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Research Article

A New Insight to Vibration Characteristics of Spans under Random Moving Load: Case Study of 38 Bridges in Ho Chi Minh City, Vietnam

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We propose a novel representative power spectrum density as a specific characteristic for showing responses of spans during a long operational period. The idea behind this method is to use the representative power spectrum density as a powerful tool to evaluate the stiffness decline of spans during their operation period. In addition, a new measurement method has been introduced to replace the traditional method of monitoring the health conditions of bridges through a periodic measurement technique. This helps to reduce costs when carrying out testing bridges. Besides, the proposed approach can be widely applied not only in Vietnam but also in many other underprivileged countries around the world. Obtained results show that, during the operational process of spans, there is not only a pure vibration evaluation such as bending vibration and torsion vibration tests but also a combination of various vibration types including bending-torsion vibration or high-level vibrations like first-mode bending and first-mode torsion. Depending on each type of structure and material properties, different types of vibrations will appear more or less during the operational process of spans under a random moving load. Furthermore, the representative power spectrum density is also suitable for evaluating and determining many different fundamental vibrations through the same measurement time as well as various measurement times.

1. Introduction

Vietnam is one of the countries having the highest economic development rate in the region as well as in the world. Meanwhile, the urbanization rate is also rapidly increasing in the southern region in general and Ho Chi Minh City in particular. According to statistical data from the Ministry of Industry and Trade (2009), there were over 95% domestic goods being transported by road. Therefore, it is extremely important and feasible to ensure that these bridge networks operate stably and in good conditions. Together with constructing more new bridges which generally cost thousands of billion Vietnam Dong every year, testing, fixing, and maintaining the existing constructions are exceptionally expensive for both urban management officials and the general public. It should also be emphasized that Vietnam has a densely interlaced river and canal network, leading to an undeniable fact that national traffic relies heavily on bridge networks [1]. Another notable issue that should be taken into consideration is that the majority of bridges were constructed years ago. In general, the bridge network in this city is currently in degradation or even very serious degradation. There is no denying that the degradation rate of bridge constructions is always in inverse proportion to the rate of economic growth, inevitably entailing the increasing demands for traffic and transportation of goods accordingly. Consequently, the pressing problem at present is that, besides constructing more new bridges, we should make great efforts to maintain and fix the integrity of existing ones. At the moment, the only basis for determining the priority level of repair plans is testing. The testing procedure is strictly regulated in accordance with TCVN (National Institute of Standards and Technology) [2–6]. According to this regulation, this kind of testing mainly depends on the static bearing strength condition of spans [5, 6]. Load is created based on the initial estimated load level that bridges have to bear. The bridges' capability of bearing load is illustrated through some measurement figures such as deflection of spans, deformation, impact coefficient, fundamental frequency, and moving level of some parts like abutment, pillow, and cylinder. This method determines the real values of mechanical parameters at the time of testing. If the testing takes place in accordance with prescribed cycle, the activity of evaluating the working capability and predicting the degradation progress can be controlled. However, one of the major drawbacks of the current procedure for testing bridges is that all the measurement data are collected on a static basis, except for fundamental frequency value and impact coefficient which are determined by creating a vibration condition through a pulse.

Nevertheless, in actual fact, the real working conditions of bridges are dynamic. Almost all motor vehicles moving on bridges can make them vibrate excluding pedestrians and bicycles. The vibration amplitude will be different compared to moving values when load is placed statically on bridges. Furthermore, the period between 2 testing times is quite far. Due to the fact that it is extremely expensive for testing activities, most medium-sized and small-sized bridges are only tested twice during their operational process. The first time is when bridges have just been completed and another time is when they are scheduled to be demolished for constructing a new one. As a result, the real condition of bridges between such two times is undefined, making it difficult for technical management agencies to make decisions on the time and specific parts for repairing or maintaining. Thenceforth, they can determine the appropriate technical methods for maintenance. Nonetheless, periodic testing activities for bridges have become less frequent since the national budget is constantly reduced. Consequently, managing staff or administrators need to propose appropriate testing methods for bridges with low costs in order to address the current issue of having little information about bridges' health. In the meantime, they should also recommend some methods for classifying damage levels of main load-bearing parts like abutment, pillow, and cylinder. The purpose is to concentrate appropriate expenditure on repairing bridges, which is an urgent need for Ho Chi Minh City at the moment as well as in the future. From the year 2000 onwards, there is a greater volume of studies regarding systems of structural health monitoring (SHM) of bridges which have been carried out to evaluate and monitor their working states as well as their responses during operational periods [7, 8]. The most striking features of these monitoring systems are that they will monitor the operational process of bridges 24 hours per day. Meanwhile, they also warn us about possible unexpected situations which may happen to bridges during the moving process of vehicles. However, during the process of building such these systems, the construction cost would ever be the biggest issue for each nation. If we implement these SHM systems with current funded expenses, only a very limited number of bridges can be installed. Consequently, this method cannot be applied for all bridges on a large scale, but for some major or priority ones, especially cross-sea bridges and cable-stayed bridges. There is an undeniable fact that medium and small bridges do not receive proper attention and care in monitoring and evaluating their quality. Therefore, the main objective of this article is to evaluate the mechanical response of bridges during their operational period and to figure out the relationship between hardness deterioration and other factors such as moving load and environmental elements. Consequently, the researchers have come up with the idea of using realistic vibration measurement data of spans as a parameter to evaluate their quality.

In this paper, we investigate the experimental vibration characteristics of bridges by a new method and propose a representative power spectral density approach. The biggest advantage of new monitoring and periodic inspection methods proposed, recognized by collecting data of bridges' responses during the practical moving process, is that it can detect the overloaded practical situations (situations that exceed the allowed bearing capacity). Furthermore, this method does not require much money invested in SHM systems like many major bridges that are currently using those systems. In addition, the new method can record the movements of mechanical parameters and collect the recorded figures, which then helps open up possibilities of figuring out locations where the bearing capacity of bridges deteriorates as well as the deterioration speed. This can help us make decisions and plans for testing the quality of projects and for repairing them in a proper manner. In years to come, this method can be applied for developing countries which have similar natural conditions like Vietnam, specifically Southeast Asian and African countries. In these regions, the quality of medium and small bridges is always monitored and evaluated in order to ensure the safety of travelers and vehicles. Besides, we propose using a representative power spectral density as an effective characteristic which demonstrates different constituent vibrations concurrently occurring on vibration spectrums of spans. The representative power spectral density characterizes the vibration process of spans during a certain measurement period. In order to see the changing process of responses of spans, we should examine the representative power spectral density through different measurement periods.

2. Theoretical Background

2.1. Basic Vibration Types. To most structures, spans are composed mainly of steel and concrete, in which the major part of bearing capacity is the steel structure inside the spans. Besides, spans usually have much greater length compared to their thickness; therefore, we often model structures of spans to steel beam types [9–13]. Consequently, they can show specific characteristics of mechanical responses which are totally similar to the theoretical beam simulation. This means that, during the operational process of spans, fundamental states of beams will be fully shown such as vertical-shaft vibration, bending vibration, torsional vibration, and bending-torsion vibration. Depending on the load-bearing capacity of beams at measured points, one certain type of vibration will be shown to a greater degree than the others, or all will be dominant. When spans are under the effect of direction x of vertical direction running along the length of spans shown in Figure 1, responses of bridges at that moment are longitudinal vibration. The natural frequencies of longitudinal axial vibration of shaft are shown in equation (1).

Natural frequency:

$$\omega_{\mathrm{Ln}} = \frac{n\pi}{l}\sqrt{\frac{E}{\rho}}, \tag{1}$$

where l is the length of shaft, E is the elastic module, ρ is the density of beams, and A is the section area.

Similar to the situations of vertical-shaft vibration of beams, the bending vibration of beams is caused by force of direction z (direction z), when the main force affecting beams is the cutting force on each cross section of structures. The chief impact of bending vibration is that such vibration often causes cutting force which will cut or break structures under vibration shown in Figure 2 and equation (2) is the natural frequencies of bending vibrating.

Natural frequency:

$$\omega_{\mathrm{Bn}} = (n\pi l)^2\sqrt{\frac{EJ_z}{\rho A}}, \tag{2}$$

where w is the axial displacement y and J_z is the moment of inertia section of shaft z.

The last situation of a single vibration state to be taken into consideration is torsional vibrations. They are created by forces under effects of directions being in quadrature with beams and torsional moments which are the two major factors leading to eccentricity and unbalance compared to initial positions, in which torsional vibration of beams at bearing load states is pure vibrations, with deformation of shafts which is caused by the movement among cross-section rotating neutral shafts which is torsional vibrations [14]. In terms of technology, especially techniques of designing beams, the shafts are under effects of pure torsions or complicated forces but values of torsional moments of the shafts are much greater than other internal-force members which will create torsional vibrations. At this time, torsional vibrations of beams are simulated as in Figure 3. We can identify the values of natural frequencies and mode shape functions of beams under the effects of bending-torsional vibration by equation (3).

Natural frequency:

$$\omega_{\mathrm{Tn}} = \frac{n\pi}{l}\sqrt{\frac{GJ_x}{J_0}}, \tag{3}$$

where G is the slip elastic module, J_0 is the moment of inertia masses, and J_x is the moment of inertia section of shaft x.

Shafts under bending-torsional vibrations are those with bearing capacity on the condition that, on each cross section of theirs, there are only internal-force members including bending moments M_x and M_y and torsional moments M_z. The problems of compound vibrations for this model are less commonly used. According to the proposals of Timoshenko for the bending-torsional vibration model corresponding to beams, spans and beam structures are shaped and illustrated as in Figure 4.

Natural frequency:

$$\omega_{\mathrm{BTn}}^2 = \frac{\left(\omega_{\mathrm{Tn}}^2 + \omega_{\mathrm{Bn}}^2\right) \pm \sqrt{\left(\omega_{\mathrm{Tn}}^2 - \omega_{\mathrm{Bn}}^2\right)^2 + 4\lambda\omega_{\mathrm{Tn}}^2\omega_{\mathrm{Bn}}^2}}{2(1-\lambda)}. \tag{4}$$

At this time, values of natural frequencies in bending-torsional situations are determined by weighted average values (λ) of equation (11) between bending natural frequencies and torsional natural frequencies. If either of them is 0 (or $\lambda = 0$), bending-torsional natural frequencies become equations (2) and (3).

2.2. Forced Vibration. The mode of vibration at each natural frequency is generally called the mode shape function $\phi_n(z)$ of mechanical and structural systems. The vibrating equation at each mode shape of structures having damping is as follows:

$$\ddot{q}_j + 2\omega_j\xi_j\dot{q}_j + \omega_j^2 q_j = \int_0^l f(x,t)\phi_j(x)\mathrm{d}x = F_j(t), \tag{5}$$

where q_j is the principal j^{th} coordinates, $f(x,t)$ is the operating force on strength unit of shafts, ω_j and ξ_j are the natural frequencies and damping coefficients of j^{th} mode shape function, and $F_j(t)$ is the principal natural load of j^{th} mode shape function in the generalized coordinating system. As shown in equation (5), the left side of the equation is fixed for similar vibration structures. However, depending on applied force members $F_j(t)$ of the right side, there will be the formation of natural vibrations and special characteristics of each structure. Regarding practical vibrations of bridges, members $F_j(t)$ are usually determined by harmonic force function or random force function.

2.2.1. Harmonic Force Function. In the case in which responses of bridge structures are created by the effect of the harmonic forced force, $F_j(t) = F_j \sin\omega t$, and $q_j(t)$ in the equation can be described by the function of the frequency of such a forced force and its amplitude depends on the ratio between forced frequency and natural frequency of the mechanical-structural system, which can be expressed as

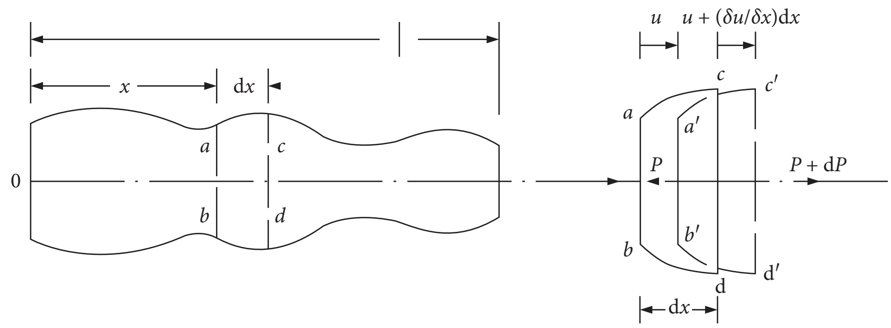


FIGURE 1: Longitudinal axial vibration of shaft.

$$q_j = q_{0j} \sin(\omega t - \varphi), \tag{6a}$$

$$q_{0j} = F_j H_j(\omega) = \frac{F_j/k}{\sqrt{\left(1-\left(\omega/\omega_j\right)^2\right)^2 + \left(2\xi_j \omega/\omega_j 2\xi_j\right)^2}}, \tag{6b}$$

$$\tan\varphi = \frac{2\xi\omega/\omega_j}{1-\left(\omega/\omega_j\right)^2}, \tag{6c}$$

where q_{0j} is the vibrating amplitude, H is the transfer function, and φ is the angle of phase difference. We consider δQ as an amplification coefficient of vibrating amplitude compared to static displacement:

$$\delta Q = \frac{q_0 k}{f_0} = \frac{1}{\sqrt{\left(1-\left(\omega/\omega_j\right)^2\right)^2 + \left(2\xi_j \omega/\omega_j\right)^2}}. \tag{7}$$

The phenomenon in which the amplitude increases suddenly at forced frequency values which are located at the same or in the neighborhood of natural frequencies is often called resonance phenomenon as in Figure 5. In order to ensure the safety, the deign procedure requires that natural frequencies of each component, especially spans, should not coincide with the frequencies of force caused by traffic. According to the regulations of the above-mentioned testing agencies, the condition to avoid resonance phenomenon caused by traffic load is that the natural frequency of bridges is not located in the frequency domain 1.6 Hz–3.3 Hz.

2.2.2. Random Force Function. Situations in which applied force function is harmonic vibration function are least likely to happen in practice. This is due to the fact that the process of moving load on bridges is a random one (random moving load, random velocity, random time, etc.), in which subjects moving on bridges will create forced forces that make them vibrate. Vibrating frequency relies on many factors such as roughness of bridge surface, velocity, and inertia force of vehicles. At that time, the root of the equation is as follows:

$$q_j(t) = \frac{1}{\omega_{\mathrm{d}j}} \int_0^l \phi_j(x) \left[\int_o^t e^{-\xi\omega_j(t-\tau)} f(x,t) \sin\omega_{\mathrm{d}j}(t-\tau)\,\mathrm{d}\tau\right] \mathrm{d}z, \tag{8a}$$

with damping frequency

$$\omega_{\mathrm{d}j} = \omega_j \sqrt{1-\xi^2}. \tag{8b}$$

At this point, the vibration of the mechanical and structural systems is shown as follows:

$$w(x,t) = \sum_{j=1}^{\infty} \phi_j(x) q_j(t) = \sum_{j=1}^{\infty} \frac{\phi_j(x)}{\omega_{\mathrm{d}j}} \int_0^l \phi_j(x) \left[\int_o^t e^{-\xi\omega_j(t-\tau)} f(x,t) \sin\omega_{\mathrm{d}j}(t-\tau)\,\mathrm{d}\tau\right] \mathrm{d}x. \tag{9}$$

In simpler situations, vehicles can create harmonic vibration with different n^{th} frequency ω_k and $(k = 1, \ldots, n)$. Then, the root of the vibrating equation of the mechanical-structural system is given as follows:

$$w(x,t) = \sum_{j=1}^{\infty} \phi_j(x) q_j(t) = \sum_{j=1}^{\infty} \sum_{k=1}^{n} \phi_j(x) F_{\mathrm{jk}} H_j(\omega_k) \sin\omega_k t. \tag{10}$$

Nevertheless, in practice, we frequently evaluate and identify the values of frequencies of beam structures which are under the effects of random forces, in which data obtained from practical measuring processes do not completely comply with laws of a certain function or identifying a random function in practice is still difficult. As a result, many researchers have adopted the Fourier Transform for the purpose of creating a spectrum (which is often called an amplitude-frequency spectrum).

2.3. Spectrum Analysis. In the time domain, the correlation function R_w of continuous signal samples and discrete signal samples is calculated, in which N is sample points and τ is

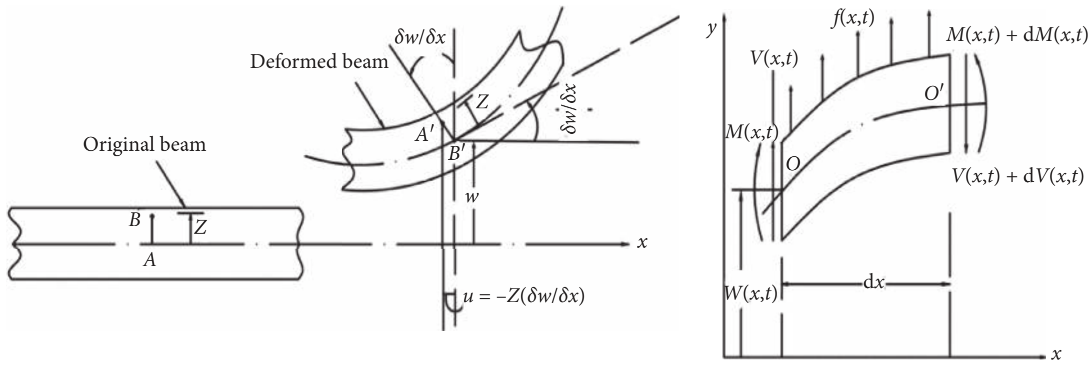


Figure 2: Bending vibration of shaft.

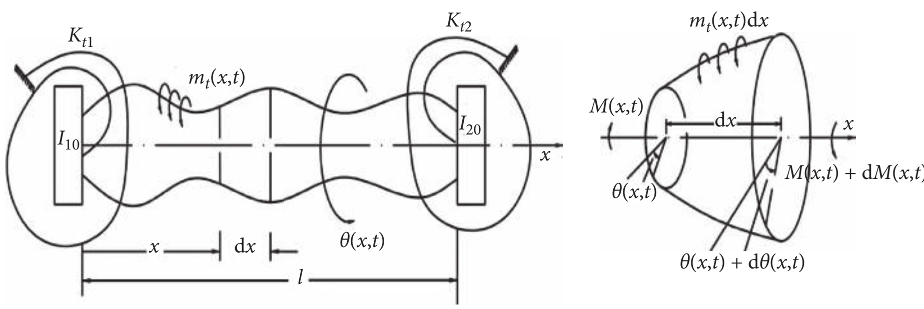


Figure 3: Torsional vibration of shaft.

the time variable ($\tau = n\Delta t$). We recommend changing the correlation function R_w of continuous signal samples into the multiplication of vibration functions as in equation (10) with respect to time t and vibration functions with respect to time $(t + \tau)$. Hence, equation (11) is given as follows:

$$R_w(\tau) = E[w(t)w(t+\tau)] = \begin{cases} \lim \dfrac{1}{2T} \displaystyle\int_{-T}^{T} w(t) \cdot w(t+\tau)\mathrm{d}t \longrightarrow \text{continuous}, \\ \dfrac{1}{N-n} \displaystyle\sum_{i=1}^{N-n} w_i \cdot w_{i+n}(\tau = n\Delta t) \longrightarrow \text{discrete}. \end{cases} \tag{11}$$

Autocorrelation function R_w measures the correlation between two vibration states at two different times of responses. If correlation function R_w shows an upward trend when τ is bigger, random signals vary quickly and contain high-frequency components. Thus, R_x is used to measure the random variation of signals.

Power spectrum S_w of a simple random process is determined from Fourier transfer of corresponding autocorrelation function R_w. Power spectrum and correlation function are a pair of Fourier transfer:

$$S_w(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_w(\tau)e^{-i\omega\tau}\mathrm{d}\tau \Longleftrightarrow R_w(\tau) = \int_{-\infty}^{\infty} S_w(\omega)e^{i\omega\tau}\mathrm{d}\omega. \tag{12}$$

At that time, when analyzing the vibration spectrum of bridges, we can see that the chart contains most of its peaks at frequency harmonics if ω overlaps natural frequencies of mechanical-structural systems or the amplitude of force F_j is significant.

However, all domains which satisfy the peaks occur at some certain frequency harmonics. Frequency values obtained from practical signals always change. As we can see in Figure 6, extreme amplitudes of these frequency harmonics depend not only on corresponding frequency locations compared to natural frequencies but also on different forced amplitudes of traffic load on bridges.

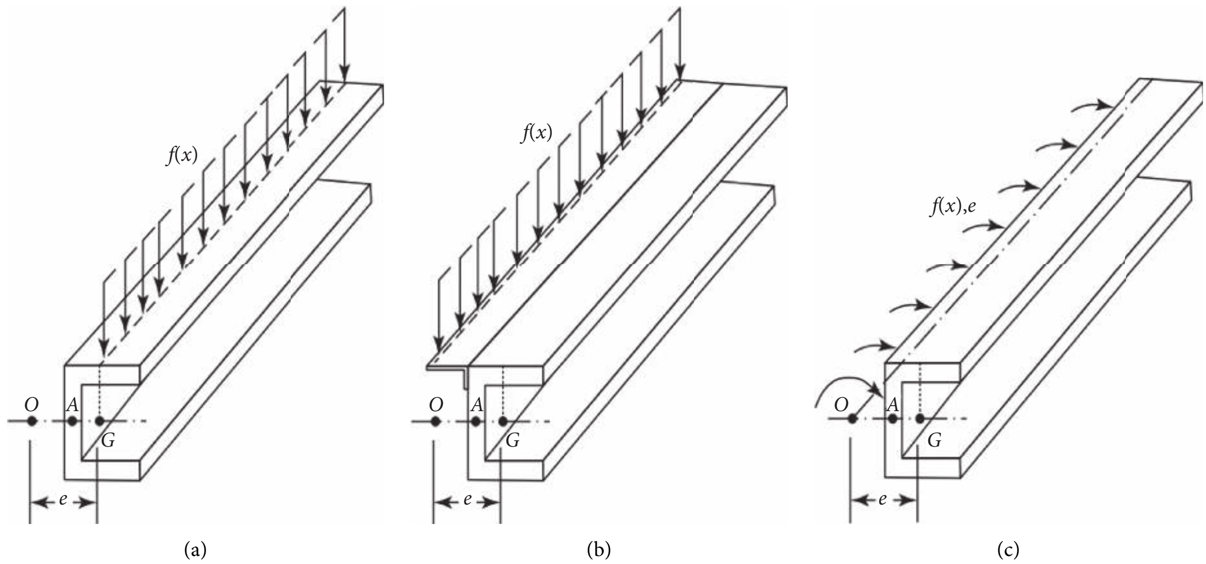


Figure 4: Types of bearing capacity of shaft, (a) central axial force, (b) axial force at slip center, and (c) moment at slip center.

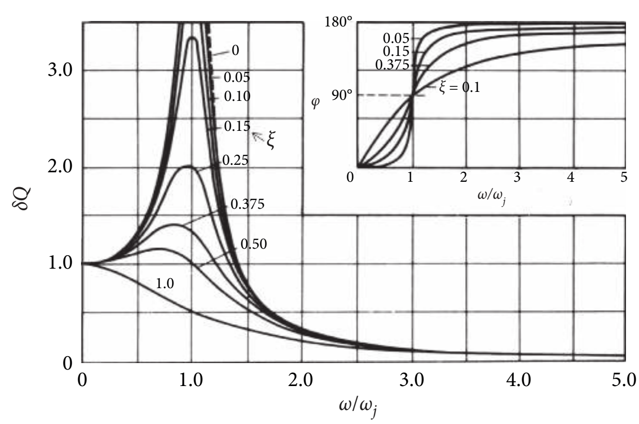


Figure 5: Amplitude amplification coefficient and phase shift angle of mode shapes.

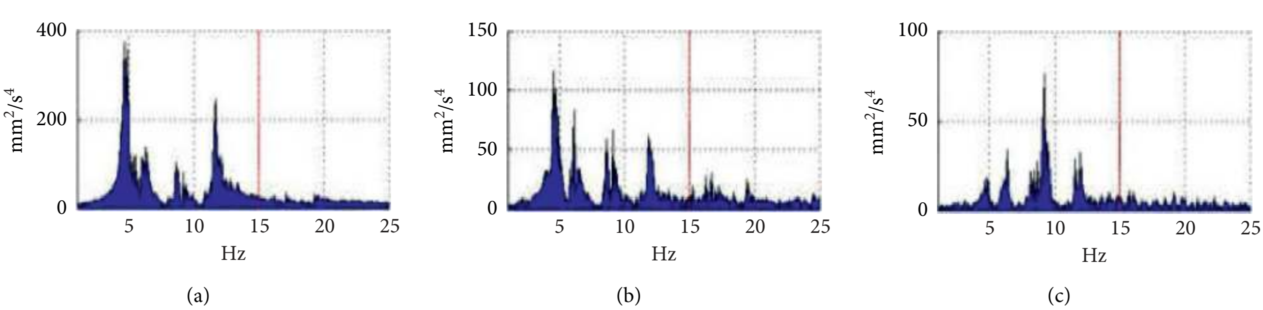


Figure 6: Vibration spectrum of spans under traffic load.

3. Procedure of Practical Measurement

The model of the testing construction site is carried out in accordance with the experimental project which tests practical vibration [7, 8, 15, 16] on the majority of bridges in Ho Chi Minh City from October 2011 to 2014 by Ho Chi Minh City University of Technology, Laboratory of Applied Mechanics (LAM). LAM has practically tested 38 bridges with numerous structures and different construction period. The sample testing method is carried out on Ben Noc bridge, which is constructed on Le Van Viet Street, Tang Nhon Phu A Ward, District 9, Ho Chi Minh City, as shown in Figure 7. The structure of this bridge is prestressed concrete with 3 single spans in Figure 8. We have carried out testing

Figure 7: Picture of Ben Noc bridge in practice.

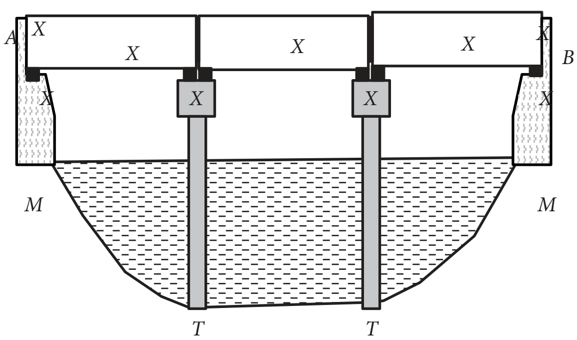


Figure 8: Simulation of cross section of Ben Noc bridge.

vibration with deformation and deflection concurrently at each similar time. The most distinguishing feature of Ben Noc bridge is that when the width of its spans is narrow, there is only one single vehicle moving each time.

The procedure surveying data at such these bridges encompasses measurement signals of deformation, of deflection, and of vibration, in which measurement signals of deflection and of deformation are measured under the beams as in Figure 9, with measurement signals of deformation being surveyed on 4 measurement points as shown by Figure 10 and measurement signals of deflection being surveyed on an intermediate point of spans. Furthermore, measurement signals of vibration are surveyed at 3 points on spans in such a way that each measurement point will be measured in 3 respective axes *Ox, Oy,* and *Oz* in Figure 11 and the plan for arranging location of measurement points as Figure 12. The process of receiving signals from parameters is illustrated in Figure 13 and sample parameters are shown in Table 1.

4. Validation and Discussion

A practical survey at Ca Tre 1 in district 9 in November 2009 showed that there were 3 different measurement signal files at the same measurement point.

Corresponding to each measurement signal file (original signals), through Fourier analysis method, we will receive a spectrum which is often called a spectrum between

Figure 9: Practical measuring deformation signals.

amplitude and frequency. The procedure to perform this Fourier transformation method is shown in Figure 14. Through this transformation method, some situations may appear as follows:

(i) Situation 1: from one original signal file, we will receive an amplitude-frequency spectrum at which there is only one frequency appearing on the spectrum as Figure 15. Theories of vibration show that, in this situation, when spans are under the action of load caused by road vehicles, spans at this moment only show a single vibration type which could be bending, torsion, or bending-torsion vibration. Sharing the same ideas as the above-mentioned results, the researchers [8–11, 16] have revealed that the probability of occurrence of such these natural vibration types can be different corresponding to different groups of bridges. As shown in Table 1, the probability of bending frequencies is always much higher than torsional frequencies with respect to spans having structures in prestressed concrete or composite concrete as in Tables 2 and 3 and the simulation results are performed by finite element method as shown by Table 4 [17, 18]. Conversely, with regard to spans having box structures or the distance between the length points and width points which is not far, bending-torsion frequencies are highest see in Tables 5 and 6. The manuscript compared this result with the numerical simulation using finite element method as shown in Table 7 [19, 20]. As shown in the survey on 15 flyover spans (Binh Phuoc 1-2, Linh Xuan, Thu Duc 1-2, Song Than 1-2, etc.) and some spans having specific structures like Phu My cable-stayed bridge as shown in Table 8 and suspended spans of Sai Gon 1 bridge, the probability of occurrence of dual frequencies like bending-torsion (showing both bending state and torsional state) is always higher than other single frequencies; see Tables 5, 6 and 9.

(ii) Situation 2: from one original signal file, there are two different frequencies concurrently occurring in the amplitude-frequency spectrum as shown in Figure 15. We can see that these two frequencies can be either extremely close to each other in Figure 15(a) or really far from each other in

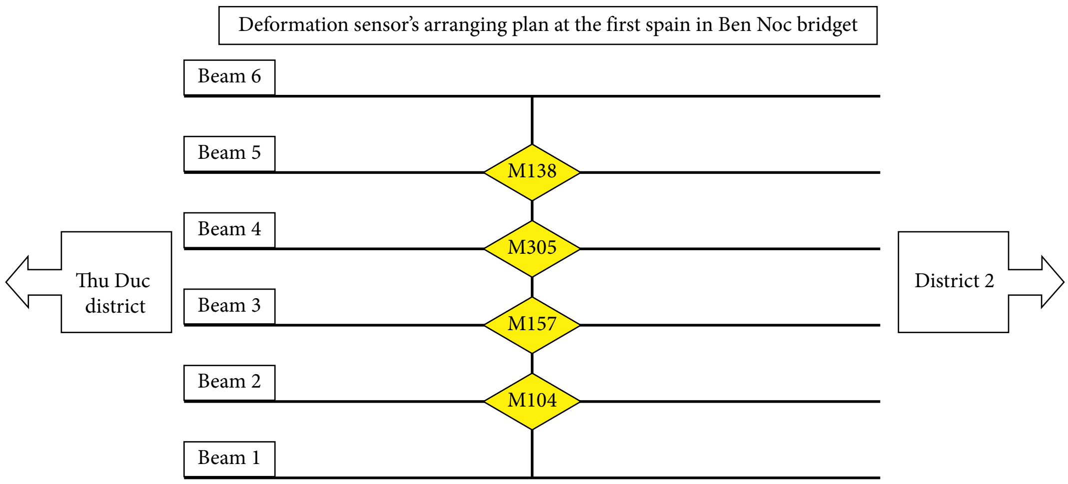


Figure 10: Plan for arranging location of deformation measurement points at Ben Noc bridge.

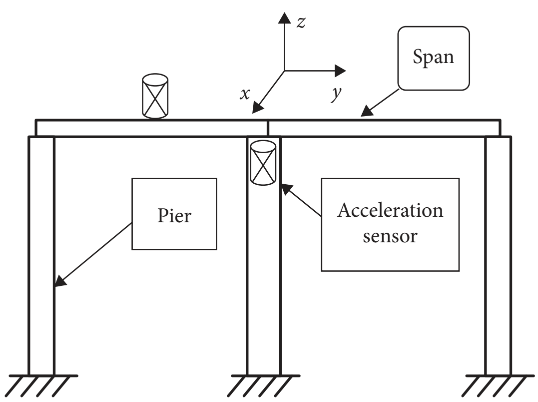


Figure 11: Plan for arranging location of vibration measurement points at Ben Noc bridge.

Figure 12: Measuring vibrating signals at Ben Noc bridge.

Figure 15(b). With regard to this situation, we are unable to categorize these occurred frequency values into a specific situation such as bending, torsion, or combined vibrations as in situation 1 because the nature of these two frequencies is only one vibration type (which could be bending, torsion, or bending-torsion). Therefore, we have to admit that the simultaneous occurrence of these two frequencies is due to the deterioration of natural frequency values during their operating process. If locations of two vibration frequencies are close to each other like the second situation, they are frequently categorized into three groups as in Figure 15, when the amplitudes of both frequencies are nearly equal or they are opposite (one frequency with very high amplitude and the other with very low amplitude). According to [21], the above-mentioned phenomenon shows that changes in mechanical features of structures, particularly beams of spams, are always affected by the aging of materials during the operating process. Researchers' results have been validated on a majority of bridges during consecutive time periods (2 years or over 5 years for Sai Gon bridge and Rach Chiec bridge, resp.). At this time, researchers have recognized that, during the operating process of spans, they are under the effects of many factors including moving load, a warm wet environment, construction process, and aging of materials. Hardness deterioration and bearing capacity of spans are most clearly and stably shown through natural frequency values or fundamental frequency values as shown in Table 10. This study also shows the stability of frequencies, especially the first natural frequency values in Table 11. This is the foundation for many national managers and scientists including [22–26] or foreign ones such as [27–33] to choose this value as a measuring scale for general deterioration of spans (through testing standards, design criteria, etc.). According to [34], researchers have figured out parameters that are often used to evaluate the reduction of bearing capacity of spans like deflection values, deformation values, damping coefficient values, and impact coefficient values. The survey on hardness deterioration on Tang Long bridge in district 9, Ho Chi Minh City [17, 35], during two consecutive years shows that, during its operating

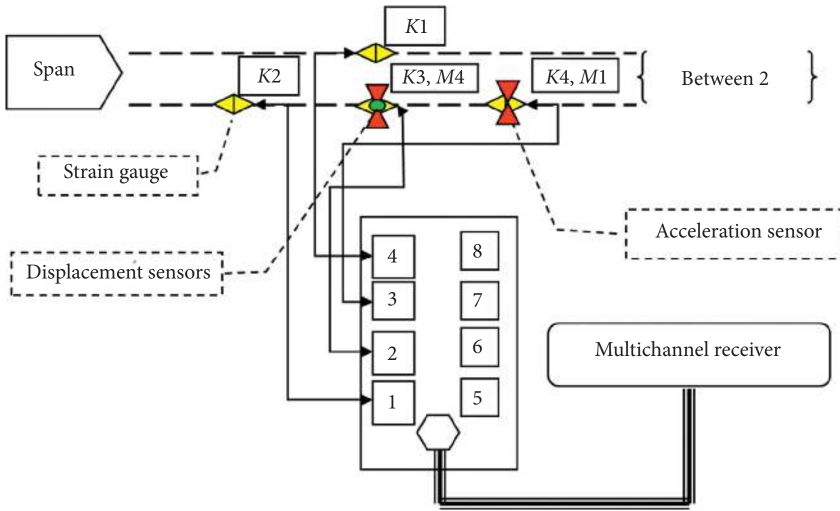


Figure 13: The process of receiving signals and processing deformation data.

Table 1: Signals' sample parameters.

Sample frequency	100 sample (seconds)
The time to collect a file	1920 seconds
Total of measurement points	4 measurement points
The number of files in a measurement point	96
The number of samples in a measurement point	192000 sample
Total of samples in 4 measurement points	960000 sample

process, the reduction of natural frequency values is not linear and discontinuous. In practice, changes in natural frequency values always create some certain discontinuous gaps within a certain hardness range before changing completely into another hardness state. This means that when surveying spans, we should take notice of transitional stages from one hardness level to another.

(iii) The last situation: vibration spectrums obtained from one signal file can contain many different frequencies. In practical situations, during the surveying process of 38 bridges (around 208 spans) of different structures, it is less likely that there is an occurrence of only one frequency in one measured signal file, accounting for less than 15% as seen in Table 12. Tables 13 and 14 are survey results of vibration frequencies of some spans in the measuring projects of 38 bridges in district 2, district 9, and Thu Duc district. These tables show that there is only a minority of measured signal files containing vibration frequency; the majority will have two or three vibration frequencies. Specifically, spans of flyover bridges with their length are equal to or shorter than their width which can have four or five different vibration frequencies as shown in Table 14. Hence, spans under the effects of random forces caused by traffic load will contain many different vibrations. A general tendency is that there is an incorporation of many different vibrations into one such as bending-torsion, bending-bending, and torsion-torsion. In the meantime, spans can show many high-level mode shapes of these vibrations.

When evaluating changes in the hardness of bridges, we should always have the most overall perspective on their response process in different vibration situations. However, each measured file only shows one single vibration state of spans in a certain situation. Using only one data file to evaluate the overall responses of the operating process of spans is completely inaccurate especially in practice; the applied force is random with different response states. Therefore, in order to evaluate these responses most objectively and accurately, the number of measured signal files should be sufficient enough. In addition, the measurement state should be large enough to be able to evaluate the responses of spans in an overall manner. Effects of moving load on bridges are a random function; hence, when measuring, we have to survey the number of signal files in such a way that they are large enough to evaluate responses of spans. According to [36], we can conclude that, depending on different types of bridges and traffic flows on bridges, measurement time can range from 7 minutes to 15 minutes accordingly. This amount of measurement time can be divided into 10 to 30 different measured files. Based on [37], researchers have suggested many measures in order to evaluate the overall responses of spans under the effects of the random load. The most significant thing that should be mentioned is the definition of "representative power spectral density." Figure 16 is the construction process of representative power spectral density.

The representative power spectral density is shown by three fundamental elements including frequency values in the acceptable frequency range of measurement, amplitudes of frequency values, frequency of occurrence of frequency values in these measured files. A representative power spectral density is a new term and never appears in any of the

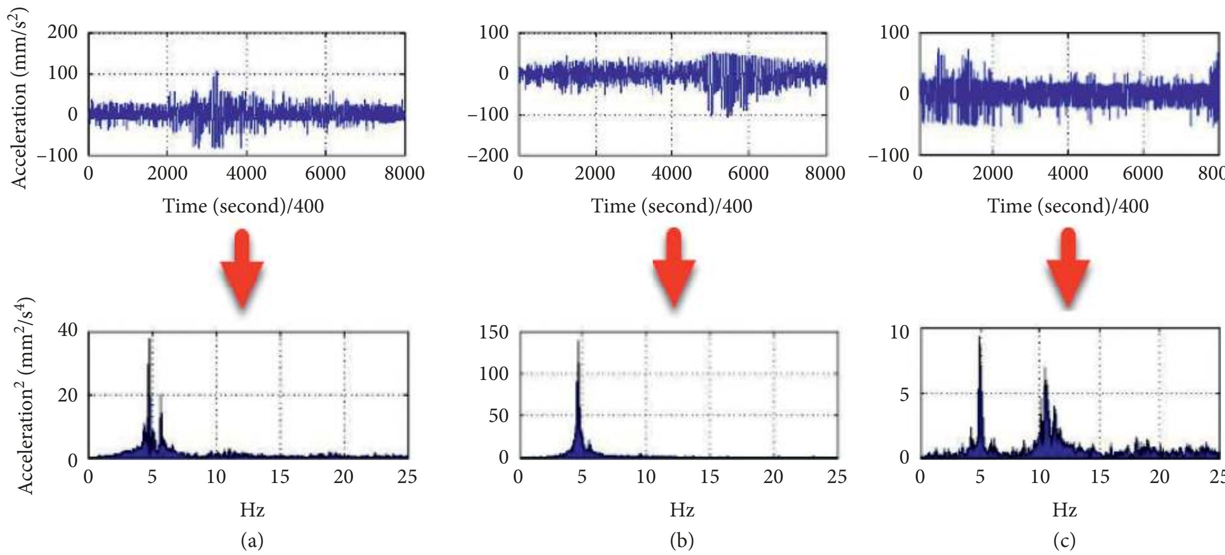


FIGURE 14: Some practical spectral types frequently appear in the measurement project of 38 bridges in district 2, district 9, and Thu Duc district.

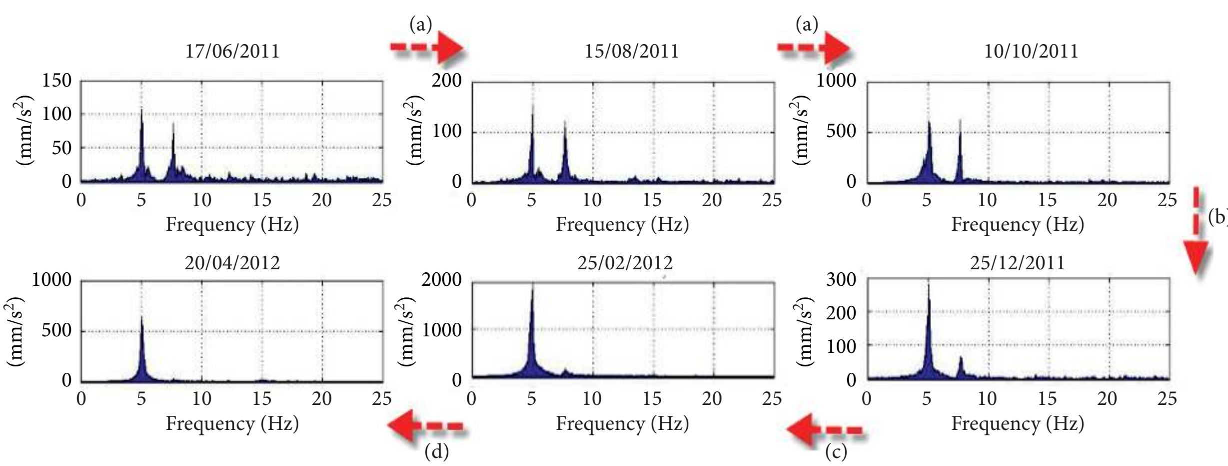


FIGURE 15: The changing process of two close frequencies.

TABLE 2: Statistics of frequencies of Ben Noc bridge (obtained from LAM).

<table>
<tr><th>Order of measurement files</th><th>Bending (Hz)</th><th>Torsion (Hz)</th><th>Bending-torsion (Hz)</th></tr>
<tr><td>1</td><td>5.1</td><td>||*</td><td>||</td></tr>
<tr><td>2</td><td>5</td><td>7.0</td><td>||</td></tr>
<tr><td>3</td><td>5.1</td><td>||</td><td>||</td></tr>
<tr><td>4</td><td>5</td><td>7.0</td><td>||</td></tr>
<tr><td>5</td><td>5</td><td>||</td><td>||</td></tr>
<tr><td>6</td><td>5</td><td>7.1</td><td>||</td></tr>
<tr><td>7</td><td>5</td><td>||</td><td>||</td></tr>
<tr><td>8</td><td>5.2</td><td>7.0</td><td>||</td></tr>
<tr><td>9</td><td>5</td><td>||</td><td>||</td></tr>
<tr><td>10</td><td>5.3</td><td>7.2</td><td>||</td></tr>
</table>

||* No appearance.

TABLE 3: The probability of occurrence of frequencies on Ben Noc bridge (obtained from LAM).

Number	Vibration types	Simulation results		Measurement results	
		Frequency (Hz)	Errors compared to measurement	Dominant frequency (Hz)	Probability of occurrence
1	Bending	5.02	0.4	5.0	100
2	Bending-torsion	6.57	0	0	0
3	Torsion	7.15	2.14	7.0	50

Table 4: The numerical simulation using finite element method at Ben Noc bridge.

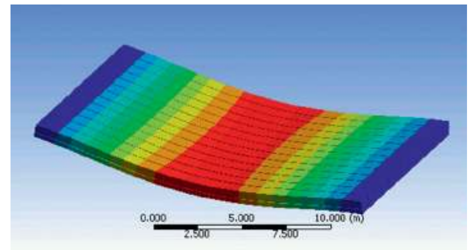
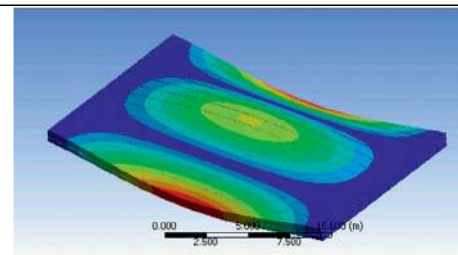
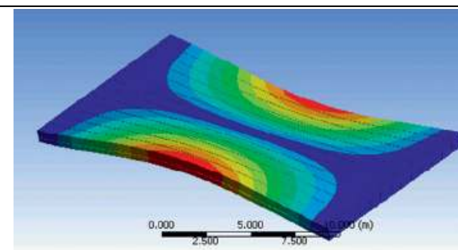
Number	Vibration types	Simulation results by FEM		
		Frequency (Hz)	Errors compared to measurement	Figure
1	Bending	5.02	0.4	
2	Bending-torsion	6.57	0	
3	Torsion	7.15	2.14	

Table 5: Statistics of frequencies of Binh Phuoc 2 flyover from different measurement times (obtained from LAM).

Order of measurement files	Bending (Hz)	Torsion (Hz)	Bending-torsion (Hz)
1	5.1	7.1	6.1
2	||*	7.3	6.4
3	5.1	||	||
4	||	7.4	6.5
5	||	7.5	||
6	||	7.4	||
7	5	7.3	||
8	5.2	7.5	6.2
9	||	7.3	6.5
10	5.3	7.5	6.4
11	5	7.3	||
12	||	7.5	||
13	5.1	7.4	6.6
14	5.1	7.5	||
15	5.1	7.3	6.4
16	5	7.4	6.3

||*No appearance.

Table 6: Results of frequencies, vibration types, comparison, and probability of occurrence (obtained from LAM).

Number	Types of vibration	Simulation results		Measurement results	
		Frequencies (Hz)	Errors compared to measurement	Dominant frequencies (Hz)	Probability of occurrence
1	Bending	5.211	0.211	5.2	62.25
2	Bending-torsion	6.37	0.469	6.4	50
3	Torsion	7.332	2.24	7.3	93.75

Table 7: The numerical simulation using finite element method at Binh Phuoc 2 flyover.

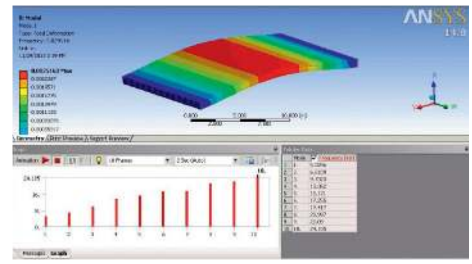
Number	Vibration types	Simulation results by FEM		
		Frequency (Hz)	Errors compared to measurement (%)	Figure
1	Bending	5.211	0.211	
2	Bending-torsion	6.37	0.469	
3	Torsion	7.332	2.24	

Table 8: Statistics of frequencies of Phu My cable-stayed bridge (obtained from LAM).

Order of measurement files of suspended spans	Bending (Hz)	Torsion (Hz)	Bending-torsion (Hz)
1	0.91; 1.43	4.83; 6.37	2.34; 3.37
2	0.93; 1.50	||; 6.15	2.14; 3.26
3	0.90; 1.50	||; ||	2.16; 3.30
4	0.87; 1.51	||; 5.99	2.27; 3.27
5	0.87; 1.54	||; 5.96	2.18; 3.26
6	0.93; 1.48	||; 6.28	2.26; 3.25
7	0.91; 1.53	||; 6.56	2.34; 3.36
8	0.86; 1.40	4.62; 6.39	2.27; 3.22
9	0.87; 1.49	||; 6.53	2.16; 3.42
10	0.90; 1.52	||; ||	2.14; 3.55
11	0.91; 1.43	4.83; 6.37	2.34; 3.37
12	0.93; 1.50	||; 6.15	2.14; 3.26
13	0.90; 1.50	||; ||	2.16; 3.30
14	0.91; 1.43	4.83; 6.37	2.34; 3.37
15	0.93; 1.50	||; 6.15	2.14; 3.26
16	0.90; 1.50	||; ||	2.16; 3.30

||*No appearance.

Table 9: Frequency of occurrence of frequencies on Phu My cable-stayed bridge (obtained from LAM).

Number	Types of vibration	Simulation results		Measurement results	
		Frequencies (Hz)	Errors compared to measurement	Dominant frequencies (Hz)	Probability of occurrence (%)
1	Bending	0.89	2.197	0.9	100
2	Bending-torsion	2.12	0.935	2.1	50
3	Torsion	4.78	6.024	4.7	100

Table 10: Natural frequencies of Ong Nhieu bridge.

Ong Nhieu				Frequencies (Hz)
Spans	Measurement times	Measurement directions	Traffic states	
1	1	x	Minivans, cars, dumper trucks,. . ., 8 h 35 : 9 h 10 am, 17/6/2011	4.8
		z		10.3
	2	x	Trucks, buses,. . ., 9 h 5 : 11 h am, 10/10/2011	4.8
		z		10.5
	3	x	Minivans, cars, dumper trucks,. . ., 11 h: 12 h 20 am, 25/12/2011	4.9
		z		10.3
	4	x	Minivans, cars, dumper trucks,. . ., 8 h: 90 h 45 am, 20/4/2012	4.8

reference materials, except the ones from the Laboratory of Applied Mechanics (LAM), Ho Chi Minh City University of Technology [38]. This is a new proposal in order to evaluate the overall responses of spans during their operating process. The most remarkable feature in this proposal for a representative power spectral density is given as follows.

Table 11: Lowest natural frequencies of some prestressed concrete bridges.

Bridges	Spans	Frequencies (Hz)
Ca Tre 1	1; 2; 3	4.6; 4.8; 4.9
Giong Ong To 2	1; 2; 3	4.8; 4.7; 4.6
Giong Ong To Moi	1; 2; 3	5; 4.8; 4.5
My Thuy 2	1; 2; 3	4.8; 4.9; 4.7
Sai Gon	1; 2; 3	3.8; 3.5; 3.4
Ben N c	1; 2; 3	5
Ong Nhieu	1; 2; 3	4.8; 3.3; 4.9
Binh Phuoc 1	1; 2; 3	4.9; 4.7; 4.8
Song Than 2	1; 2; 3	5

Table 12: Statistical quantities of natural frequency values of 38 bridges in district 2, district 9, and Thu Duc district.

	Spans of prestressed concrete bridges	Steel spans	Total	Percentage (%)
One natural frequency value	20 spans	11 spans	31 spans	14.90
Two natural frequency values	35 spans	29 spans	85 spans	40.86
More than two natural frequency values	102 spans	11 spans	92 spans	44.24
Total	157 spans	51 spans	208 spans	100

Table 13: Vibration frequencies corresponding to different axes of some contiguous spans and piers of Giong Ong To 2 bridge.

<table>
<tr><th rowspan="2">Spans</th><th colspan="3">Frequencies (Hz)</th></tr>
<tr><th>f_1 (Hz)</th><th>f_2 (Hz)</th><th>f_3 (Hz)</th></tr>
<tr><td>1</td><td>5.3</td><td>8.2</td><td>18.5</td></tr>
<tr><td>2</td><td>5.2</td><td>16.6</td><td>28.9</td></tr>
<tr><td>3</td><td>4.8; 5.2</td><td>18.5</td><td>||</td></tr>
<tr><td>4</td><td>5.3</td><td>15.7</td><td>19.3</td></tr>
<tr><td>5</td><td>5.3; 4.7</td><td>18.1</td><td>||</td></tr>
<tr><td>6</td><td>5.2</td><td>18.2</td><td>||</td></tr>
<tr><td>7</td><td>5.4; 4.8</td><td>23.9</td><td>||</td></tr>
<tr><td>8</td><td>3.2</td><td>5.3</td><td>20.2</td></tr>
<tr><td>9</td><td>5.3</td><td>12.4</td><td>27.8</td></tr>
<tr><td>10</td><td>5.2</td><td>8</td><td>15.9</td></tr>
<tr><td>11</td><td>5.3</td><td>8.3</td><td>25</td></tr>
</table>

||*No appearance.

Table 14: Natural frequency values of some spans of flyover bridges.

<table>
<tr><th rowspan="2">Bridges</th><th rowspan="2">Spans</th><th colspan="4">Frequencies (Hz)</th></tr>
<tr><th>f_1 (Hz)</th><th>f_2 (Hz)</th><th>f_3 (Hz)</th><th>f_4 (Hz)</th></tr>
<tr><td rowspan="6">Linh Xuan flyover bridge</td><td>Span 1</td><td>5</td><td>6.3</td><td>8.7</td><td>20.9</td></tr>
<tr><td>Span 2</td><td>5.4</td><td>6.5</td><td>7.7</td><td>10.6</td></tr>
<tr><td>Span 3</td><td>5.3</td><td>6.5</td><td>9</td><td>10.7</td></tr>
<tr><td>Span 4</td><td>5.1</td><td>6.4</td><td>9</td><td>11</td></tr>
<tr><td>Span 5</td><td>5.2</td><td>6.3</td><td>||</td><td>10.6</td></tr>
<tr><td>Span 6</td><td>5.2</td><td>6.3</td><td>8.5</td><td>11.2</td></tr>
<tr><td rowspan="6">Binh Phuoc 1 flyover bridge</td><td>Span 1</td><td>5.1</td><td>7.3</td><td>10.9</td><td>||</td></tr>
<tr><td>Span 2</td><td>4.8</td><td>10.1</td><td>11.1</td><td>||</td></tr>
<tr><td>Span 3</td><td>4.9</td><td>9.4</td><td>11.3</td><td>||</td></tr>
<tr><td>Span 4</td><td>4.9</td><td>10.9</td><td>||</td><td>||</td></tr>
<tr><td>Span 5</td><td>4.8</td><td>7.1</td><td>10.1</td><td>11.6</td></tr>
<tr><td>Span 6</td><td>5.1</td><td>7.3</td><td>11.5</td><td>||</td></tr>
</table>

||*No appearance.

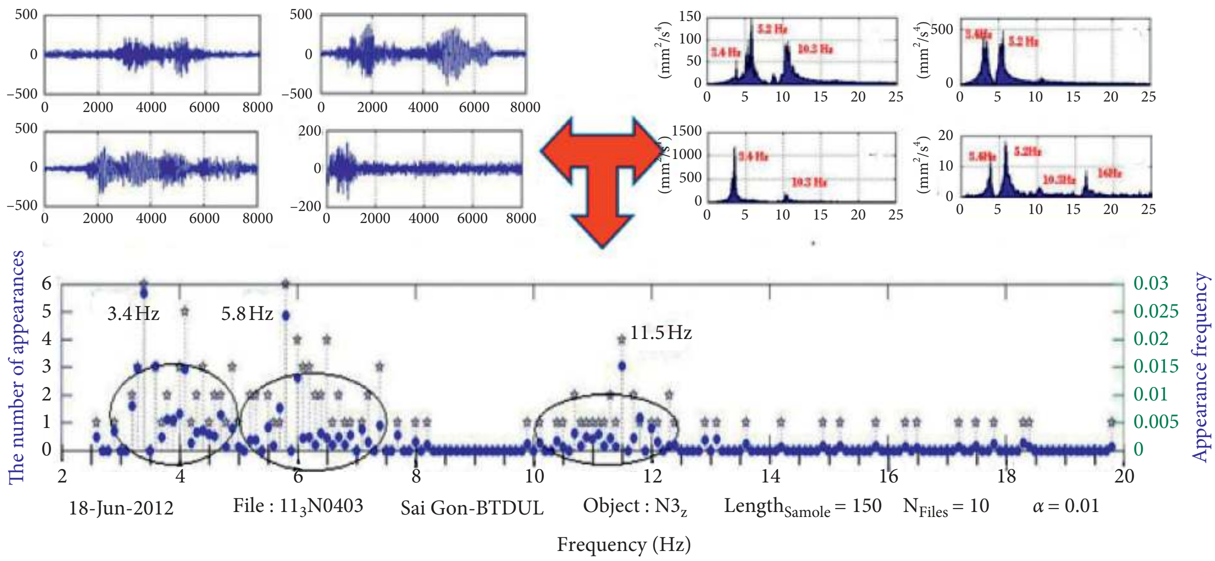


Figure 16: The construction process of representative power spectral density of Sai Gon bridge.

4.1. Stable Characteristics of a Representative Power Spectral. Figure 17 is a representative power spectral density of different measurement times at the second span of Giong Ong To Moi bridge.

Consequently, during 2-year time of observing the second span of Giong Ong To Moi bridge, we can see that the shapes of spectrums are unchanged, and dominant frequency values (highest frequencies) always occur in each spectrum of different measurement times. Besides, the survey of Sai Gon 1 bridge indicates that, after 4 measurement times (the distance between the first measurement time and the fourth time is about 8 months), frequency values of each subject which is surveyed by a representative power spectral density often displace no more than 0.2 HZ. Table 15 illustrates the stability level of the representative power spectral density of fundamental vibration frequency values with respect to vertical axes (f_z). These data are collected from 4 different measurement times of Sai Gon bridge at its first and second spans (prestressed concrete beams) and the 16th span (composite concrete beams).

The cause of disparity among measurement times is the major changes in traffic load at a certain measurement time since this is the high priority bridge of the whole city with complicated traffic flows. With regard to bridges with normal or fewer traffic flows, natural frequency values from 4 measurement times are equal. Table 16 is fundamental natural frequency values from 4 measurement times at its first span of Ben Noc bridge.

4.2. Can Show Many Different Vibrations within the Same Spectrum. All survey results of a representative power spectral density of some spans in the measurement project of 38 bridges in Ho Chi Minh City indicate that creating a representative power spectral density can identify different vibrations of spans during their operating process, in which some spans only show a single vibration type like the responses of My Thuy bridge as in Figure 18 or show two single vibration types concurrently including bending and torsion vibrations as shown by Figure 19 of spans of Giong Ong To bridge. Bridges with very special structures will show different vibration types including single low-level mode vibrations, compound vibrations, and single high-level mode vibrations as Figures 20 and 21.

4.3. Frequency of Occurrence of Natural Frequency Values. In the situation in which all natural frequency values are almost unchanged, in order to evaluate responses of bridges caused by moving load, we have recommended an AF parameter [38] (Appearance Frequency) which is called “the frequency of occurrence of respective natural frequencies.” This is considered an important recommendation which has never been recorded in any published reference materials before. Each natural frequency value occurring on spans will be characterized by an AF value. These values will show us the occurrence of frequencies from different measurement times within the same or different measurement periods. However, because the number of measured files in those measurement periods is virtually different, we have decided to convert AF values into percentage (%) so that our evaluation is appropriate. Table 17 is AF values of the 4th span of Thu Duc 2 flyover bridge in 3 different measurement periods during 2 consecutive years of periodical monitoring.

Through 3 different measurement times during 2 years of monitoring, we understand that AF values can significantly change while natural frequency values are almost constant. From Figures 22(a)–22(c), AF values of respective natural frequencies in the first measurement period are nearly close to each other as in Figure 22(a). Nonetheless, AF values of 2 low frequencies will gradually increase (from 36% in the first time to 75% in the third time), whereas AF values of 2 high frequencies will decrease in accordance with monitoring

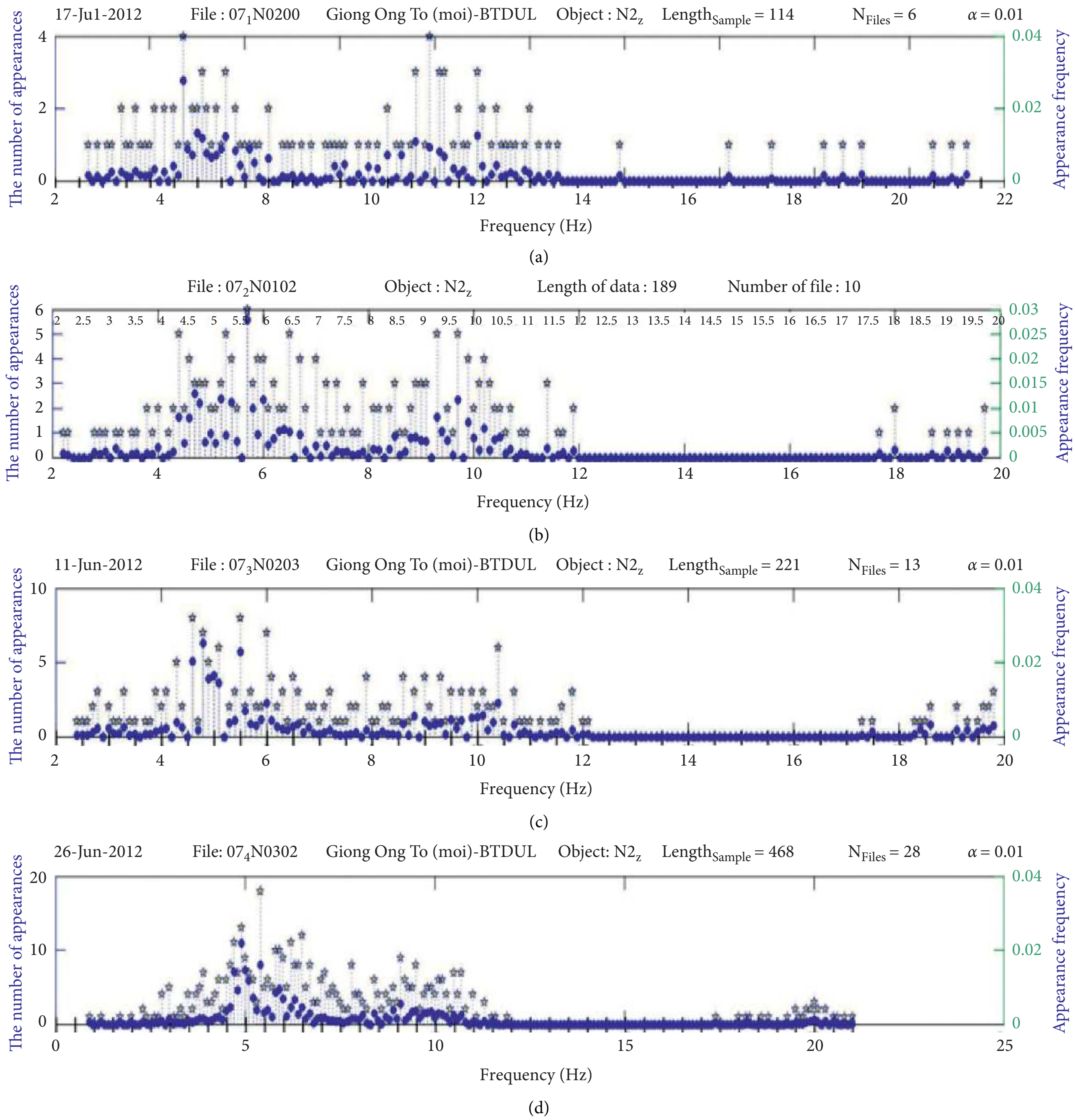


Figure 17: A representative power spectral density of Giong Ong To Moi bridge. (a) The first time (July 19, 2011). (b) The second time (December 22, 2011). (c) The third time (June 11, 2012). (d) Fourth time (July 11, 2012).

time. The higher the frequencies are, the faster AF values decline, and AF values will be 0 when those frequencies disappear as in Figure 22(c). Conducting a similar survey on the 1st span of Thu Duc 2 flyover, we obtain results as shown in Figures 23(a)–23(b).

The frequency of occurrence of the lowest natural frequencies always accounts for the largest proportion and this proportion will gradually decrease in subsequent frequencies. Over a course of time, the highest natural frequencies will decline and disappear; they will make way for other subsequent natural frequencies (they will give preference for subsequent higher natural frequencies). In terms of charts, the tendency of AF values towards respective natural frequencies is likely to separate into a linear line as in Figure 23(a). Meanwhile, natural frequency values are almost constant through different measurement times. The whole declining process of the frequency of occurrence of natural frequencies is illustrated in Figure 24 [39].

$f_{i\_}i^{th}$ is the frequency range, T_i denotes time, N_i is the number of occurrence times, and ω_i is the natural frequency values.

Table 15: Values f_z at different spans of Sai Gon bridge.

Order of spans	Measurement times	Traffic states	f_z (Hz)
1	1	In the morning, traffic vehicles are mainly buses and minivans	3.8
	2	In the morning, traffic vehicles are mainly buses and minivans	3.9
	3	In the evening, traffic vehicles are mainly container lorries	3.8
	4	In the evening, traffic vehicles are mainly container lorries	3.8
2	1	In the morning, traffic vehicles are mainly buses and minivans	3.5
	2	In the morning, traffic vehicles are mainly buses and minivans	3.4
	3	In the evening, traffic vehicles are mainly container lorries	3.5
	4	In the evening, traffic vehicles are mainly container lorries	3.4
...	...	....	...
16	1	In the morning, traffic vehicles are mainly buses and minivans	2.3
	2	In the morning, traffic vehicles are mainly buses and minivans	2.1
	3	In the evening, traffic vehicles are mainly container lorries	2.3
	4	In the evening, traffic vehicles are mainly container lorries	2.2

Table 16: Frequencies f_z of Ben Noc bridge.

Ben Noc bridge			Frequencies (Hz)
Spans	Measurement times	Measurement directions	
1	1	X	5
		Z	5
	2	X	5
		Z	5
	3	X	4.9
		Z	5.1
	4	X	5
		Z	4.9

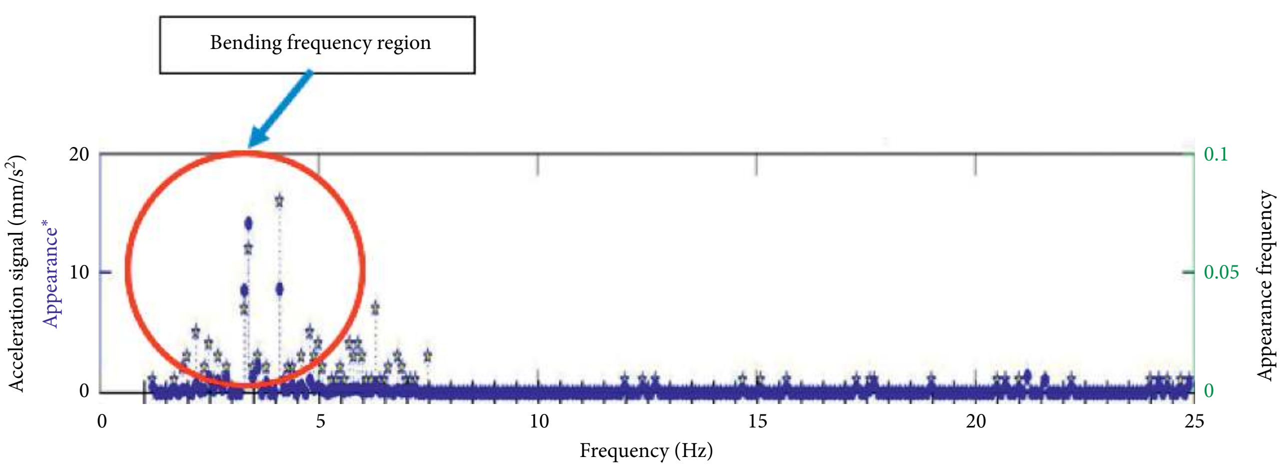


Figure 18: The representative power spectral density of the first span of My Thuy 2 bridge.

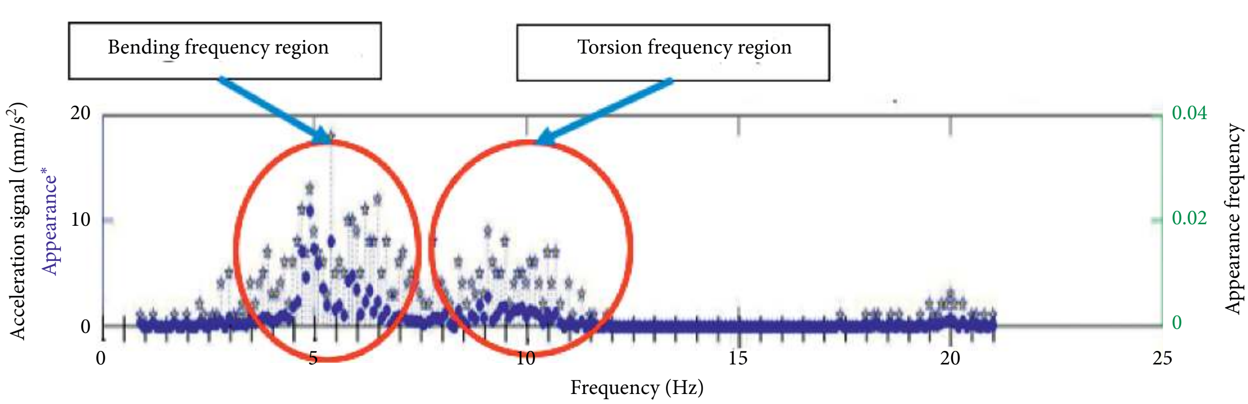


Figure 19: The representative power spectral density of Giong Ong To 2 bridge.

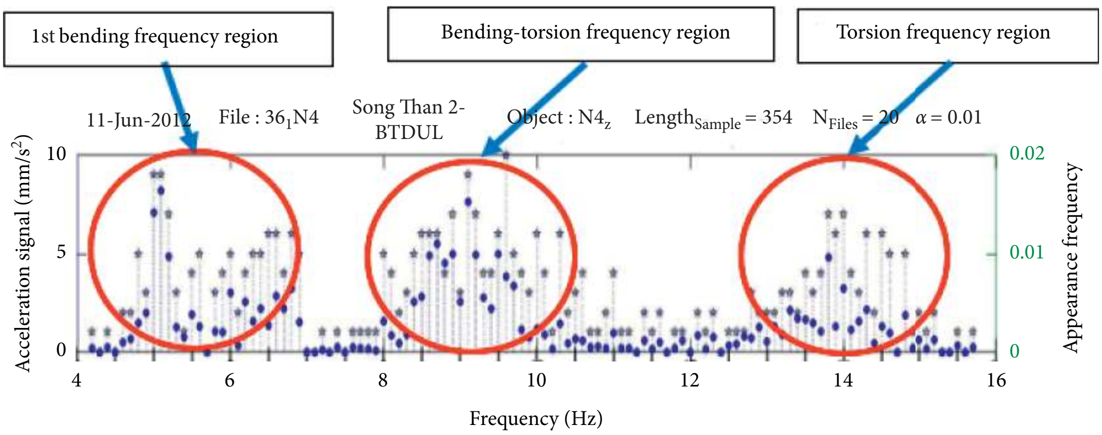


FIGURE 20: The representative power spectral density of Song Than 2 bridge.

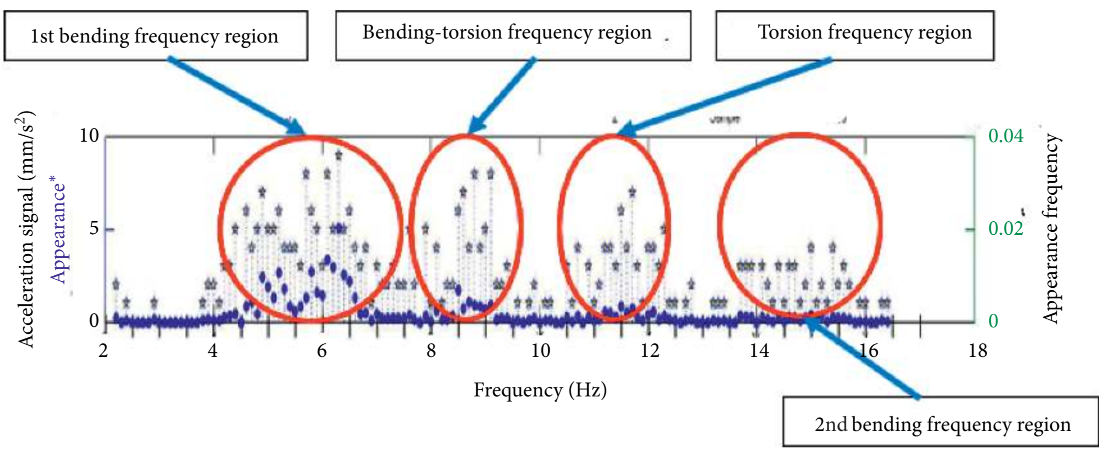


FIGURE 21: The representative power spectral density of Linh Xuan flyover bridge.

TABLE 17: Characteristics of natural frequencies of the 4th span of Thu Duc 2 flyover bridge through 3 different measurement periods.

Spans	Natural frequency values	AF values, 1st measurement time (%)	AF values, 2nd measurement time (%)	AF values, 3rd measurement time (%)
4	$f_1 = 4.9$	36	60	75
	$f_2 = 6$	32	40	50
	$f_3 = 8.4$	41	40	41
	$f_4 = 9.8$	32	30	31
	$f_5 = 12.3$	27	13	0

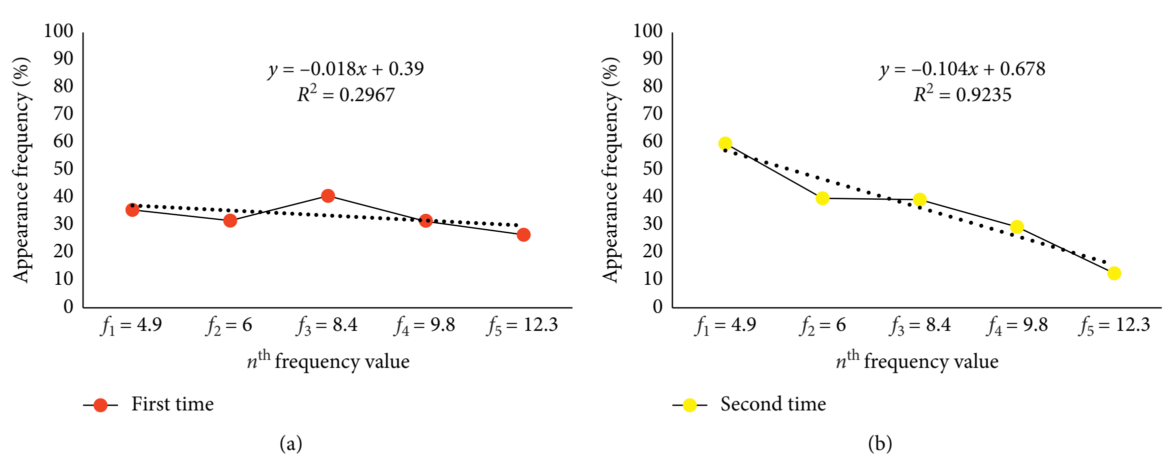
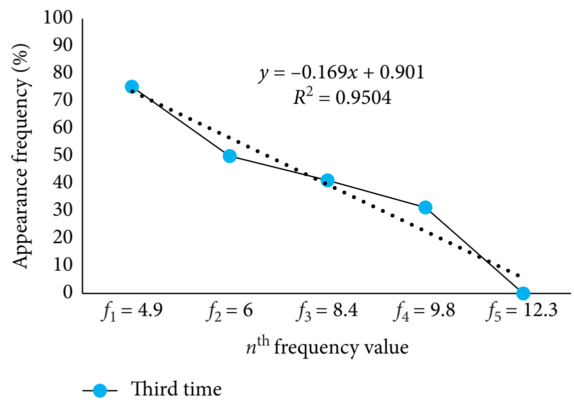


FIGURE 22: Continued.



(c)

Figure 22: Changes in AF values of the 4th span of Thu Duc 2 flyover bridge through 3 different measurement periods.

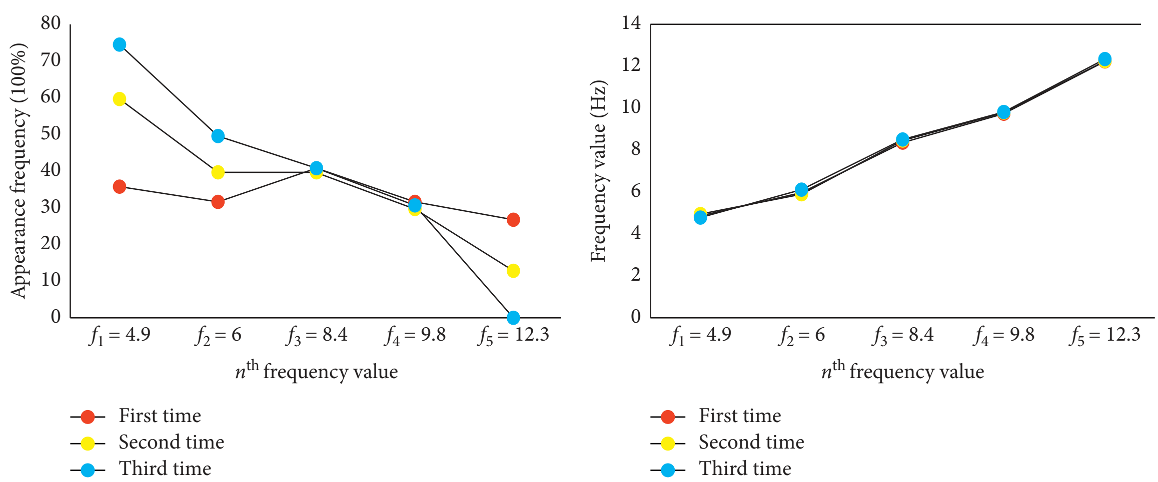


Figure 23: Changes in AF values and natural frequencies of the 1st span of Thu Duc 2 flyover bridge in 3 different measurement periods.

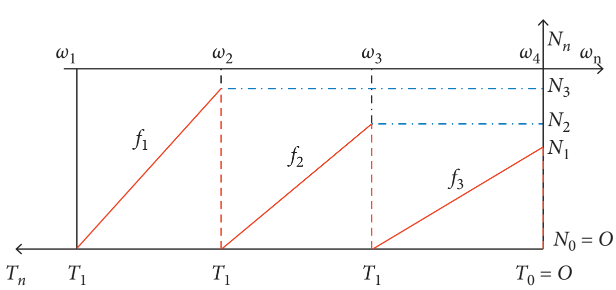


Figure 24: The description of the degradation process of spans throughout a defined time.

5. Conclusion

In this paper, we have investigated 38 bridges (corresponding to 208 spans) with various different types of structures, construction time, and the number of vehicles moving. Studying vibration characteristics of spans under random moving load is a pilot project in order to apply the method of testing into evaluating the quality and working capacity of these bridges. Through results obtained, the following remarks are addressed:

(i) During the spans' operating process, under the effects of random moving load, spans' responses not only have a single vibration type but also have a combination of different vibration types and compound vibration types (the combination of 2 or many different vibration types). Depending on each structure and mechanical characteristics of materials, one vibration member will be more sensitive than the others and vice versa. Particularly, with regard to spans having complicated structures like flyover bridges and cable-stayed bridges, their responses will become even more complicated due to the occurrence of many combined vibration types.

(ii) From practical surveys, researchers in this article have recommended using a representative power spectral density as an effective characteristic which demonstrates different constituent vibrations concurrently occurring on vibration spectrums of spans. The representative power spectral density characterizes the vibration process of spans during a certain measurement period. In order to see the changing process of responses of spans, we should survey the representative power spectral density through different measurement periods.

(iii) A representative power spectral density illustrates the probability of occurrence of each vibration type through many measurement periods. This is a new discovery which is going to be further and deeply studied in subsequent articles.

Data Availability

All data generated or analyzed during this study are included in this work.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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