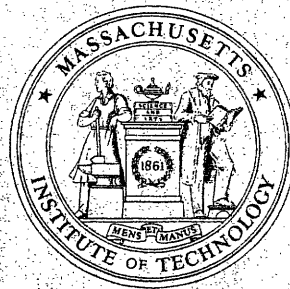


OPERATIONS RESEARCH CENTER

working paper



**MASSACHUSETTS INSTITUTE
OF TECHNOLOGY**

**A New M.I.T. Graduate Course:
Analysis of Urban Service Systems***

by

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During the 1971 spring term, we offered at MIT a new graduate level course, "Analysis of Urban Service Systems". The course was jointly registered in three departments:

Department of Civil Engineering--Prof. Amedeo Odoni

Department of Electrical Engineering--Prof. Richard C. Larson

Department of Urban Studies and Planning--Prof. Aaron Fleisher

There were two basic goals for this course:

1. to present through lectures and problem sets analytical tools useful for analyzing a large class of urban service systems--police and fire departments, emergency ambulance services, rapid transit systems, urban health care facilities, airports.
2. to have the student structure a comprehensive quantitative approach to a set of problems actually experienced by one agency. Wherever possible, this would entail interaction with the agency. The result of this effort would be written up in a formal report and presented to the class orally.

Basically the course was an attempt to bridge the gap between "case study" courses in which the student may not grasp the subtleties and limitations of the methods used and "methods" courses which are often cited by students as lacking in relevance and which do not often convey the complexities of the "real world". The course is a direct outgrowth of MIT's work on operations research and public systems, much of this originating at

the MIT Operations Research Center under the pioneering guidance of Prof. Alvin W. Drake.

Administratively, this was an experimental course. Although we listed courses in probability and calculus as prerequisites, we solicited participation by a wide variety of students, some without a strong mathematics background. These students were aided somewhat by tutorial sessions held by our teaching assistant, Gerry Banner. Also, these students usually had a strong background in sociology, urban economics, or other topics which would complement the talents of the more quantitatively oriented students. The credit hours were flexible and separately arranged for each student.

The course operated simultaneously on two levels. The first level involved student projects. At the beginning of the term, we handed out about ten "for instance" problems that were written up as suggestions for projects during the term. Originally we had projected an enrollment of about ten, thinking that each student would choose a separate project. However, our first day attendance exceeded 50, so we were forced somewhat to revise operating procedures. (About 30 stayed with the course through to completion.) We helped the students organize into groups of 2 - 4, each group focusing on a particular problem area. To indicate progress on this level of the course, we asked each group to submit a "progress report" every two weeks.

The second level was more traditional, with formal class lectures and problem sets. The problems in each set ranged from the drill variety to

quite difficult ones involving considerable expansion on the ideas presented in class. Each student was asked to submit whatever problems he was motivated to work on, but no particular problems were assigned. In fact, we didn't expect anyone to do more than, say, 75% of the problems. The worked problems were submitted for grading every two weeks (along with the project progress reports).

The evaluation of each student's performance was largely subjective. There was no formal quiz. The project report, both oral and written, the problem set submissions and our personal interactions with each student provided the necessary information for "grades".

In the following paragraphs we will briefly outline the course content, the experience with the projects, and the students' evaluation of the course.

LECTURE TOPICS

0. Introduction (Larson)

Our introductory lecture consisted of a general discussion of mathematical models, their use in decision-making applications and their limitations, especially in the complex environments of urban governments. We also discussed several of the "for instance" problems, four of which are stated below:

Rapid Transit

The RTA (Rapid Transit Authority) has been experiencing difficulties in handling rush hour traffic. So far, two operational problems have been identified.

The first involves the assignment of men to eight-hour shifts of duty to best reflect the time-varying demand rates. The Authority has to know the properties of several alternative shift structures for its next round of negotiations with the Transit Workers Union.

The second stems from the voluminous complaints received from passengers who claim that the delays incurred during rush hours are too great (both on the bus routes and on the train routes). Too often a potential passenger will wait for twenty minutes or so and then find two or three buses (or trains) arriving nearly simultaneously. This phenomenon has been labeled "clumping".

As an outside consultant, you have been hired to examine these problems. Depending on the quality of your recommendations, your consultantship may be continued for examination of a wide class of operational and managerial problems. How do you proceed?

Police Department

Since the State Legislature passed the "Three Tour Statute" in 1922, the Police Department has been legally constrained to allocate an equal number of men to each of the three tours of duty (midnight to 8:00 am, 8:00 am - 4:00 pm, 4:00 pm to midnight). In recent years this constraint has been particularly troublesome, since near-saturation loads occurred during predictable periods, but to relieve the congestion additional men would have to be added to the force around the clock. This was prohibited by budgetary pressures.

Examining this situation, the in-house planning and research group sees an opportunity to use simulation and analytical models in a very important way. Instead of accepting the existing statute as a "given" constraint, the group plans to examine how the patrol force would function if the law were modified to allow tours with non-equal manning. The group is convinced that the current total number of men available is sufficient to handle the needs of the city if the tours could be restructured (perhaps even allowing overlapping tours) to reflect the widely varying call-for-service rates and the needs for preventive patrol.

The group initially plans to use queuing models to get a rough idea of the number of men required by place and by time of day to achieve a "reasonable level of service". Then response and patrol models will be used to structure thinking about sector design, workloads, preventive patrol coverage, etc. Finally, several detailed simulation tests will be performed to determine the extent of improvement obtained by reallocating the men. If the results are sufficiently promising, the group plans to make the findings of the study publicly available. Eventually, it is hoped that this may cause a revision in the current law.

Working as the study group, how do you proceed?

Taxicabs

The Bonded Taxi Company has been losing revenue recently because of the overambitiousness of some of its own drivers. The radio dispatcher usually assigns that cab which claims to be closest to the scene of a call for cab service. However, some of the more ambitious drivers have been claiming to be "right around the block" while actually they might be at a distant location. Such a car takes a long time to reach the scene; upon arrival, the driver finds either a very dissatisfied customer who will never again use Bonded or no customer at all (because the customer called another cab company).

The Urbtronics Corporation has offered to sell Bonded a $\frac{1}{4}$ -mile-resolution car-locator system which, it is claimed, would provide the dispatcher with accurate position information, thereby eliminating distant dispatches due to overly ambitious drivers. Other advantages of the car-locator system would include safety to drivers and better tracking of actual passenger mileage.

You have been hired to evaluate the advantages (and disadvantages) of the car-locator system. What do you do?

Ambulance Services

Currently the city's emergency ambulance needs are handled by several private companies. These companies have been facing a deteriorating financial situation, with labor and maintenance costs increasing and an increasing number of indigents unable to

pay the cost of ambulance service. The companies cannot afford the expense of highly trained drivers; but a recently passed state law requires that all drivers have substantial paramedical training by January 1 of next year. Given this situation, it is highly probable that the private companies will go out of business and that some other means of providing ambulance service will have to be provided.

The Mayor's office has requested a study group to examine and evaluate alternative proposals for providing ambulance service:

1. Incorporate ambulance service into police department operations;
2. Have a separate city-sponsored ambulance fleet;
3. Subsidize current companies or a merged version of those companies.

As the appointed study group, structure and analyze this problem, paying particular attention to operational and economic issues.

Also in the general introduction, we surveyed the allocation problems experienced by urban emergency service systems and assigned as outside reading, "Methods for Allocating Urban Emergency Units", (Chaiken and Larson, New York City Rand Institute, 1971).

1. Geometrical Probability (Larson)

This topic is relatively unknown in recent operation research literature. It deals with random points in a plane; random planes

in 3 space; geometrical interrelationships among points and/or planes; problems of coverage; invariances to translations, reflections and rotations. Kendall and Moran have a fascinating monograph (Geometrical Probability, Griffin and Co., London, 1963) which surveys the important theoretical results and provides applications in astronomy, atomic physics, biology, crystallography, sampling theory and virology. In an urban environment these ideas are particularly relevant to obtaining travel time relationships, given the spatial distributions of service units and incidents which may occur anywhere in the city.

In our course we were limited to three 90-minute lectures on these topics, and these focused on the following:

- Review of functions of random variables
- Spatial Poisson processes
- Crofton's theorems (for computing mean values, cumulative distribution functions, etc.)
- Robbins' theorem on random sets (dealing with probabilistic aspects of coverage)
- Applications to urban services (e.g., travel distances and travel times)

Our 10-problem problem set covering this material provided enough diversity to be interesting to nearly all the students in the class. Several examples follow:

Crofton's Theorem on Mean Values

Here we wish to apply Crofton's method for finding mean values to the problem of finding the mean Euclidean distance,

$$E[D] = E \left[\sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} \right]$$

where (X_1, Y_1) and (X_2, Y_2) are uniformly and independently distributed over a circle of radius R.

(a) By arguments similar to those used in the lecture, show that

$$\delta\mu = \frac{4(\mu_1 - \mu)}{R} \delta R$$

where

$$\mu = E[D]$$

$$\mu_1 = E[D / \text{one of the points is in the infinitesimal ring of width } \delta R \text{ on the circle circumference}]$$

$$\mu + \delta\mu = \text{mean travel distance in a circle of radius } R + \delta R.$$

(b) Show that

$$\mu_1 = \frac{1}{\pi R^2} \int_0^{2R} 2x^2 \cos^{-1} \frac{x}{2R} dx = \frac{32R}{9\pi}$$

(c) Use your results in (a) and (b) to obtain

$$\mu = E[D] = \frac{128}{45\pi} R$$

Coverage; Robbins' Theorem on Random Sets.

Imagine a square region of a city having unit area. Suppose there are N ambulettes whose positions are independently and

uniformly distributed over a region T consisting of all points in the city whose distance from the square is not greater than a . The area of T is $1 + 4a + \pi a^2$. A point in the unit square is said to have sufficient ambulance coverage if at least one ambulance is within a (Euclidean) distance a of the point. Find the expected area within the square which is sufficiently covered.

Coverage of a square lattice by a rectangle.

The city's geographical structure is being placed on a computer. All coordinate positions are being quantized, where the unit of quantization is 500 feet. The quantization points comprise a lattice which runs East-West and North-South. The police department wishes to know how many quantization points will be contained in an arbitrary rectangular sector of dimension l (East-West) and m (North-South).

Assume that the location of the sector on the lattice can be modeled as random (but the sides are parallel to the two directions of the lattice). Let N be the number of lattice points contained within the sector.

(a) Show that

$$E[N] = lm$$

(b) Let $l = p + q$, $m = P + Q$ ($0 \leq q, Q < 1$). Show that

$$\sigma_N^2 = q(1 - q)Q(1 - Q) + l^2Q(1 - Q) + m^2q(1 - q).$$

Spatial Poisson Process

Suppose response units are distributed throughout the city as a homogeneous spatial Poisson process, with an average of λ response units per square mile. Assume that the travel time between (x_1, y_1) and (x_2, y_2) is

$$t = \frac{|x_1 - x_2|}{v_x} + \frac{|y_1 - y_2|}{v_y} ,$$

where v_x and v_y are travel speeds in the directions of the abscissa and ordinate, respectively.

Assume an incident occurs somewhere in the city, independent of the locations of the response units. Find the pdf for T_k , where

$$T_k = \text{travel time to the } k\text{'th nearest response unit, } k = 1, 2, \dots$$

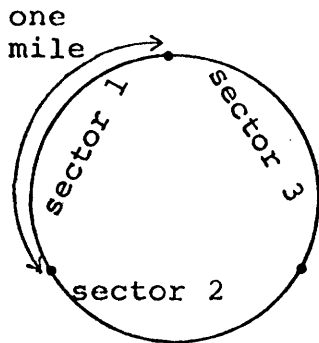
2. Multi-Server Queuing Systems (Odoni)

This 2-lecture sequence covered the queues M/G/1, M/M/S, multi-server priority queues, and queues with blocking. M/G/1, in particular, provides much insight when applied to urban service systems, where the service time is the sum of travel time to and from the scene and service time at the scene. In attempting to convey the material to as many students as possible, no transform analysis was used in deriving the queuing results. For instance, the well-known algebraic treatment of M/G/1 was used to derive mean in-queue waiting time.

3. Spatially Distributed Queues (Larson)

This 2-lecture sequence built on the ideas of geometrical probability and multi-server queuing theory to examine some special properties of multi-server spatially distributed queues. A sample from the problem set is given below:

A Three-Server Queue.



A certain circular highway is patrolled by three public safety cars. Each car patrols a one-mile sector of the three-mile highway. (see Figure 1)

(Figure 1)

Calls for assistance occur along the highway. A dispatcher assigns a car to each call, if a least one is available. We wish to examine various operating properties of this system.

The system parameters are as follows:

1. Call positions are uniformly, independently distributed over the circular highway.
2. The call arrival process is a homogenous Poisson process with rate parameter λ calls per hour.
3. Service time at the scene of the call has a negative exponential distribution with mean $\mu^{-1} = 1/2$ hour.
4. Travel time is negligibly small compared to service time at the scene.
5. Speed of response is always 15 mph.
6. U-turns are permissible everywhere.

For parts (a) - (c), assume the dispatching strategy is as follows:

Given a call from sector i ($i = 1, 2, 3$),

1. Assign car i , if available.
2. Otherwise, randomly choose some car j ($j \neq i$), and assign it, if at least one other car is available.
3. Otherwise, the call is lost.

- (a) Find the steady-state probability that i cars are busy ($i = 0, 1, 2, 3$).
- (b) Find the steady-state probability that car 1 is busy and car 2 is free.
- (c) Find the average travel time to δ call for this system. Evaluate for $\lambda \approx 0$, $\lambda = 3$, $\lambda = 1000$.
- (d) It has been proposed that the Public Safety Bureau should purchase a perfect resolution car locator system. With such a system, the dispatching strategy is changed as follows:

Given a call from sector i ($i = 1, 2, 3$),

1. Assign the closest available car, if at least one is available;
2. Otherwise, the call is lost.

Find the average travel time to a call for this system. Evaluate for $\lambda \approx 0$, $\lambda = 3$, $\lambda = 1000$.

4. The Response Areas for Emergency Units (Guest Lecturer, Jan Chaiken)

Jan Chaiken kindly agreed to present to the class a 90-minute version of the paper, "Response Areas for Two Emergency Units", (Carter, Chaiken, Ignall, New York City Rand Institute, 1971).

In talking with students afterwards, they seemed to be particularly interested in the ideas of "dominance" illustrated in that paper which provide a convenient way of dealing with multiple objectives.

5. Current Models of the Metropolis (Fleisher)

This 3-lecture sequence attempted to construct a bridge between the operationally-oriented models discussed earlier and more global urban planning models, as illustrated in the work of Lowry, Icard,

and, more recently, Forrester. Several of these global models were discussed in detail, with emphasis on their scope, decision-making applications, and limitations.

6. Network Techniques (Odoni)

This 2-lecture sequence introduced some of the classical results in network analysis that are applicable to certain urban service systems. Among the problems discussed were the minimal spanning tree problem, the shortest route problem, and the maximal flow problem. The homework problems developed these ideas further in situations relating to urban service systems.

7. Location and Redistricting Theory: Algorithmic Formulations

(Guest Lecturer, Dave Marks)

Dave Marks gave two very interesting lectures on algorithmic formulations of many of the problems discussed elsewhere in the course from a geometrical and probabilistic point of view. Much of the material is from the paper "An Analysis of Private and Public Sector Location Models" (Revelle, Marks and Liebman, Management Science, Vol. 16, No. 11, July, 1970).

8. Multi-Attributed Utility Theory (Guest Lecturer, Ralph Keeney)

Ralph Keeney presented some of his interesting results applying multi-attributed utility theory to determining the relative desirability of various response patterns to fires. For instance, is it more desirable for the first engine to have a one minute response time and the first ladder a three minute response time, or for both to have a two minute response time?

9. Simulation (Odoni, Larson, Fleisher)

Finally, the formal lecture part of the course ended with three lectures on simulation. The topics ranged from general aspects of Monte Carlo Simulation (Odoni), to special problems encountered within urban service systems (Larson), to simulating large planning models involving population mobility, economic growth and other factors (Fleisher).

PROJECTS

As indicated previously, the class worked in groups of 2 - 4 on particular problems of urban services concurrently with the lecture presentations and problem sets. The oral class presentations were given in May, following the formal lecture sequence. Final reports were submitted on the last day of class. The topics varied widely with respect to the topics addressed and the roles played by quantitative methods. A partial list of report titles reads as follows:

- The Location of Emergency Ambulances in Boston
- On **Outpatient** Scheduling and the No-show Problem
- Public Sector Vehicle Routing Problems: Optimization and Heuristic Techniques
- East Boston is Not an Airport
- Jury Selection and Management in Suffolk County Court
- The Urban Open Space System
- Frequency Allocation/Channel Sharing
- Towards the Quantification of Transit Scheduling Procedures
- MBTA, Some Deficiencies and Suggestions for Improvements

It is interesting to observe that this list is quite different from the original set of "for instance" problems we gave the class on day number one. More so than we had expected, each group seemed to be able to identify their own problem area and structure a coherent approach to a class of problems in that area.

To take an example, in the ambulance study, two students had originally expressed an interest in police department problems. The Director of Planning and Research at the Boston Police Department briefed us on the City's current transition from a police-run emergency ambulance service to a system operated out of Boston City Hospital. The current police system utilizes standard police wagons and modified station wagons as ambulances for the great majority of emergencies reported in the City of Boston. The modified system operated out of City Hospital will utilize well-equipped ambulances with highly trained drivers and attendants. These could still be dispatched by the police dispatcher or by the ambulance dispatcher at City Hospital. The transition to the new system is still in the early phases, there being less than five new ambulances operating. There are many questions now being faced concerning the organizational structure of the new system, including the total number of ambulances to purchase, where to put them, and how the system should operate.

To assist the students in addressing these problems, the Boston Police Department kindly agreed to perform some special computer runs indicating the time and space distribution of ambulance calls in the City of Boston. The students used these and other data to focus on the short-term problems associated with the location and operational procedures of the next

several ambulances to be purchased. One of the students (Jan Hoey) now plans continued study of the broader aspects of this problem as a Master's thesis.

The jury selection and management study originated with suggestions by Mr. J. Romanow, Legal Systems Analyst for the Suffolk County Court. He had been conducting a study with an aim toward proposing legislation to modify the way juries were selected in order to achieve a more representative sample of the population. Also, his project involved the management of particular jurors once within the courthouse. Three students worked on various parts of these problems from a quantitative point of view--examining city-wide demographic characteristics, obtained from the city census, and comparing to a sample of jurors in one year. The jury management part of the study included an examination (using queuing theory where appropriate) of the number of jurors needed, given the demand currently generated by trials and pre-trial activity. One of these students has been employed by the courts as a part-time consultant to continue the study through the summer. Another is continuing along to a Doctorate, developing quantitative models of other parts of the criminal justice system.

STUDENT EVALUATION

At the end of the term we held an informal 1-hour "cider and donuts" meeting to review the course from the students' point of view. In addition to this discussion, we asked each of them to fill out a two-page course evaluation sheet.

We were particularly interested in their reaction to the experimental format in which technical topics were presented in class, not to be later "tested" in a traditional sense, but rather in a project format which allowed each student to focus on whatever sub-topics he found relative to his project.

By and large, the response to the course and its format was very favorable. As one student said, "It's nice for a change to get something in a course which is useful." Another remarked that the problem of having to formulate and structure a real world problem had been the most valuable single feature of his graduate education. Others were happy to have interacted with administrators in operating agencies, several thereby obtaining thesis topics or consulting jobs or the satisfaction of having had some impact. As possible revisions, the students suggested:

- A guest lecture from a city mayor or other administrator
- More emphasis on model validation and parameter estimation
- More discussion of actual experiences resulting from applying quantitative techniques

We hope to incorporate these suggestions into next year's course.