



A new IMM algorithm using fixed coefficients filters (fastIMM)

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ABSTRACT

In this paper we present a new approach based on two filters $\alpha\beta$ and $\alpha\beta\gamma$ using Interacting Multiples Models (IMM) design instead of a Kalman filter second and third order for the tracking a single maneuver target. The comparison between the proposed filter and the IMM improves the computing time amount about 60% while having a high accuracy.

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1. Introduction

Target tracking and predicting is realized in track while scan systems, which are sampled data filters, based on previously observed positions containing measurement noise. The performance of these filters is function of their noise smoothing behavior and their transient system response. A filter developed in the mid 1950s, the $\alpha\beta$ tracker, is popular because of its simplicity and consequently inexpensive computational requirements. This permits its use in limited power capacity applications' like passive sono-buoys. The $\alpha\beta$ filter performance has been analyzed by Sklansky [1].

Particularly in radar applications, having a single model to capture the dynamics of a system (target) is not enough, and therefore algorithms based on several models (modes) may be necessary for tracking the behavior of un-predictable target. The Interacting Multiples Models (IMM) is such an algorithm. In this algorithm several filters are run in parallel, each filter matching a specific model for the target's dynamic. A particularity of the IMM is that these models interact. The state estimates and their covariances, obtained from different filters, are computed and combined to form the overall state estimate and its covariance. To reduce the complexity, the filters used in the proposed IMM algorithm are the $\alpha\beta$ and $\alpha\beta\gamma$ filters. We here after describe these filters before showing how they are incorporated into the IMM algorithm.

2. $\alpha\beta$ and $\alpha\beta\gamma$ filter

2.1. $\alpha\beta$ filter

The $\alpha\beta$ filter is probably the most extensively applied fixed coefficient filter. It may be viewed as the steady state second order Kalman filter. This filter is defined by the following [2,3]:

$$\hat{x}(k) = x_p(k) + \alpha(x_0(k) - x_p(k)) \quad (1)$$

$$\hat{v}(k) = \hat{v}(k-1) + \frac{\beta}{T}(x_0(k) - x_p(k)) \quad (2)$$

$$x_p(k+1) = \hat{x}(k) + T\hat{v}(k) \quad (3)$$

where $\hat{x}(k)$ is the coordinate of the smoothed (estimated) target's position, $x_0(k)$ is the coordinate of the measured target's position at the k th scan, $x_p(k)$ is the coordinate of the predicted target's position at the k th scan, $\hat{v}(k)$ is the smoothed target's velocity at the k th scan, T is the radar scan time or the sample interval, and α, β are the fixed coefficients filter parameters. Finally, the usual initialization procedure is

$$x_p(1) = \hat{x}(0) \quad \text{and} \quad \hat{v}(0) = 0 \quad (4)$$

and

$$\hat{v}(1) = \frac{[\hat{x}(1) - \hat{x}(0)]}{T} \quad (5)$$

According to [4], the $\alpha\beta$ estimator is optimal the two coefficients α, β verify the following equation:

$$\beta = \frac{\alpha^2}{2 - \alpha} \quad (6)$$

Further details on the steady state of the second order Kalman filter can be found in [5].

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2.2. $\alpha\beta\gamma$ filter

The $\alpha\beta\gamma$ filter is also a fixed coefficients filter. It is the steady state of the third order Kalman filter. This filter is defined by the following [2,3]:

$$\hat{x}(k) = x_p(k) + \alpha(x_0(k) - x_p(k)) \quad (7)$$

$$\hat{v}(k) = \hat{v}(k-1) + \frac{\beta}{T}(x_0(k) - x_p(k)) \quad (8)$$

$$\hat{a}(k) = \hat{a}(k-1) + \frac{\gamma}{2T^2}(x_0(k) - x_p(k)) \quad (9)$$

$$x_p(k+1) = \hat{x}(k) + T\hat{v}(k) + \frac{1}{2}T^2\hat{a}(k) \quad (10)$$

where $\hat{a}(k)$ represent the smoothed target's acceleration, γ an additional parameter of the filter, and the other quantities are defined previously.

Further details on the steady state of the third order Kalman filter have been reported in [3].

2.3. Computation of the coefficients α, β and α, β, γ

According to [3], the $\alpha\beta\gamma$ coefficients are function of the target maneuvering index defined by

$$\lambda = \left(\frac{\sigma_v \cdot T^2}{\sigma_w} \right) \quad (11)$$

where σ_v is the standard deviation of the system noise and σ_w is the standard deviation of the measurement noise. The α, β coefficients can be calculated using the following equations [3]:

$$\beta = (\lambda^2 + 4\lambda - \lambda \sqrt{\lambda^2 + 6\lambda})/4 \quad (12)$$

$$\alpha = -(\lambda^2 + 8\lambda - (\lambda + 4)\sqrt{\lambda^2 + 8\lambda})/8 \quad (13)$$

while the α, β, γ coefficients are computed using the following equations:

$$\alpha = 1 - s^2$$

$$\beta = 2(1 - s)^2$$

$$\gamma = 2\lambda s \quad (14)$$

where

$$s = z - \left(\frac{p}{3z} \right) - \frac{b}{3} \quad (15)$$

In the previous equation

$$b = \frac{\lambda}{2} - 3$$

$$p = c - \frac{b^2}{3} \quad (16)$$

$$z = -\sqrt[3]{-q - \sqrt{\left(q^2 + \frac{4p^2}{27}\right)}}/2 \quad (17)$$

With

$$c = \frac{\lambda}{2} + 3$$

and

$$q = \frac{2b^3}{27} - \frac{bc}{3} - 1$$

3. IMM and FastIMM Algorithm

In the IMM algorithm a number of models are used to describe the dynamic of a target. The overall target state estimate is the combination of all the state estimates obtained from the filters matched to different models. The probability of transition between different models is assumed to be governed by a Markov chain. The diagram of the IMM is shown in Fig. 1 for a number of models equal to two. In the standard IMM filter, the filters N1 and N2 are usually chosen to be a second order and a third order Kalman filters, respectively [3,5]. The first one is suitable for a target that moves at a nearly constant velocity, while the second one is appropriate for a target that manoeuvres [6,7]. In the proposed algorithm, we have replaced these filters by their steady states filters, the $\alpha\beta$ and the $\alpha\beta\gamma$ filters.

Step 1: Calculation of the probabilities for models mixing. The probability that model i was effective at instant $k-1$, given that model j is effective at instant k and conditioned on measurements received until time $k-1$ is calculated from:

$$\begin{aligned} \mu_{i/j}(k-1|k-1) &= P(M_i(k-1)|M_j(k), Z^{k-1}) \\ &= \frac{1}{\bar{c}_j} P(M_j(k)|M_i(k-1), Z^{k-1}) P(M_i(k-1)|Z^{k-1}) \\ &= \frac{1}{\bar{c}_j} p_{ij} \mu_i(k-1), \quad i, j = 1, \dots, r \end{aligned} \quad (18)$$

where p_{ij} is the a priori probability of transition from model i to model j , $\mu_i(k-1)$ is the probability that model i is effective at the time $k-1$ and \bar{c}_j is a normalization constant calculated from:

$$\bar{c}_j = \sum_{i=1}^r p_{ij} \mu_i(k-1), \quad j = 1, \dots, r \quad (19)$$

with r representing the number of models in interaction.

Step 2: Mixing of the model conditioned estimates. The model conditioned state estimates are mixed as follows, to yield the initial state for filter j :

$$\hat{x}^{0j}(k-1|k-1) = \sum_{i=1}^r \hat{x}^i(k-1|k-1) \mu_{i/j}(k-1|k-1) \quad (20)$$

Step 3: Mode conditioned state estimation. Using the initial estimated state and the validated measurements as input, the mode conditioned state estimates are calculated via the $\alpha\beta$ filter for $j=1$ and the $\alpha\beta\gamma$ filter for $j=2$.

Step 4: Likelihood function computation. The likelihood function $\mathcal{A}^j(k)$ of model $M_j(k)$ is calculated using the following equations:

$$\begin{aligned} \mathcal{A}^j(k) &= P[z(k)|M_j(k), Z^{k-1}] = P[z(k)|M_j(k), \hat{x}^{0j}(k-1|k-1), \hat{P}^{0j}(k-1|k-1)] \\ &= \frac{1}{\sqrt{|2\pi S^j(k)|}} \exp(-0.5(v^j(k))^T (S^j(k))^{-1} v^j(k)) \end{aligned} \quad (21)$$

In the above equation

$$v(k) = z(k) - H^j(k) \hat{x}^{0j}(k-1|k-1) \quad (22)$$

is the innovation of the filter matched to the model $M_j(k)$ and $S^j(k)$ is its covariance given by

$$S^j = \frac{\sigma_w^2}{(1 - \alpha_j)} \quad (23)$$

In which σ_w^2 is the measurement variance and α_j is the coefficient α of the fixed coefficients filter j .

Step 5: Updating the models probabilities. The probability of the model $M_j(k)$ at instant k is computed using the

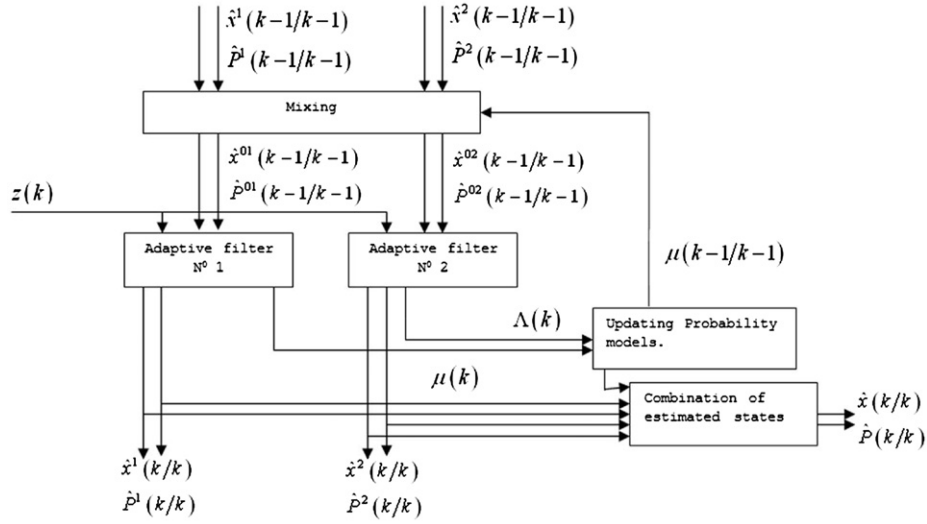


Fig. 1. The filter interacting multiples models.

following equation:

$$\mu_j(k) = P(M_j(k)|Z^k) = \frac{1}{c} \Lambda^j(k) \bar{c}_j$$

with $j = 1, \dots, r$ (24)

where \bar{c}_j is defined in Eq. (19) and c is the normalization constant defined as

$$c = \sum_{j=1}^r \Lambda^j(k) \bar{c}_j$$

(25)

Step 6: Combining the estimates conditioned on different models.
The estimated states, based on different models, are combined to yield the overall state estimate $\hat{x}(k|k)$, according to

$$\hat{x}(k|k) = \sum_{j=1}^r \hat{x}^j(k|k) \mu_j(k)$$

(26)

4. Simulation results

To test the performance of the proposed algorithm, we use the generic ATC tracking problem, defined in [8]. The radar, located at $[0, 0]$ m, provides direct position only measurements (after polar-to-Cartesian conversion) with RMS errors of 100 m in each of the two Cartesian coordinates. The interval between samples is $T = 5$ s. In the scenario under consideration, starting from $[25\,000, 10\,000]$ m at time $t = 0$ s, the aircraft flies westward for 125 s at 120 m/s, before executing a $1^\circ/\text{s}$ coordinates turn (which amounts to an acceleration of $0.2g$ at this speed) for 90 s, then it flies southward for another 125 s, followed by a $3^\circ/\text{s}$ turn (an acceleration of $0.6g$ at this speed) for 30 s. After the turn, it continues to fly westward at a constant velocity. The target trajectory is shown in Fig. 2. This scenario lends to a maneuvering index that is quite high (almost 1.5) and thus very little noise reduction can be achieved by a single model based state estimator, which will have to be, by necessity, conservative, since designed for maximum acceleration. To generate the measurements a Gaussian white noise with a standard deviation equal to 100 m has been added to the Cartesian coordinates of the target's position. The performances of the proposed algorithm are compared to those of the standard IMM estimator using a second order linear kinematic model with a

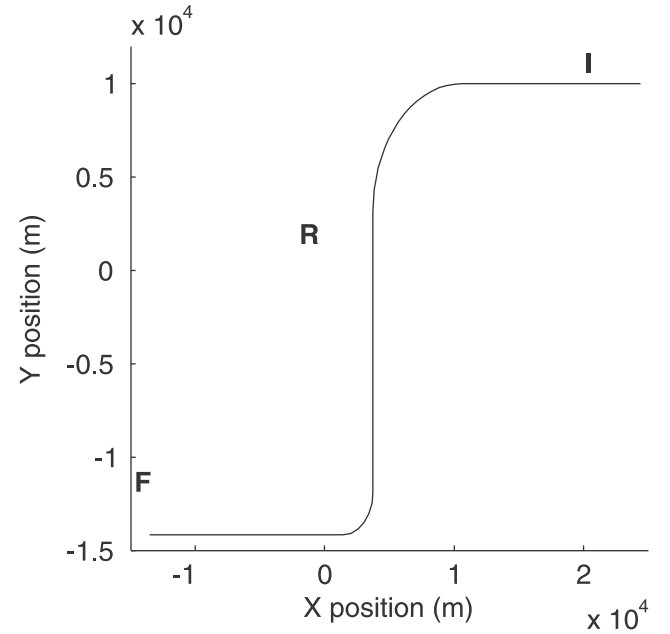


Fig. 2. Target trajectory (I: initial point, F: final point, R: radar location).

lower process noise level (0.1 m/s^2 to model the uniform motion and a third kinematic model with a 0.6 m/s^2 for the process noise standard deviation to handle the maneuvers. The following mode transition probability matrix was used in both, the proposed and standard IMMs:

$$\pi = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

The values of the parameters σ_v and σ_w that intervene in the calculation of the coefficients α, β , through the maneuvering index, were chosen to be 0.1 m/s^2 and 100 m.

In Fig. 3, the combined root mean square error in position, obtained over 1000 Monte-Carlo runs is displayed. We can observe that the filtering quality of the FastIMM during uniform motion is better than that of the standard IMM filter and that the filtering quality of both filters are similar during maneuvering

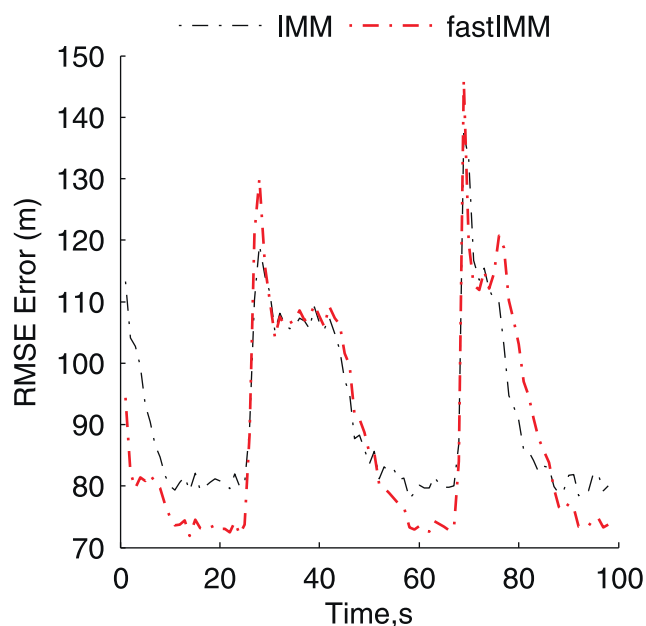


Fig. 3. Comparison between the RMSE in position of the standard IMM and FastIMM for $T = 5$ s.

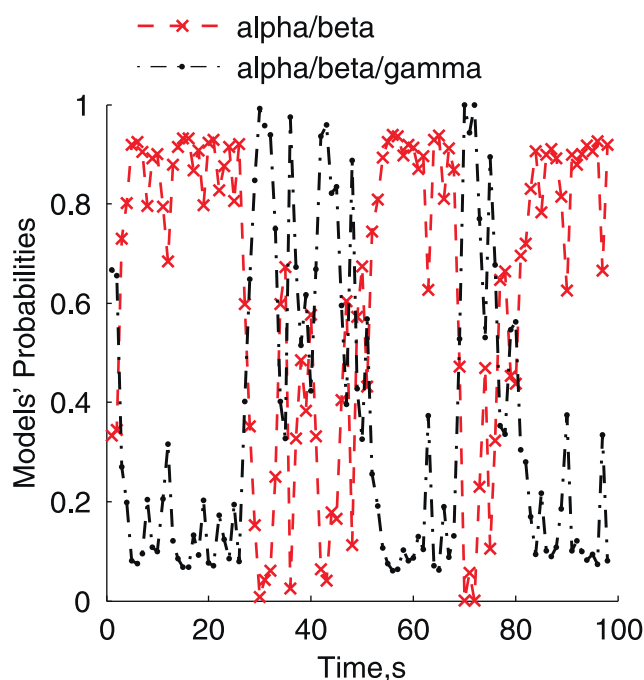


Fig. 5. Models' probabilities evolution for FastIMM filter.

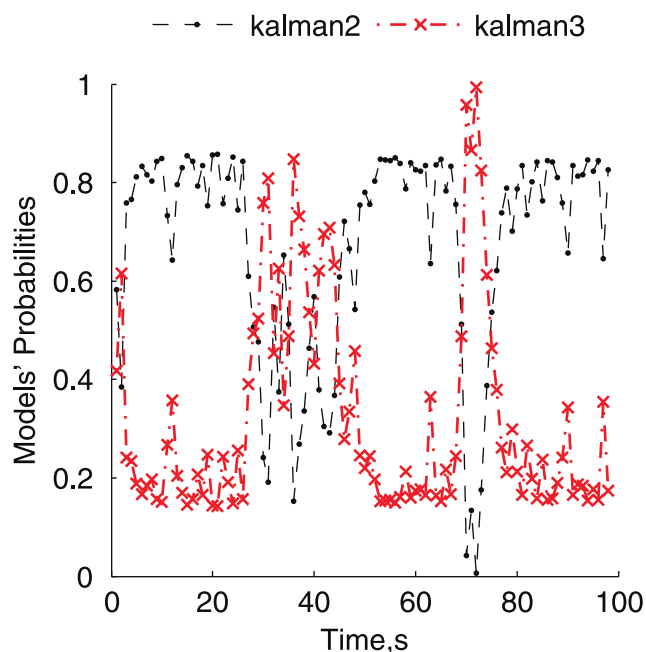


Fig. 4. Models' probabilities evolution for the standard IMM filter.

segments, except at the end of the last one, where the standard IMM outperforms the proposed FastIMM.

Figs. 4 and 5 plot the evolution of the models probabilities for the IMM and the FastIMM. It can be observed from these figures that the difference between these probabilities is more pronounced in the FastIMM.

To complete our analysis we have conducted another simulation of 1000 Monte-Carlo runs with an update time reduced to 3 s. The obtained RMSE in position are presented in Fig. 6. They show as expected, that an improvement in performance is obtained when the update time is reduced. They also show that the

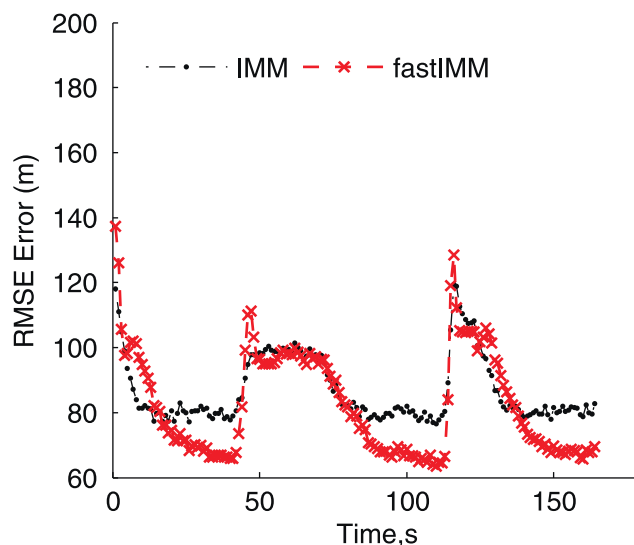


Fig. 6. Comparison between the RMSE in position of the standard IMM and the FastIMM for $T = 3$ s.

difference in performance between the standard IMM and the FastIMM is smaller when the update time is reduced.

In the proposed (FastIMM) algorithm, the computation of the gain is not necessary in both filters, second and third order Kalman filters, hence the step of the matrix inversion computation is eliminated in each run of the algorithm, reducing considerably the computation load. In order to evaluate the performance of the proposed algorithm in terms of complexity, we have used the execution time given by the Matlab software; we have noticed that the FastIMM reduce de execution time by amount of approximately 60% while having a high accuracy. we have also observed that this is scenario independent, to some extent.

5. Conclusion

In this paper, we have proposed a new design of the IMM filter for tracking a single maneuvering target, named FastIMM. In this algorithm, the second and the third order Kalman filters are replaced by their steady state filters, the $\alpha\beta$ and $\alpha\beta\gamma$ filters that were suitably designed. This algorithm not only greatly decreases the computational burden but also keeps a high accuracy. These are promising results for the application of the proposed algorithm in real time.

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