A new formulation technique to model materials and operations planning: the generic materials and operations planning (GMOP) problem

Jose P. Garcia-Sabater*, Julien Maheut and Juan A. Marin-Garcia

ROGLE, Departamento de Organización de Empresas,
Universitat Politecnica de Valencia,
Camino de Vera S/N, 46022 Valencia, Spain
E-mail: jpgarcia@omp.upv.es
E-mail: juma2@doctor.upv.es
E-mail: jamarin@omp.upv.es
*Corresponding author

Abstract: This paper presents a technique that mathematically models relationships between operations and materials, which amends the usual technique used to model materials and resources requirement planning through mathematical programming. This technique represents operations and materials requirement planning by extending the bill of materials concept beyond the Gozinto structure. This so-called generic materials and operations planning (GMOP) problem is based on the 'stroke' concept. The decision variables are the operations (strokes) each resource is capable of, and not materials or resources. This form extends modelling capacity to transformation operations, resource and product substitution, and material transportation. It considers most conventional bills of materials types (direct, alternative and reverse BOMs, alternative resources and routings) with the same data structure. It contemplates multi-level problem modelling, and even packaging and alternative transport modes. The same data structure represents these characteristics. The problem, its mathematical modelling approach and examples illustrating its use are provided. [Received 31 March 2011; Revised 26 May 2011; Accepted 21 June 2011].

Keywords: alternative bill of materials; alternative operations; alternative routings; generic materials and operations planning; GMOP; material requirement planning; packaging; product substitution; reverse bill of materials; stroke; supply chain management.

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Biographical notes: José P. Garcia-Sabater is a Professor of Operations Management in the Departamento de Organización de Empresas at the Universidad Politécnica de Valencia, Spain. He received his Ingeniero Industrial degree and his PhD from the Universidad Politécnica de Valencia. He also received a Combined Engineering degree from Coventry University, UK. His research and teaching interests are in the areas of supply chain management, operations management, and management science. His work has appeared in journals such as *European Journal of Operational Research, International Journal of Production Economics, Fuzzy Sets and Systems, Discrete Applied Mathematics*, and others.

Julien Maheut is a Researcher at Research Group ROGLE at the Universidad Politécnica de Valencia, Spain. He is currently coursing a PhD programme in Operational Research and Supply Chain Management Advanced Models at the Universidad Politécnica de Valencia sponsored by a VALi+d grant from the Generalitat Valenciana. He received his Master in Science in Supply Chain Management and Logistics from UPV and a Diplôme d'ingénieur and a Master's degree from Arts et Métiers ParisTech. His research interests include industrial engineering, supply chain management, collaborative planning, modelling and simulation, and multiagent system.

Juan A. Marin-Garcia is a Lecturer at the Departamento de Organización de Empresas in the Universidad Politécnica de Valencia, Spain. He lectures on management, teamwork and human resources management. With regard to these areas he has also worked as a Consultant for some companies in Spain and El Salvador, Central America. He is a foundation member of Research Groups ROGLE and i-GRHUP, where he develops his activities in different projects. He develops pedagogic activities inside IEMA group. His main research fields are lean manufacturing systems, participative management, continuous improvement, performance evaluation and active learning in higher education.

1 Introduction

In the 1970s when multi-level materials management started, the tools/methodologies used became known as material requirement planning (MRP). Later on, the need to plan the resources to produce the materials was incorporated. In the proposed structures, all the materials and resources needed to manufacture a product were associated with it. This origin probably marked a lock-in (Arthur, 1989; David, 1985) to tackle MRP and resources requirement planning.

This paper proposes an alternative modelling technique that places emphasis on planning what is known to be done rather than the result of the action (the product). The proposed modelling method is useful given its simplicity and generality. Furthermore, its proposal is feasible since the mathematical programming of problem-solving technology has considerably improved in the last ten years (Bixby and Rothberg, 2007).

The need for this new modelling approach emerged when design production planning in the automobile sector. Given the pressure to continuously improve (Lamming, 1993), it is usual to come across manufacturing processes that do not strictly adapt to an assembly-type structure (Garcia-Sabater and Vidal-Carreras, 2010), and alternative products and resources, or deliberate co-production circumstances, may arise (Vidal-Carreras and Garcia-Sabater, 2009). These features, together with the relevance of alternative processes, ought to be taken into account in the planning process. This work specifies and analyses in detail a modelling process which has been outlined in papers, like that of Calderon-Lama et al. (2009) or that by Garcia-Sabater et al. (2009), and has been implemented in different tools of current use.

The rest of the paper is arranged as so: the evolution of materials and operations planning has been briefly reviewed by considering resources constraints. Some works relating to materials and resources planning under the name of multi-level capacitated lot-sizing problem (MLCLSP), or others, have been analysed. Special stress is placed on works which present variants in modelling instead of on those that present variants in ways of solving. Next, the proposal is presented and how this proposal includes the cited variants in a more compact formulation. Then there are two cases in which the concept has been successfully applied, along with some general observations to implement the tool in a practical context. Finally, some conclusions and future research lines are offered and proposed, and a simplified case study is provided as an appendix.

2 Materials and operations planning

2.1 Introduction to the MRP logic and its evolution

Many authors indicate that Orlicky (1975) was the first to successfully apply the logic of the so-called MRP, although others like Mabert (2007) highlight former experiences in the 1950s. In any case, they all acknowledge that Orlicky greatly boosted the main technique known to plan the demand of materials of demand-dependent products.

Although MRP is a well-studied technique, it is relatively frequent to encounter firms that do not apply it at all or that do not apply it fully (Lin et al., 2009). More often than not, the generated plan is more descriptive of the activity itself than it is prescriptive (Shapiro, 2010).

2.2 Basic MRP data

The basic input data in an MRP system are the master production schedule (MPS), the bill of materials (BOM) and initial stock levels (Slack et al., 2010). Executing the planning process generates production and purchase orders that will feedback the next MRP execution. The quantity and quality of the information that the MRP requires to run is the main problem to be faced (Chase et al., 2004).

So, as the MPS plan attempts to satisfy due dates and customer demand, increases in demand uncertainty have mixed effects on the MPS due date performance (Enns, 2002). The problem of demand uncertainty has different origins, and this problem is tackled from various perspectives: improving forecast processes (Poler et al., 2008), optimising safety stocks (Molinder, 1997), or developing solutions that consider uncertainty when planning (Mula et al., 2008).

Another equally important problem is the lack of certainty of the inventory data. As this is evidently an operative-type problem, firms of all sizes with non-reliable and accurate data at the inventory level are frequently found. Improving the quality of the registered data (Kang and Gershwin, 2005) is probably the only reasonable way forward.

If the problem of both the MPS and stocks levels is considerable, then the BOM structure problem is no better. From the data structures viewpoint, this theme has been extensively covered. The structure of the products to be planned has been complicated by both technological development and mass customisation-type strategies (Pine, 1993). Thus, representation of BOMs has become an ever increasingly complex problem (Hegge and Wortmann, 1991). So, it may be stated that research into how to acquire, store and manage BOMs is far from complete (Stapic et al., 2009).

The basic structure of conventional BOM has always been to relate a parent item with one or several child items, which only takes place in pure convergent product structures (Perez Perales et al., 2002). Yet, even in these cases, once one of the firm's subsystems has defined a structure (e.g., the design department), an algorithm will have to be performed to transform its BOM structure into the structure required by the logistics department (Chang et al., 1997; Olsen et al., 1997) proposed that such a process is part of defining the BOM.

Concepts like the generic bill of materials and operations (GBOMO) by Jiao et al. (2000) and phantom items (Clement et al., 1995; Luszczak, 2010) are ways of structuring BOMs to facilitate the consideration of product variants, transportation processes and co-products or substitution products. However, merely considering 'alternate BOMs' (Escudero, 1994) or 'reverse BOMs' (Gupta and Taleb, 1994; Lambert and Gupta, 2002), as two examples, is a complicated issue.

2.3 Considering resources and their capacity constraints

One of the most obvious constraints of classic MRP systems is not considering capacity limits. Although one work (Mize et al., 1979) had already considered this problem, its real circumstances had not actually been considered until MRPII had been introduced, whose authorship is attributed to Wight (1984). When it came about, MRPII was seen as a closed loop system. After performing the materials explosion, the capacity constraints required an adjustment of the input data and a subsequent launch of the MRP to restart the analysis. The process was repeated until an acceptable result was obtained (Voss and Woodruff, 2005). In this way, the database required to calculate material requirements must be extended by incorporating routing (Plenert, 1999) and, of course, the capacity data. Thus, to implement an MRPII system properly, both product structures and process structures are required (what is known as generic routing).

For different reasons, both structures have evolved separately in enterprise resource planning (ERP). Tatsiopoulos (1996) indicates three reasons why this situation comes about: avoiding the inflation of part numbers, the existence or non-existence of production stages with intermediate warehouses, and the need to maintain different attributes for materials and operations. Yet, the same author points out that a unified structure is better understood in small firms than a separate structure.

2.4 The MRPII mathematical formulation

Initially, those applications dedicated to MRP emerged mainly from perspectives based on data processing rather than from a mathematical or an optimisation perspective. The development of operations planning applications that consider capacity constraints occupies a relatively common place in the literature (Drexl and Kimms, 1997). The consideration of multi-level systems creates numerous ways of covering capacity constraints and, should they be required, the need for not exceeding this capacity (Rong et al., 2006). Yet, tackling the problem from the mathematical programming perspective is more likely to be a good mechanism, as suggested by Segerstedt (1996b), who indicates that formulas are the "supreme methods for communication".

It is worth stressing that this author's notion of assigning a BOM and a bill of resources to each product that is susceptible to being assembled has been maintained since it was proposed, and has not been since amended. The matrix linking each parent item with its child items required for its assembly appears in this formulation, and is in accordance with the Gozinto structure presented by Vazsonyi (1954). Mize et al. (1979) already presented a matrix-based calculation method, although the proposal of considering an MRP enabled by mathematical programming can be attributed to Billington et al. (1983). Obviously, existing technological constraints (both hardware and software) prevented these authors in 1983 from stating that the model itself was applicable. So, their work proposes alternative methods to solve it.

This work included some concepts such as the lead time, which is also associated with the product, plus a yield for production and a bill of resources in a matrix form. Likewise, the objective function of the model contemplated in (Billington et al., 1983) incorporates stock holding and setup costs, as well as production costs relating to overtime and idle time.

Billington briefly considers one of the most important problems in the practical formulation of any of these models; that of setting the coefficient values of the objective function. Later, Segerstedt (1996a) considers a variant of the model and justifies why it is not to be put into practice by explicitly associating it with the user's incapacity to understand marginal costs.

3 MLCLSP and the extensions required to adapt it to reality

The commonest name with which to consider the mathematical model that simultaneously solves the materials and operations planning problem is the MLCLSP. Other authors ascertain that this is a mathematical version of the more general supply chain operations planning problem (de Kok and Fransoo, 2003), or they include other adjectives when defining it; for example, dynamic (Buschkühl et al., 2009).

The model representing the problem is a simple one, but solving it in a reasonable time has always proved a complex matter. Therefore, other authors like Stadtler (1996) have developed more sophisticated models which, using more constraints, help solve the mathematical programming model faster. Some authors like Pochet and Wolsey (2006) suggest that optimisation tools cannot tackle real problems. Nevertheless, increased computing capacity of late, and not just in computers, but also optimisation technology itself (Bixby and Rothberg, 2007), offers hope.

All in all, most works on the MLCLSP still assume that the BOM entails assembly products. A series of problem variants based on amending the structure of BOMs may also be found in both the practice and the literature. Some interesting ones are provided below.

3.1 Gozinto matrix and resources matrix

As previously mentioned, the conventional way of representing the BOM is the Gozinto matrix (goes into) A_{ij} in which products i (parent item) relate to products j (child items). The products structure is assumed to always be convergent. The values of matrix A_{ij} are normally positive integers. BOMs are, therefore, represented in denominated direct BOMs.

In association with each product *i*, the quantity of resource *r* required to produce a unit of product *i* by means of matrix U_{ir} is also constituted, and this structure was considered by Mize et al. (1979).

3.2 Alternative products and resources

The existence of alternative components was contemplated by, for example, the work of Escudero (1994), which offers obvious advantages thanks to the addition of both risks in components availability and demand (Balakrishnan and Geunes, 2000). This problem is sometimes called requirement planning with substitution (RPS) as, for example, in Lang (2010), which requires long computing times. Attempts have been made to overcome this computing cost problem by means of alternative and ever increasingly complex formulations (Geunes, 2003), even though the problem is not a multi-level one. Lang and Domschke (2010) proposed extending this problem by considering the limited constraint for one resource, or for many.

Ram et al. (2006) proposed an interesting variant, the so-called flexible BOM, in which the BOM depends on the availability of materials. However, these authors maintain that this concept cannot be applied to most production systems. Lin et al. (2009) suggests that the existence of alternative products could be the manufacturer's decision, basically as a result of product binning; however, it may seem inappropriate for certain clients who recognise the difference. In this way, the number of alternative products would grow in accordance with the alternative components employed. If as Balakrishnan and Geunes (2000) suggest the number of alternative elements which may be simultaneously considered for a given product is large, then the use of the phantom products concept is an interesting one: "Phantoms are items produced in the manufacturing process and thus are definable parent items, but they are not typically stocked" (Clement et al., 1995).

Lot-sizing problems, which include the suppliers' selection or multiple manufacturing alternatives, are also related with products substitution (Aissaoui et al., 2007) both in terms of considering them and how to solve them. Likewise, the lateral transshipments proposed by Tagaras (1999) are assimilable to products substitution.

One situation which, to the best of our knowledge, has not been modelled is that of considering different lead-times with different costs, but using the same resource; this would once again imply the use of alternative resources. Some authors suggest that the definition of lead time is exogenous to the problem (de Kok and Fransoo, 2003), and that it would be interesting to develop a costs model according to which a supplier could commit itself in a short time at a higher cost (with the same resources and the same resources utilisation).

3.3 Reverse BOM

Reverse BOMs are needed when a product gives way to two products or more through the transformation process. One of the reasons behind this is that the so-called co-products, or by-products, appear. Segerstedt (1996a) terms these structures 'divergent structures', and indicates that the way to model them is to assume that the a_{ij} value determines the amount of each *i* obtained from the transformation of *j*. This type of modelling is, however, very limited to specific kinds of divergent processes.

Another special type of problems with divergent structures occurs in the so-called reverse MRP (Gupta and Taleb, 1994). This problem does not consider products that are not assembled, but those that are disassembled or separated into pieces. These structures tend to be represented inversely to the conventional structure (Inderfurth and Langella, 2006). Spengler et al. (1997) introduced the phenomenon for the dismantling process with buildings, and consider different activity alternatives that generate varying amounts of finished products. These authors propose an MILP model; however they believed that the commercial software programmes available in 1997 could not solve it in a reasonable time. Apparently the reverse lot-sizing problem is considerably more complex than the direct lot-sizing problem (Barba-Gutierrez et al., 2008).

In general terms, direct BOMs are never mixed with reverse ones in mathematical problem modelling. Schutz et al. (2009) incorporates reverse BOMs along with direct ones and, in parallel, state that: "In addition, even when it comes to pure operational models, we do not know any alternative model that handles a combination of splitting processes and combining processes". In order to solve the problem of including them in the same model, two different matrices are established for each one. The direct ones are known as BOMs, while the reverse ones are called r-BOMs.

3.4 Several inputs and outputs in the same process

Indeed, processes can be found in which one operation involves disassembly and assembly (or processes that may be considered simultaneous). These simultaneous assembly and disassembly processes are quite usual in the chemical industry. Pantelides (1994) presents a bipartite graph called the state task network, which was later extended to the resource task network (Barbosa-Póvoa and Pantelides, 1997), and is widely used in scheduling-related works in the chemical industry.

Sousa et al. (2008) considered an integrated planning and scheduling model for a network of chemical products firms. Their proposal includes two solution stages using MILP models. Having considered the existence of this type of processes (with several inputs and outputs) in the chemical industry, the authors simplify them with a conventional Gozinto structure by contemplating their models which, incidentally, include transport between plants.

Co-production, a normal feature in the process industry (Crama et al., 2001), is not often considered in the discrete production theory. In general, the existence of co-products (or by-products) is generally considered 'non-deliberate', although it could well be 'deliberate' (Vidal-Carreras and Garcia-Sabater, 2009), in other words, a decision is made to co-produce two or more products simultaneously in the same operation.

This co-production problem may also vary, this being the aforementioned product binning problem (Lyon et al., 2001), where various product qualities were obtained during the operation, but always after analysing the result.

3.5 Transport between plants

Transport between plants has been considered by a number of authors, including Sousa et al. (2008) and Schutz et al. (2009). In general, the problem of incorporating new sites tends to be solved by including a new sub index with the variables. In any case, and as suggested by de Kok and Fransoo (2003), and by Pires et al. (2008) later, basically, a product at another site is just another product.

Caner Taskin and Tamer Ünal (2009) contemplates a planning model that simultaneously considers substitutable products, yield production, co-production and multisites. This work examines a multi-level problem, and the multisite concept is present, although sites eventually overlap. Thus, the problem in this work boils down to an alternative resources analysis (located at different sites).

Pires et al. (2008) work out the bill of materials and movements (BOMM) in a virtual enterprise (VE) setting. In fact, the proposal put forward by Carvalho et al. (2005) states that this structure is defined as the central piece of VEs' production and control planning systems. According to these authors, only one materials structure, which also includes products sites, will enable the coordination of the so-called autonomous production systems. This paper considers the materials and movements structure to be a dynamic entity, and proposes the IDEF0 diagrams of the processes to amend and maintain the BOMM throughout the VE's life. Moreover, this work does not consider coordination at all, but assumes that the proposed structure must be taken into account. A complementary problem appears with the alternative transports considered in, for example, Calderon-Lama et al. (2009).

3.6 Packaging

Should a product be packaged with different packaging, it might be defined as different entities; this fact is mentioned in Pinto et al. (2007). Furthermore, Voss and Woodruff (2005) consider stock-keeping units (SKUs) as the minimum unit to be planned. Caner et al. (2009) believes that a product packaged for one client is a different product if it is packaged for another client. Along these lines, changing packaging could be viewed as a substitution activity (Lang and Domschke, 2010). Thus, transferring one product between packaging must be considered another operation.

Our experience, based mainly in the automobile sector, reaches a higher level as it assumes that end product packaging are an input of the process and that, similarly, raw materials packaging become the output of the same operation. Having contemplated packaging, considering returnable packaging becomes unavoidable. However, returnable packaging pose a cyclic structure problem which, despite being habitual in the chemical industry (Scheer, 1994), is not usual in discrete manufacturing, which therefore poses problems in most approaches, as in Ball et al. (2003) and Sahling et al. (2009).

Once packaging has been considered to be different components of a specific product, the possibility of determining a transport plan for empty packages becomes an obvious option, if required.

The full truck load strategy may be adopted using the same argument in certain sectors that have imposed the use of complete packaging as a means of transport (Puig-Bernabeu et al., 2010). The use of complete packaging entails the appearance of over-deliveries (or negative backlogs); that is, those products delivered before they are required for complete packaging delivery.

4 Modelling the GMOP problem

4.1 Definition for the concept of 'stroke'

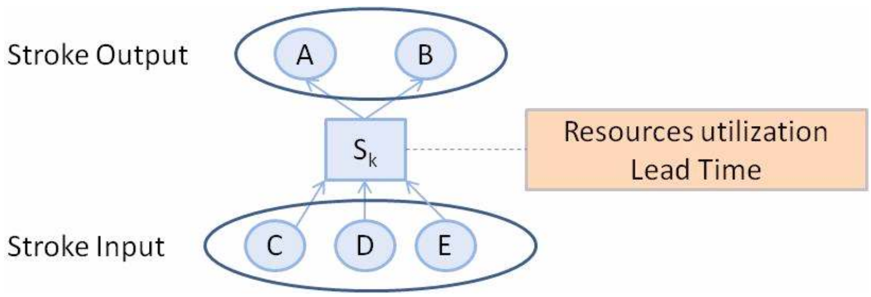
To consider this proposal, it is compulsory to specify some basic assumptions. Not only the place where products are stored should be considered, as proposed in Pires et al. (2008), but also the type of packaging to be used. The products contemplated in this approach should always be SKU which, within the frame of this work, are products defined with both their packaging and site. Such data can be ignored if there is no possibility or need to consider packaging or sites in a given problem.

The series of products that the operation input consists in will be known as 'stroke input (SI)'. Kitting is the name given by Jiao et al. (2000) to a very similar concept. The series of products of an operation output will be called 'stroke output (SO)'.

A stroke represents any operation that transforms (or transports) a series of products (measured as SKUs) into another series of products (also measured as SKUs). This operation and, therefore the stroke representing it, has an associated cost and lead time, and consumes a certain amount of resources during the first of the planning periods; however, this aspect could be reconsidered in accordance with the specific case. Figure 1 proposes a conceptual representation of a stroke. The due date in the proposed deterministic model is taken into account. Should it be a stochastic model, the approaches of Hnaien et al. (2008) could be employed to diminish the due date uncertainty problem.

Resources are associated with each stroke, but not with the product (or the series of products) obtained. In general terms, it is possible to obtain this data from the bill of routing (Tatsiopoulos, 1996).

Figure 1 Conceptual representation of a S_k stroke (see online version for colours)



4.2 Mathematical formulation of the MLCLS problem using strokes: the GMOP model

This section contemplates the mathematical formulation of the problem model, and the name put forward for this model is the GMOP Problem. To mathematically formulate the problem, it is necessary to define the nomenclature presented in Table 1.

Table 1 Nomenclature

Indices	
i	Index set of products (includes product, packaging and site)
t	Index set of planning periods
r	Index set of resources
k	Index set of strokes
Parameters	
$D_{i,t}$	Demand of product i for period t
$h_{i,t}$	Cost of storing a unit of product i in period t
$CO_{k,t}$	Cost of stroke k in period t
$CS_{k,t}$	Cost of the setup of stroke k in period t
$CB_{i,t}$	Cost of purchasing product i in period t
$SO_{i,k}$	Number of units i that generates a stroke k
$SI_{i,k}$	Number of units i that stroke k consumes
LT_k	Lead time of stroke k
$KAP_{r,t}$	Capacity availability of resource r in period t (in time units)
M	A sufficiently large number
$TO_{k,r}$	Capacity of the resource r required for performing one unit of stroke k (in time units)
$TS_{k,r}$	Capacity required of resource r for setup of stroke k (in time units)
Variables	
$z_{k,t}$	Amount of strokes k to be performed in period t
$\delta k,t$	= 1 if stroke k is performed in period t (0 otherwise)
$w_{i,t}$	Purchase quantity for product i in period t
$x_{i,t}$	Stock level of product i on hand at the end of period t

The linear GMOP programming model may be formulated as so:

$$Z : \min \sum_t \sum_i \left(h_{i,t} \cdot x_{i,t}\right) + \sum_t \sum_k \left(CS_{k,t} \cdot \delta_{k,t} + CO_{k,t} \cdot z_{k,t}\right) + \sum_t \sum_i \left(CB_{i,t} \cdot w_{i,t}\right) \tag{1}$$

Subject to:

$$x_{i,t} = x_{i,t-1} - D_{i,t} + w_{i,t} - \sum_k \left(SI_{i,k} \cdot z_{k,t}\right) + \sum_k \left(SO_{i,k} \cdot z_{k,t-LT_k}\right), \quad \forall i \text{ and } t, \tag{2}$$

$$z_{k,t} - M \cdot \delta_{k,t} \le 0, \qquad \forall k \text{ and } t, \tag{3}$$

$$\sum_k \left(TS_{k,r} \cdot \delta_{k,t}\right) + \sum_k \left(TO_{k,r} \cdot z_{k,t}\right) \le KAP_{r,t} \qquad \forall r \text{ and } t, \tag{4}$$

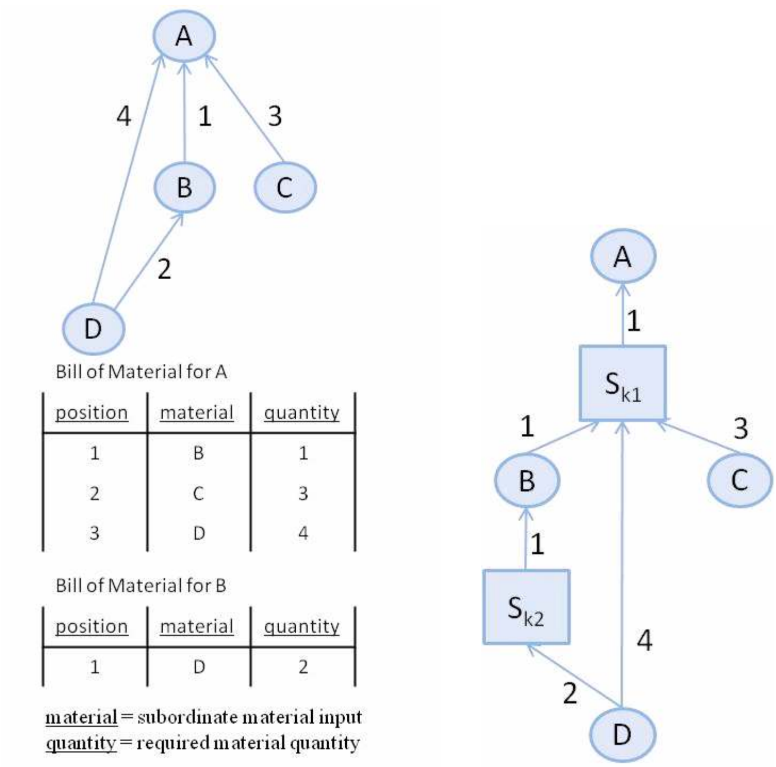
$$x_{i,t} \ge 0;\ w_{i,t} \ge 0 \qquad \forall i \text{ and } t, \tag{5}$$

$$z_{k,t} \in \mathbb{Z}^+;\ \delta_{\mathrm{k,t}} \in \{0,1\} \qquad \forall k \text{ and } t \tag{6}$$

The objective function (1) attempts to minimise the costs involved in storing and purchasing materials, and in performing operations by considering both setup and storage costs. Constraint (2) is a stock continuity constraint where that obtained by the planned strokes is added to the stock of the former period, with the associated lead-time, or it is compared externally and demand is deducted since this is what is consumed in the planned strokes for the considered time instant. Constraint (3) is introduced to know if stroke k is produced in t by employing the capacity associated with the setup (setup forcing). Constraint (4) is a capacity constraint that limits the use of resource r in period t by considering both setup and operations times. Constraints (5) and (6) define the range of variables.

For simplicity reasons, the following have not been incorporated: the initial level of stocks, planned receipts of goods, details about the lead time consideration [a similar application can be found in Clark and Armentano (1993)]. Moreover, other variants, such as the possibility of delays, over-deliveries, or the use of additional capacity, have not been specified.

Figure 2 Gozinto graph vs. stroke graph and BOM vs. stroke matrices, (a) bill of material and associated Gozinto graph (b) stroke graph (c) corresponding bill of operations and materials (d) strokes matrices $SI_{i,k}$ and $SO_{i,k}$ (see online version for colours)



(a) (b)

Figure 2 Gozinto graph vs. stroke graph and BOM vs. stroke matrices, (a) bill of material and associated Gozinto graph (b) stroke graph (c) corresponding bill of operations and materials (d) strokes matrices $SI_{i,k}$ and $SO_{i,k}$ (continued) (see online version for colours)

Stroke Matrix SO

Stroke	position	Material output	quantity
S_{k1}	1	A	1
S_{k2}	1	B	1

Stroke Matrix SI

Stroke	Position	Material input	quantity
S_{k1}	1	B	1
S_{k1}	2	C	3
S_{k1}	3	D	4
S_{k2}	1	D	2

(c)

$SI_{i,k}$	S_{k1}	S_{k2}
A	0	0
B	1	0
C	3	0
D	4	2

$SO_{i,k}$	S_{k1}	S_{k2}
A	1	0
B	0	1
C	0	0
D	0	0

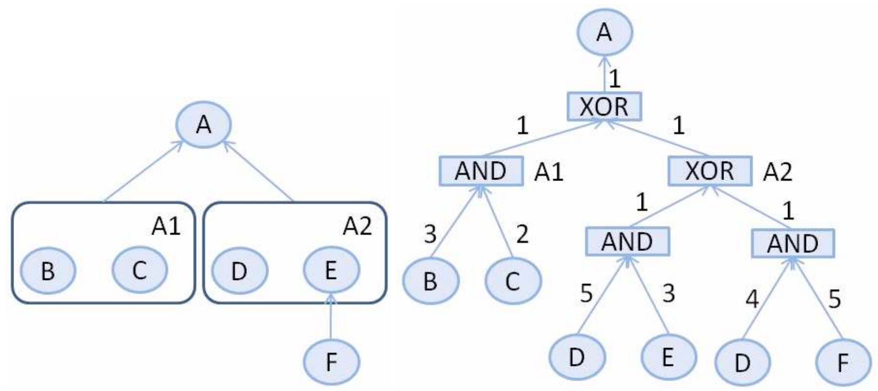
(d)

4.3 How does the proposal solve the extensions to materials and operations planning?

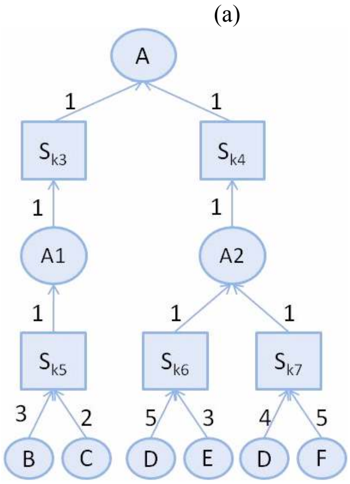
Next, the different transformations required between the classic Gozinto structures and the structures that the strokes use as a planning method are established. The so-called direct BOMs are conventional ones. One or several products give(s) way to a single product. In this case, the operation representing the stroke is of an assembly or transformation kind. Traditionally, this is the type of BOM that has been represented with a Gozinto graph which, in the proposed mathematical representation, requires two similarly sized matrices based on the BOM (Figure 2). To understand our proposal, the 'stroke graph' and the 'stroke matrices' associated with the same BOM are introduced.

The fact that there are substitution components does not amend the formal structure of the problem; it is merely a matter of creating an additional operation. The same structure applies to the existence of alternative operations, or even to different ways (with different costs) of doing the same operation. Thus, that which for Sahling et al. (2009) is a very useful future work for the case of parallel machines is actually included very simply in the representation. The structure conventionally employed to express products substitution is a substitution hypergraph (Lang, 2010). An example of a simple substitution hypergraph with six products, two assemblies and the corresponding Gozinto factors is depicted in Figure 3(a). The corresponding AND-XOR graph representation (Özturan, 2004) is also depicted in Figure 3(b). The corresponding stroke graph is proposed in Figure 3(c) and its associated matrices are presented in Figure 3(d). It is worth mentioning that we do not consider '*abstract products*' (a similar concept to phantom items) here, but we contemplate two alternative operations (see Sk_6 and Sk_7).

Figure 3 Representations of alternative operations with substitution products, (a) substitution hypergraph without abstract product (b) corresponding AND-XOR graph representation without abstract product (c) stroke graph with substitution and without abstract product (d) stroke matrices in case of products substitution and without abstract product (see online version for colours)



(a) (b)

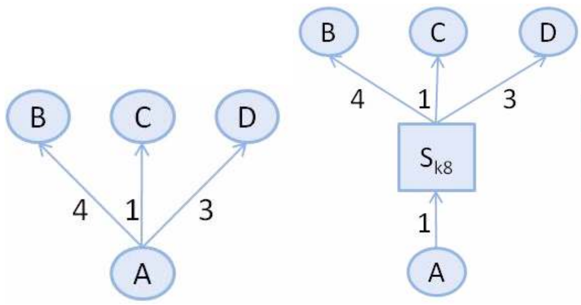


$SI_{i,k}$	S_{k3}	S_{k4}	S_{k5}	S_{k6}	S_{k7}
A	0	0	0	0	0
A1	1	0	0	0	0
A2	0	1	0	0	0
B	0	0	3	0	0
C	0	0	2	0	0
D	0	0	0	5	4
E	0	0	0	3	0
F	0	0	0	0	5

$SO_{i,k}$	S_{k3}	S_{k4}	S_{k5}	S_{k6}	S_{k7}
A	1	1	0	0	0
A1	0	0	1	0	0
A2	0	0	0	1	1
B	0	0	0	0	0
C	0	0	0	0	0
D	0	0	0	0	0
E	0	0	0	0	0
F	0	0	0	0	0

(c) (d)

Figure 4 Representation of the stroke graph and stroke matrices from the reverse or divergent BOMs, (a) divergent graph (b) stroke graph (c) corresponding stroke matrices (see online version for colours)



$SO_{i,k}$	S_{k8}
A	0
B	4
C	1
D	3

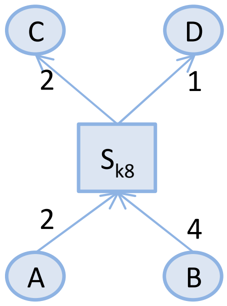
$SI_{i,k}$	S_{k8}
A	1
B	0
C	0
D	0

(a) (b) (c)

The so-called reverse or divergent structures may also be represented simply if the same concept is used (Figure 4).

Besides, the operations that generate two products simultaneously are represented in a similar way (Figure 5).

Figure 5 Representation of the stroke graph and stroke matrices from complex processes (transfers, transports, etc.), (a) stoke graph (b) corresponding stroke matrices (see online version for colours)



(a)

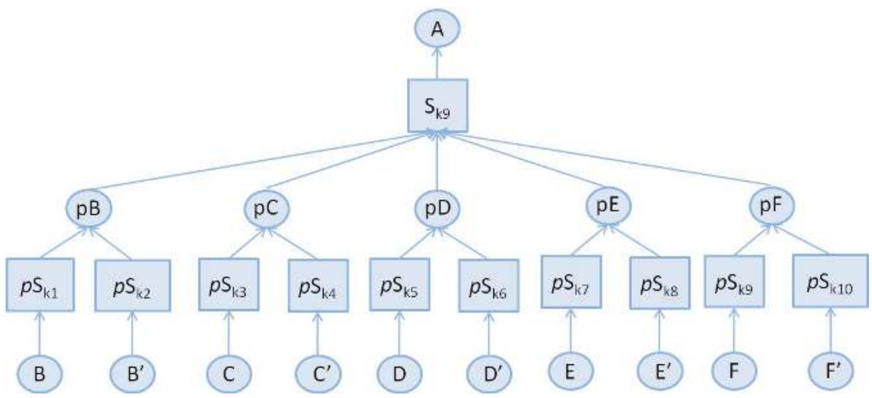
$SI_{i,k}$	S_{k9}
A	2
B	4
C	0
D	0

$SO_{i,k}$	S_{k9}
A	0
B	0
C	2
D	1

(b)

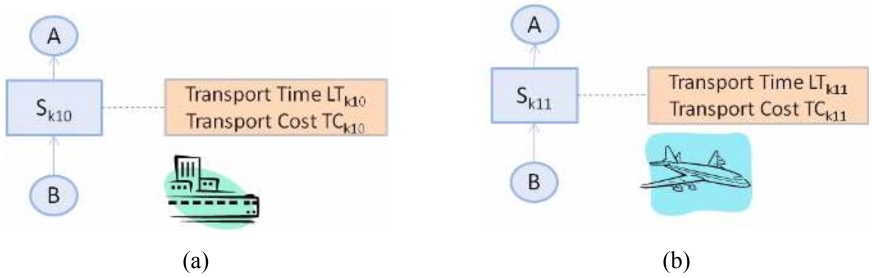
The fact that there are multiple substitution products (owing to, for instance, product binning) in a given step of the process, could pose a problem which involves excessive growth in the number of strokes required. To avoid this problem, the use of phantom items, '*pi*', and phantom strokes, 'pS_k', is advised. Phantom strokes neither consume resources nor have lead times, and phantom products are not stored. An example of a stroke graph with six products, five substitution products, five phantom items, ten phantom strokes and stroke factors assumed to be 1 is depicted in Figure 6.

Figure 6 A stroke graph with phantom items and phantom strokes to produce a single product (see online version for colours)



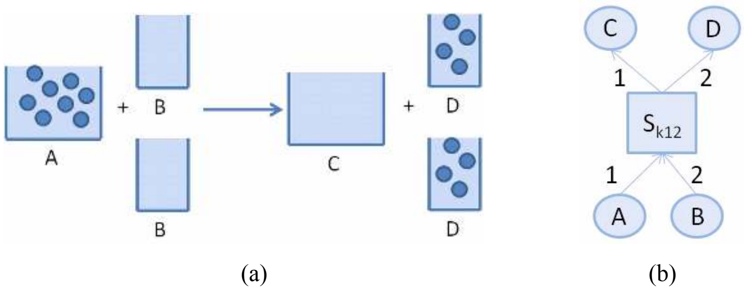
The problem structure needs no amendments if there are alternative forms of transports (with different costs and transit times), as shown in Figure 7.

Figure 7 Representation of stroke graphs showing alternative forms of transport, (a) stroke of transport by boat (b) stroke of transport by plane (see online version for colours)



Considering packaging (represented in Figure 8) as a necessary element to perform an operation, and occasionally as the element resulting from the same operation (empty packaging), does not entail having to amend the problem formulation.

Figure 8 Representation of the strokes of the operations involving change of packaging, (a) scheme about operations of (un)packaging (b) corresponding stroke graph (see online version for colours)



In general and as previously indicated, the proposal would involve products being identified not only by their individual characteristics, but by them bearing the associated packaging and site. In this way, transformation operations would entail product change, but not necessarily packaging or site; transport operations would entail site, but not necessarily product or packaging; finally, packaging change operations would entail neither product change nor site.

5 The GMOP in practice

5.1 A practical application: the Segura case

The intention here is to generate a production and operations plan for a network of firms which produces and assembles metal elements, basically for the automobile sector. As they are the global supplier of some parts, this entails having to send the same reference on different packaging types depending on whether they are to be returned or not, or if the destination installations have certain, more or less, automated processes.

Furthermore, some products involve more than seven processing stages (including several stamping, welding, chemical treatment, painting stages, etc.). Some stages are ‘convergent’ (welding or assembly), while others are ‘divergent’ (cutting); in a given case with four components, two different products are obtained. The structure consists in approximately 500 end products and some 2,500 intermediate products in any of its stages. Seven plants located in a radius of roughly 30 kilometres are considered. Products and semi-finished goods are transported inside complete containers. This means having to manage over-deliveries, delays in deliveries and movement of empty containers among plants, among other aspects.

The developed tool not only plans production operations, but also movement of materials and the packaging requirements in all seven plants. Budget limits did not allow the use of professional software to solve the real problem. Therefore, a multi-agent system-based heuristics is implemented which employ the stroke concept in (Garcia-Sabater et al, 2006). Appendix 1 presents a simplified example of the Segura problem for three end products (with variations among them) for two clients (with different quality requirements) at two sites.

5.2 A practical application: the engines case

The case described in (Garcia-Sabater et al., 2009) is another application where this form of modelling has been successfully used. Here attempts have been made to plan the assembly and transformation operations of an engine manufacturing plant with 40 engine derivatives, and with a similar number of components and mechanised raw material.

One important aspect of the system is that there are some components that are classified into two categories given their quality features. Some clients accept engines with components from both categories, while others do not. Seeing as there are 40 engines and that each engine has five different components, if attempts had been made to build a Gozinto matrix for each combination or way of producing engines, this would have resulted in 1,280 different engines in accordance with the components that may be produced. The use of phantom strokes and phantom components leads to a lower number of 80 different engines being produced, and also enables the inclusion of some components that act as substitution products. This problem not only handles complete packaging, but also fills trucks with packaging or considers sequence-dependent setup costs. Presently this tool is able to generate a feasible 42-day horizon plan by considering capacity constraints in just over ten minutes.

5.3 A preliminary analysis of the advantages and disadvantages of this proposal

The main advantage of this way of formulating the problem is that it represents materials and resources requirements planning in a compact, intelligible fashion as a result of the decision variable, this being the amount of operations (strokes) to be performed in each period. Another advantage is that it proves easier to incorporate alternative processes and products, and it enables the consideration of cyclic materials structures.

With the materials and resources structures of conventional ERP systems, with which attempts have been made to implement them (SAP, BaaN, MfgPro, Movex, etc.), it is reasonably simple to generate an application that converts data structures into the data structures required to implement the application. Furthermore, the presented formulation enables the data that is generally available to be used (as alternative routes), but which the MRP or the MRPII explosion does not generally consider. The fact that the tool in use at the time of implementation does not consider them actually poses an additional problem, that of the data being incorrect: 'as they are not used, no-one checks them'. A data control protocol needs to be prepared to avoid this problem.

Another advantage that the proposal offers is that it separates the availability of the materials from the operations employing them. Strokes enable other operations to be modelled; for instance, programmed maintenance procedures, to which a range of periods may be assigned in which they are to take place. It may also absorb the purchase process as a stroke with no raw material.

Perhaps the main problem encountered when introducing these structures is that when former constraints are released, the production department's 'wish list' is triggered, requiring a new and more difficult consideration.

If the complete packaging concept is in use, two fundamental difficulties emerge. The first entails the required incorporation of over-deliveries if orders are not contained in complete packaging units. The second involves the genuine existence of packaging fractions, which the system must somehow deal with. At the mathematical level, using this structure poses certain problems. The first is that the Gozinto matrix has always been used in the MLCLSP problem; thus the considered ways to solve the problem are concentrated in this representation, so there is not a handful of algorithms ready to be used.

On the other hand, it is obvious that the new form of representation could consume more memory for simple problems and use a higher number of variables than its conventional formulation. This larger memory consumption and the higher number of variables could imply longer computing times. Nevertheless, the GMOP model may be easily decomposed by separating sites and through unions by means of transport processes, thus allowing a simple heuristics to be done.

6 Conclusions and future research lines

A form of modelling the relationship between operations and the materials required to manufacture a product has been considered. This way of defining the relationships between operations and materials suggests a compact mathematical programming model to plan operations in a supply chain. Apart from capacity constraints, this GMOP model also takes into account: direct and reverse BOMs, multisite production, alternative products and resources, co-products, by-products and yields, transport – including alternative forms of transport – and packaging.

The literature relating to considering problems from both the mathematical and data structure viewpoints has been reviewed. Attempts have been made to define why BOMs and bills of resources were structured from the materials obtained. One suggestion is that this decision subsequently acted as a lock-in. The literature about multi-level lot-sizing problems has also been reviewed by acknowledging how different requirements have been suggested in data structure terms, and by discovering a significant increase in relation to the combination of characteristics in recent years.

The proposed structure has been verified to indeed support the variations analysed in the MLCLSP, which have been included in a single structure. Two cases to which the tool has been applied have been briefly described.

After accepting this modelling approach, many new research lines will open, and will have to be deployed in the near future. First and more relevant, although considerable work has been done to solve the classical production planning problem and its variants to optimality in a reasonable time [see, for example, the work done by Grubbström et al. (2010) during decades (Grubbström and Thu Thuy Huynh, 2006)], the new formulation presented herein requires changing and adapting the different methodologies.

The incorporation of variants into the demand and/or production parameters such as uncertainty (Mula et al., 2007, 2008) is another future research line.

The incorporation of the stroke concept for modelling and solving the distributed problem in a distributed way is a very interesting line. Adapting methods like those described in Dudek and Stadtler (2007) is something that should be done in the near future.

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Appendix

A simplified case study

Let us take a firm that sells three products (A, H, J) to two different clients (α and β). Product A is sold in two different formats: in disposable cardboard boxes (U) with 50 units and in returnable racks (V) with 100 units. Client α purchases product A in the disposable boxes format, whereas client β buys product A in returnable boxes with 100 units. The packaging of this product may change, if required. Disposable boxes cannot be reused, but returnable racks can. Product A is made by welding components B, C and D (the last of which needs two units). Component B is obtained from a stamping process using steel that comes as a spool named E. By stamping spool E, 5,000 units of B are obtained, which are kept in boxes-pallets (W) holding 250 units each. By using slightly different matrices and the same spool E, 2,500 B parts and 2,500 C parts may be obtained. Each box-pallet (W) holds 500 C parts. From a certain F spool, 2,000 D units are obtained. Unfortunately, the manufacturing process is not capable and produces 25% of the parts of inferior quality, which are called D'. Component D' cannot be employed to manufacture product A. Each pallet (W) holds 250 D units. The welding of one B unit, one G unit and one D unit produces product H. If we use component D', we obtain product H'. Component G is purchased directly and only the main plant (π) may purchase it.

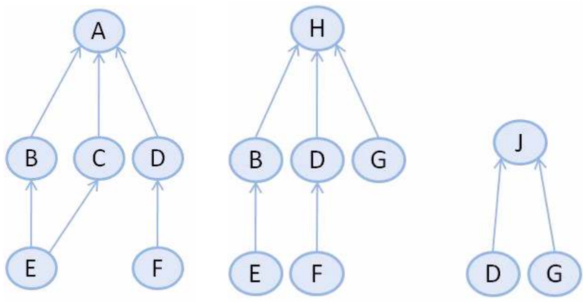
Product H is sold in disposable boxes (U) to both clients and each box (U) holds 125 units. However, while H' or H may be sold to client α, client β only accepts product H. Product J is manufactured by welding component D or D' with component G, and it makes no difference if this product is made with either of these components. Product J is sold in returnable racks (V) that hold 50 units.

The firm has plant π that works on stamping processes and another plant working on welding processes. The firm also uses subcontractor σ that welds, and this arrangement works out considerably less expensive for some operations. The product may be sent to clients from either the firm's main warehouse or the subcontractor. However, the subcontractor does not weld product D'.

There is a form of two-way transport between the firm and the subcontractor. Returnable packaging can be returned to the firm's installations from clients at a given cost, or can be acquired at a different cost. To simplify the analysis of the results, a very important capacity constraint has been incorporated into the transport of product G between the main plant and the subcontractor. Random setup and operation costs have been assigned to each stroke. The cost of storage is equivalent for all the parties, except for the empty packaging with the client, thus allowing them to be immediately returned. To simplify the results analysis, all the operations have a lead time of two time units.

Figure 9 represents the BOM. For simplicity reasons, neither the problems relating to the second qualities associated with the existence of D' nor the different packaging in which A may be served have been represented.

Figure 9 Representation of the BOM, (a) BOM of A (b) BOM of H (see online version for colours)



(a) (b)

Figure 9 represents the movement of materials among installations. The route from the facilities of clients α and β and the π facilities refers to empty packaging.

Figure 10 Representation of the distribution network structure (see online version for colours)

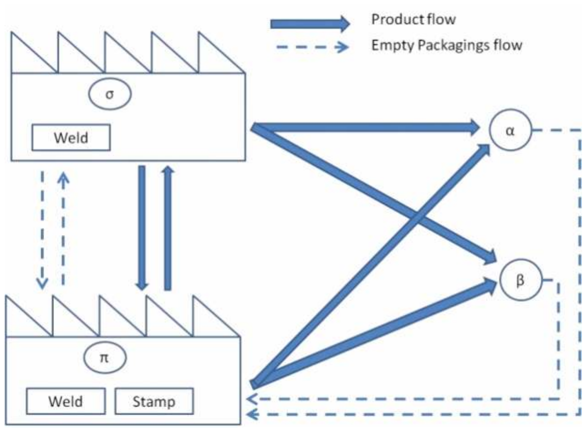


Table 2 presents a list of the different products considered (reference + site + packaging). They have been coded using three characters: the first refers to the product (an underscore indicates empty packaging), the second indicates site (*α*, *β*, *π*, *σ*), while the third represents the packaging type used (U, V and W, and an underscore indicates that there is no packaging or that packaging is irrelevant).

Table 2 Coding products (reference + site + packaging)

Packaging	_πU	_πV	_πW	_σU	_σV	_σW	_αV	_βV					
Raw material	Eπ_	Fπ_	Gπ_	Gσ_									
Semi-finished products	BπW	BσW	Cπ W	CσW	DπW	D’π w	Dσw						
End products (origin)	AπU	AπV	AσU	AσV	HπU	H’π U	HσU	JπV	JσV	Jα_	Jαv	Jβ_	JβV
End products (destination)	Aα_	AαU	Aβ	AβV	Hα_	HαU	H’αU	Hβ_	HβU				

As shown in Table 3, a problem with the demand of a few products in a few periods has been designed to analyse how the model performs (neither the initial level of stocks nor planned receptions have been simultaneously introduced). Since each stage is considered to have two lead time days and some processes have five stages, demand has been left empty until period ten.

Table 3 Demand for end products

$D_{i,t}$	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Aα_	0	0	0	0	0	0	0	0	0	1,000	0	1,000	0	1,000	0
Aβ_	0	0	0	0	0	0	0	0	0	0	2,000	0	2,000	0	0
Hα_	0	0	0	0	0	0	0	0	0	500	0	2,000	0	6,000	0
Hβ_	0	0	0	0	0	0	0	0	0	0	3,000	0	3,000	0	0
Jα_	0	0	0	0	0	0	0	0	0	3,000	0	3,000	0	3,000	0
Jβ_	0	0	0	0	0	0	0	0	0	0	500	0	500	500	0

Although SI and SO are different series, it is useful to represent them in a combined manner as a single matrix *S*, where $S = SO - SI$, which allows each stroke to be analysed in a more compact manner. Table 4 represents the strokes performed in plant *π*. Similarly, the strokes performed in *σ* are represented in Table 4. Tables 6 and 7 respectively represent transport strokes and transformation strokes.

Table 4 Matrix *S* for the strokes performed in *π*

	Unpacks AπU into AπV	*Unpacks AπV into AπU*	*Stamps E into B*	*Stamps E into B and C*	*Stamps F into D and D'*	*Welds AπU*	*Welds AπV*	*Welds HπU*	*Welds H'πU*	*WeldsHsU*	*Welds JπV with D*	*Welds JπV with D'*
_πU	0	–2	0	0	0	–10	0	–2	–2	0	0	0
_πV	–1	1	0	0	0	0	–5	0	0	0	–5	–5
_πW	0	0	–20	–15	–8	4	4	2	2	0	1	1
AπU	–2	2	0	0	0	10	0	0	0	0	0	0
AπV	1	–1	0	0	0	0	5	0	0	0	0	0
CπW	0	0	0	5	0	–1	–1	0	0	0	0	0
DπW	0	0	0	0	6	–4	–4	–1	0	0	–1	0
D'πW	0	0	0	0	2	0	0	0	–1	0	0	–1
Eπ_	0	0	–1	–1	0	0	0	0	0	0	0	0
Fπ_	0	0	0	0	–1	0	0	0	0	0	0	0
Gπ_	0	0	0	0	0	0	0	–250	–250	0	–250	–250
HπU	0	0	0	0	0	0	0	2	0	0	0	0
H'πU	0	0	0	0	0	0	0	0	2	0	0	0
JπV	0	0	0	0	0	0	0	0	0	0	5	5

Table 5 Matrix *S* for the strokes performed in *σ*

	Welds AσU	*Welds AσV*	*Welds HσU*	*Welds JσV with D*
_sU	–10	0	–2	0
_sV	0	–5	0	–5
_sW	4	4	2	1
AsU	10	0	0	0
AsV	0	5	0	0
BsW	–2	–2	–1	0
CsW	–1	–1	0	0
DsW	–4	–4	–1	–1
Gs_	0	0	–250	–250
HsU	0	0	2	0
JsV	0	0	0	5

Table 6 Matrix *S* for the transport strokes

	_πV	_πW	_σV	_σW	_αV	_βV	AπU	AπV	AσU	AσV	AαU	AβV	BπW	BσW	CπW	CσW	DπW	DσW	Gπ_	Gσ_	HπU	H'πU	HσU	HαU	HβU	JπV	JσV	JαV	JβV
Transports _σW to _πW	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports _πV to _σV	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports _πW to _σW	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports _αV to _πV	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports _βV to _πV	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports BπW to BσW	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports CπW to CσW	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports AπU to AαU	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports AπV to AβV	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports AσU to AαU	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports AσV to AβV	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports DπW to DσW	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0
Transports Gπ_ to Gσ_	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0
Transports HπU to HαU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0
Transports H'πU to HαU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0
Transports HπU to HβU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0
Transports HσU to HβU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0
Transports JπV to JαV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0
Transports JπV to JβV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1
Transports JσV to JαV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0
Transports JσV to JβV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1

Table 7 Matrix *S* for the transformation strokes

	Consumes Aα_ from AαU	*Consumes Aβ_ from AβV*	*Consumes Hα_ from HαU*	*Consumes Hα_ from H'αU*	*Consumes Hβ_ from HβU*	*Consumes Jα_ from JαV*	*Consumes Jβ_ from JβV*
_αV	0	0	0	0	0	1	0
_βV	0	1	0	0	0	0	1
Aα_	50	0	0	0	0	0	0
AαU	–1	0	0	0	0	0	0
Aβ_	0	100	0	0	0	0	0
AβV	0	–1	0	0	0	0	0
Hα_	0	0	125	125	0	0	0
HαU	0	0	–1	0	0	0	0
H'αU	0	0	0	–1	0	0	0
Hβ_	0	0	0	0	125	0	0
HβU	0	0	0	0	–1	0	0
Jα_	0	0	0	0	0	50	0
JαV	0	0	0	0	0	–1	0
Jβ_	0	0	0	0	0	0	50
JβV	0	0	0	0	0	0	–1

After executing the model (which, in this case, takes tenths of a second with Gurobi Optimizer 4.5), all the operations that must be done are obtained, including the transport of components, end products and even empty packagings, as represented in the next table.

Table 8 Planned strokes

$z_{k,t}$	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Consumes Aα_ from AαU	0	0	0	0	0	0	0	20	0	20	0	20	0	0	0
Consumes Aβ_ from AβV	0	0	0	0	0	0	0	0	20	0	20	0	0	0	0
Consumes Hα_ from HαU	0	0	0	0	0	0	0	4	0	16	0	48	0	0	0
Consumes Hα_ from H'αU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Consumes Hβ_ from HβU	0	0	0	0	0	0	0	0	24	0	24	0	0	0	0
Consumes Jα_ from JαV	0	0	0	0	0	0	0	60	0	60	0	60	0	0	0

Table 8 Planned strokes (continued)

$z_{k,t}$	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Consumes Jβ_ from JβV	0	0	0	0	0	0	0	0	10	0	20	0	0	0	0
Unpacks AπU into AπV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Unpacks AπV into AπU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Stamps E into B	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0
Stamps E into BβC	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Stamps F into DβD′	15	0	0	0	5	0	0	0	0	0	0	0	0	0	0
Welds AπU	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
Welds AπV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WeldsAσU	0	0	0	0	0	2	0	2	0	0	0	0	0	0	0
WeldsAσV	0	0	0	0	4	0	4	0	0	0	0	0	0	0	0
Welds HπU	0	0	14	0	0	0	26	0	0	0	0	0	0	0	0
Welds H'πU	0	0	8	0	0	0	10	0	0	0	0	0	0	0	0
WeldsHσU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Welds JπV with D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Welds JπV with D′	0	0	22	0	0	0	0	0	0	0	0	0	0	0	0
WeldsJσV with D	0	0	0	0	8	4	4	4	0	0	0	0	0	0	0
Transports _σW to _πW	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports _πV to _σV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports _πW to _σW	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports _αV to _πV	0	0	0	0	0	0	0	0	0	60	0	60	0	60	0
Transports _βV to _πV	0	0	0	0	0	0	0	0	0	0	30	0	40	0	0
Transports BπW to BσW	0	0	8	4	12	0	0	0	0	0	0	0	0	0	0
Transports CπW to CσW	0	0	4	2	6	0	0	0	0	0	0	0	0	0	0
Transports AπU to AαU	0	0	0	0	0	20	0	0	0	0	0	0	0	0	0
Transports AπV to AβV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 8 Planned strokes (continued)

$z_{k,t}$	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Transports AσU to AαU	0	0	0	0	0	0	0	20	0	20	0	0	0	0	0
Transports AσV to AβV	0	0	0	0	0	0	20	0	20	0	0	0	0	0	0
Transports DπW to DσW	0	0	24	44	0	0	0	0	0	0	0	0	0	0	0
Transports Gπ_ to Gσ_	0	1,000	1,000	1,000	1,000	1,000	0	0	0	0	0	0	0	0	0
Transports HπU to HαU	0	0	0	0	0	0	0	4	0	28	0	0	0	0	0
Transports H'πU to HαU	0	0	0	0	0	16	0	0	0	20	0	0	0	0	0
Transports HπU to HβU	0	0	0	0	24	0	0	0	24	0	0	0	0	0	0
Transports HσU to HβU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Transports JπV to JαV	0	0	0	0	0	100	0	0	0	0	0	0	0	0	0
Transports JπV to JβV	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0
Transports JσV to JαV	0	0	0	0	0	0	20	0	0	60	0	0	0	0	0
Transports JσV to JβV	0	0	0	0	0	0	0	0	20	0	0	0	0	0	0