

A new hybrid approach based on discrete differential evolution algorithm to enhancement solutions of quadratic assignment problem

Asaad Shakir Hameed^{a*}, Burhanuddin Mohd Aboobaider^a, Modhi Lafta Mutar^a and Ngo Hea Choon^a

^a*Faculty of Information and Communication Technology, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100, Durian Tunggal, Melaka, Malaysia*

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ABSTRACT

The Combinatorial Optimization Problem (COPs) is one of the branches of applied mathematics and computer sciences, which is accompanied by many problems such as Facility Layout Problem (FLP), Vehicle Routing Problem (VRP), etc. Even though the use of several mathematical formulations is employed for FLP, Quadratic Assignment Problem (QAP) is one of the most commonly used. One of the major problems of Combinatorial NP-hard Optimization Problem is QAP mathematical model. Consequently, many approaches have been introduced to solve this problem, and these approaches are classified as Approximate and Exact methods. With QAP, each facility is allocated to just one location, thereby reducing cost in terms of aggregate distances weighted by flow values. The primary aim of this study is to propose a hybrid approach which combines Discrete Differential Evolution (DDE) algorithm and Tabu Search (TS) algorithm to enhance solutions of QAP model, to reduce the distances between the locations by finding the best distribution of N facilities to N locations, and to implement hybrid approach based on discrete differential evolution (HDDETS) on many instances of QAP from the benchmark. The performance of the proposed approach has been tested on several sets of instances from the data set of QAP and the results obtained have shown the effective performance of the proposed algorithm in improving several solutions of QAP in reasonable time. Afterwards, the proposed approach is compared with other recent methods in the literature review. Based on the computation results, the proposed hybrid approach outperforms the other methods.

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1. Introduction

The emergence of Combinatorial Optimization Problems (COPs) from theory and practice poses a great challenge that has continued to attract the attention of practitioners, researchers and academicians globally for the last five decades. Facility Layout Problem is an example of such combinatorial problems and finding a solution to this problem has remained a major challenge (Scalia et al., 2019). The main purpose of finding a solution to this problem is to enable the arrangement of departments within the boundaries of the predefined facility such that the functions can efficiently interact with one another, while the total cost of mobility is reduced. Many studies have been carried out in the area of facility

* Corresponding author
E-mail: asaadutem@yahoo.com (A. S. Hameed)

layout problems, but most of them have only focused on studying facility layout problems in manufacturing facilities, with just a few of them analyzing this problem within hospital domain. The modelling of FLP was first carried out as a quadratic assignment problem (QAP) by (Koopmans & Beckmann, 1957). According to Samanta et al. (2018), these COPs emerge from real-life situations. The use of discrete formulations is employed in layout problems which involve determining possible positions of facilities prior to their optimization. QAP is commonly used for this kind of problem. The QAP is regarded as a problem of NP-Hard combinatorial optimization (Şahinkoç & Bilge, 2018), which serves as a model for many real-life applications such as hospital layout, backboard wiring, campus layout, scheduling and designing of keyboard typewriter, etc. ever since the QAP was formulated, the attention of researchers has been drawn to it because of its importance in theory and practice, and most importantly because of how complex it is (Duman et al., 2012; Benlic & Hao, 2013; Kaviani et al., 2014; Abdel-Basset et al., 2018a, Cela et al., 2018). The FLP has been introduced as a QAP in order to identify the ideal allocation of N facilities to N locations, where there must be equality between the number of locations and number of facilities. Researchers around the world have accepted the complexity associated with finding a solution, but now, there is no available polynomial time algorithm that can be used to solve QAP. In recent times, the approximate algorithms have been used more than the exact algorithms, because it can find the optimal solution with unreasonable time. However, most of the times it is impossible to solve a problem that is more than 20 within a reasonable period of time (Abdel-Baset et al., 2017). Therefore, researchers are more interested in employing the use of meta-heuristic and heuristic approaches to solve huge QAP problems. The motivation of this paper is proposing a novel approximate meta-heuristic algorithm that can enable the most efficient allocation of N facilities to N locations ($N > 30$) of QAP. It is hoped that this approach will, in turn, enhance the reduction of cost while the problem is solved within the shortest time possible. The use of different methods, which are classified as a heuristic, meta-heuristic and exact methods has been employed in solving this challenging problem. Out of the three categories of methods, researchers are paying more attention to meta-heuristic methods, and this is evident in its increased usage in solving problems associated with optimization. Regardless of the inability of these methods to solve problems optimally, their efficiency is guaranteed especially when the models are complex. One of the meta-heuristic methods that are widely used in models of healthcare facility location is Tabu search (TS) (Zhang et al., 2010). Apart from Tabu, there are other methods that are used in solving such problems, such as Genetic Algorithm (GA) (Radiah Shariff & Noor Hasnah Moin, 2012). Pareto Ant Colony Optimization (P-ACO) (Doerner et al., 2007), and Simulate Annealing (SA) (Syam & Côté, 2010). One of the greatest problems associated with the exact methods is their cost of computation with more time, and for this reason, this study is carried out to find the best solutions for QAP. In order to achieve this, a new method is proposed in this study. This study seeks to achieve more objectives as follows: (i) The major objective of this study is to propose a hybrid approach which combines Discrete Differential Evolution (DDE) algorithm and Tabu Search (TS) algorithm for enhancing solutions of QAP model, (ii) To minimize the cost through reducing the distances between the locations by finding the best distribution of N facilities from N locations, and (iii) To implement HDETS on many instances of QAP from the benchmark.

The other sections of this paper are as follows. Section 2 introduces the Quadratic Assignment Problem QAP. In Section 3 the Review of Literature is provided. In Section 4, the algorithm that has been proposed (HDETS) has been examined and discussed. The Computational Results are discussed in Section 5. Lastly, the conclusions and some recommendations for future studies are given in Section 6.

2. Quadratic Assignment Problem QAP

The QAP has several real-life applications, which makes it an interesting area of study for researchers since its inception (Czapiński, 2013; Abdelkafi et al., 2015; Çela et al., 2017). The QAP mathematical model has been presented as follows:

$$\min f(\pi) = \sum_{i=1}^n \sum_{j=1}^n F_{ij} D_{\pi(i)\pi(j)} \quad (1)$$

Overall permutations $\pi \in P_n$.

The model of QAP consists of two matrices each of them size $N \times N$, $N=1, 2, \dots, n$.

- The F refers to the flow or weight between each pair of facilities is represented by F_{ij} denoting the flow from facility i to facility j ;
- The D connotes the distance that exist between each pair of locations being represented by D_{ij} , which denotes the distance from location i to location j ;
- π is the best way through which a solution to a QAP problem can be represented.
- The aim is to allocate N facilities to N locations at a low cost.

3. Literature Review

QAP remains a major problem that is yet to have an exact solution. To this end, many researchers have invested so many resources into finding the most appropriate solution to this problem, and they have as well used several methods with different techniques to solve the problem. In this section, the review of literature is presented to show some of the several techniques that other researchers have used to solve the QAP. The Discrete Particle Swarm Optimization (DPSO) algorithm was introduced by Pradeepmon et al. Sridharan (2016). In a study carried out by Pradeepmon (2018), the DPSO algorithm was modified and named Modified DPSO. This development was also aimed at solving the QAP. Also, in (Shukla, 2015) the Bat Algorithm (BA) was used for the same purpose. Similarly, the study conducted by Riffi et al. (2017) aimed at enhancing the BA search strategy by introducing a new method. In their proposed method, the Discrete Bat Algorithm (DBA) was combined with BA, an enhanced uniform crossover, and a 2-exchange neighborhood method. The Ant Colony Optimization (ACO) algorithm has been suggested by Xia and Zhou (2018). In the research conducted by Abdel-Basset et al. (2018b), a new approach known as the WAITS was introduced. The WAITS is integration between meta-heuristic whale optimization and the tabu search, hence the name. Similarly, Ahmed (2018) carried out a study in which the lexsearch and genetic algorithms were combined to form a hybrid algorithm (LSGA) that can be used in solving the QAP effectively. A hybrid method in which the Ant Colony Algorithm was combined with Tabu Search algorithm, was proposed by Lv (2012). The experimental data for this proposed hybrid algorithm indicated that the smallest average error value was obtained using the proposed hybrid algorithm. In research carried out by Da Silva et al. (2012), another hybrid algorithm was proposed. The proposed algorithm was an integration of Tabu search meta-heuristics and greedy randomized adaptive search procedure (GRASP). Their results showed that the proposed algorithm produced low-cost solutions for 50 instances. Similarly, another hybrid algorithm, which is a combination of Simulated Annealing and Tabu Search was introduced by Kaviani et al. (2014) as a solution to the QAP. In the proposed algorithm, memory structures were used through Tabu search as a means of explaining the user-provided set of rules. In contrast to other studies, in a research carried out by (Said et al., 2014) the Genetic algorithm, Simulated Annealing and Tabu Search were compared in terms of execution time. The study results revealed that the performance of the Tabu search was better than that of other meta-heuristic algorithms in terms of execution time for solving practical QAP instances and the algorithm demonstrated faster execution time. Another integration was performed by Harris et al. (2015), and in their study, they integrated the Tabu Search with Memetic algorithm. Through the restarts, the solution space is explored, and the problem of convergence is avoided by the algorithm. Furthermore, the search for local optima is intensified using Tabu Search. Findings of their study revealed that the proposed algorithm was less time consuming and outperformed other methods in terms of solving real-life instances and random instances with high quality. In order to solve the QAP, Lim et al. (2016) proposed another hybrid algorithm which is formed by combining the Biogeography-Based Optimization Algorithm and Tabu Search. With the use of the proposed hybrid algorithm, the best solutions were found for 36 instances out of 37 instances. This shows that the performance of the hybrid algorithm was good.

In other studies, attempts were made by researchers to solve the problems of discrete optimization. In such studies, modifications were made to the Differential Evolution (DE). An algorithm associated with discrete differential evolution (DDE) was proposed by (Pan et al., 2008) for the purpose of computing differences in the flow-shop preparation problem. Results of their study showed that the efficiency of the proposed algorithm was lower than that of other methods, and this was perceived to be caused using probability of low mutation (0.2). However, the DDE algorithm operation is more successful and efficient when the local search is used. In a study earlier conducted by Kushida et al. (2012) the DE was modified to a discrete optimization problem and afterward used in solving the QAP. Similarly, the use of insertion and swap was employed by Tasgetiren et al. (2013) in modifying DDE with the local search-based modification. With the use of DDE alongside local search, improvements were observed in the results of two kinds of dense and sparse instances of QAPLIB.

4. Methods

Three phases are involved in this section. In the first phase, discrete differential evolution algorithm (DDE) is included, the second phase includes the Tabu search algorithm TS, and finally, in the third phase, the proposed hybrid, which is a combination of both TS and DDE is introduced.

4.1 Discrete Differential Evolution Algorithm (DDE)

One of the most recently introduced Evolutionary Algorithm is the Differential Evolution (DE) optimization method, which was first introduced by Storn and Price (1997). The Evolutionary Algorithm is regarded as a category of efficient optimization techniques used worldwide to solve a wide range of hard problems. DE is known as a global optimizer that is constantly dependent on random space and population (Lampinen, 2005). The DE has proven to be more efficient and powerful, and for this reason, it is rapidly emerging as a popular optimizer that is used in different areas like the function of continuous real value and for solving a combinatorial optimization problem with a discrete decision. In this study, the discrete differential algorithm DDE which has been modified by (Tasgetiren et al., 2013) is used. The Discrete Differential algorithm DDE is illustrated in the flowchart in Fig. 1. and the steps of it have been introduced as follows:

I. Initialization initialize population matrix $\pi = \{\pi_1, \pi_2, \pi_3, \dots, \pi_{NP}\}$ randomly. Matrix size $NP \times ND$ where NP is number of population and ND dimension of problem space. All population individuals should be unique.

II. Evaluate fitness: find the best solution πb^{t-1} from population π .

III. Mutation: obtain the mutant individual, the following equation can be used:

$$v_i^t = \begin{cases} \text{insert}(\pi_b^{t-1}) & \text{if}(r < P_m) \\ \text{swap}(\pi_b^{t-1}) & \text{otherwise} \end{cases} \quad (2)$$

where πb^{t-1} is the best solution from the previous generation in the target population; P_m is the perturbation probability; and swap are simply the single insertion and swap moves, r is a uniform random number belong to $[0,1]$.

IV. Crossover: obtain the crossover, the following equation can be used:

$$u_i^t = \begin{cases} CR(v_i^t, \pi_i^{t-1}) & \text{if}(r < P_c) \\ v_i^t & \text{otherwise} \end{cases} \quad (3)$$

where π_b^{t-1} is the best solution from the previous generation in the target population; Pc is the crossover probability; and CR is crossover operation. then the crossover operator is applied to generate the trial individual u_i^t Otherwise the trial individual is chosen as $u_i^t = v_i^t$.

V. Selection: selection is based on fitness function; the following equation can be used:

$$\pi_i^t = \begin{cases} u_i^t & \text{if } (f(\pi_i^t) < f(\pi_i^{t-1})) \\ \pi_i^{t-1} & \text{otherwise} \end{cases} \quad (4)$$

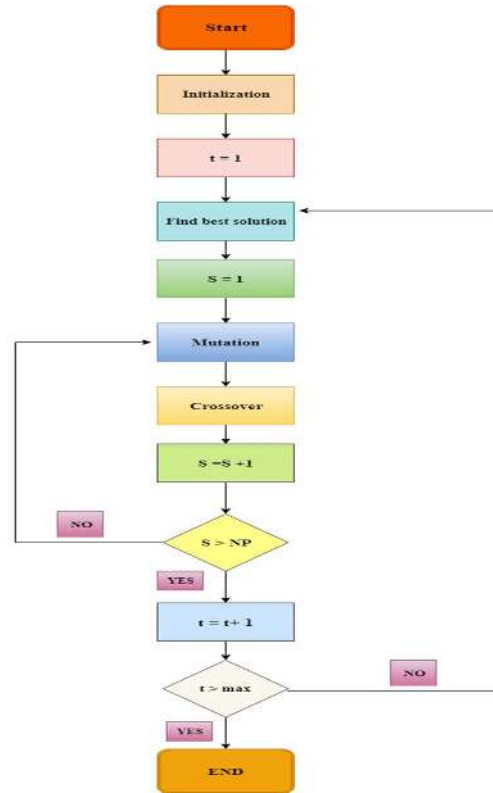


Fig. 1. Flowchart of DDE algorithm

4.2 Tabu Search Algorithm (TS)

In order to solve the large combinatorial optimization problem, the use of Tabu search (TS) has been employed with great success (Van Luong et al., 2010). Despite the efficiency and the meta-heuristic strength demonstrated by the TS, it is usually combined with other solutions like evolutionary computation. The central idea behind TS involves the specification of a set of moves or a neighborhood which can be used in a specific solution so as to enable the generation of a new solution (Taillard, 1991). The neighborhood solution that is considered by TS is to have the best evaluation. In an event that improving moves are absent, TS selects the neighborhood solution that has minimal effect in terms of degrading the objective function. It is possible to avoid the return to a local optimum that has just been visited by using a list of tabu. In an event that tabu moves are perceived as fascinating, the introduction of an aspiration criterion is made so that these tabu moves can be selected.

4.3 The proposed algorithm HDETS

In Fig. 2 below, the HARDEST algorithm flow chart is presented. The basic steps of the HDETS are addressed as follows:

1- Initialization: initialize population matrix $\pi = \{\pi_1, \pi_2, \pi_3, \dots, \pi_{NP}\}$ randomly. Matrix size $NP \times ND$ where NP is several population and ND dimension of problem space. All population individuals should be unique. Initialize set Solution Wait for each solution $SW =$ array of NP with zeros and maximum wait, and ht (iteration of tabu search).

2- Evaluate fitness: to the fined best solution based on the Eq. (1)

3- Mutation: use the Eq. (2)

4- Crossover: the crossover has been introduced by Eq. (3). The central idea of crossover is to leverage the best benefits from the parent algorithm during the production of the new one, which is often known as the hybrid. A wide range of crossover operators are found in the literature, and such crossover operators have been proposed by researchers with the aim of solving quadratic assignment problem. In this study, the crossover which has been used is referred to as the uniform-like crossover (ULX) which was introduced by (Tate & Smith, 1995). The crossover was obtained as follows:

- The offspring inherits any facility which is has been allocated to the same location in both parents
- The selection of every unallocated facility is carried out randomly so as to ensure that each facility that is unassigned is chosen just once. Here, a random selection of one of the parents is made. In a situation whereby the location of the chosen facility is unoccupied, the offspring inherits it. However, if the location is occupied in the first parent, then an attempt is made to allocate the location of the facility from the second parent.
- Once a location has been allocated to a facility, it is marked. If the facility which is allocated to this location in the parent that was used in the previous rule is not allocated, the offspring inherits it.

5- apply the TS for a hybrid: TS used to an enhancement of the solution based on some characteristics as follows:

- i. Intensification:** In Intensification the promising area is explored more fully in the hope to find the best solutions by using neighborhood search, the size of a neighborhood is $n(n-1)/2$ and calculated through the following:

$$\Delta\text{cost}(\pi, i, j) = (a_{ii} - a_{jj}) (b_{\pi(j)\pi(j)} - b_{\pi(i)\pi(j)}) + (a_{ij} - a_{ji}) (b_{\pi(j)\pi(i)} - b_{\pi(i)\pi(j)}) + \sum_{k=1, k \neq i, j}^n (a_{ik} - a_{jk}) (b_{\pi(j)\pi(k)} - b_{\pi(i)\pi(k)}) + (a_{ki} - a_{kj}) (b_{\pi(k)\pi(j)} - b_{\pi(k)\pi(i)}) \quad (4)$$

where $a_{ii}, a_{jj} = 0, i=1, 2, \dots, n, k=1, 2, 3, \dots, n$ such that $k \neq i, k \neq j$

- ii. Tabu list:** The tabu list has been used to avoid the solution which visited in the past.

6- Selection: selection is based on fitness function; the following equation can be used:

$$\pi_i^t = \begin{cases} u_i^t & \text{if } (f(\pi_i^t) < f(\pi_i^{t-1})) \\ \pi_i^{t-1} & \text{otherwise} \end{cases} \quad (5)$$

7- Update solution waiting:

$$SW^i = \begin{cases} 0 & \pi_i^t = u_i^t \\ SW^i + 1 & \pi_i^t = \pi_i^{t-1} \end{cases} \quad (6)$$

4.3.1 Pseudo-code of HDDETS algorithm

Generate population matrix $\pi = \{\pi_1, \pi_2, \pi_3, \dots, \pi_{NP}\}$ randomly. Matrix size is $NP \times ND$ where NP is the number of population and ND is the dimension of problem space. Max_t = number of maximum iterations. Set Solution Wait for each solution $SW =$ array of NP . Max_ht = number of maximum iterations of TS.

```

While t < max_t
  For each solution
    Evaluate fitness: Equation (1)
    Mutation: Equation (2)
    Crossover: Equation (3)
    If (r < 0.5)
      ht = 1
      While ht < max_ht
        For each solution
          Great neighborhood
          Evaluate the neighborhood solutions.
          Choose best admissible solutions  $\pi_i^h$  which not exist in tabu list.
          Update tabu list.
          If best tabu solution is better than current solution update current
            solution
          else
            Great a new neighborhood
          end if
        end For
      end while
    else
      Selection: Equation (5)
      Update solution waiting  $SW^i$ : Equation (6)
    end if
    if  $SW^i$  reach to maximum waiting W
      regenerate the current solution.
    end
  end for
  t = t + 1
end while

```

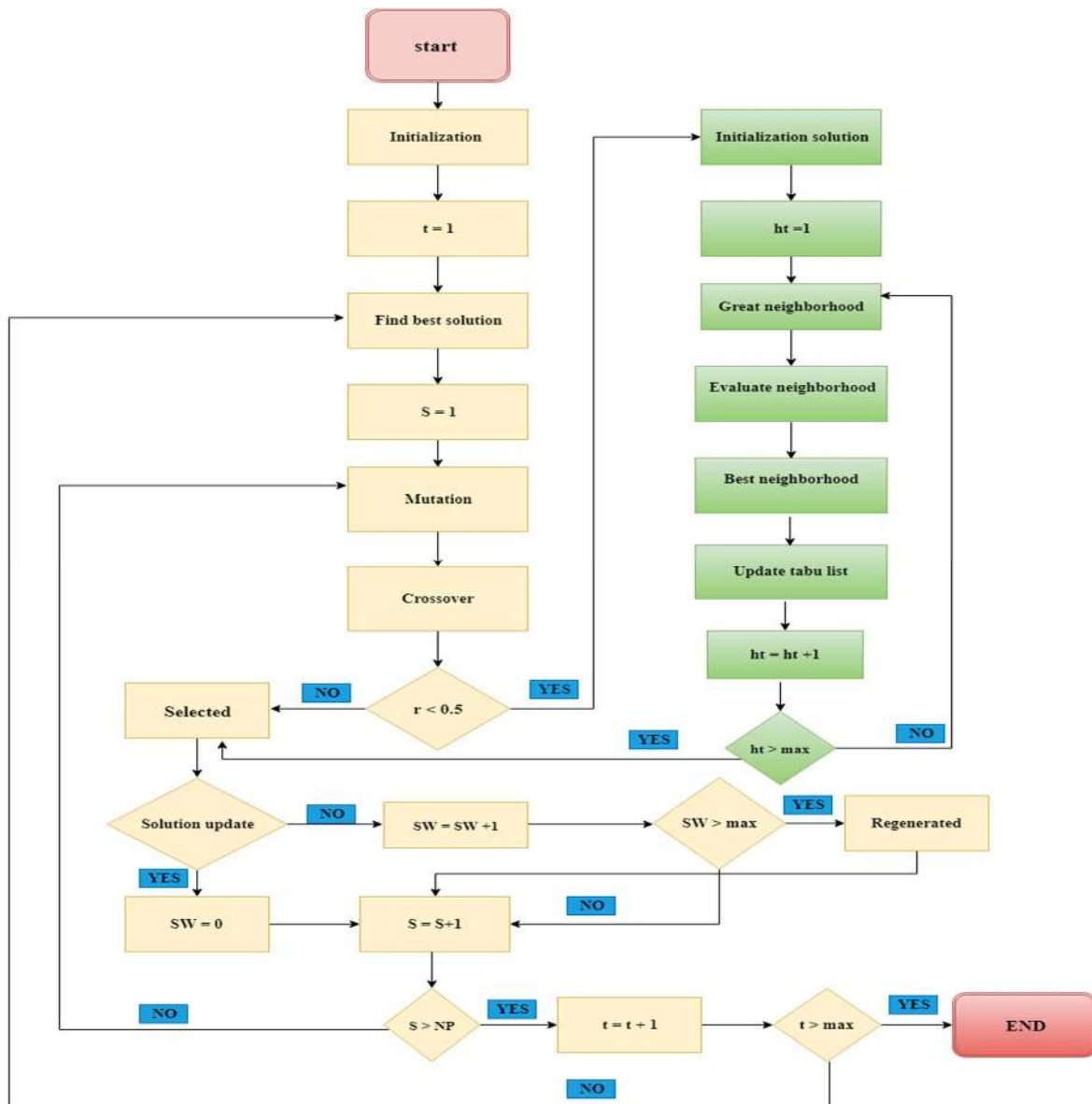


Fig. 2. Flowchart of HDETS algorithm

5. Computational Results

In this section, the efficiency of the proposed algorithm is presented. In order to encode the proposed algorithm, MATLAB was employed on a PC with Intel (R) Core (TM) i7-3770 CPU @ 3.40 GHz. Also, the PC which was used operates under MS Windows 10 and has a RAM of 4GB. This section consists of two parts, and the first part highlights the parameters used for the proposed algorithm, while in the second part the results of the study are discussed. The results were obtained using the proposed algorithm. The proposed algorithm has been applied to seven categories of instances from QAPLIB as Table 1.

Table 1
Instances of QAP from QAPLIB

Category 1	Category 2	Category 3	Category 4	Category 5	Category 6	Category 7
Tai12a	Nug12	Chr12a	Esc16a	Lipa20a	Had12	Sko42
Tai12b	Nug14	Chr12b	Esc16b	Lipa20b	Had14	Sko49
Tai15a	Nug15	Chr12c	Esc16c	Lipa30a	Had16	Sko56
Tai15b	Nug16a	Chr15a	Esc16d	Lipa30b	Had18	Sko64
Tai17a	Nug16b	Chr15b	Esc16e	Lipa40a	Had20	Sko72
Tai20a	Nug17	Chr15c	Esc16f	Lipa40b	-	Sko81
Tai20b	Nug18	Chr18a	Esc16g	Lipa50a	-	Sko90
Tai25a	Nug20	Chr18b	Esc16h	Lipa50b	-	Sko100a
Tai25b	Nug21	Chr20a	Esc16i	Lipa60a	-	Sko100b
Tai30a	Nug22	Chr20b	Esc16j	Lipa60b	-	Sko100c
Tai30b	Nug24	Chr20c	Esc32a	Lipa70a	-	Sko100d
Tai35a	Nug25	Chr22a	Esc32b	Lipa70b	-	Sko100e
Tai35b	Nug27	Chr22b	Esc32c	Lipa80a	-	Sko100f
Tai40a	Nug28	Chr25a	Esc32d	Lipa80b	-	-
Tai40b	Nug30	-	Esc32e	Lipa90a	-	-
Tai50a	-	-	Esc32g	Lipa90b	-	-
Tai50b	-	-	Esc32h	-	-	-
Tai60a	-	-	Esc64a	-	-	-
Tai64c	-	-	Esc128a	-	-	-
Tai80a	-	-	-	-	-	-
Tai80b	-	-	-	-	-	-
Tai100a	-	-	-	-	-	-
Tai100b	-	-	-	-	-	-
Tai150b	-	-	-	-	-	-
Tai256c	-	-	-	-	-	-

5.1 Parameter setting

In order to determine the most appropriate parameter settings, extensive experiments, as well as many runs of the algorithm, were performed. The set values of the parameters for the three algorithms were presented in Table 2. The quality of the solutions obtained by using the proposed algorithm can be influenced by the set algorithm parameters. To identify the most suitable set of parameter values that produce desirable outcomes, numerous tests were performed.

Table 2
Parameter setting

Parameter	Value
NP Number of Population	200
Maximum Iterations	100
P _m Perturbation Probability of Mutation	0.7
P _c Perturbation Probability of Crossover	0.8
P _h Probability of Hybrid	0.5
Maximum waiting for solutions updates	10
Tabu list length	10
Maximum iterations of TS	25
Number of runs	10

5.2 Results and Discussions

This section shows the computational results of the efficiency of the proposed algorithm. The suggested algorithm HDETS has been run on 10 different instances made up of problems that are referred to as follows: Tai, Nug, Chr, Esc, Lipa, Had, and Sko. Table3 shows the instances which have been used in this study. The QAP size falls within the range of 12 to 256. Many statistical analyses have been carried out for every instance which include the best solution, worst solution, average solution, best gap, worst gap, average gap, standard deviation, and time. The experiment show the effect of integrating the tabu search algorithm TS with the discrete differential evolution algorithm DDE. The performance of the

algorithm which is proposed in this study HDETS was evaluated by comparing it with other algorithms. Specific criteria which include quality of solution and measured running times were used in comparing the algorithms. The use of quality of solution criterion for comparison of algorithms is more appropriate in heuristic and estimation methods, especially (in optimization). On the other hand, the running time comparison criterion is the most appropriate for exact algorithms. However, in a case where the produced solutions are similar in terms of quality, comparison of running times of approximation algorithms and heuristics will be suitable. This work focused on solution quality. The accuracy of an algorithm is calculated using a percentage deviation or gap. In this study, the solution quality criterion was used in calculating the accuracy, which is calculated through the question below:

$$\text{Gap} = (C_{\text{Best}} - C^*) / C^* \times 100, \quad (7)$$

where C_{Best} is the best objective value found over 10 runs, while C^* is the best-known value taken from QAPLIB. The results of the proposed algorithm (HDETS) are presented in Table 3. The results are discussed using three scenarios as follows:

Scenario 1:

The proposed algorithm was applied to the cases shown in Table 1. All the numerical results were excellent and have been presented in Table 3. It was found that the proposed algorithm achieved an accuracy of 100 % in 83 test instances out of 105 test instances. These excellent results can be attributed to the use of an algorithm feature that can continuously improve all the solutions in each iteration until the best solution is reached. The strength of this algorithm is due to the integration of the diversification property of the algorithm DDE with the intensification feature of TS algorithm, as well as the use of tabu-list which prevents the recurrence of solutions that have been visited in the past.

Table 3
Results of the HDETS algorithm for some instances from QAPLIB

Name of problem	Solution in QAPLIB	Proposed algorithm	Best Solution	Worst Solution	Average Solution	Best Gap	Worst Gap	Average Gap	Time (seconds)	Stander Division
Nug12	578	HDETS	578	578	578	0	0	0	0.514	0
Nug14	1014	HDETS	1014	1014	1014	0	0	0	0.432	0
Nug15	1150	HDETS	1150	1150	1150	0	0	0	0.378	0
Nug16a	1610	HDETS	1610	1610	1610	0	0	0	0.573	0
Nug16b	1240	HDETS	1240	1240	1240	0	0	0	0.461	0
Nug17	1732	HDETS	1732	1732	1732	0	0	0	4.204	0
Nug18	1930	HDETS	1930	1930	1930	0	0	0	0.526	0
Nug20	2570	HDETS	2570	2570	2570	0	0	0	1.889	0
Nug21	2438	HDETS	2438	2438	2438	0	0	0	2.275	0
Nug22	3596	HDETS	3596	3596	3596	0	0	0	1.672	0
Nug24	3488	HDETS	3488	3488	3488	0	0	0	1.99	0
Nug25	3744	HDETS	3744	3744	3744	0	0	0	3.201	0
Nug27	5234	HDETS	5234	5234	5234	0	0	0	1.299	0
Nug28	5166	HDETS	5166	5166	5166	0	0	0	43.434	0
Nug30	6124	HDETS	6124	6148	6126	0	0.391	0.039	3.273	0.123
Chr12a	9552	HDETS	9552	9552	9552	0	0	0	0.518	0
Chr12b	9742	HDETS	9742	9742	9742	0	0	0	0.281	0
Chr12c	11156	HDETS	11156	11156	11156	0	0	0	0.558	0
Chr15a	9896	HDETS	9896	9896	9896	0	0	0	1.077	0
Chr15b	7990	HDETS	7990	7990	7990	0	0	0	0.369	0
Chr15c	9504	HDETS	9504	9504	9504	0	0	0	2.026	0
Chr18a	11098	HDETS	11098	11098	11098	0	0	0	1.01	0
Chr18b	1534	HDETS	1534	1534	1534	0	0	0	0.522	0
Chr20a	2192	HDETS	2192	2192	2192	0	0	0	2.057	0
Chr20b	2298	HDETS	2298	2298	2298	0	0	0	50.772	0
Chr20c	14142	HDETS	14142	14142	14142	0	0	0	0.849	0
Chr22a	6156	HDETS	6156	6156	6156	0	0	0	51.954	0
Chr22b	6194	HDETS	6194	6194	6194	0	0	0	64.016	0
Chr25a	3796	HDETS	3796	3796	3796	0	0	0	9.588	0

Table 3

Results of the HDDETS algorithm for some instances from QAPLIB (Continued)

Name of problem	Solution in QAPLIB	Proposed algorithm	Best Solution	Worst Solution	Average Solution	Best Gap	Worst Gap	Average Gap	Time (seconds)	Stander Division
Sko42	15812	HDDETS	15812	15818	15814	0	0.037	0.015	20.864	0.019
Sko49	23386	HDDETS	23386	23440	23403	0	0.23	0.072	27.001	0.063
Sko56	34458	HDDETS	34458	34580	34503	0	0.354	0.131	623.89	0.132
Sko64	48498	HDDETS	48498	48902	48622	0	0.833	0.255	858.975	0.241
Sko72	66256	HDDETS	66316	66626	66429	0.09	0.558	0.261	368.841	0.136
Sko81	90998	HDDETS	91060	91524	91313	0.068	0.578	0.346	1624.424	0.18
Sko90	115534	HDDETS	115756	116498	116046	0.192	0.834	0.443	1727.96	0.197
Sko100a	152002	HDDETS	152316	154014	152725	0.206	1.323	0.475	2969.776	0.382
Sko100b	153890	HDDETS	154168	155054	154600	0.18	0.756	0.461	1490.758	0.189
Sko100c	147862	HDDETS	148148	149426	148753	0.193	1.057	0.602	2930.167	0.325
Sko100d	149576	HDDETS	149762	150512	150217	0.124	0.625	0.428	1472.923	0.15
Sko100e	149150	HDDETS	149514	151034	150024	0.244	1.263	0.585	6488.738	0.341
Sko100f	149063	HDDETS	149714	150464	149919	0.454	0.958	0.592	1265.821	0.144
Tai12a	224416	HDDETS	224416	224416	224416	0	0	0	0.508	0
Tai12b	39464925	HDDETS	39464925	39464925	39464925	0	0	0	0.684	0
Tai15a	388214	HDDETS	388214	388214	388214	0	0	0	0.847	0
Tai15b	51765268	HDDETS	51765268	51765268	51765268	0	0	0	0.81	0
Tai17a	491812	HDDETS	491812	491812	491812	0	0	0	5	0
Tai20a	703482	HDDETS	703482	706786	704026	0	0.469	0.077	5.653	0.167
Tai20b	122455319	HDDETS	122455319	122455319	122455319	0	0	0	0.511	0
Tai25a	1167256	HDDETS	1167256	1174422	1170285	0	0.613	0.259	7.848	0.209
Tai25b	344355646	HDDETS	344355646	344355646	344355646	0	0	0	13.46	0
Tai30a	1818146	HDDETS	1818146	1818146	1818146	0	0	0	57.4	0
Tai30b	637117113	HDDETS	637117113	637117113	637117113	0	0	0	29.794	0
Tai35a	2422002	HDDETS	2422002	2431810	2423613	0	0.404	0.066	29.589	0.131
Tai35b	283315445	HDDETS	283315445	283315445	283315445	0	0	0	57.4	0
Tai40a	3139370	HDDETS	3141431	3151727	3148060	0.065	0.393	0.276	138.357	0.087
Tai40b	637250948	HDDETS	637250948	650062131	638532066	0	2.01	0.201	416.445	0.635
Tai50a	4938796	HDDETS	4989160	5010958	5002852	1.019	1.461	1.297	768.214	0.162
Tai50b	458821517	HDDETS	458821517	460726849	459656699	0	0.415	0.182	48.479	0.194
Tai60a	7205962	HDDETS	7281638	7338518	7309055	1.05	1.839	1.43	921.089	0.263
Tai60b	608, 215, 054	HDDETS	7205962	608501817	640242782	0.047	5.265	1.526	42.188	1.616
Tai64c	1855928	HDDETS	1855928	1855928	1855928	0	0	0	8.308	0
Tai80a	13499184	HDDETS	13642148	13749540	13690956	1.059	1.854	1.42	1195.736	0.215
Tai80b	818415043	HDDETS	818415043	831997039	824550128	0	1.659	0.749	1399.883	0.561
Tai100a	21125314	HDDETS	21269898	21395720	21342495	1.069	1.667	1.414	2740.755	0.202
Tai100b	1185996137	HDDETS	1187179912	1212182931	1191632007	0.099	2.208	0.475	1553.481	0.624
Tai150b	498896643	HDDETS	501892435	508173332	505261057	0.6	1.859	1.275	9402.76	0.442
Tai256c	44759294	HDDETS	44786418	44838798	44813276	0.06	0.177	0.12	41014.57	0.041
Esc16a	68	HDDETS	68	68	68	0	0	0	0.533	0
Esc16b	292	HDDETS	292	292	292	0	0	0	0.634	0
Esc16c	160	HDDETS	160	160	160	0	0	0	0.577	0
Esc16d	16	HDDETS	16	16	16	0	0	0	0.532	0
Esc16e	28	HDDETS	28	28	28	0	0	0	0.55	0
Esc16f	0	HDDETS	0	0	0	0	0	0	0.426	0
Esc16g	26	HDDETS	26	26	26	0	0	0	0.472	0
Esc16h	996	HDDETS	996	996	996	0	0	0	0.473	0
Esc16i	14	HDDETS	14	14	14	0	0	0	0.629	0
Esc16j	8	HDDETS	8	8	8	0	0	0	0.737	0
Esc32a	130	HDDETS	130	130	130	0	0	0	7.953	0
Esc32b	168	HDDETS	168	168	168	0	0	0	1.924	0
Esc32c	642	HDDETS	642	642	642	0	0	0	2.276	0
Esc32d	200	HDDETS	200	200	200	0	0	0	2.184	0
Esc32e	2	HDDETS	2	2	2	0	0	0	1.835	0
Esc32g	6	HDDETS	6	6	6	0	0	0	1.907	0
Esc32h	438	HDDETS	438	438	438	0	0	0	1.896	0
Esc64a	116	HDDETS	116	116	116	0	0	0	9.927	0
Esc128a	64	HDDETS	64	64	64	0	0	0	55.614	0

Table 3

Results of the HDETS algorithm for some instances from QAPLIB (Continued)

Name of problem	Solution in QAPLIB	Proposed algorithm	Best Solution	Worst Solution	Average Solution	Best Gap	Worst Gap	Average Gap	Time (seconds)	Stander Division
Lipa20a	3683	HDETS	3683	3683	3683	0	0	0	1.351	0
Lipa20b	27076	HDETS	27076	27076	27076	0	0	0	1.011	0
Lipa30a	131178	HDETS	131178	131178	131178	0	0	0	9.965	0
Lipa30b	151426	HDETS	151426	151426	151426	0	0	0	5.141	0
Lipa40a	31538	HDETS	31538	31844	31684	0	0.97	0.461	16.246	0.487
Lipa40b	476581	HDETS	476581	476581	476581	0	0	0	6.946	0
Lipa50a	62093	HDETS	62093	62629	62451	0	0.863	0.576	42.387	0.398
Lipa50b	1210244	HDETS	1210244	1210244	1210244	0	0	0	32.891	0
Lipa60a	107218	HDETS	107897	108019	107959	0.633	0.747	0.69	463.998	0.034
Lipa60b	2520135	HDETS	2520135	2969956	2742733	0	17.849	8.832	433.066	9.311
Lipa70a	169755	HDETS	170787	170858	170824	0.607	0.649	0.629	1056.769	0.014
Lipa70b	4603200	HDETS	4603200	5475784	5285704	0	18.956	14.8267	470.74	7.818
Lipa80a	253195	HDETS	254506	254695	254590	0.517	0.592	0.551	719.781	0.023
Lipa80b	7763962	HDETS	7763962	9293826	9131465	0	19.7047	17.613	176.991	6.189
Lipa90a	360630	HDETS	362307	362601	362480	0.465	0.546	0.513	2000.489	0.026
Lipa90b	12490441	HDETS	12490441	15002587	14479768	0	20.112	15.926	1851.828	8.395
Had12	1652	HDETS	1652	1652	1652	0	0	0	0.784	0
Had14	2724	HDETS	2724	2724	2724	0	0	0	0.583	0
Had16	3720	HDETS	3720	3720	3720	0	0	0	0.437	0
Had18	5358	HDETS	5358	5358	5358	0	0	0	0.674	0
Had20	6922	HDETS	6922	6922	6922	0	0	0	0.837	0

Scenario 2:

All solutions for all cases mentioned in the database of QAP are divided into two types:

- Optimal Solution (OPT)
- Best Known Solution (BKS)

In this study, the number of instances that have the Optimal Solution is 77 instances and the number of instances that have the Best-Known Solution is 28 instances. An Optimal Solution can be obtained by the proposed algorithm in 73 instances out of 77 instances and it can produce Best Known Solution in 10 instances out of 28 instances. The first comparison was done in this study to evaluate the effectiveness of the proposed algorithm HDETS. The proposed algorithm was compared with TS and DDE. In Table 4, the results of the comparison are presented, and it can be observed from the results that the HDETS outperformed DDE and TS in all instances. Afterward, another comparison has been carried out between the proposed algorithm and another algorithm in the literature. Prior to the proposal of a hybrid algorithm in this study, a new approach called whale algorithm integrated with Tabu search for quadratic assignment problem (WAITS) had been introduced by (Abdel-Basset et al., 2018a). A comparison was done between the WAITS and the algorithm proposed in this study. Based on the outcome of the comparison, the performance of WAITS is better than that of other algorithms in terms of solving QAP. More so, it can produce an optimal solution for many instances of QAP.

Table 4 shows the comparison between our proposed HDETS and WAITS. The main contribution of this study is providing an improved solution for QAP, especially that which has not produced an optimal solution. For instance, in the case of (Tai50a, Tai80b, Tai100a, and Tai150b) the best gap of this instance was reached at (1.57 %, 1.20 %, 2.04 %, and 1.76 % respectively) compared with the solution in a dataset of QAP. By applying our proposed algorithm to solve the instance (Tai50a, Tai80b, Tai100a, and Tai150b) this gap was reduced to (0 %, 0 %, 1.146 %, and 0.6 % respectively). Table 4 shows our contribution in terms of providing improved solutions for QAP. In the instances of (Sko49, Sko56, Sko64, Sko72, Sko100b, and Sko100e), many researchers have developed several optimization methods

to improve the solutions of these instances so that they can reach the best or the same values within the database for QAP. So far, the best gap has been found for these cases by WAITS as follows: (0.13 %, 0.08 %, 0.07 %, 0.27 %, 0.74 %, and 0.76 %, respectively). Another contribution of the algorithm HDETS is enhancing the solutions of these instances; the results produced by HDETS were found to be better than those of WAITS. More so, HDETS reached the best gap of (0 %, 0 %, 0 %, 0.09 %, 0.18 %, and 0.124 %, respectively). Below are the figures (Figs. 3-9) that show the gaps obtained from the algorithms in Table 4. In Table 5 below, a summary of the comparison results between HDETS and WAITS is presented.

Table 4

Comparative results between DDE, TS, HDETS, and WAITS algorithms for QAP

No.	Problem	Best-Known Solution	DDE	TS	HDETS	WAITS
			Best gap	Best gap	Best gap	Best gap
1	Chr12a	9552	0	3.810	0	0
2	Chr12b	9742	0	0	0	0
3	Chr12c	11156	2.312	2.312	0	0
4	Chr15a	9896	0	9.256	0	0
5	Chr15b	7990	14.167	23.329	0	0
6	Chr18a	11098	27.967	26.872	0	0
7	Chr18b	1534	30.365	11.155	0	0
8	Chr20a	2192	1.825	7.561	0	0
9	Chr20b	2298	13.594	20.255	0	1.56
10	Chr20c	14142	15.665	13.838	0	0
11	Chr22a	6156	40.347	35.384	0	0.16
12	Chr22b	6194	9.096	8.219	0	0
13	Chr25a	3796	8.653	7.426	0	0
14	Esc16a	68	0	0	0	0
15	Esc16b	292	0	0	0	0
16	Esc16c	160	0	0	0	0
17	Esc16d	16	0	0	0	0
18	Esc16e	28	0	0	0	0
19	Esc16f	0	0	0	0	0
20	Esc16g	26	0	0	0	0
21	Esc16h	996	0	0	0	0
22	Esc16i	14	0	0	0	0
23	Esc16j	8	0	15.3846	0	0
24	Esc32a	130	20	14.2857	0	0
25	Esc32b	168	19.047	0	0	0
26	Esc32c	642	0	0	0	0
27	Esc32d	200	0	0	0	0
28	Esc32e	2	0	0	0	0
29	Esc32g	6	0	0.91324	0	0
30	Esc32h	438	0.913	0	0	0
31	Esc64a	116	0	0	0	0
32	Esc128a	64	34.375	0	0	0
33	Lipa20a	3683	1.710	2.1721	0	0
34	Lipa20b	27076	14.791	0	0	0
35	Lipa30a	131178	1.844	1.7529	0	0
36	Lipa30b	151426	15.766	15.7998	0	0
37	Lipa40a	31538	1.417	1.4554	0	0
38	Lipa40b	476581	19.009	18.2678	0	0
39	Lipa50a	62093	1.3673	1.3705	0	0
40	Lipa50b	1210244	19.278	19.2295	0	0
41	Lipa60a	107218	1.221	1.2759	0.633	0
42	Lipa60b	2520135	21.013	21.2654	0	0
43	Lipa70a	169755	1.122	1.1611	0.607	0
44	Lipa70b	4603200	22.022	22.1949	0	0
45	Lipa80a	253195	1.029	1.0861	0.517	0.55
46	Lipa80b	7763962	23.047	23.4897	0	0
47	Lipa90a	360630	0.963	1.0559	0.465	0.50
48	Lipa90b	12490441	23.243	24.0423	0	0

Table 4
Comparative results between DDE, TS, HDETS, and WAITS algorithms for QAP (Continued)

No.	Problem	Best-Known Solution	DDE	TS	HDETS	WAITS
			Best gap	Best gap	Best gap	Best gap
49	Nug12	578	1.73	2.422	0	0
50	Nug14	1014	2.366	0	0	0
51	Nug16a	1150	2.782	0.173	0	0
52	Nug16b	1610	2.608	0.993	0	0
53	Nug17	1240	3.225	1.774	0	0
54	Nug18	1930	0.923	1.732	0	0
55	Nug20	2570	0.310	0.932	0	0
56	Nug21	2438	1.400	1.4	0	0
57	Nug22	3596	1.230	2.297	0	0
58	Nug24	3488	1.724	0.166	0	0
59	Nug25	3744	2.216	2.867	0	0
60	Nug27	5234	1.442	1.121	0	0
61	Nug28	5166	1.528	3.248	0	0
62	Nug30	6124	3.832	4.065	0	0.52
63	Sko42	15812.0	2.567	4.3638	0	0
64	Sko49	23386	1.599	4.8833	0	0.13
65	Sko56	34458	2.704	5.2876	0	0.08
66	Sko64	48498	3.365	5.2909	0	0.07
67	Sko72	66256	3.595	6.7164	0.09	0.27
68	Sko81	90998	3.356	6.2815	0.068	0.19
69	Sko90	115534	3.661	7.1183	0.192	0.56
70	Sko100a	152002	3.326	7.2039	0.206	0.76
71	Sko100b	153890	3.184	6.6723	0.18	0.74
72	Sko100c	147862	3.907	7.3799	0.193	0.99
73	Sko100d	149576	3.866	7.3394	0.124	0.98
74	Sko100e	149150	3.886	7.3067	0.244	0.76
75	Sko100f	149036	3.616	6.899	0.454	0.95
76	Had12	1652	0.121	0	0	0
77	Had14	2724	0.22	0	0	0
78	Had16	3720	0.86	0	0	0
79	Had18	5358	0.298	0.074	0	0
80	Had20	6922	1.126	0.086	0	0
81	Tai12a	224416	0	3.842	0	0
82	Tai12b	39464925	2.8496	4.263	0	0
83	Tai15a	388214	2.043	0.16898	0	0
84	Tai15b	51765268	0.339	2.4024	0	0
85	Tai20a	491812	2.983	0.90165	0	0
86	Tai20b	703482	4.592	4.3505	0	0
87	Tai25a	122455319	1.743	1.6315	0	0
88	Tai25b	1167256	4.216	4.0909	0	0
89	Tai30a	344355646	2.039	6.5651	0	0.48
90	Tai30b	1818146	4.548	5.5332	0	0
91	Tai35a	637117113	3.502	4.0868	0	0.06
92	Tai35b	2422002	4.777	6.3761	0	0
93	Tai40a	3139370	2.157	0.092568	0	0.52
94	Tai40b	637250948	4.748	6.9324	0.065	0.005
95	Tai50a	637250948	0.0769	11.4704	0	1.57
96	Tai50b	4938796	5.246	12.7767	1.019	0.05
97	Tai60a	458821517	3.388	13.2447	0	1.93
98	Tai60b	7205962	4.609	1.974	0.047	0.74
99	Tai64c	1855928	0.4175	2.4024	0	0
100	Tai80a	13499184	5.665	0.90165	1.059	1.90
101	Tai80b	818415043	7.486	4.3505	0	1.20
102	Tai100a	21125314	5.743	1.6315	1.146	2.04
103	Tai100b	1185996137	7.116	4.0909	0.099	0.50
104	Tai150b	498896643	8.6079	6.5651	0.6	1.76
105	Tai256c	44759294	1.5643	2.1909	0.06	0.26

Below the figures which show the gaps obtained from the performance of the algorithms in Table 4.

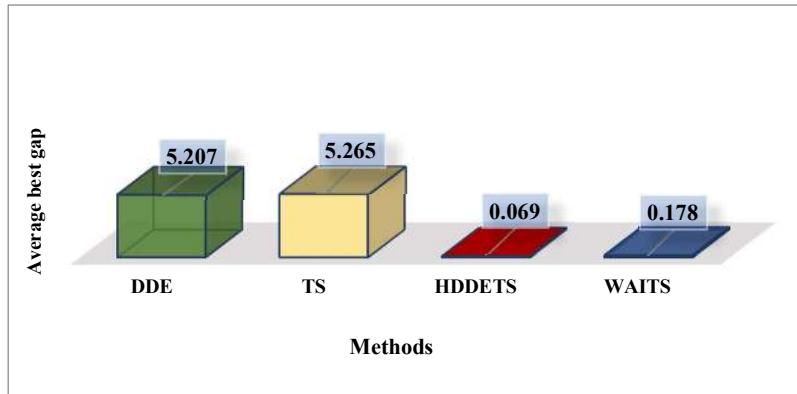


Fig. 2. comparison among DDE, TS, HDETS, and WAITS

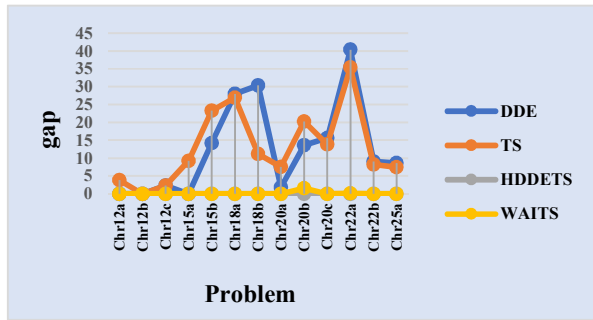


Fig. 3. Comparison on instance Chr

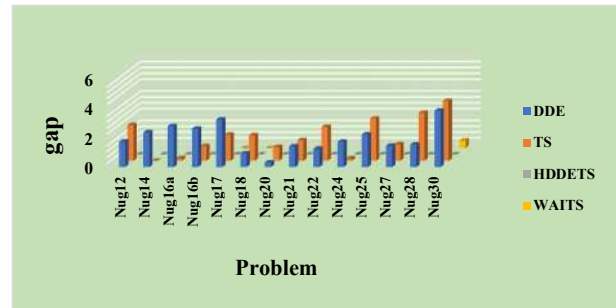


Fig. 4. Comparison on instance Nug

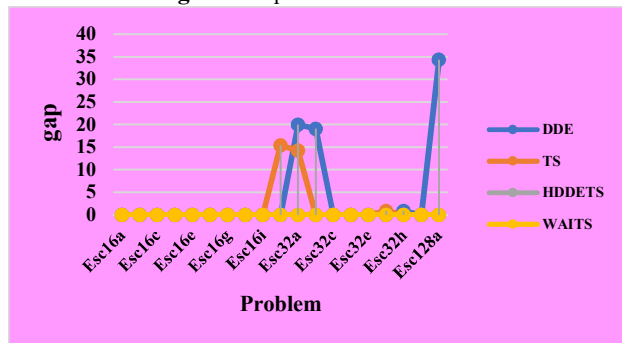


Fig. 5. Comparison on instance Esc

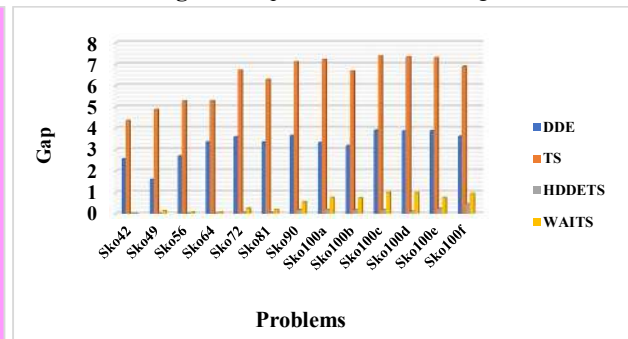


Fig. 6. Comparison on instance Sko

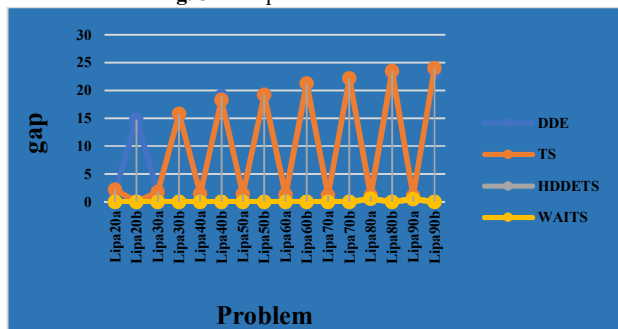


Fig. 7. Comparison on instance Lipa

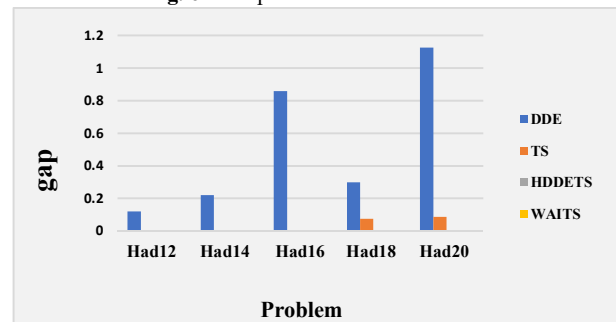


Fig. 8. Comparison on instance Lipa

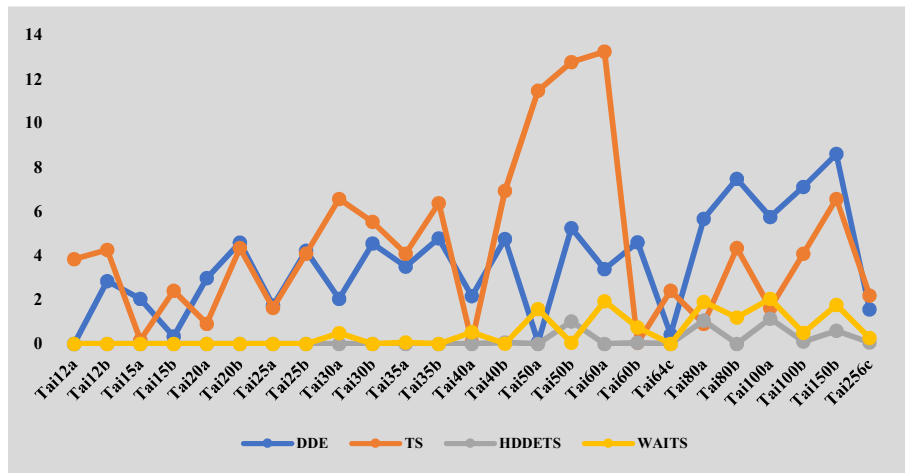


Fig. 9. Comparison on instance Tai

Table 5 presents a summary of the comparison results between HDETS and WAITS.

Table 5
Summary of the comparison results between HDETS and WAITS

Category	Name of Problem	Number of Instances	Type of Solution		HDETS		WAITS	
			OPT	BKS	OPT	BKS	OPT	BKS
1	Tai	25	10	15	10	7	8	3
2	Nug	14	14	-	14	-	13	-
3	Chr	13	13	-	13	-	11	-
4	Esc	19	19	-	19	-	19	-
5	Lipa	16	16	-	12	-	14	-
6	Had	5	5	-	5	-	5	-
7	Sko	13	-	13	-	3	-	1
Sum		105	77	28	73	10	70	4

Scenario 3:

In the next step, the effect and validation of the proposed algorithm HDETS are presented. This is achieved by comparing the proposed algorithm with other algorithms. The most robust and latest algorithms were used for the comparison. Table 6 shows the results of the comparison between HDETS and four other algorithms. Comparisons between HDETS and the following algorithms were done:

- Discrete Bat Algorithm (DBA) (Riffi et al., 2017)
- Development of modified discrete particle swarm (DPSO) (Pradeepmon, 2018)
- Biogeography-Based Optimization Algorithm Hybridized with Tabu Search (BBOTS) (Lim et al., 2016)
- A hybrid algorithm combining lexisearch and genetic algorithms (LSGA) (Ahmed, 2018)

For the compared cases in Table 6, the first comparison which was between HDETS and DBA, it was found that the DBA can reach the optimal solution for 35 out of 54 instances and reach to Best Known Solution for 5 out of 21 instances. While the HDETS has been solved 54 optimal solutions out of 54 instances, this implies that the gap of the best value found was 0 %. On another hand, it was observed that the HDETS can reach the Best-Known Solution for 12 out of 21 instances. The results obtained by the DBA algorithm are as follows: the optimal solution was achieved for (8 instances from case Bur out of 8 instances, 5 instances from the case Chr out of 5 instances, 10 instances from the case Esc out of 10 instances, 3 instances from the case Nug out of 15 instances, and 9 instances from the case Tai out of 10). For the best-known solution in case Tai, the DBA can reach 4 instances out of 14 instances, the best

value for the best average gap report is 0.872 %. HDETS can found 8 best-known solutions out of 14 instances with the best value is 0.333 % of the average gap. Next test for the best-known solution has been applied on a case Sko, the results show DBA found 1 best-known solution out for 7 instances with the best average gap value is 0.208 %. Whereas HDETS has been reached to 4 best-known solutions out for 7 instances and the best average gap is 0.05 %.

The next comparison was between HDETS and DPSO; DPSO has been tested on 23 instances of QAP which produced optimal solutions. The results have shown that one optimal solution was found, and the results recorded the best value for an average gap for the rest of the instances at 0.618 %. When the HDETS was applied to these instances, it was found that HDETS has the capability of improving all the 23 instances, while reducing the gap to 0 % for all these instances. Similarly, the proposed algorithm has been compared with BBOTS, and this algorithm was applied in 5 cases (Bur, Chr, Esc, Nug, and Tai) of QAP. The results of these comparisons are as follows: in the case of Bur, the best value of the average gap was found to be 0.003 %. On the other hand, results obtained from the proposed algorithm HDETS achieved an average gap of 0 %. For cases Chr, the difference between the results was obvious, where the performance of HDETS was better than BBOTS; the average gap obtained by HDETS was 0.185 %, while the best average gap was 0 %. The results of the comparison were equal to an average gap for both BBOTS and HDETS algorithms in case Esc. For the case of Nug, the results for BBOTS in terms of the best value for the average gap was 0.019 %, while it was found that the HDETS can lower the average gap to 0 %. On the other hand, for instances (tai12a, tai15a, tai17a, tai20a, tai30a, and tai80a) the BBOTS algorithm was used to solve these cases, and the average gap of 0.892 % was achieved, while the use of HDETS to solve these instances enhanced the reduction of the best average rate to 0 %.

Table 6
Comparison among DBA, DPSO, BBOTS, and HDETS

No.	Problem	Type of Solve		DBA		DPSO		BBOTS		HDETS	
		OPT	BKS	Best Solve	Gap	Best Solve	Gap	Best Solve	Gap	Best Solve	Gap
1	bur26a	5,426,670	-	5,426,670	0	5434783	0.150	5426670	0.028	5,426,670	0
2	bur26b	3,817,852	-	3,817,852	0	3824420	0.172	3817852	0	3,817,852	0
3	bur26c	5,426,795	-	5,426,795	0	5428396	0.030	5426795	0	5,426,795	0
4	bur26d	3,821,225	-	3,821,225	0	3821419	0.005	3821225	0	3,821,225	0
5	bur26e	5,386,879	-	5,386,879	0	5387320	0.008	5386879	0	5,386,879	0
6	bur26f	3,782,044	-	3,782,044	0	3783123	0.029	3782044	0	3,782,044	0
7	bur26g	10,117,172	-	10,117,172	0	10118542	0.014	10117172	0	10,117,172	0
8	bur26h	7,098,658	-	7,098,658	0	7099677	0.014	7098658	0	7,098,658	0
9	chr12a	9552	-	9552	0	-	-	9552	0	9552	0
10	chr12b	9742	-	7990	-	-	-	9742	0	9742	0
11	chr12c	11156	-	-	-	-	-	11156	0	11156	0
12	chr15a	9896	-	-	-	-	-	9896	0	9896	0
13	chr15b	7990	-	-	0	-	-	7990	0.298	7990	0
14	chr15c	9504	-	-	-	-	-	9504	0	9504	0
15	chr18a	11098	-	11,098	0	-	-	11098	0.079	11098	0
16	chr18b	1534	-	-	-	-	-	1534	0	1534	0
17	chr20a	2192	-	-	-	-	-	2192	0.876	2192	0
18	chr20c	14142	-	14,142	0	-	-	14142	0.604	14142	0
19	chr25a	3796	-	3796	0	-	-	-	-	3796	0
20	esc16a	68	-	68	0	-	-	68	0	68	0
21	esc16b	292	-	292	0	-	-	292	0	292	0
22	esc16c	160	-	160	0	-	-	160	0	160	0
23	esc16d	16	-	16	0	-	-	-	-	16	0
24	esc16e	28	-	28	0	-	-	-	-	28	0
25	esc16f	0	-	0	0	-	-	-	-	0	0
26	esc32a	130	-	130	0	-	-	-	-	130	0
27	esc32e	2	-	2	0	-	-	-	-	2	0
28	esc32g	6	-	6	0	-	-	-	-	6	0
29	esc64a	116	-	116	0	-	-	-	-	116	0

Table 6
Comparison among DBA, DPSO, BBOTS, and HDETS (Continued)

No.	Problem	Type of Solve		DBA		DPSO		BBOTS		HDETS	
		OPT	BKS	Best Solve	Gap	Best Solve	Gap	Best Solve	Gap	Best Solve	Gap
30	nug12	578	-	-	-	582	0.692	578	0	578	0
31	nug14	1014	-	-	-	1016	0.197	1014	0	1014	0
32	nug15	1150	-	-	-	1164	1.217	1150	0	1150	0
33	nug16a	1610	-	-	-	1630	1.242	1610	0	1610	0
34	nug16b	1240	-	-	-	1240	0.000	1240	0	1240	0
35	nug17	1732	-	-	-	1750	1.039	1732	0.012	1732	0
36	nug18	1930	-	-	-	1936	0.311	1930	0	1930	0
37	nug20	2570	-	2570	0	2570	0	2570	0	2570	0
38	nug21	2438	-	2438	0	2444	0.246	2438	0	2438	0
39	nug22	3596	-	-	-	3602	0.167	3596	0	3596	0
40	nug24	3488	-	-	-	3578	2.580	3488	0	3488	0
41	nug25	3744	-	-	-	3766	0.588	3744	0	3744	0
42	nug27	5234	-	-	-	5294	1.146	5234	0	5234	0
43	nug28	5166	-	-	-	5228	1.200	5166	0.209	5166	0
44	nug30	6124	-	6124	0	6206	1.339	6124	0.065	6124	0
45	tai12a	224,416	-	224,416	0	-	-	224416	0	224416	0
46	tai12b	39,464,925	-	39,464,925	0	-	-	-	-	39464925	0
47	tai15a	388,214	-	388,214	0	-	-	388214	0	388214	0
48	tai15b	51,765,268	-	51,765,268	0	-	-	-	-	51765268	0
49	tai17a	491,812	-	491,812	0	-	-	491812	0.093	491812	0
50	tai20a	703,482	-	703,482	0	-	-	705622	0.677	703482	0
51	tai20b	122,455,319	-	122,455,319	0	-	-	-	-	122455319	0
52	tai25a	1,167,256	-	1,172,754	0.47	-	-	-	-	1167256	0
53	tai25b	344,355,646	-	344,355,646	0	-	-	-	-	344355646	0
54	tai30a	-	1,818,146	1,831,272	0.72	-	-	1843224	1.795	1818146	0
55	tai30b	637,117,113	-	637,117,113	0	-	-	-	-	637117113	0
56	tai35a	-	2,422,002	2,438,440	0.67	-	-	-	-	2422002	0
57	tai35b	-	283,315,445	283,315,445	0	-	-	-	-	283315445	0
58	tai40a	-	3,139,370	3,139,370	1.3	-	-	-	-	3150391	0
59	tai40b	-	637,250,948	637,250,948	0	-	-	-	-	637250948	0.065
60	tai50a	-	4,938,796	5,042,654	2.10	-	-	-	-	4965748	0
61	tai50b	-	458,821,517	458,830,119	0	-	-	-	-	458821517	1.019
62	tai60a	-	7,205,962	7,387,482	2.5	-	-	-	-	7266970	0
63	tai60b	-	608,215,054	608,414,385	0.03	-	-	-	-	1855928	1.05
64	tai64c	-	1,855,928	1,855,928	0	-	-	-	-	13616880	0
65	tai80a	-	13,499,184	13,810,130	2.30	-	-	13841214	2.788	818415043	1.059
66	tai80b	-	818,415,043	819,367,807	0.11	-	-	-	-	818415043	0
67	tai100a	-	21,052,466	21,541,326	2.3	-	-	-	-	21285950	1.146
68	tai100b	-	1185996137	1188168753	0.18	-	-	-	-	1187179912	0.099
69	sko42	-	15,812	15,812	0	-	-	-	-	15812	0
70	sko49	-	23,386	23,421	0.14	-	-	-	-	23386	0
71	sko56	-	34,458	34,524	0.19	-	-	-	-	34458	0
72	sko64	-	48,498	48,656	0.32	-	-	-	-	48498	0
73	sko72	-	66,256	66,422	0.25	-	-	-	-	66256	0.09
74	sko81	-	90,998	91,252	0.27	-	-	-	-	91008	0.068
75	sko90	-	115,534	115,874	0.29	-	-	-	-	115578	0.192

Finally, the performance of the proposed algorithm HDETS was compared with another algorithm contained in the literature review of this study. This algorithm is a hybrid algorithm which is a combination of lexisearch and genetic algorithms (LSGA) proposed by (Ahmed, 2018). The results of this comparison have been presented in table 7. It was found that in the instances (Tai20a, Tai30a, Tai40a, Tai50a, Tai60a, Tai80a, Tai100a, Tai20b, Tai30b, Tai40b, Tai50b, Tai60b, Tai80b, Tai100b, Tai150b), the LSGA algorithm was able to solve this case with the best value of average gap of 0.665 %, while the proposed algorithm HDETS reduced this value to 0.004 %. On the other hand, the LSGA algorithm solved the instances (sko42, sko49, sko81, sko90, sko100a, sko100d), and the algorithm was able to find the best average gap which was 0.191%, while the HDETS reinforced the solutions of these instances and it obtained an average gap of 0.093 % for these instances. Below Fig. 11 and Fig. 12 show the best gaps obtained from the performance of the algorithms in Table 7.

Table 7
Comparison among LSGA and HDEETS

Instance	BKS	LSGA		HDEETS		Instance	BKS	LSGA		HDEETS	
		Gap %	Gap %	Gap %	Gap %			Gap %	Gap %		
Tai20a	703,482	0.48	0	sko42	15,812	0	0				
Tai30a	1,818,146	1.06	0	sko49	23,386	0.14	0				
Tai40a	3,139,370	1.62	0	sko81	90,998	0.1	0.068				
Tai50a	4,938,796	1.49	0	sko90	115,534	0.33	0.192				
Tai60a	7,205,962	1.53	0	sko100a	152,002	0.26	0.206				
Tai80a	13,499,184	1.53	0	-	-	-	-				
Tai100a	21,052,466	1.53	0	-	-	-	-				
Tai20b	122,455,319	0	0	-	-	-	-				
Tai30b	637,117,113	0	0	-	-	-	-				
Tai40b	637,250,948	0	0	-	-	-	-				
Tai50b	458,821,517	0	0	-	-	-	-				
Tai60b	608,215,054	0	0	-	-	-	-				
Tai80b	818,415,043	0.01	0	-	-	-	-				
Tai100b	1,185,996,137	0.01	0.065	-	-	-	-				
Tai150b	498,896,643	0.72	0	-	-	-	-				
AVERAGE gap		0.665	0.004			0.191	0.093				

Below Figures have been shown the gaps obtained from the performance of the algorithms in Table 7.

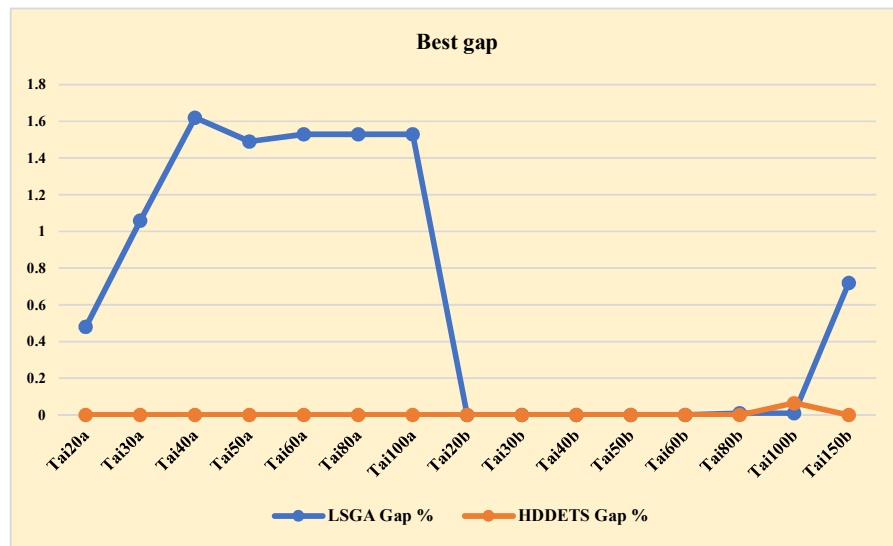


Fig. 10. Comparative study 1 between LSGA and HDEETS

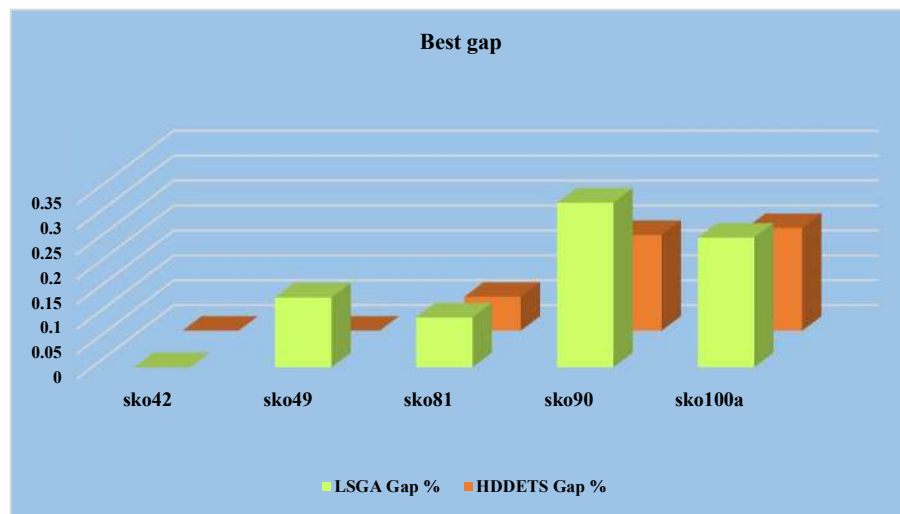


Fig. 11. Comparative study 2 between LSGA and HDEETS

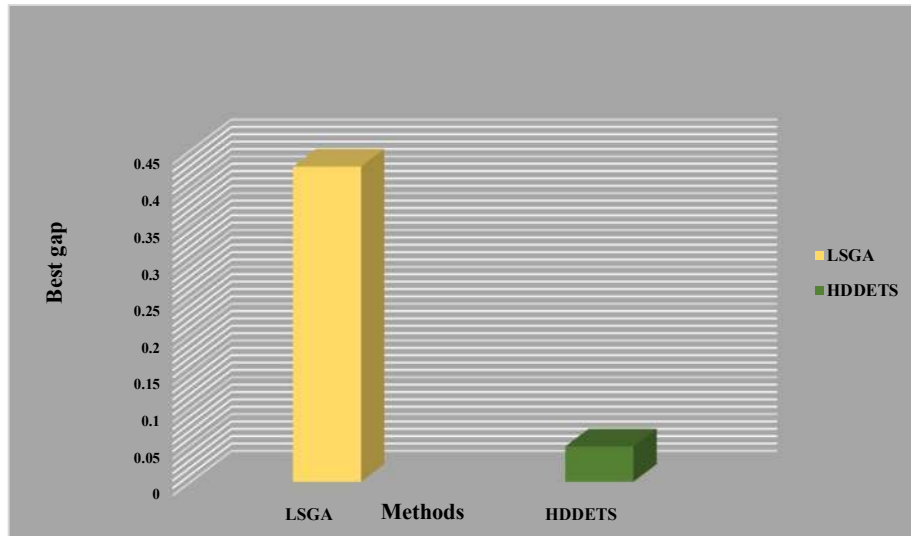


Fig. 12. Comparison on Average best for QAP

6. Conclusion

In this paper, a Discrete Differential Evolution algorithm hybrid with Tabu Search HDETS has been proposed with the aim of enhancing the solution of QAP. The limitation of the standard Discrete Differential Evolution algorithm is the low level of accuracy of solutions obtained for QAP problems, and this limitation has been alleviated by the proposed approach. The comparative results have shown that HDETS algorithm outperforms the classic DDE and TS. The HDETS algorithm has enhanced the solutions of QAP. Seven different case studies including 105 instances have been tested and used in analyzing the performance of the proposed HDETS. The effect of the HDETS algorithm on improving solutions was clear and has been discussed in the results and discussions section of this paper. The results showed the contribution of HDETS to improving solutions of QAP. The HDETS produced 73 optimal solutions out of 77 and has reached up to 10 best-known solutions out of 28. These are the best values obtained by the HDETS compared to other recently proposed algorithms in the literature review in this paper. It is recommended that future research focus on the application of HDETS algorithm in a real-world application such as Campus Layout or Hospital Layout. Another future work can focus on applying our proposed algorithm in other combinatorial optimization problems such as scheduling models or vehicle routing problem.

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