

A New Mathematical Model for Multi Product Location-Allocation Problem with Considering the Routes of Vehicles

Hany Seidgar, Sahar Tadayoni Rad and Hamed Fazlollahtabar

Abstract--- Facility location–allocation (FLA), which has been proved to be a valuable method in siting service facility, is widely used in real life, such as emergency service systems, telecommunication net works, public services, etc. FLA provides a valuable method in deciding where to place facilities coupled with determining how to assign demand to the located facilities in order to utilize resources effectively. This paper presents a new mathematical model for locating and allocating in transportation system in which each production center can produce multi type of products. The objective is to minimize the total costs in the production and transportation systems such as: fixed cost of opening the production centers, total production cost and total transportation cost that are entered to system by vehicles.

Keywords--- Location Allocation Problem, Vehicle Routing Problem, Multi Products System, Optimization

I. INTRODUCTION AND LITERATURE REVIEW

LOCATION a facility into the best place is a decision making problem. The best place depends to the optimal distance, the capacity of the facility, optimal cost etc. Location allocation can be based on one criterion like optimal distance or adding various combinations of criteria, for example: distance and capacity of the facility together or capacity of the facility or optimal cost together and so on. Therefore, the goal of the location allocation problem is to obtain the best location or locations to fit one or more facilities.

Facility location–allocation (FLA) provides a valuable method in deciding where to place facilities coupled with determining how to assign demand to the located facilities in order to utilize resources effectively. This problem received much attention recently and was formulated as several kinds of models, such as continuous location models, network location models and mixed integer programming models. The vehicle routing problem (VRP) can be defined as finding optimal routes for a set of vehicles to serve some geographically scattered customers to minimize the total

operation cost, with each route starting and ending at the depot. VRP at first was discussed by Dantzig et al., [1]. In the traditional VRP, the vehicles are the servers for the customer locations and a customer requires only one task (or visit) by one vehicle. Min [2] was the first to introduce vehicle routing problem with simultaneous pickup and delivery (VRPSPD) for minimizing the total travel time of the route by considering the vehicle capacity as the problem constraint. As Perl and Daskin [3] pointed out, location routing problems involve three inter-related, fundamental decisions: where to locate the facilities, how to allocate customers to facilities, and how to route the vehicles to serve customers. The difference of the location routing problem from the classic vehicle routing problem is that not only routing must be designed but also, the optimal depot location must be simultaneously determined. The main difference between the location routing problem and the classical location – allocation problem is that, once the facility is located, the former requires a visitation of customers through tours while the latter assumes that the customer will be visited from the vehicle directly and, then, it will return to the facility without serving any other customer (Min *et al.*, [2]). The location routing problem is very difficult to solve with the use of exact algorithms, especially if the number of customers or the candidate for location facilities are very large due to the fact that this problem belongs to the category of NP-hard problems, i.e. no polynomial time algorithms are known for their solution. However, a number of exact algorithms have been proposed for the solution of this problem solving mainly problems with a small number of locations and customers (Prins *et al.*, [4]; Russell *et al.*, [5]).

Dethloff [6] and Tang and Galvano [7] (2006) then, contributed on mathematical reformulations. Berbeglia *et al.*, [8] also introduced a general framework to model static pickup and delivery problems. Jin Ai and Kachitvichyanukul [9] generalized the three existing formulation and reformulated the VRPSPD as a direct extension of basic VRP. In solution technique areas, Moshvio [10] studied PDP with divisible demands, in which each customer can be served by more than one vehicle, and presented greedy constructive algorithms based on tour partitioning. Salhi and Nagy [11] proposed four insertion-based heuristics, in which partial routes are constructed for some customers in basic steps and then the remaining customers will be inserted to the existing routes. Dohn *et al.*, [12] studied the scheduling problem of ground handling tasks in some European airports. The problem requires manpower allocation with the consideration of time windows, job-teaming constraints, and a limited number of teams in order to

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maximize the total number of assigned tasks. Kim *et al.*, [13] presented a combined vehicle routing and staff scheduling problem where a certain number of tasks has to be performed in a fixed sequence by a set of teams. The teams should be moved by a set of vehicles and the objective is to find an efficient schedule for the teams. They developed a mixed integer programming model for the problem that could not be solved optimally due to out of memory errors. Anyway, they developed a dispatching based heuristic algorithm to find feasible solutions for the generated problems. Ho and Leung [14] studied a manpower scheduling problem with job time windows and job-skills constraints. They considered an airline servicing operations before the flights take-off. Given the jobs to be serviced and the roster of workers for each shift, the problem is to form teams and assign them to the jobs, to service as many flights as possible.

This paper studies the location-allocation and vehicle routing problem. Each customer has several products. Demand of each customer for each product satisfied by production centers and rule of the vehicles that are existed in the each production center is to move the product to each customer.

II. PROBLEM DEFINITION AND MATHEMATICAL MODEL

This paper considers location-allocation problem with multi product and routing of vehicles. Each production center can produce type of products. A new mathematical model for location-allocation problem with multiple productions is presented. The objective of the problem is to minimize the sum of location-allocation and vehicle routing costs.

The assumptions, indices, parameters and decision variables are introduced before developing the proposed mathematical model.

A. Assumptions

- There are number customers locating in known points.
- The travel distance between two nodes (customer-production center and customer-customer) is calculated using the Euclidean distance.
- Customers may require multi products and it is possible for them to give of one or more production center in the system.
- Numbers of candidate production centers are pre-defined and there are several known vehicles to serve the customers in the system.
- Vehicles are different types based on their capacities and limitation on move the products to customers.
- Each vehicle can used in more than one tour and will be back to its own depot at the end of the tour.
- Demand of each customer is predefined for each product.

The structure of problem graphically illustrated in Figure 1.

B. Indices

- j index of customers ($j=1,2,\dots,NC$)
- l index of production centers ($l=1,2,\dots,NPC$)
- s index of type of productions ($s=1,2,\dots,NP$)
- k index of vehicles ($k=1,2,\dots,NV$)

C. Parameters

- NC Number of customers.
- NPC Number of production centers.
- NP Number of products (goods).
- NV Number of vehicles in transportation system.
- D_{sj} Demand of customer j for product s .
- Ca_l Production capacity of production center l .
- Cv_k Capacity of vehicle k .
- LB_{sl} Lower bound of product s in production center l if l th production center opens for product s .
- UB_{sl} Upper bound of product s in production center l if l th production center opens for product s .
- $a_{sl} =$
1: if l th production center can be produced s th product, otherwise 0.
- $b_{kl} =$
1: if k th vehicle exists in l th production center, otherwise 0.
- $c_{ks} =$
1: if k th vehicle can be transport s th product, otherwise 0.
- F_{sl} Fixed cost of production s if it produces in production center l .
- $Cost1_{sl}$ Production cost of product s in production center l .
- $Cost2_k$ Routing cost of vehicle k .

D. Decision Variables

- $x_{klsj} =$
1: if s th product transport from l th production center to j th customer by k th vehicle, otherwise 0.
- y_{klsj} Amount of s th product that are transported from l th production center to j th customer by k th vehicle.
- $z_{jlk} =$
1: if k th vehicle loaded from l th production center to j th customer, otherwise 0.
- $w_{ijlk} =$
1: if k th vehicle loaded of l th production center moved from customer i to customer j , otherwise 0.
- $w'_{jlk} =$
: if k th vehicle came from l th production center to customer j , otherwise 0.
- $w''_{jlk} =$
1: if k th vehicle backed from customer j to production center l , otherwise 0.
- $N_{sl} =$
1: if l th production center produced s th product, otherwise 0.

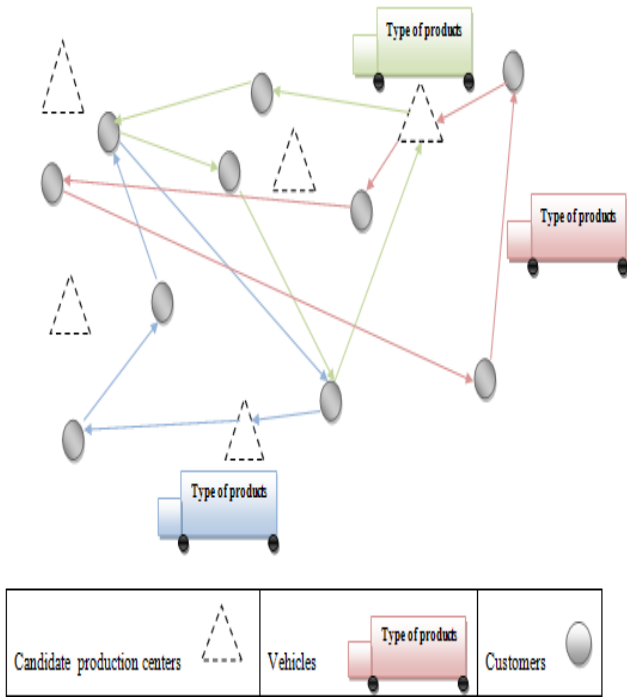


Figure 1: Structure of Location-Allocation Problem with Multi Products and Vehicle Routing Problem

E. Mathematical Model

$$\begin{aligned}
 & \text{Minz} \\
 & = \sum_{l=1}^{NPC} \sum_{s=1}^{NP} (Cost1_{sl} \cdot [\sum_{j=1}^{NC} \sum_{k=1}^{NV} y_{klsj}] + N_{sl} \cdot F_{sl}) \\
 & + \sum_{i=1}^{NC} \sum_{j=1}^{NC} \sum_{k=1}^{NV} (w'_{ijk} + w''_{jlk}) \cdot Cost2_k \cdot dis_{ij} \\
 & + \sum_{i=1}^{NC} \sum_{j=1}^{NC} \sum_{k=1}^{NV} \sum_{l=1}^{NPC} w_{ijkl} \cdot Cost2_k \cdot dis_{ij} \tag{1} \\
 & x_{klsj} \leq a_{sl} \cdot b_{kl} \cdot c_{ks} \quad \forall k, l, s, j \tag{2} \\
 & y_{klsj} \geq x_{klsj} \quad \forall k, l, s, j \tag{3} \\
 & y_{klsj} \leq M \cdot x_{klsj} \quad \forall k, l, s, j \tag{4} \\
 & \sum_{k=1}^{NV} \sum_{l=1}^{NPC} y_{klsj} = D_{sj} \quad \forall s, j \tag{5} \\
 & \sum_{j=1}^{NC} \sum_{k=1}^{NV} \sum_{s=1}^{NP} n'_s \cdot y_{klsj} \leq Ca_l \quad \forall l \tag{6} \\
 & \sum_{j=1}^{NC} \sum_{l=1}^{NPC} \sum_{s=1}^{NP} n_s \cdot y_{klsj} \cdot b_{kl} \leq Cv_k \quad \forall k \tag{7} \\
 & N_{sl} \leq a_{sl} \quad \forall s, l \tag{8} \\
 & \sum_{j=1}^{NC} \sum_{k=1}^{NV} y_{klsj} \geq LB_{sl} \cdot N_{sl} \quad \forall s, l \tag{9} \\
 & \sum_{j=1}^{NC} \sum_{k=1}^{NV} y_{klsj} \leq UB_{sl} \cdot N_{sl} \quad \forall s, l \tag{10}
 \end{aligned}$$

$$\sum_{j=1}^{NC} \sum_{k=1}^{NV} y_{klsj} \leq \sum_{j=1}^{NC} D_{sj} \cdot N_{sl} \quad \forall s, l \tag{11}$$

$$z_{jlk} \geq \sum_{s=1}^{NP} y_{klsj} \quad \forall j, k, l \tag{12}$$

$$z_{jlk} \leq M \cdot \sum_{s=1}^{NP} y_{klsj} \quad \forall j, k, l \tag{13}$$

$$w_{ijkl} \leq 0.5(z_{ilk} + z_{jlk}) \quad \forall i, j, k, l \tag{14}$$

$$w_{ijkl} \geq (z_{ilk} + z_{jlk} - 1) \quad \forall i, j, k, l \tag{15}$$

$$w'_{ijk} \leq z_{jkl} \quad \forall j, k, l \tag{16}$$

$$w'_{ijk} + \sum_{i \neq j}^{NC} w_{ijkl} = z_{jlk} \quad \forall j, k, l \tag{17}$$

$$w''_{jlk} + \sum_{i \neq j}^{NC} w_{ijkl} = z_{jlk} \quad \forall j, k, l \tag{18}$$

$$s_{jlk} \geq s_{ilk} + 1 + M \cdot (w_{ijkl} - 1) \quad \forall i, j, k, l \tag{19}$$

$$x_{klsj}, z_{jlk}, w_{ijkl}, w'_{jlk}, w''_{jlk}, N_{sl} = \{0,1\} \tag{20}$$

$$y_{klsj}, s_{jlk} \geq 0 \tag{21}$$

Objective value defines by Eq (1) that the objective is to minimize the total costs (Routing and production costs).Eq (2)causes to earn a feasible solution space for seeking the good solution based on ability of vehicles and production center for moving and producing them respectively. Esq (3)and(4) ensure that amount of products that are assigned to each customer and moved by each vehicle when are exist that there is created connection. Eq(5) guaranties each demand of each customer for each product satisfy by production centers that are in the production system. Eq (6) ensures that cannot produce at each production center more than their capacities. Eq(7) ensures that each vehicle cannot move bigger than their capacities at a tour of its route. Esq (8)-(11) ensure when a production line for a product can be ran that lower and upper bounds satisfied. Esq (12) and (13) help to identify the connection between customer-customer and customer – production center. Esq (14)-(18) determine the route of each vehicle at each production center(tour of each vehicle). Restriction (19) is sub-tour elimination constraint set. Constraints (20) and (21) provide limits on the decision variables.

III. NUMERICAL EXAMPLE

In order to confirm that the mathematical model works as intended, an example is provided and the results of the execution with details of location-allocation of customers by vehicles are presented. Suppose that there are 5 customers, 4 vehicles, 2 products, 3 production centers, the demand of each customer for each product, cost of vehicle, capacity of each vehicle and production capacity of each production center are shown in Tables 1-13 and $n_s = n'_s = 1$.

Table 1: Location of Customer of Example

Number of customer	1	2	3	4	5
(X,Y)	(10,20)	(15,30)	(20,15)	(18,25)	(35,40)

Table 2: Location of Candidate Production Center of Example

Number of candidate production center	1	2	3
(X,Y)	(10,20)	(15,30)	(20,15)

Table 3: Capacity of Production Centers of Example

Number of production center	1	2	3
Production capacity	200	300	500

Table 4: Capacity of Vehicles of Example

Number of vehicle	1	2	3	4	5	6
Capacity of vehicle	50	100	150	50	200	150

Table 5: Fixed Cost for Opening Production Line of a Product at a Production Center of Example

	Production center 1	Production center 2	Production center3
Product 1	500	400	300
Product 2	400	350	300

Table 6: Possibility for Producing a Product at a Production Center

	Production center 1	Production center 2	Production center3
Product 1	1	1	0
Product 2	1	1	1

Table7: Lower Bound of a Product at a Production Center of Example

	Production center 1	Production center 2	Production center3
Product 1	40	30	50
Product 2	20	40	30

Table 8: Upper Bound of a Product at a Production Center of Example

	Production center 1	Production center 2	Production center3
Product 1	200	150	100
Product 2	300	350	200

Table 9: Demand of Each Customer for Each Product

	Custome r 1	Custome r 2	Custome r 3	Custome r 4	Custome r 5
Produc t 1	80	40	35	50	90
Produc t 2	100	30	25	35	20

Table 10: Producing Cost of a Product at a Production Center

	Production center 1	Production center 2	Production center3
Product 1	2	4	1
Product 2	2	3	2

Table 11: Cost of Vehicle

Number of vehicle	1	2	3	4	5	6
Capacity of vehicle	15	20	30	40	5	15

Table12: Exiting Possibility for Each Vehicle of each Production Center

	Production center 1	Production center 2	Production center3
Vehicle 1	1	0	0
Vehicle 2	0	0	1
Vehicle3	1	0	0
Vehicle4	0	1	0
Vehicle 5	0	1	0
Vehicle 6	1	0	0

Table 13: Ability of Each Vehicle for Moving Each Product

	Product1	Product2
Vehicle 1	1	0
Vehicle 2	1	1
Vehicle3	0	1
Vehicle4	1	1
Vehicle 5	1	0
Vehicle 6	1	1

The objective is to minimize the total costs (Routing and production costs).The following Figure illustrates local solution obtained of solution space in details from the mathematical model. Figure2 illustrates locations 1, 2 and 3 open to satisfy demands of the customers in the system production. Vehicles 1 and 6 at the first production center, vehicles 4 and 5 at the second production center and finally vehicle 2 at the third production center moved of the production centers to satisfy the demands of the existing customers. Vehicles1,4,5 and 6 load 50, 40, 200 and 5 units respectively of product 1 to sent them to their customers and vehicles 2 and 6 load 65 and 145 units of product 2 to satisfy the demand of the customers in the system production. Amount of the demands that are satisfied by each vehicle at the each production center are shown in Table 14. Also, based on the solutions that are obtained by solving the mathematical model, production line for producing product 1 is activated at the centers 1 and2. At the production center 3 only activates production line2.

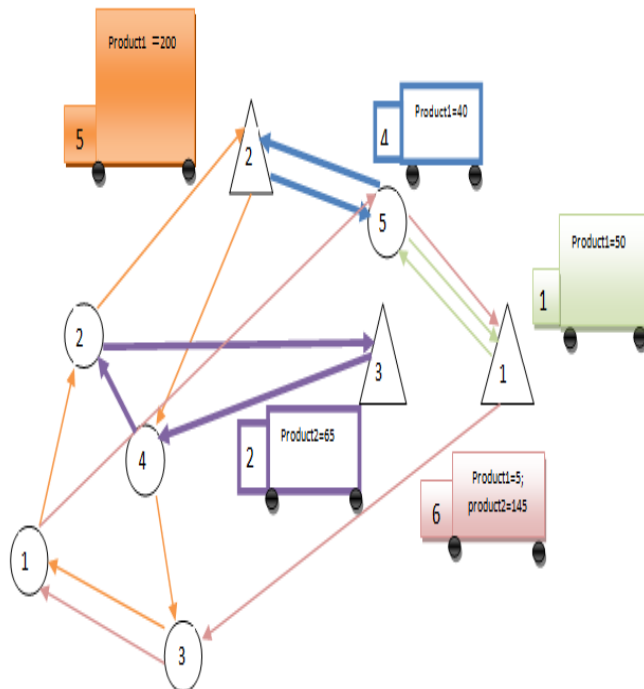


Figure 2: Presentation of the Solution Graphically

As it can be seen in Figure 2 and the Table 14 , the objective value is equal to 5923.785.

Table 14: Amount of the Each Product that are Transported by Each Vehicle to Each Customer

Decision Variable	Value of the decision variable	Decision Variable	Value of the decision variable
y_{1115}	50	y_{5213}	30
y_{2322}	30	y_{5214}	50
y_{2324}	35	y_{6113}	5
y_{4215}	40	y_{6121}	100
y_{5211}	80	y_{6123}	25
y_{5212}	40	y_{6125}	20

IV. CONCLUSION AND FUTURE ENHANCEMENT

In this paper a new mathematical model is presented for locating and allocating candidate production center with considering the route of vehicles. The objective was to minimize the total costs in the transportation and production systems. This problem is Np-hard, therefore it is worthwhile to apply meta-heuristic algorithms to find the good solution in the solution space. Drivers have significant rule for traveling the products by vehicles, the competency of drivers can be considered for future research.

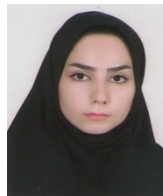
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