. Since

$$\begin{aligned} h_i(\mu) &= \int_0^\omega f_i(s, \widehat{\mu})\, ds \\ &= \int_0^\omega r_i(t)\, dt - \sum_{i=1}^n \int_0^\omega A_{ij}(t)\, dt e^{\mu_j} \\ &\quad - \sum_{i=1}^n \int_0^\omega B_{ij}(t)\, dt e^{\mu_j} - \sum_{i=1}^n \int_0^\omega C_{ij}(t)\, dt e^{\mu_j} \\ &\quad - \int_0^\omega [e_i(t) + f_i(t)]\, dt e^{\mu_i} \\ &= \Bigg\{ \overline{r_i} - \sum_{j=1}^n \Big[\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}} e^{\mu_j} \\ &\quad + \left(\overline{e_i} + \overline{f_i}\right) \Big] \Bigg\} \omega \end{aligned} \tag{71}$$

and $M > \sum_{i=1}^n |\ln \mu_i^*|$, we have $h(\mu) \neq 0$ for any $\mu \in \partial B_M(R^n)$. That is, condition (ii) in Lemma 1 holds. At last, we verify that condition (iii) of Lemma 1 also holds. By assumption (1) of Theorem 10 and the formula for the Brouwer degree (see Theorem 2.2.3 in [35, 36]), a straightforward calculation shows that

$$\begin{aligned} \deg(h, B_M(R^n)) &= \sum_{\mu \in h^{-1}(0) \bigcap B_M(R^n)} \operatorname{sign} \det Dh(\mu) \\ &= \operatorname{sign} \Big\{ (-1)^n \det \\ &\quad \times \Big[\left(\overline{A_{ij}} + \overline{B_{ij}} + \overline{C_{ij}} \right) e^{\sum_{j=1}^n \mu_j^*} \\ &\quad + \left(\overline{e_i} + \overline{f_i} \right) e^{\mu_i^*} \Big] \Big\} \neq 0. \end{aligned} \tag{72}$$

By now, all the assumptions required in Lemma 1 hold. It follows from Lemma 1 and Remark 2 that system (34) has an ω-periodic solution. Returning to $y_i(t) = e^{x_i(t)}$, we infer that systems (18) and (19) have at least one positive ω-periodic solution. By Lemmas 7 and 8, $(N^*(t), u^*(t))^T = (N_1^*(t), \ldots, N_n^*(t), u_1^*(t), \ldots, u_n^*(t))^T$ is the unique positive periodic solution of the system (1) and (2), where $N_i^*(t) = \prod_{0<t_k<t} \Delta_{ik} y_i^*(t)$ $(i = 1, 2, \ldots, n)$. The proof of Theorem 10 is complete. □

Consider the following:

$$\begin{aligned} N_i'(t) &= N_i(t) \Bigg[r_i(t) - \sum_{j=1}^n a_{ij}(t) N_j(t) \\ &\quad - \sum_{j=1}^n b_{ij}(t) \int_{-\infty}^t k_{ij}(t-s) N_j(s)\, ds \\ &\quad - \sum_{j=1}^n c_{ij}(t) N_j\left(t - \gamma_{ij}(t)\right) \\ &\quad - \sum_{j=1}^n d_{ij}(t) N_j'\left(t - \tau_{ij}(t)\right) - e_i(t) u_i(t) \\ &\quad - f_i(t) u_i\left(t - \delta_i(t)\right) \Bigg], \\ u_i'(t) &= -\alpha_i(t) u_i(t) + \beta_i(t) N_i(t) \\ &\quad + \theta_i(t) N_i\left(t - \sigma_i(t)\right), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{73}$$

which is a special case of system (1) without impulse. We get easily the following result. Here, we have the following notations:

$$\rho_{ij}^* = \frac{\Gamma_{ij}^{*L} \left(1 - \gamma_{ij}'\right)^L}{\left(1 - \gamma_{ij}'\right)^L + \left|d_{ij}\right|_0}, \qquad \overline{R_i} = \frac{1}{\omega} \int_0^\omega |r_i(t)|\, dt,$$

$$L^* = \max \left\{ \sum_{i=1}^n \sum_{j=1}^n \left|d_{ij}\right|_0, \sum_{i=1}^n \sum_{j=1}^n \left|d_{0,ij}\right|_0 \right\},$$

$$M^* = \max \left\{ \sum_{i=1}^n \left|\ln \mu_i^*\right|_0, H^*, \frac{1}{2} \omega \Theta^* + \sum_{i=1}^n \Delta_i \right\},$$

$$H^* = \max_{i \in [1,n]} \{H_i^*\}, \qquad H_i^* = \ln \frac{\overline{r_i}}{\rho_{ii}^*} + \sum_{j=1}^n \frac{\overline{r_i}}{\rho_{ij}^*} + \left(\overline{R_i} + \overline{r_i}\right) \omega,$$

$$\begin{aligned} \Theta^* &= \Bigg(\sum_{i=1}^n |r_i|_0 + \sum_{i=1}^n \sum_{j=1}^n \left|a_{ij}\right|_0 e^{H_j^*} \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \left|b_{ij}\right|_0 e^{H_j^*} \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \left|c_{ij}\right|_0 e^{H_j^*} + \left(|e_i|_0 + |f_i|_0\right) \frac{|\beta_i|_0 + |\theta_i|_0}{\alpha_i^L} e^{H_i^*} \Bigg) \\ &\quad \times \left(1 - \sum_{i=1}^n \sum_{j=1}^n \left|d_{0,ij}\right|_0 e^{H_j^*} \right)^{-1}, \end{aligned}$$

$$\Theta_i = \max\left\{\left|\ln\frac{r_i}{\overline{a_{ii}}+\overline{b_{ii}}+\overline{c_{ii}}}\right|, \left|\ln\frac{\overline{r_i}-\sum_{j=1,j\neq i}^{n}\left(\overline{a_{ij}}+\overline{b_{ij}}+\overline{c_{ij}}\right)e^{H_j^*}}{\overline{a_{ii}}+\overline{b_{ii}}+\overline{c_{ii}}+\left(\overline{e_i}+\overline{f_i}\right)\left(|\beta_i|_0+|\theta_i|_0/\alpha_i^L\right)}\right|\right\},$$

$$\Gamma_{ij}^*(t) = a_{ij}(t)+b_{ij}(t)+\frac{c_{ij}\left(\vartheta_{ij}(t)\right)}{1-\gamma_{ij}'\left(\vartheta_{ij}(t)\right)} - \frac{d_{ij}'\left(\nu_{ij}(t)\right)}{1-\tau_{ij}'\left(\nu_{ij}(t)\right)},$$

$$\Gamma_i^1(t) = e_i(t)\left(\psi_i 1\right)(t),$$

$$\Gamma_i^2(t) = f_i(t)\left(\psi_i 1\right)\left(t-\sigma_i(t)\right),$$

$$d_{0,ij}(t) = d_{ij}(t)\left(1-\tau_{ij}'(t)\right), \quad (74)$$

and $\vartheta_{ij}(t), \nu_{ij}(t)$ represent the inverse function of $t-\gamma_{ij}(t), t-\tau_{ij}(t)$ $(i, j = 1, 2, \ldots, n)$, respectively.

Corollary 13. *Suppose that the following conditions hold;*

(1) *the system of algebraic equations*

$$f^*(\mu) = \left(\overline{r_i}-\sum_{j=1}^{n}\left(\left(\overline{a_{ij}}+\overline{b_{ij}}+\overline{c_{ij}}\right)\mu_j + \left(\overline{\Gamma_i^1}+\overline{\Gamma_i^2}\right)\mu_i\right)\right)_{n\times 1} = 0 \quad (75)$$

has a unique positive solution $\mu^* = (\mu_1^*, \ldots, \mu_n^*)$;

(2) $\overline{a_{ij}}+\overline{b_{ij}}+\overline{c_{ij}} > 0, \overline{r_i} > \sum_{j=1,j\neq i}^{n}(\overline{a_{ij}}+\overline{b_{ij}}+\overline{c_{ij}})e^{H_j^*}$, $\tau_{ij}'(t) < 1$, $\gamma_{ij}'(t) < 1$ *and* $\Gamma_{ij}^*(t) > 0$;

(3) $K^* =: L^* e^{M^*} < 1$. *Then,* (73) *has at least one positive* ω*-periodic solution.*

Proof. Its proof is similar to the proof of Theorem 10. Here, we omit it. □

Similarly, we can get the following results.

Theorem 14. *Assume that conditions of Theorem 10 hold, and then, the conclusion of Theorem 10 holds for the following system:*

$$N_i'(t) = -N_i(t)\left[r_i(t)-\sum_{j=1}^{n}a_{ij}(t)N_j(t) - \sum_{j=1}^{n}b_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)N_j(s)\,ds - \sum_{j=1}^{n}c_{ij}(t)N_j\left(t-\gamma_{ij}(t)\right) - \sum_{j=1}^{n}d_{ij}(t)N_j'\left(t-\tau_{ij}(t)\right)-e_i(t)u_i(t) - f_i(t)u_i\left(t-\delta_i(t)\right)\right],$$
$$i = 1, 2, \ldots, n, \; t\neq t_k,$$
$$u_i'(t) = -\alpha_i(t)u_i(t)+\beta_i(t)N_i(t) + \theta_i(t)N_i\left(t-\sigma_i(t)\right), \quad t\geq 0,$$
$$\Delta N_i(t_k) = \left(p_{ik}+q_{ik}\right)N_i(t_k),$$
$$i = 1, 2, \ldots, n, \; k = 1, 2, \ldots. \quad (76)$$

Proof. Its proof is similar to the proof of Theorem 10. Here, we omit it. □

Corollary 15. *Assume that conditions of Corollary 13 hold, and then, the conclusion of Corollary 13 holds for the following system:*

$$N_i'(t) = -N_i(t)\left[r_i(t)-\sum_{j=1}^{n}a_{ij}(t)N_j(t) - \sum_{j=1}^{n}b_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)N_j(s)\,ds - \sum_{j=1}^{n}c_{ij}(t)N_j\left(t-\tau_{ij}(t)\right) - \sum_{j=1}^{n}d_{ij}(t)N_j'\left(t-\gamma_{ij}(t)\right)\right], \quad (77)$$
$$u_i'(t) = -\alpha_i(t)u_i(t)+\beta_i(t)N_i(t) + \theta_i(t)N_i\left(t-\sigma_i(t)\right), \quad i = 1, 2, \ldots, n.$$

Proof. Its proof is similar to the proof of Theorem 10. Here, we omit it. □

Remark 16. When $\alpha_i(t) = \beta_i(t) = \theta_i(t) = 0$, we can derive some immediate corollaries from Theorems 10 and 14; thus, our results generalize the corresponding results in [14].

4. Applications

In order to illustrate some features of our main result, in the following, we will apply Theorem 10 to some special cases, which have been studied extensively in the literature.

Application 17. We consider an n-species neutral delay competition system in a periodic environment with impulse:

$$\begin{aligned} N_i'(t) = \pm N_i(t)\Bigg[& a_i(t) - \sum_{j=1}^{n} b_{ij}(t) N_j(t) \\ & - \sum_{j=1}^{n} c_{ij}(t) N_j(t-\tau_{ij}(t)) \\ & - \sum_{j=1}^{n} d_{ij}(t) N_j'(t-\gamma_{ij}(t)) \\ & - e_i(t) u_i(t) - f_i(t) u_i \\ & \times (t-\delta_i(t)) \Bigg], \\ & i = 1,2,\ldots,n,\ t \neq t_k, \\ u_i'(t) = & -\alpha_i(t) u_i(t) + \beta_i(t) N_i(t) \\ & + \theta_i(t) N_i(t-\sigma_i(t)), \quad t \geq 0, \\ \Delta N_i(t_k) = & (p_{ik} + q_{ik}) N_i(t_k), \\ & i = 1,2,\ldots,n,\ k = 1,2,\ldots, \end{aligned} \tag{78}$$

where $b_{ij}, c_{ij}, e_i, f_i, \alpha_i, \beta_i, \theta_i \in C(R,[0,+\infty))$, $d_{ij} \in C^1(R,[0,+\infty))$, $\gamma_{ij} \in C^1(R,R)$, and $\tau_{ij} \in C^2(R,R)$ are continuous ω-periodic functions. And $a_i \in C(R,R)$ are continuous ω-periodic functions with $\int_0^{\omega} a_i(t)dt > 0$. Here, we give the following notations:

$$\begin{aligned} \overline{A_i} &= \frac{1}{\omega}\int_0^{\omega} |a_i(t)|\,dt, \\ B_{ij}(t) &= b_{ij}(t) \prod_{0<t_k<t} (1 + p_{ik} + q_{ik}), \\ C_{ij}(t) &= c_{ij}(t) \prod_{0<t_k<t-\tau_{ij}(t)} (1 + p_{ik} + q_{ik}), \\ D_{ij}(t) &= d_{ij}(t) \prod_{0<t_k<t-\gamma_{ij}(t)} (1 + p_{ik} + q_{ik}), \\ D_{0,ij}(t) &= D_{ij}(t)\left(1-\gamma_{ij}'(t)\right), \\ \beta_i^*(t) &= \beta_i(t) \prod_{0<t_k<t} (1 + p_{ik} + q_{ik}), \\ \theta_i^*(t) &= \theta_i(t) \prod_{0<t_k<t} (1 + p_{ik} + q_{ik}), \\ \Gamma_{ij}(t) &= A_{ij}(t) + B_{ij}(t) \\ &\quad + \frac{C_{ij}(\vartheta_{ij}(t))}{1-\gamma_{ij}'(\vartheta_{ij}(t))} - \frac{D_{ij}'(\nu_{ij}(t))}{1-\tau_{ij}'(\nu_{ij}(t))}, \\ \Gamma_i^1(t) &= e_i(t)(\psi_i 1)(t), \\ \Gamma_i^2(t) &= f_i(t)(\psi_i 1)(t-\sigma_i(t)), \\ L_1 &= \max\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}|D_{ij}|_0, \sum_{i=1}^{n}\sum_{j=1}^{n}|D_{0,ij}|_0\right\}, \\ M_1 &= \max\left\{\sum_{i=1}^{n}|\ln \mu_i^*|_0, H^*, \frac{1}{2}\omega\Lambda^* + \sum_{i=1}^{n}\Lambda_i\right\}, \\ H^* &= \max_{i\in[1,n]}\{H_i^*\}, \\ H_i^* &= \ln\frac{\overline{a_i}}{\rho_{ii}^*} + \sum_{j=1}^{n}\frac{\overline{a_i}}{\rho_{ij}^*} + \left(\overline{A_i} + \overline{a_i}\right)\omega, \\ \rho_{ij}^* &= \frac{\Gamma_{ij}^{*L}\left(1-\gamma_{ij}'\right)^L}{\left(1-\gamma_{ij}'\right)^L + |D_{ij}|_0}, \\ \Lambda^* &= \Bigg(\sum_{i=1}^{n}|a_i|_0 + \sum_{i=1}^{n}\sum_{j=1}^{n}|B_{ij}|_0 e^{H_j^*} \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}|C_{ij}|_0 e^{H_j^*} \\ &\quad + \left(|e_i|_0 + |f_i|_0\right)\frac{|\beta_i^*|_0 + |\theta_i^*|_0}{\alpha_i^L} e^{H_i^*}\Bigg) \\ &\quad \times \left(1 - \sum_{i=1}^{n}\sum_{j=1}^{n}|D_{0,ij}|_0 e^{H_j^*}\right)^{-1}, \\ \Lambda_i &= \max\left\{\left|\ln\frac{a_i}{\overline{B_{ii}} + \overline{C_{ii}}}\right|, \right. \\ &\quad \left. \left|\ln\frac{\overline{a_i} - \sum_{j=1,j\neq i}^{n}\left(\overline{B_{ij}} + \overline{C_{ij}}\right)e^{H_j^*}}{\overline{B_{ii}} + \overline{C_{ii}} + \left(\overline{e_i} + \overline{f_i}\right)\left(\left(|\beta_i^*|_0 + |\theta_i^*|_0\right)/\alpha_i^L\right)}\right|\right\}, \\ & \qquad i, j = 1,2,\ldots,n, \end{aligned} \tag{79}$$

where $\vartheta_{ij}(t), \nu_{ij}(t)$ represent the inverse function of $t - \gamma_{ij}(t), t - \tau_{ij}(t)$, respectively. Applying Theorem 10 to (78), we can obtain the following theorem.

Theorem 18. *Assume that the following conditions are satisfied:*

(1) *the system of algebraic equations*

$$f^*(\mu) = \left(\overline{a_i} - \sum_{j=1}^{n} \left(\left(\overline{B_{ij}} + \overline{C_{ij}} \right) \mu_j + \left(\overline{\Gamma_i^1} + \overline{\Gamma_i^2} \right) \mu_i \right) \right)_{n\times 1} = 0 \tag{80}$$

has a unique positive solution $\mu^* = (\mu_1^*, \ldots, \mu_n^*)$;

(2) $\overline{B_{ij}} + \overline{C_{ij}} > 0$, $\overline{a_i} > \sum_{j=1, j\neq i}^{n} (\overline{B_{ij}} + \overline{C_{ij}}) e^{H_j}$, $\gamma_{ij}'(t) < 1$, $\tau_{ij}'(t) < 1$, *and* $\Gamma_{ij}(t) > 0$;

(3) $K_1 =: L_1 e^{M_1} < 1$.

Then, (78) *has at least one positive ω-periodic solution.*

Remark 19. When $\alpha_i(t) = \beta_i(t) = \theta_i(t) = u_i(t) = 0$, $p_{ik} + q_{ik} = 0$, we can derive some immediate corollaries of Theorem 18, then, Theorem 18 generalizes the corresponding results in [9, 10]. On the other hand, when $p_{ik} + q_{ik} = 0$, we can derive an immediate corollary of Theorem 18, then, Theorem 18 generalizes the corresponding results in [11].

Application 20. We consider the single specie neutral delay logistic equation with impulse:

$$\begin{aligned} N'(t) = \pm N(t) \Bigg[& a(t) - b(t) N(t) \\ & - \sum_{i=1}^{n} c_i(t) \int_{-\infty}^{t} k_i(t-s) N(s)\, ds \\ & - \sum_{j=1}^{m} d_j(t) N\left(t - \gamma_j(t)\right) \\ & - \sum_{l=1}^{p} e_l(t) N'\left(t - \tau_l(t)\right) - f(t) u(t) \\ & - g(t) u(t - \delta(t)) \Bigg], \quad t \neq t_k, \\ u'(t) = & -\alpha(t) u(t) + \beta(t) N(t) \\ & + \theta(t) N(t - \sigma(t)), \quad t \geq 0, \\ N\left(t_k^+\right) = & \left(p_k + q_k\right) N\left(t_k\right), \quad k = 1, 2, \ldots, \end{aligned} \tag{81}$$

where $b, c_i, d_j, f, g, \alpha, \beta, \theta \in C(R, [0, +\infty))$, $e_l \in C^1(R, [0, +\infty))$, $\gamma_j \in C^1(R, R)$, and $\tau_l \in C^2(R, R)$ are continuous ω-periodic functions. And $a \in C(R, R)$ are continuous ω-periodic functions with $\int_0^\omega a(t) dt > 0$. Here, we have the following notations:

$$\begin{aligned} \overline{A} &= \frac{1}{\omega} \int_0^\omega |a(t)|\, dt, \\ B(t) &= b(t) \prod_{0<t_k<t} \left(1 + p_k + q_k\right), \\ C_i(t) &= c_i(t) \prod_{0<t_k<t} \left(1 + p_k + q_k\right), \\ D_j(t) &= d_j(t) \prod_{0<t_k<t-\sigma_j(t)} \left(1 + p_k + q_k\right), \\ E_l(t) &= e_l(t) \prod_{0<t_k<t-\tau_l(t)} \left(1 + p_k + q_k\right), \\ E_{0,l}(t) &= E_l(t) \left(1 - \tau_l'(t)\right), \\ \beta^*(t) &= \beta(t) \prod_{0<t_k<t} \left(1 + p_{ik} + q_{ik}\right), \\ \theta^*(t) &= \theta(t) \prod_{0<t_k<t} \left(1 + p_{ik} + q_{ik}\right), \\ \rho &= \frac{\Gamma_1^L \left(1 - \tau_l'\right)^L}{\left(1 - \tau_l'\right)^L + \left|E_l\right|_0}, \\ L_2 &= \max \left\{ \sum_{l=1}^{p} \left|E_l\right|_0, \sum_{l=1}^{p} \left|E_{0,l}\right|_0 \right\}, \\ M_2 &= \max \left\{H^*, \Delta^*\right\}, \\ H^* &= \ln \frac{\overline{a}}{\rho} + \frac{\overline{a}}{\rho} + \left(\overline{A} + |\overline{a}|\right) \omega, \\ \Delta^* &= \frac{\omega}{2} \Bigg(|a|_0 + \Bigg[|B|_0 + \sum_{i=1}^{n} \left|C_i\right|_0 + \sum_{j=1}^{m} \left|D_j\right|_0 + \left(|f|_0 + |g|_0\right) \times \frac{\left|\beta^*\right|_0 + \left|\theta^*\right|_0}{\alpha^L} \Bigg] e^{H^*} \Bigg) \\ &\quad \times \left(1 - \left|E_{0,l}\right|_0 e^{H^*}\right)^{-1} + \left| \ln \frac{\overline{a}}{\overline{B} + \sum_{i=1}^{n} \overline{C_i} + \sum_{j=1}^{m} \overline{D_j} + \overline{\Gamma_2} + \overline{\Gamma_3}} \right|, \\ \Gamma_1(t) &= B(t) + \sum_{i=1}^{n} C_i(t) + \sum_{j=1}^{m} \frac{D_j\left(\mu_j(t)\right)}{1 - \sigma'\left(\mu_j(t)\right)} - \sum_{l=1}^{p} \frac{E_l'\left(\nu_l(t)\right)}{1 - \tau_l'\left(\nu_l(t)\right)}, \end{aligned}$$

$$\Gamma_2(t) = f(t)(\psi 1)(t),$$
$$\Gamma_3(t) = g(t)(\psi 1)(t - \sigma(t)), \tag{82}$$

and $\mu_j(t), \nu_l(t)$ represent the inverse function of $t - \gamma_j(t), t - \tau_l(t)$, respectively. Applying Theorem 10 to (81), we can obtain the following theorem.

Theorem 21. *Assume that the following conditions are satisfied:*

(1) $\overline{B} + \sum_{i=1}^{n} \overline{C_i} + \sum_{j=1}^{m} \overline{D_j} + \overline{\Gamma_2} + \overline{\Gamma_3} > 0$, $\gamma_j'(t) < 1$, $\tau_l'(t) < 1$ *and* $\Gamma_1(t) > 0$;

(2) $K_2 =: L_2 e^{M_2} < 1$.

Then, system (81) *has at least one positive ω-periodic solution.*

Remark 22. When $i = 0$, $j = l = 1$, $\alpha(t) = \beta(t) = \theta(t) = u(0) = 0$, $\sigma_j(t) = \tau_l(t)$, and $p_k + q_k = 0$, we can derive an immediate corollary of Theorem 21, which is also an answer to the open problem 9.2 due to Kuang [2]. On the other hand, when $i = 0$, $\alpha(t) = \beta(t) = \theta(t) = u(t) = 0$, and $p_k + q_k = 0$, we can derive some immediate corollaries of Theorem 21 (that is the corresponding results in [7, 8]), therefore, our result improves and generalizes the corresponding result in [7, 8]. Moreover, when $l = 0$, $\alpha(t) = \beta(t) = \theta(t) = u(t) = 0$, we can see that our Theorem 21 can hold without the assumption $\overline{a} > 0$. When $\overline{a} < 0$, Wang's main result (see Theorem 3.1 in [13]) cannot be applied. Therefore, in comparison with [13], our result improves and generalizes the result in [13].

Acknowledgments

This research is supported by NSF of China (nos. 10971229, 11161015, and 11371367), PSF of China (no. 2012M512162), NSF of Hunan Province (nos. 11JJ900, 12JJ9001, and 13JJ4098), the Education Foundation of Hunan province (nos. 12C0541, and 13C084), and the construct program of the key discipline in Hunan province.

References

[1] K. Gopalsamy, X. Z. He, and L. Z. Wen, "On a periodic neutral logistic equation," *Glasgow Mathematical Journal*, vol. 33, no. 3, pp. 281–286, 1991.

[2] Y. Kuang, *Delay Differential Equations with Applications in Population Dynamics*, vol. 191 of *Mathematics in Science and Engineering*, Academic Press, Boston, Mass, USA, 1993.

[3] Y. K. Li, "Positive periodic solutions for a neutral delay model," *Acta Mathematica Sinica. Chinese Series*, vol. 39, no. 6, pp. 789–795, 1996 (Chinese).

[4] Q. Li and J. D. Cao, "On positive periodic solutions for neutral delay model," *Journal of Mathematical Research and Exposition*, vol. 20, no. 4, pp. 562–565, 1999.

[5] H. Fang and J. B. Li, "On the existence of periodic solutions of a neutral delay model of single-species population growth," *Journal of Mathematical Analysis and Applications*, vol. 259, no. 1, pp. 8–17, 2001.

[6] S. P. Lu and W. G. Ge, "Existence of positive periodic solutions for neutral functional differential equations with deviating arguments," *Applied Mathematics. A Journal of Chinese Universities Series B*, vol. 17, no. 4, pp. 382–390, 2002.

[7] L. Y. Kun, "Periodic solution of a periodic neutral delay equation," *Journal of Mathematical Analysis and Applications*, vol. 214, no. 1, pp. 11–21, 1997.

[8] S. P. Lu and W. G. Ge, "Existence of positive periodic solutions for neutral population model with multiple delays," *Applied Mathematics and Computation*, vol. 153, no. 3, pp. 885–902, 2004.

[9] Y. K. Li, "On a periodic neutral delay Lotka-Volterra system," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 39, no. 6, pp. 767–778, 2000.

[10] Z. J. Liu and L. S. Chen, "On positive periodic solutions of a nonautonomous neutral delay *n*-species competitive system," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 68, no. 6, pp. 1409–1420, 2008.

[11] F. D. Chen, "Positive periodic solutions of neutral Lotka-Volterra system with feedback control," *Applied Mathematics and Computation*, vol. 162, no. 3, pp. 1279–1302, 2005.

[12] H.-F. Huo, "Existence of positive periodic solutions of a neutral delay Lotka-Volterra system with impulses," *Computers & Mathematics with Applications*, vol. 48, no. 12, pp. 1833–1846, 2004.

[13] Q. Wang and B. X. Dai, "Existence of positive periodic solutions for a neutral population model with delays and impulse," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 69, no. 11, pp. 3919–3930, 2008.

[14] Z. G. Luo, B. X. Dai, and Q. Wang, "Existence of positive periodic solutions for a nonautonomous neutral delay *n*-species competitive model with impulses," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 5, pp. 3955–3967, 2010.

[15] X. X. Chen, "Almost periodic solutions of nonlinear delay population equation with feedback control," *Nonlinear Analysis: Real World Applications*, vol. 8, no. 1, pp. 62–72, 2007.

[16] F. D. Chen, "Permanence of a discrete *N*-species cooperation system with time delays and feedback controls," *Applied Mathematics and Computation*, vol. 186, no. 1, pp. 23–29, 2007.

[17] F. D. Chen and X. H. Cao, "Existence of almost periodic solution in a ratio-dependent Leslie system with feedback controls," *Journal of Mathematical Analysis and Applications*, vol. 341, no. 2, pp. 1399–1412, 2008.

[18] Z. J. Zeng and Z. C. Zhou, "Multiple positive periodic solutions for a class of state-dependent delay functional differential equations with feedback control," *Applied Mathematics and Computation*, vol. 197, no. 1, pp. 306–316, 2008.

[19] W. Qi and B. X. Dai, "Almost periodic solution for *n*-species Lotka-Volterra competitive system with delay and feedback controls," *Applied Mathematics and Computation*, vol. 200, no. 1, pp. 133–146, 2008.

[20] L. F. Nie, J. G. Peng, and Z. D. Teng, "Permanence and stability in multi-species non-autonomous Lotka-Volterra competitive systems with delays and feedback controls," *Mathematical and Computer Modelling*, vol. 49, no. 1-2, pp. 295–306, 2009.

[21] H. X. Hu, Z. D. Teng, and S. J. Gao, "Extinction in nonautonomous Lotka-Volterra competitive system with pure-delays and feedback controls," *Nonlinear Analysis: Real World Applications*, vol. 10, no. 4, pp. 2508–2520, 2009.

[22] Y.-H. Fan and L.-L. Wang, "Global asymptotical stability of a logistic model with feedback control," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 4, pp. 2686–2697, 2010.

[23] Z. Zhao, T. Y. Wang, and L. S. Chen, "Dynamic analysis of a turbidostat model with the feedback control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 4, pp. 1028–1035, 2010.

[24] J. R. Yan and A. M. Zhao, "Oscillation and stability of linear impulsive delay differential equations," *Journal of Mathematical Analysis and Applications*, vol. 227, no. 1, pp. 187–194, 1998.

[25] X. Z. Liu and G. Ballinger, "Boundedness for impulsive delay differential equations and applications to population growth models," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 53, no. 7-8, pp. 1041–1062, 2003.

[26] X. Y. Zhang, J. R. Yan, and A. M. Zhao, "Existence of positive periodic solutions for an impulsive differential equation," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 68, no. 10, pp. 3209–3216, 2008.

[27] Q. Wang, B. X. Dai, and Y. M. Chen, "Multiple periodic solutions of an impulsive predator-prey model with Holling-type IV functional response," *Mathematical and Computer Modelling*, vol. 49, no. 9-10, pp. 1829–1836, 2009.

[28] S. Ahmad and G. T. Stamov, "Almost periodic solutions of N-dimensional impulsive competitive systems," *Nonlinear Analysis: Real World Applications*, vol. 10, no. 3, pp. 1846–1853, 2009.

[29] Z. J. Liu, J. H. Wu, Y. P. Chen, and M. Haque, "Impulsive perturbations in a periodic delay differential equation model of plankton allelopathy," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 1, pp. 432–445, 2010.

[30] H. L. Wang, "Dispersal permanence of periodic predator-prey model with Ivlev-type functional response and impulsive effects," *Applied Mathematical Modelling*, vol. 34, no. 12, pp. 3713–3725, 2010.

[31] D. Baĭnov and P. Simeonov, *Impulsive Differential Equations: Periodic Solutions and Applications*, vol. 66 of *Pitman Monographs and Surveys in Pure and Applied Mathematics*, Longman Scientific & Technical, Harlow, UK, 1993.

[32] A. M. Samoĭlenko and N. A. Perestyuk, *Impulsive Differential Equations*, vol. 14 of *World Scientific Series on Nonlinear Science. Series A: Monographs and Treatises*, World Scientific Publishing, River Edge, NJ, USA, 1995.

[33] M. Benchohra, J. Henderson, and S. Ntouyas, *Impulsive Differential Equations and Inclusions*, vol. 2 of *Contemporary Mathematics and Its Applications*, Hindawi Publishing Corporation, New York, NY, USA, 2006.

[34] L. H. Erbe, W. Krawcewicz, and J. H. Wu, "A composite coincidence degree with applications to boundary value problems of neutral equations," *Transactions of the American Mathematical Society*, vol. 335, no. 2, pp. 459–478, 1993.

[35] W. Krawcewicz and J. H. Wu, *Theory of Degrees with Applications to Bifurcations and Differential Equations*, John Wiley & Sons, New York, NY, USA, 1996.

[36] H. Fang, "Positive periodic solutions of n-species neutral delay systems," *Czechoslovak Mathematical Journal*, vol. 53(128), no. 3, pp. 561–570, 2003.