A New Hot-Tearing Criterion

M. RAPPAZ, J.-M. DREZET, and M. GREMAUD

A new criterion for the appearance of hot tears in metallic alloys is proposed. Based upon a mass balance performed over the liquid and solid phases, it accounts for the tensile deformation of the solid skeleton perpendicular to the growing dendrites and for the induced interdendritic liquid feeding. This model introduces a critical deformation rate ($\dot{\varepsilon}_{p,\max}$) beyond which cavitation, *i.e.*, nucleation of a first void, occurs. As should be expected, this critical value is an increasing function of the thermal gradient and permeability and a decreasing function of the viscosity. The shrinkage contribution, which is also included in the model, is shown to be of the same order of magnitude as that associated with the tensile deformation of the solid skeleton. A hot-cracking sensitivity (HCS) index is then defined as $\dot{\varepsilon}_{p,max}^{-1}$. When applied to a variable-concentration aluminum-copper alloy, this HCS criterion can reproduce the typical ''Λ curves'' previously deduced by Clyne and Davies on a phenomenological basis. The calculated values are in fairly good agreement with those obtained experimentally by Spittle and Cushway for a non-grain-refined alloy. A comparison of this criterion to hot cracks observed in ring-mold solidification tests indicates cavitation depression of a few kilo Pascal and tensile stresses in the coherent mushy zone of a few mega Pascal. These values are discussed in terms of those obtained by other means (coherency measurement, microporosity observation, and simulation). Even though this HCS criterion is based only upon the appearance of a first void and not on its propagation, it sets up for the first time a physically sound basis for the study of hot-crack formation.

I. INTRODUCTION

DURING solidification, a metal experiences temperature differences which induce convection in the liquid region and deformation in the solid. The first phenomenon involves very large displacements but low stresses, whereas the opposite occurs for the second one. This drastically different behavior is due to the very large change of viscosity of a metal when it undergoes the liquid-to-solid transition (twenty orders of magnitude or more[1]). The transition is made more complicated for an alloy because the presence of the mushy zone somehow ''mixes'' the two behaviors: the deformation of the dendritic network strongly depends on its coherency state and the flow of liquid now occurs in a porous solid phase.

Two major defects related to a lack of feeding can be encountered in alloys: porosity and hot tears. As pointed out clearly by Campbell,[2] the first defect is associated with a hydrostatic depression in the mushy zone combined with segregation of gaseous solute elements (hydrogen, nitrogen, and carbon monoxide). This depression is associated with the suction of the liquid in the porous dendritic region due to shrinkage. The models developed for the prediction of microporosity formation are, therefore, based on the solution of the Darcy equation coupled with a mass balance and a microsegregation model of gaseous elements.[2–5] The formation of hot tears is also linked to a lack of feeding in the mushy zone, but only for specific regions where the dendritic network is submitted to shear or tensile stresses.[2,6–14] These stresses are induced by differential thermal contraction upon cooling. When the dendritic network is coherent,* it can sustain and, as a matter of fact, also transmit stresses. Above the coherency temperature, liquid still present in between the dendrites is continuous, since the solid dendrite arms have not yet coalesced. Deformation induced by thermal stresses can, therefore, pull these arms apart quite easily. If the interdendritic liquid flow can feed such regions, almost nothing is noticed, except maybe some local inverse segregation (''healed'' hot tears).[2] However, deep in the mushy zone, where the permeability of the mush is very small, an opening of the non-coherent dendritic network by tensile deformation cannot be compensated for by the liquid, and hot tears form.

Due to the complexity of the mechanisms involved in hot-tearing formation, the models developed so far are relatively simple. Most of them are based upon the consideration of the solidification interval:[2] the larger the solidification interval of the alloy, the more sensitive it is to hot tearing. Using a lever-rule approximation, a binary alloy exhibits a maximum hot-cracking sensitivity (HCS) for a nominal composition equal to that of the maximum solubility of the solid. With a Scheil approximation, the eutectic temperature is always reached and, thus, the HCS is a monotonically decreasing function of the composition. The most sophisticated models use a back-diffusion

*Coherency of the dendritic network is related to the links between the fragments of solid. The notion of coherency is a function of the deformation mode (shearing, compression, or tension). Therefore, the coherency temperature, *i.e.*, the temperature at which coherency is reached, strongly depends upon the mechanical test being used. For some authors,[15] coherency, as measured by a shearing torque device, is attained when the dendritic mush fills the space. Other authors have used an indentation test to determine coherency, *i.e.*, mainly by compression.[16] In the present case, the word ''coherency'' is used for a dendritic network that can oppose some mechanical resistance to tensile stresses. It has been measured, for example, by Ackermann *et al.*[17] using an *in situ* tensile experimental device plunged into a molten alloy.

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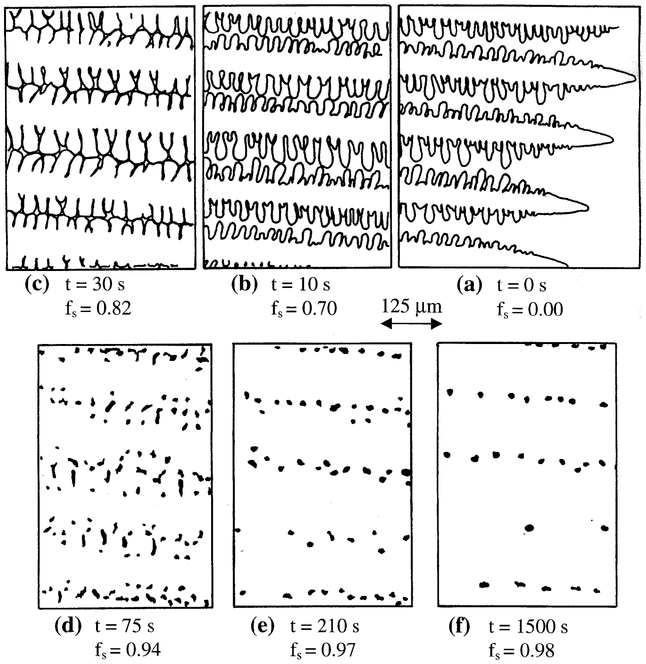

Fig. 1—(*a*) through (*f*) Microscopy observations of a succinonitrile-acetone alloy, which has been directionally solidified in a Bridgman-type apparatus (redrawn from a micrograph of Kurz[18]).

model,[1] for which the maximum HCS is dictated by the Fourier coefficient in the solid phase. Clyne and Davies[6] have recognized that hot cracking is due to an opening of the mushy zone in a ''vulnerable'' region where the dendrite arms can be pulled apart easily. They introduced a HCS criterion equal to t_v/t_r, where t_v is the time spent by the mushy zone in the vulnerable region and t_r is a normalization time during which stresses in the mushy zone can be relaxed. These authors have chosen t_v as the time difference separating the instants at which a volume fraction of solid of 0.9 and 0.99 is reached. Using a totally different approach, Feurer[8] has focused mainly on the feeding of the mushy zone: similarly to what has been done for microporosity formation, he has calculated the depression in the mushy zone associated with shrinkage. As pointed out by Campbell,[2] feeding is also important in hot-tearing formation, but the driving mechanism is uniaxial tensile deformation and not hydrostatic depression due to shrinkage.

In the present contribution, a simple two-phase model is derived for the formation of hot tears. It tries to combine the deformation of the coherent solid network, feeding of the mushy zone, and a criterion at which initiation of a hot tear in the interdendritic liquid region occurs.

II. CRITICAL REGION OF THE MUSHY ZONE

In order to illustrate the problem of hot-cracking formation, the aspect of a mushy zone of a succinonitrile-acetone organic alloy is shown in Figure 1.[18] This alloy was directionally solidified between two glass plates using a Bridgman-type experiment,[19] and the formation of the dendrites was observed by transmission light microscopy at various stages. The time indicated at the top of each figure has been calculated from the relationship $(x^* - x)/v_T$, where v_T is the solidification rate or the speed of the isotherms, x^* is the dendrite tip position, and x is the location of the figure. For each figure, the volume fraction of solid (f_s) has been estimated by visual inspection. Various stages can be distinguished. In Figure 1(a), the dendritic network is very open and the interdendritic liquid can flow without difficulty (this is the region of importance for convection-induced macrosegregation). The fraction of solid rapidly increases due to microsegregation (Figure 1(b)), but the dendrite arms have not yet coalesced or bridged, even at $f_s = 0.82$ (Figure 1(c)). At much longer times and, thus, deeper in the mushy zone,* the volume fraction of solid has increased only slightly, but isolated pockets of liquid now remain in between dendrite arms which have coalesced (Figures 1(d) through (f)). This region of the mush can resist and deform plastically when subject to tensile stresses.

*The succinonitrile-acetone system does not form a eutectic.

As pointed out by Clyne and Davies,[6] the critical region for hot tearing in an alloy corresponds to the zone of Figure 1(c): in this region, the film of interdendritic liquid is continuous and can open easily if thermal stresses are induced and transmitted by the coherent mush located underneath. The only resistance this film can oppose is the pressure of ''cavitation,'' *i.e.*, the pressure at which nucleation of a first void, possibly leading to a crack, will start. On the other hand, any opening of this continuous interdendritic liquid film in this zone can hardly be compensated for by feeding from the upper region of the mush, because of the high volume fraction of solid (*i.e.*, low permeability).

In the following section, a simple hot-tearing criterion based upon deformation of the mush and liquid feeding is derived.

III. HOT-TEARING MODEL

Figure 2 is a schematic diagram of the columnar dendritic growth seen in Figure 1. The dendrites are assumed to grow in a Bridgman-type configuration, *i.e.*, in a given thermal gradient (G) and with a velocity (v_T) equal to the speed of the liquidus isotherm. This velocity points toward the right and, therefore, the liquid has to flow from right to left in order to compensate for shrinkage, the specific mass of the solid being larger than that of the liquid for most metallic alloys. If the dendritic network is submitted to a tensile deformation rate perpendicular to the growth direction ($\dot{\varepsilon}_p$), the flow should also compensate for that deformation if no hot tears form. The pressure in the interdendritic liquid is schematically represented at the bottom of Figure 2: it decreases from the metallostatic pressure (p_m) near the dendrite tips. If the pressure falls below a cavitation pressure (p_c), a void may form (black region in Figure 2) and give rise to a crack. Therefore, a hot tear will form at the critical pressure ($p_{\min}$),

$$p_{\min} = p_m - \Delta p_\varepsilon - \Delta p_{sh} = p_c \quad \text{or} \qquad [1]$$
$$\Delta p_{\max} = \Delta p_\varepsilon + \Delta p_{sh} = \Delta p_c = p_m - p_c$$

The terms Δp_ε and Δp_{sh} are the pressure drop contributions associated with deformation and shrinkage, respectively (taken as positive values). In order to calculate these two

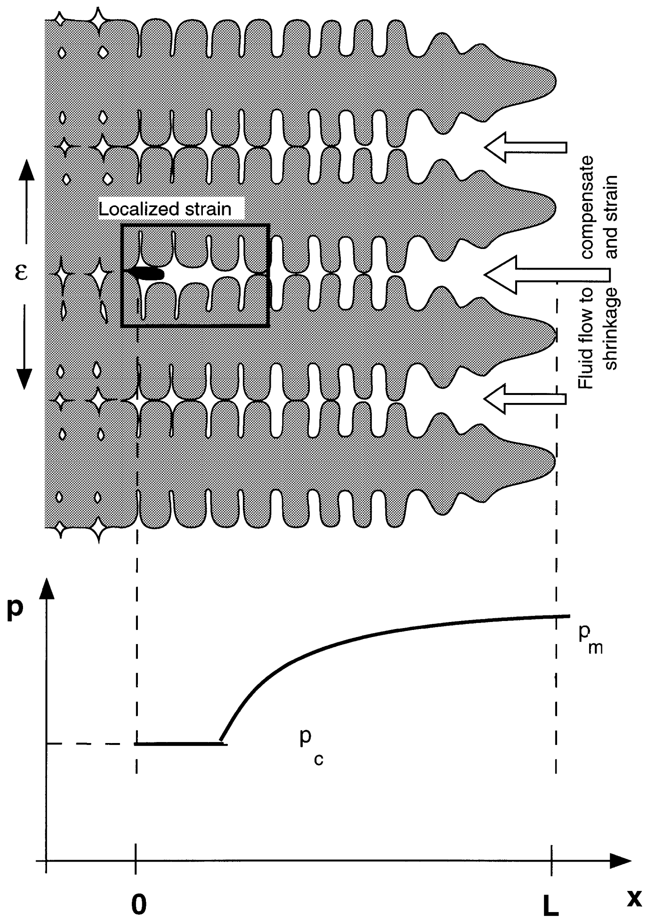

Fig. 2—Schematics of the formation of a hot tear in between columnar dendrites as a result of a localized strain transmitted by the coherent dendrites below. The pressure in the interdendritic liquid is also indicated.

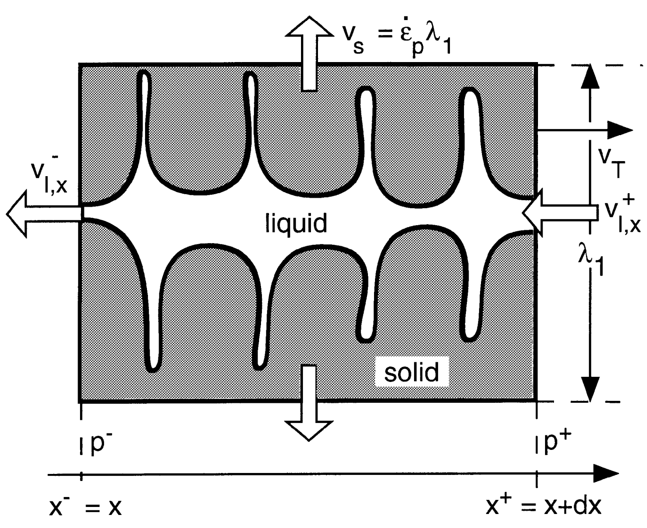

Fig. 3—Schematics of the mass balance performed at the scale of a small volume element over which liquid can enter through the vertical faces in order to compensate shrinkage and uniaxial deformation of the solid skeleton along the vertical direction.

contributions, a mass balance is performed at the scale of a small volume element shown in Figure 3. In a reference frame attached to the isotherms and under steady-state conditions, this mass balance can be written as

$$\mathrm{div}\, \langle \rho v \rangle - v_T \frac{\partial \langle \rho \rangle}{\partial x} = 0 \qquad [2]$$

where the notation "$\langle . \rangle$" is used to indicate values locally averaged over the liquid and solid phases.[20] The average specific mass $\langle \rho \rangle = \rho_s f_s + \rho_l f_l$ is the mean specific mass of the solid and liquid phases, and $\langle \rho v \rangle = \rho_s f_s v_s + \rho_l f_l v_l$ is the average mass flow. The volume fraction of liquid (f_l) is equal to $(1 - f_s)$, and the specific masses of the two phases (ρ_s and ρ_l) are assumed to be constant, but not equal. Considering that the fluid moves along the x-axis only, whereas the solid deforms in the transverse direction, one has

$$\frac{\partial(\rho_l f_l v_{l,x})}{\partial x} + \frac{\partial(\rho_s f_s v_{s,y})}{\partial y} - v_T \left[\frac{\partial(\rho_s f_s)}{\partial x} + \frac{\partial(\rho_l f_l)}{\partial x} \right] = 0 \qquad [3]$$

Taking f_s as a function of x only, Eq. [3] can be rewritten in the form

$$\frac{d\,(f_l v_{l,x})}{dx} + (1 + \beta) f_s \dot{\varepsilon}_p - v_T \beta \frac{df_s}{dx} = 0 \qquad [4]$$

where the deformation rate of the solid along the y-direction ($\dot{\varepsilon}_p = \frac{\partial v_{s,y}}{\partial y}$) has been introduced, together with the shrinkage factor $\beta = \frac{\rho_s}{\rho_l} - 1$ ($\beta > 0$). The integration of Eq. [4] over the distance x gives

$$f_l v_{l,x} + (1 + \beta) \int f_s \dot{\varepsilon}_p\, dx - v_T \beta f_s = f_l v_{l,x} + (1 + \beta) E(x) - v_T \beta f_s = C \qquad [5]$$

The entity $E(x)$ is the deformation rate times the volume fraction of solid, cumulated over the distance x of the mushy zone. The integration constant C, appearing in Eq. [5], can be determined from the velocity of the fluid at $x = L$, where L is the length of the mushy zone (Figure 2). Indeed, at the tip of the dendrites (*i.e.*, $f_l = 1$), the velocity of the fluid ($[v_{l,x}]_{x=L}$) must compensate for the shrinkage and the cumulated deformation of the whole mushy zone if no void forms. The shrinkage of the whole mushy zone, given by the mass conservation equation, is identical to that of a planar front and equals $-v_T\beta$. The cumulated deformation within the mushy zone is equal to $-(1 + \beta)\, E(L)$. Introducing these two expressions in Eq. [5] gives the following condition for the constant C:

$$[v_{l,x}]_{x=L} = -v_T\beta - (1 + \beta)\, E(L) = -(1 + \beta)\, E(L) + C \quad \text{or} \quad C = -v_T\beta \qquad [6]$$

Replacing, then, C* in Eq. [5] finally gives the velocity of the liquid at any point of the mushy zone;

$$f_l v_{l,x} = -(1 + \beta)\, E(x) - v_T \beta f_l \qquad [7]$$

*Please note that this value of C is consistent with the boundary conditions imposed at the roots of the dendrites: at $f_s = 1$, $v_{l,x} = 0$, and $E = 0$. And, thus, $C = -v_T\beta$. However, the presence of eutectic near $f_s = 1$ might make this condition more complicated. This is why C has been deduced from the boundary condition at the tip of the dendrites.

In the absence of deformation ($E = 0$), it is interesting to note that the actual velocity of the fluid ($v_{l,x}$) is constant and equal to $-v_T\beta$ at any point of the mushy zone. The left-hand-side term of Eq. [7] can be related to the pressure gradient in the liquid *via* the Darcy equation,[3–5,8]

$$f_l\, v_{l,x} = -\frac{K}{\mu}\frac{dp}{dx} \qquad [8]$$

where K is the permeability of the mushy zone and μ is the viscosity of the liquid. Please note that the contribution of gravity has been neglected in this equation. Combining Eqs. [7] and [8] and integrating over the whole length of the mushy zone finally gives the pressure drop between the tips and roots of the dendrites,

$$\begin{aligned}\Delta p_{\max} &= \Delta p_\varepsilon + \Delta p_{sh} = p_L - p_0 \\ &= (1+\beta)\mu \int_0^L \frac{E}{K}\,dx + v_T\,\beta\mu \int_0^L \frac{f_l}{K}\,dx\end{aligned} \qquad [9]$$

$$\text{with } E(x) = \int f_s \dot{\varepsilon}_p\, dx \qquad [10]$$

The first term on the right-hand side of Eq. [9] is the contribution to the pressure drop associated with the deformation of the solid skeleton, whereas the second one is the shrinkage contribution also found in microporosity models.[3,4,5] Please note that the permeability, is also a function of x or f_s. Using the Carman–Kozeny approximation,[3,4,5]

$$K = \frac{\lambda_2^2}{180}\frac{(1-f_s)^3}{f_s^2} \qquad [11]$$

where λ_2 is the secondary dendrite arm spacing, one finally gets

$$\begin{aligned}\Delta p_{\max} = p_L - p_0 = \\ \frac{180}{\lambda_2^2}\frac{(1+\beta)\mu}{G}\int_{T_S}^{T_L}\frac{E(T)\,f_s(T)^2}{(1-f_s(T))^3}\,dT \\ + \frac{180}{\lambda_2^2}\frac{v_T\,\beta\mu}{G}\int_{T_S}^{T_L}\frac{f_s(T)^2}{(1-f_s(T))^2}\,dT\end{aligned} \qquad [12]$$

Please note that the integrals over x have been replaced by integrals over the temperature, thus introducing the temperature gradient, G, f_s and E being two functions of T. The terms T_L and T_S are the liquidus temperature and the temperature at the end of solidification, respectively. As can be seen, the contribution of shrinkage to the pressure is proportional to the ratio v_T/G, a factor which was already deduced by Niyama *et al.*[21] for his criterion of microporosity formation.* Furthermore, the volume fraction of solid has been assumed to be related only to the temperature field, even though it can depend also on the cooling rate due to back-diffusion.[1] The expression used for the relationship $f_s(T)$ in Eq. [13] is due to Kurz and Fisher:[1]

$$f_s(T) = \frac{1}{1-2\alpha_s^* k}\left[1 - \left[\frac{T_m - T}{T_m - T_L}\right]^{\frac{1-2\alpha_s^* k}{k-1}}\right] \qquad [13]$$

where k is the partition coefficient, T_m is the melting point of the pure metal, and α_s^* is related to the Fourier number α_s *via* the relationship

$$\begin{aligned}\alpha_s^* = \alpha_s\,[1 - \exp(-\alpha_s^{-1})] - 0.5\exp(-0.5\,\alpha_s^{-1}) \\ \text{with } \alpha_s = D_s t_f/\lambda_2^2\end{aligned} \qquad [14]$$

where D_s is the diffusion coefficient in the solid that t_f is the solidification time.

In aluminum alloys, the coefficient α_s is small (0.01 to 0.03) and $\alpha_s^* \approx \alpha_s$ (*i.e.*, a situation close to a Scheil-type model). Under these conditions, the model of Clyne and Kurz is similar to that of Brody and Flemings.[1]

Combining Eqs. [1] and [12] allows one, then, to obtain the maximum deformation rate ($\dot{\varepsilon}_{p,\max}$) that can be sustained by the mushy zone before a hot tear nucleates at the root of the dendrites,

$$F(\dot{\varepsilon}_{p,\max}) = \frac{\lambda_2^2}{180}\frac{G}{(1+\beta)\mu}\Delta p_c - v_T\frac{\beta}{1+\beta}H \qquad [15]$$

$$\begin{aligned}\text{with}\quad F(\dot{\varepsilon}_p) = \int_{T_S}^{T_L}\frac{E(T)\,f_s(T)^2}{(1-f_s(T))^3}\,dT; \\ E(T) = \frac{1}{G}\int f_s(T)\,\dot{\varepsilon}_p(T)\,dT\end{aligned} \qquad [16a]$$

$$\text{and}\quad H = \int_{T_S}^{T_L}\frac{f_s(T)^2}{(1-f_s(T))^2}\,dT \qquad [16b]$$

IV. RESULTS AND DISCUSSION

In the previous section, the expressions for the pressure drop contributions associated with deformation and shrinkage were derived (Eqs. [9] and [12]). In a first step, it is assumed that the deformation rate is homogeneous over the entire length of the mushy zone, *i.e.*, that $\dot{\varepsilon}_p$ in Eq. [5] is independent of T (or x). Under these conditions, the cumulated deformation rate ($E(T)$) and the two other integrals appearing in Eq. [16] depend only on the solidification path $f_s(T)$, this last relationship being given by Eq. [13]. Assuming a constant cooling rate ($\dot{T} = -1$ K/s) and a uniform strain rate of 10^{-4} s^{-1}, the shrinkage and mechanical contributions to the depression in the interdendritic liquid have been calculated for an Al-Cu 1.4 wt pct alloy. The solidification path, given by Eq. [13], has been calculated with the parameters listed in Table I. The two contributions (Δp_ε and Δp_{sh}) are plotted separately in Figure 4 as a function of the position x in the mush. For both of them, the maximum depression occurs at the roots of the dendrites ($x = 0$), as could be expected. It should be pointed out that these two contributions are on the same order of magnitude under the present conditions. It is interesting to note that a com-

*In the well-known "Niyama criterion" for microporosity formation, the velocity of the isotherm v_T is replaced by $\dot{T}/G$, where $\dot{T}$ is the cooling rate. Thus, the pressure drop Δp_{sh} becomes proportional to $\dot{T}/G^2$ and the criterion is usually expressed as a function of $G/\dot{T}^{1/2}$.

Table I. List of Parameters Used in the Calculation of Figure 4

Viscosity	μ	$1 \cdot 10^{-3}$	Pa s
Cavitation depression	Δp_c	$2 \cdot 10^{3}$	Pa
Shrinkage factor	β	0.06	—
Velocity of the isotherms	v_T	$1 \cdot 10^{-4}$	m s^{-1}
Thermal gradient	G	$1 \cdot 10^{4}$	K m^{-1}
Cooling rate	$\dot{T}$	-1	K s^{-1}
Rate of heat extraction	$\dot{H}$	$-6 \cdot 10^{3}$	J kg^{-1} s^{-1}
Secondary arm spacing	λ_2	$100 \cdot 10^{-6}$	m
Back-diffusion coefficient	α_s	0.01	—

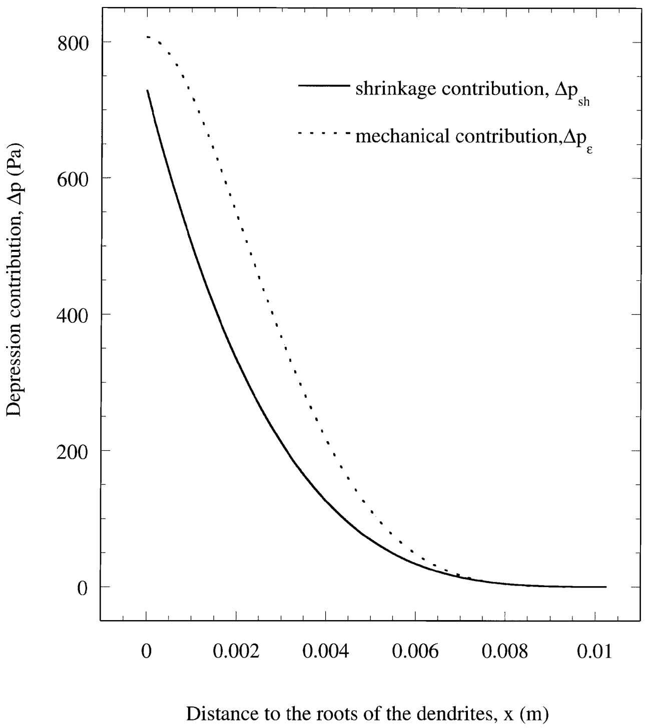

Fig. 4—Profile of the shrinkage, Δp_{sh}, and mechanical, Δp_ε, pressure drop contributions in the mushy zone for an Al-Cu 1.4 wt pct alloy ($\dot{T} = -1$ K/s and $\dot{\varepsilon}_p = 10^{-4}$ s^{-1}).

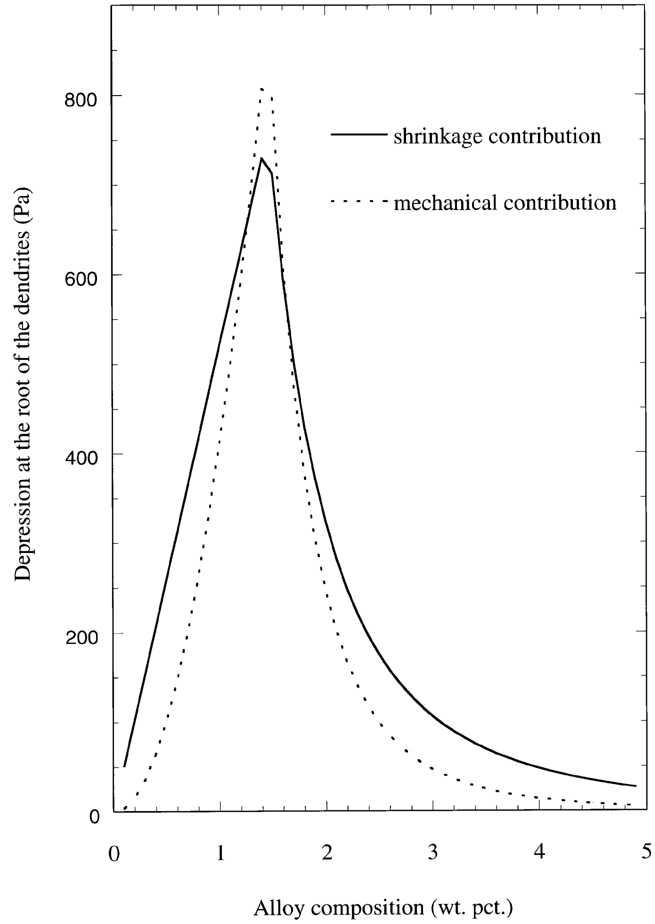

Fig. 5—Concentration dependence of the shrinkage, Δp_{sh}, and mechanical, Δp_ε, pressure drop contributions at the roots of the dendrites for the Al-Cu system ($\dot{T} = -1$ K/s and $\dot{\varepsilon}_p = 10^{-4}$s^{-1}).

pressive, instead of a tensile, strain rate of 10^{-4} s^{-1} in the mush would almost exactly compensate for the solidification shrinkage, thus eliminating liquid suction in that case.

Under the same assumptions ($\dot{T} = -1$ K/s and $\dot{\varepsilon}_p = 10^{-4}$ s^{-1}), the maximum shrinkage and mechanical contributions to the pressure drop found at the roots of the dendrites were calculated for different alloy compositions in the binary Al-Cu system. They are plotted in Figure 5. Both components of the depression exhibit a maximum at 1.4 wt pct Cu, where the solidification interval is at a maximum.

Figure 4 has clearly shown that the shrinkage and deformation contributions to the pressure drop in the interdendritic liquid are on the same order of magnitude and are both maximum at the roots of the dendrites. These maximum values have been then plotted in Figure 5. Adding the two contributions of Figure 5 would already give an indication of the HCS of the alloy, in a way similar to that used by Niyama to describe the sensitivity to pore formation. However, if one wants to separate the deformation contribution from the shrinkage part, it is better to describe the HCS index of the alloy in terms of the maximum strain rate that can be sustained by the deepest part of the mush before a void forms (Eq. [15]). This is, then, equivalent to calculate $\dot{\varepsilon}_{p,\max}$, such that $\Delta p_{\max} = \Delta p_\varepsilon + \Delta p_{sh} = \Delta p_c$, where Δp_c is the cavitation depression of the liquid and $\Delta p_{\max}$ is the total depression in the interdendritic liquid at the roots of the dendrites. The HCS index is then assumed to be proportional to $1/\dot{\varepsilon}_{p,\max}$.

This HCS index, normalized between zero and unity, was calculated for the two thermal conditions defined by Clyne and Davis:[6] mode 1 corresponds to a constant cooling rate ($\dot{T}$) and mode 2 to a constant rate of heat extraction ($\dot{H}$), ($\dot{H} = c_p\dot{T} - L\dot{f}_s$, where c_p and L are the specific heat and latent heat of fusion, respectively). The resulting HCS index is compared in Figure 6 with the measurements of Spittle and Cushway for different compositions (c) of non-grain-refined Al-Cu alloys.[22] These authors have used ''dog-bone'' shaped cylindrical molds to cast these alloys. The electrical resistance of the specimens was then measured after solidification and converted into a HCS index varying from 0 to 1. Also reported in Figure 6 are the Clyne and Davis criterion for modes 1 and 2 and a criterion which is simply proportional to the solidification interval of the mushy zone, as calculated with the help of Eq. [13] (*i.e.*, including back-diffusion). It is to be noted that the criterion of Clyne and Davis and the present one both assume that interdendritic bridging occurs at a solid fraction of about 98 pct if the eutectic temperature is not yet reached. This means that the lower integration limit in Eqs. [13] and [14] (T_S) is equal to either $T(f_s = 0.98)$ if less than 2 pct eutectic forms or to T_E at higher concentrations, where T_E is the eutectic temperature.

The Λ-shaped-curve, typical of hot tearing, is well reproduced by the present criterion for both modes; the rapid increase at a low solute content and the maximum at a composition of around 1.4 wt pct Cu, predicted by the criterion, are in relatively good agreement with the measurements of Spittle and Cushway. Please note that the maximum of the HCS curves is very close to the maximum of the solidification interval, as pointed out by Campbell.[2] The decrease between 1.4 and 3 pct is somewhat too steep in the present model, but the vanishing values obtained at concentrations higher than 3 pct are again close to the experimental ones. On the other hand, the criterion of Clyne and Davis for mode 1 surprisingly does not reproduce the increase of the HCS at a low concentration and was discarded by these authors.* The same criterion computed for mode 2 yields a too-wide Λ curve and overestimates the HCS values as compared with experiments, especially at higher concentrations. Finally, the model based simply on the solidification interval predicts an even slower decrease past the maximum.

It should be pointed out that the measurements of Spittle and Cushway correspond to an overall cracking length of the specimens; their HCS index is, therefore, an indication of the *propagation* of hot tears. The present model is only a criterion for the *appearance* (initiation) of the first hot tear in the interdendritic liquid and not of its propagation. However, in the steady-state conditions considered in the present model, there are no reasons for an initiated hot tear to stop propagating unless the deformation rate decreases.

It is also well possible that, as for microporosity formation,[3,4,5] the minimum pressure at which nucleation of a first void occurs is influenced by the initial concentration and microsegregation of dissolved gases. The influence of the Cu concentration on these phenomena has been neglected, and Δp_c was considered constant. On the other hand, the undercooling of the eutectic, which is usually substantially larger than that of the dendrite tips, has also not been considered here: it will delay the decrease of the HCS curves. The influence of a eutectic precipitation on the dendrite coherency is also unknown. Furthermore, the density of the eutectic is much larger than that of the primary phase and has been shown to induce a substantial suction of liquid deep in the mush.[23]

The assumption of a homogeneous deformation rate over the entire length of the mushy zone can also be put into question: up to which fraction of solid is the strain rate of the coherent solid skeleton effectively transmitted in the mushy zone? Two other assumptions were tried in order to test this issue: in the first one, the strain rate is homogeneous in the mush above a 40 vol pct fraction of solid and is zero for $0 \le f_s \le 0.4$; in the second, the strain rate vanishes to zero in the mushy zone as $\dot{\varepsilon}_p(f_s) = \dot{\varepsilon}_{p,o} f_s$, where $\dot{\varepsilon}_{p,o}$ is the deformation rate at the roots of the dendrites. It was verified

*This is rather surprising, since $f_s(T)$ is a weakly dependent function of T at the roots of the dendrites ($\dot{H} = c_p \dot{T}$), which means that both modes should yield comparable results.

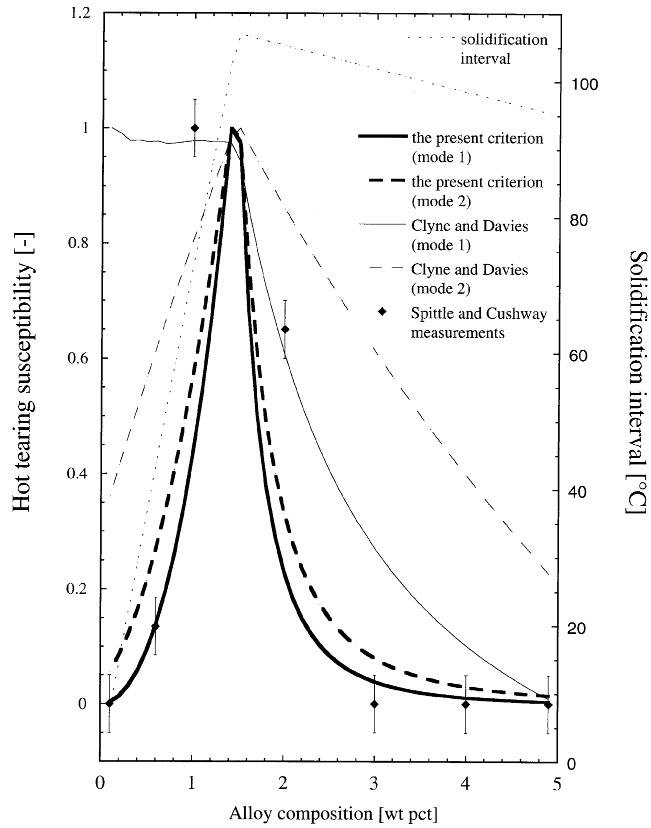

Fig. 6—Concentration dependence of the HCS criterion defined by $\dot{\varepsilon}_{p,\max}^{-1}$ and normalized to unity, where $\dot{\varepsilon}_{p,\max}$ is the maximum strain rate that can be sustained by the mushy zone before reaching a fixed cavitation depression Δp_c (Eq. [15] and list of parameters in Table I). The theoretical curves calculated for Al-Cu alloys and a constant cooling rate (mode 1) or constant heat extraction rate (mode 2) are compared with the measurements of Spittle and Cushway,[22] with the criteria of Clyne and Davis[6] and with the solidification interval criterion.[2]

that these two assumptions did not substantially affect the shape of the Λ curves. Indeed, the depression in the interdendritic liquid occurs mainly in the deepest part of the mushy zone and does not depend much on the flow near the dendrite tips. On the other hand, the fraction of solid at which interdendritic bridging is assumed to occur (0.98) has a great influence on the position of the peak of the Λ curves: when this value tends toward unity, the concentration of the HCS maximum tends toward zero. This is due to the singularity of the permeability function when $f_s \rightarrow 1$. In the present calculations, the cavitation depression was set to 2 kPa: it is an unknown key value of the model but, nevertheless, it is relatively close to the depression computed by Ampuero *et al.*[5] for microporosity formation in an Al-4.5 pct Cu alloy. It is also on the order of magnitude of the value of 1 kPa set by Drezet and Rappaz[14] in the study of ring-mold test experiments performed on an Al-4.5 wt pct Cu alloy. These authors determined this cavitation depression after estimating the value of $\dot{\varepsilon}_{p,\max}$ from a combination of thermomechanical computations of the ring-mold tests and experimental observations of the extent of hot tears.

Finally, it was also confirmed by experimental observations[2] that, for a given alloy composition and for both modes, the maximum sustainable strain rate before hot tears form is an increasing function of the thermal gradient and of the permeability and a decreasing function of the viscosity.

V. CONCLUSIONS

As a conclusion, the present criterion is only an indicator of the appearance (or initiation), and not of the propagation, of hot tears. Therefore, it is certainly not straightforward to compare $1/\dot{\varepsilon}_{p,\max}$ to the crack length measured in real specimens. Further refinements of the model can also be envisaged, such as more complex cooling conditions, accounting of gas segregation, equiaxed instead of columnar structures, *etc.* The model for the first appearance (initiation) of a hot tear is also rather crude: it corresponds to instantaneous nucleation at a critical depression. More sophisticated models, in which the time would appear (*i.e.*, nucleation rate), could also be used to predict hot-tear formation. The model was intentionally made simple so that the mechanisms clearly appear. Unlike the model of Feurer,[8] the contributions of shrinkage and uniaxial deformation are both considered in the present approach. It is believed that the physically sound framework established in the present model should allow one to extend this two-phase approach in the direction of crack propagation.

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