A New Mathematical Model for the Restoration Problem in Balanced Radial Distribution Systems

Rubén Romero, *Senior Member, IEEE*, John F. Franco, *Member, IEEE*, Fábio B. Leão, *Member, IEEE*, Marcos J. Rider, *Member, IEEE*, and Eliane S. de Souza

***Abstract*—This paper presents a comprehensive mathematical model to solve the restoration problem in balanced radial distribution systems. The restoration problem, originally modeled as mixed integer nonlinear programming, is transformed into a mixed integer second-order cone programming problem, which can be solved efficiently using several commercial solvers based on the efficient optimization technique family branch and bound. The proposed mathematical model considers several objectives in a single objective function, using parameters to preserve the hierarchy of the different objectives: 1) maximizing the satisfaction of the demand, 2) minimizing the number of switch operations, 3) prioritizing the automatic switch operation rather than a manual one, and 4) prioritizing especial loads. General and specialized tests were carried out on a 53-node test system, and the results were compared with other previously proposed algorithms. Results show that the mathematical model is robust, efficient, flexible, and presents excellent performance in finding optimal solutions.**

***Index Terms*—Distribution system optimization, mixed integer second-order cone programming, restoration problem.**

I. Introduction

THE problem of the restoration of radial electrical distribution systems (EDSs) is one of the most relevant topics related to the efficient operation of radial distribution power systems. This type of problem attempts to restore the portion of the electrical system that was de-energized as a result of switching operations to isolate a fault and the system can be re-energized through the operation of interconnection switches. In this context, the main goal is to restore the largest number of disconnected loads (out-of-service areas). The restoration must be carried out without violating the constraints of EDS operation, while still preserving the radial topology and satisfying the constraints of the problem. Such constraints include the constraints of load flow related to Kirchhoff's laws and the operational limits related to the current capacity of feeders, the voltage limits of nodes, and the capacity of the substations, among others.

Reference [1] presents a review of the key publications up to 1996 that have discussed the main characteristics of the restoration problem. The authors mention that the restoration problem must have two fundamental objectives: to restore the maximum possible demand of the region affected by the fault and to implement the restoration process as soon as possible. The authors suggest that other goals, such as minimizing power losses and load balancing between feeders, should not be considered in the restoration process. Furthermore, some operational restrictions, such as the restriction in values of voltage, can be relaxed or disregarded. They acknowledge that the restoration problem is a combinatorial nature problem and, as a result, none of the reviewed references have applied mathematically rigid optimization methodologies to the restoration problem. In [1], the authors continue to analyze the different characteristics of the problem and the constraints that may be considered in the restoration problem, including 1) the types of faults treated, i.e., line faults, bus faults, and transformer faults, 2) the portion of the distribution network that is involved in the restoration process, 3) the consideration of important consumers, i.e., important consumers found in the unaffected portion of the system must not have swapped their feeders, and those in the affected zone should be reconnected on a preferential basis; 4) after the solution of the restoration problem is found, then the sequence of switching operation for reconnection must be provided; 5) the length of the restoration process, i.e., if it takes too long, the variation of demand response should be taken into account; 6) the method of deciding the voltage constraints that can be relaxed or disregarded; and 7) the operation of the switches, i.e., switches are remotely operated or locally operated and, therefore have different times of operation. Finally, in [1] it is recognized that loss minimization is an important operational objective in the normal operation, but, in the restoration process, it generates an insignificant benefit and may conflict with the fundamental objectives. The same consideration can be applied to load balancing, and, hence, these goals should be not pursued in the restoration problem.

Another important contribution is presented in [2], which proposes an Non-dominated Sorting Genetic Algorithm-II (NSGA-II) algorithm to solve the restoration problem considered as a multi-objective optimization problem. In its formulation, the restoration problem in [2] considers most of the aspects mentioned in [1]. However, there is special emphasis on the following aspects: 1) the demand to be restored should be the maximum possible, but the minimization of power losses should also be taken into account; 2) the radial topology must be preserved, even during the operation of switches in the restoration process; 3) voltage limits and current capacity constraints must be considered; 4) the number of switching operations should be kept to a minimum, considering differently operations of

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The authors are with the Faculdade de Engenharia de Ilha Solteira, UNESP—Universidade Estadual Paulista, Departamento de Engenharia Elétrica, Ilha Solteira—SP, Brazil (e-mail: ruben@dee.feis.unesp.br; jffranco@gmail.com; bertequini@dee.feis.unesp.br; mjrider@dee.feis.unesp.br; elianesouza.col@gmail.com).

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automatically controlled switches (ACS) and manually controlled switches (MCS); and 5) priority loads should be treated preferentially, and the restoration process should be performed in the shortest amount of time possible. The authors choose four objective functions in the multi-objective optimization strategy: the maximization of the load not restored, the minimization of the ACS operations, the minimization of the MCS operations, and the minimization of power losses. A very important aspect of this work is that the authors acknowledge that these goals are not equally important and can be prioritized. As such, they consider the minimization of the load restored to be the most important. This proposal is used to identify the best solution that should be implemented in the restoration process.

In [3], the authors present an efficient heuristic method based on ranking, which considers various objective functions ranked in order of importance, i.e., maximizing the amount of priority load to be restored, maximizing the amount of total load restored, and minimizing the number of switching operations. Additionally, the mathematical model considers the three-phase power flow and allows for the analysis of real and large systems. In [4], the authors add the strategy of closing multiple switches that are normally open (multi-tier switching) as part of the search, increasing the search space of the heuristic strategy and therefore finding better quality solutions than those found in [3]. The authors call this strategy a single-tier algorithm. Also, the authors include both a geographical distance objective function and a reprogramming of the capacitors operation that are in the portion of the system not energized following the isolation of the fault. In [5], the authors add a heuristic with possibility of performing a load curtailment of in-service customers. This possibility allows, for instance, for a node that is still energized with a small demand to be turned off in order to enable the reconnection of a larger demand node that has been de-energized by the out-of-service area. The possibility of load curtailment in nodes not isolated by the fault increases the complexity of the problem, but generates better quality solutions.

In literature, few mathematical models proposed for the restoration problem are solved by exact optimization techniques. In these few studies, the models are relaxed and used as auxiliary heuristic strategies. Thus, in [6], a mixed integer linear programming model is presented. This model is relaxed (e.g., among other simplifications, it only uses the active power balance) and used only in the second phase of the optimization strategy, which accurately solves a portion of the system using a branch and bound algorithm. In the phase one, the researches separate the problem into subproblems, considering the knowledge of restoration experts.

In [7], a mathematical model is used when the initial heuristic strategy does not find a feasible solution. In this context, a portion of the system, called the "local network", is modeled through a linear programming problem using a commercial mixed integer solver. Thus, classical optimization is used as an auxiliary element to the heuristic. In addition, the mathematical modeling is also relaxed.

In [8], the restoration problem is separated into two parts. The first part deals with the classic formulation of the restoration problem, obtaining the final topology of the restored system using a genetic algorithm. The second part searches the switching operation sequence by using a dynamic programming approach.

In addition there are many publications related to the restoration problem, which are essentially devoted to the application of specific optimization techniques. Among these important publications are metaheuristics that use the multi-agent approach presented in [9], genetic algorithm, evolutionary approach and fuzzy sets, and the fuzzy non-dominated sorting evolution strategy presented in [10]–[12], heuristic optimization techniques presented in [13] and [14], a heuristic strategy based on a theoretical analysis of a search detailed in [15], a heuristic strategy that uses rules based on an operator's experience presented in [16], a heuristic strategy for the load transfer and specialized implementation of four steps in [17] and [18]. Furthermore, several methods were proposed to solve the restoration problem such as the application of expert system techniques analyzing group restoration, zone restoration and load transfer presented in [19], the heuristic generating the minimal paths to the boundary branches connecting the out-of-service area with the nearby area, presented in [20], service restoration using a fuzzy sets approach presented in [21], and the heuristic algorithm with an expert system that incorporates the network information and restoration strategies presented in [22].

After reviewing the relevant literature, we can see that there are no proposals for the complete mathematical modeling of the restoration problem, solving it using traditional optimization techniques, such as branch and bound algorithms, or commercial solvers based on exact optimization techniques. Many authors mention that this line of research is not important due to the combinatorial nature of the restoration problem. However, it should be noted that the additional problem of representing the radiality constraint through simple algebraic relations is crucial to having a complete mathematical model for problems in which the radial system topology of the optimal solution is required. This problem is solved in [23] in the formulation of model for the reconfiguration problem of radial EDSs. This formulation can be easily adapted to the restoration problem. Additionally, recent research has shown that several types of distribution network optimization problems that are originally mixed integer nonlinear programming problems can be transformed into convex optimization problems [24]–[26]. A set of conditions must be satisfied for the optimal solution of the transformed problem to be the same as that of the original problem. In this paper, the mixed integer nonlinear model developed can be transformed into a mixed integer second order cone problem as shown below.

This paper presents a comprehensive mathematical model to solve the restoration problem in radial EDSs. The original model is a problem of mixed integer nonlinear programming. However, this model is transformed into a problem of second-order cone programming, which can be solved efficiently using commercial solvers based on the efficient branch and bound optimization technique. Specifically, this paper addresses:

- The development of a general mathematical model of second-order cone programming to solve the restoration problem in balanced radial distribution systems.
- An analysis of the particular mathematical model formulations that may be more easily solved.
- A comparative analysis with other previously proposed algorithms.

II. Mathematical Model

The following assumptions were made in order to formulate the mathematical model for the restoration problem in EDSs: 1) a balanced three-phase AC power flow is considered, hence the EDS is represented by a single-phase equivalent; 2) the radial topology must be maintained; and 3) the operational limits, such as the current capacity of the feeders and the voltage magnitude limits, must be satisfied.

The objective function is modeled using five different terms. It should be noted that these terms are not necessarily equally important or conflicting. Therefore, we present a hierarchical mathematical modeling in which parts of the objective function are more important than others. This hierarchical process should account for the following criteria: 1) the most important goal is to meet the highest possible demand of the restored system and to perform all of the switching operations necessary to achieve this goal; 2) if it is necessary to open switches, then the automatic switches must be opened before the manual switches, since this kind of maneuver is possible; and 3) if it is necessary to close switches, then the automatic switches must be closed before the manual switches, since this kind of maneuver is possible. Finally, modeling the objective function makes it possible to prioritize the connection of more important loads over less important ones, yet various types of priority loads can be considered. It should also be noted that the objective function is a generalization of the proposal presented in [7]. As such, the first four terms of the objective function can be considered to be the restoration time costs or switching costs, and the last term can be considered to be the cost of non-supplied energy, as shown in [7] and [8]. However, the mathematical model has presented these terms as hierarchical, as observed in [2], and the coefficients used have been calibrated to preserve the hierarchical nature of the terms of the objective function. In addition, modeling the constraints in the new mathematical model represents an original contribution to the restoration problem. Previous studies have proposed relaxed constraints, considering only the active power balance. In our work, we have presented the full constraints of AC modeling used in distribution system operation and all of the operating limits constraints of the AC model.

The proposed mathematical formulation for the restoration problem in EDSs takes the following form:

$$\min \sum_{ij\in\Omega_{an}} \beta_{ij}x_{ij} + \sum_{ij\in\Omega_{ap}} \rho_{ij}x_{ij} + \sum_{ij\in\Omega_{fn}} \mu_{ij}(1-x_{ij}) + \sum_{ij\in\Omega_{fp}} \lambda_{ij}(1-x_{ij}) + \sum_{i\in\Omega_b} \alpha_i\left(P_i^D + Q_i^D\right)y_i \quad (1)$$

Subject to :

$$\sum_{ki\in\Omega_l} P_{ki} - \sum_{ij\in\Omega_l}\left(P_{ij} + R_{ij}I_{ij}^{\text{sqr}}\right) + P_i^G = P_i^D(1-y_i); \quad \forall i \in \Omega_b \quad (2)$$

$$\sum_{ki\in\Omega_l} Q_{ki} - \sum_{ij\in\Omega_l}\left(Q_{ij} + X_{ij}I_{ij}^{\text{sqr}}\right) + Q_i^G = Q_i^D(1-y_i); \quad \forall i \in \Omega_b \quad (3)$$

$$V_i^{\text{sqr}} - V_j^{\text{sqr}} = 2(P_{ij}R_{ij} + Q_{ij}X_{ij}) + Z_{ij}^2 I_{ij}^{\text{sqr}} + b_{ij}; \quad \forall ij \in \Omega_l \quad (4)$$

$$V_j^{\text{sqr}} I_{ij}^{\text{sqr}} \geq P_{ij}^2 + Q_{ij}^2 \quad \forall ij \in \Omega_l \quad (5)$$

$$\underline{V}^2 \leq V_i^{\text{sqr}} \leq \bar{V}^2 \quad \forall i \in \Omega_b \quad (6)$$

$$\left|I_{ij}^{\text{sqr}}\right| \leq \bar{I}_{ij}^2 x_{ij} \quad \forall ij \in \Omega_l \quad (7)$$

$$|b_{ij}| \leq (\bar{V}^2 - \underline{V}^2)(1-x_{ij}) \quad \forall ij \in \Omega_l \quad (8)$$

$$\sum_{ij\in\Omega_l\cup\Omega_h} x_{ij} = |\Omega_b| - n_s \quad (9)$$

$$\sum_{ki\in\Omega_l\cup\Omega_h} H_{ki} - \sum_{ij\in\Omega_l\cup\Omega_h} H_{ij} + H_i^G = y_i \quad \forall i \in \Omega_b \quad (10)$$

$$H_i^G = 0 \quad \forall i \in \Omega_b, i \neq S^f \quad (11)$$

$$|H_{ij}| \leq M x_{ij} \quad \forall ij \in \Omega_l \cup \Omega_h \quad (12)$$

$$P_{S^f}^G = 0 \quad (13)$$

$$Q_{S^f}^G = 0 \quad (14)$$

$$\sum_{ij\in\Omega_l\cup\Omega_h} x_{ij} + \sum_{ki\in\Omega_l\cup\Omega_h} x_{ki} \geq 1 \quad \forall i \in \Omega_b \quad (15)$$

$$|P_{ij}| \leq \bar{V}\bar{I}_{ij}x_{ij} \quad \forall ij \in \Omega_l \quad (16)$$

$$|Q_{ij}| \leq \bar{V}\bar{I}_{ij}x_{ij} \quad \forall ij \in \Omega_l \quad (17)$$

$$|y_i - y_j| \leq (1-x_{ij}) \quad \forall ij \in \Omega_l \quad (18)$$

$$x_{ij} \in \{0,1\} \quad \forall ij \in \Omega_l \cup \Omega_h \quad (19)$$

$$y_i \in \{0,1\} \quad \forall i \in \Omega_b. \quad (20)$$

The formulation presented above uses a fictitious substation at node S^f. The purpose of this substation is to maintain the connection with the part of the system that cannot be restored without modifying the switch state of that part of the EDS. In the same way, artificial circuits that link the fictitious substation to each load node are incorporated; they can only transport fictitious flow demanded by the nodes disconnected from the EDS. Also, the strategy used to model the restoration problem in EDSs is similar to the artificial node and the artificial branches in phase one of the network flow optimization method. The purpose of the fictitious substation and its artificial circuits is so that the switch state of the isolated part of the system will not change. Without the fictitious substation, unnecessary switch maneuverings could appear in the non-restored part of the electrical system in order to satisfy (9). Therefore, the fictitious substation and artificial circuits ensure that the non-restored part of the system forms a radial topology with the fictitious substation and some artificial circuits, and the operation state of the switches in the non-restored part of the system does not change. This fact is illustrated in Fig. 1. For this illustrative example, a fault at node 5 is isolated by opening the corresponding circuits and only the load at node 7 can be restored. As a result, two isolated parts are formed (one with node 6 and other with nodes 8 and 9). The fictitious substation makes it possible to form a radial topology connecting these two parts through two artificial circuits. Note that the switch at circuit 8–9 remains closed.

The decision variables of the model are x_{ij} and y_i. The binary variable x_{ij} assumes a value of 1 if the circuit ij is closed, and 0 if the circuit is open. The connection state of a load is determined by the binary variable y_i that has a value of 1 if the active and reactive power demands at node i are not supplied by the substations and therefore the node is disconnected from the EDS; otherwise, y_i is 0, implying that the demands are fully met by the substations.

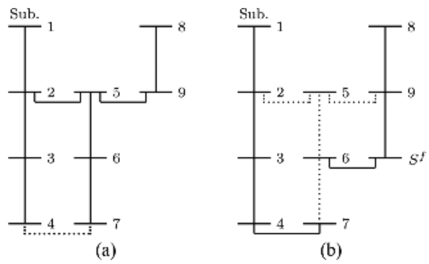

Fig. 1. Illustrative example of a fault at node 5. (a) Before the fault. (b) After the restoration process.

Other variables of the model are P_{ki}, Q_{ki}, and I_{ki}^{sqr}, which represent the active and reactive power flows and the square of the current flow magnitude in circuit ki, respectively. P_i^G and Q_i^G represent the active and reactive power generation at node i, (there is generation only at the substations). V_i^{sqr} corresponds to the square of the voltage magnitude at node i, and b_{ij} is an auxiliary variable in the calculation of the voltage drop in circuit ij that depends upon the state of that circuit. Additionally, the variables H_{ij} and H_i^G were added to the formulation in order to represent the artificial power flow in circuit ij and the artificial generation at node i, respectively.

The data and parameters of the model are as follows: α_i is the load disconnection cost, β_{ij} and ρ_{ij} are parameters related to the manual and automatic switches that are open before the restoration process, while μ_{ij} and λ_{ij} are parameters related to the manual and automatic switches that are closed; P_i^D and Q_i^D represent the active and reactive power demands at node i, respectively; R_{ij}, X_{ij}, and Z_{ij} represent the resistance, reactance, and impedance of circuit ij, respectively, and $\bar{I}_{ij}$ is the current limit of that circuit; the lower and upper voltage limits for the EDS operation are defined by $\underline{V}$ and $\bar{V}$.

The sets considered in the formulation are Ω_{ap} and Ω_{an}, which correspond to the sets of automatic and manual open switches, respectively; Ω_{fp} and Ω_{fn} represent the sets of automatic and manual closed switches, respectively; Ω_l is the set of circuits in the EDS after eliminating the circuits that cannot operate when restoring the system; Ω_b is the set of nodes; and Ω_h is the set of fictitious circuits that connect the fictitious substation to each load node. All of these sets are known before the beginning of the restoration process. Additionally, n_s is the number of substations, including the fictitious one.

In the proposed formulation, the objective function aims first at minimizing the disconnection of loads and second at reducing the operation of switches. This kind of compromise can be obtained by adequately selecting the values for the parameters $\alpha_i, \beta_{ij}, \rho_{ij}, \mu_{ij}$, and λ_{ij}. A suitable selection of these parameters makes it possible to differentiate between priority and non-priority loads, as well as to prioritize the operation of automatic switches over manual ones. The units of the parameters in the objective function must have the proper form so that each parcel of the objective function is given in monetary units. However, the values of these parameters are chosen in order to prioritize some objectives over others according to the same logic of the big-M method in LP. Thus, the objective function value is of no practical importance, since it is only used to find the values of the decision variables and to preserve the hierarchy of these variables according to the preferred options of the restoration problem. It should be noted that the coefficient of variable y_i in the objective function is $\alpha_i(P_i^D + Q_i^D)$; this value should be significantly higher than the other parameters.

Constraints (2) and (3) represent the active and reactive power balances in the nodes of the EDS, while (4) and (5) represent compliance to Kirchhoff's second law for each circuit of the network. However, in the traditional formulation, there is no additional variable b_{ij} in (4), and the equation related to (5) is an equality constraint. The variable b_{ij} is incorporated in order to satisfy (4) if circuit ij is not operating (a circuit does not need to satisfy Kirchhoff's second law if it is disconnected); the value of b_{ij} is controlled by (8) according to the state of the corresponding circuit.

In (5), the original relationship $V_j^{\text{sqr}} I_{ij}^{\text{sqr}} = P_{ij}^2 + Q_{ij}^2$ is transformed into the second order conic constraint $V_j^{\text{sqr}} I_{ij}^{\text{sqr}} \geq P_{ij}^2 + Q_{ij}^2$ according to the similar formulation presented in [27] and taking into account that the conditions presented in [25] are satisfied in the nonlinear model developed. Then, the original nonlinear problem is transformed into a second order cone model. A formal demonstration of the conditions under which this transformation is exact appears in [25]. According to [25], the transformation is valid if the objective function is linear, if there are no upper bounds on loads, if the solution assumes a radial topology, and if it complies with the following assumptions: 1) the network graph G is connected, 2) the objective function is convex, 3) the objective function strictly increases with the square of the current, non-increasing with the load and independent of branch power flow, and 4) the problem is feasible. A detailed analysis showed that the model (1)–(20) meets all of the requirements outlined in [25] (the sufficient conditions for optimality). Additionally, it was observed that (5) was active in the optimal solution for all of the performed tests. Since the proposed method is a relaxation of an exact model, it is important to check if the solution obtained is feasible and if the error in the approximation is not large. For the tests showed in the Tests and Results Section, it was verified that the obtained solutions satisfied (5) as a stricter equality constraint and there was no error in the approximation.

Constraints (6) and (7) represent the operational limits of voltage and current, respectively. Constraint (8) is used to ensure the feasibility of (4). So, if $x_{ij} = 1$ (circuit ij is operating), then $b_{ij} = 0$, which means that Kirchhoff's second law must be satisfied in that circuit, otherwise b_{ij} takes any value that satisfies (4). The radial topology of the EDS after the restoration is guaranteed by (2), (3), (9), and (10) as was proven in [23].

Constraint (10) represents the artificial power flow balance in each node of the EDS and, as such, forces the topology of the buses not restored to be radial. Thus, if $y_i = 0$, then node i has been restored. Therefore, the artificial flows are equal to zero because there is no connection between this node and the fictitious substation, which is the only node that supplies artificial flow. In this case, (10) becomes irrelevant. However, if y_i is equal to 1, then node i has not been restored and there is a demand of a fictitious flow of unitary value in node i that needs to be supplied by the artificial substation. Therefore, there must be a path between node i and the substation through closed circuits or an active fictitious branch between the fictitious substation and node i. Thus, this strategy ensures that the not restored portion of the system is radial and connected, and, more importantly, does not require switch maneuvers in the non-restored portion of the system. Constraint (11) establishes that only the artificial substation generates artificial flow. The artificial power flow in a circuit is limited by (12) according to the operation state of the circuit, while (13) and (14) define that there is not active and reactive generation at the fictitious substation.

Additionally, a set of constraints is added to the proposed formulation in order to improve the solution process. The constraint (15) is a surrogate constraint indicating that at least one circuit must be connected to a load node. This condition is not necessary in the original model, but it can be important for accelerating the optimization process in the branch and bound algorithm. Constraints (16) and (17) force the active and reactive power in one branch to be equal to zero when there is no closed branch and limit these values to be smaller than the maximum apparent power in the branch. Constraint (18) ensures that if circuit ij is operating, then the variables y_i and y_j have the same value. Finally, the binary nature of the decision variables x_{ij} and y_i is represented by (19) and (20). Additionally, a quadratic constraint limiting the apparent power generated at each substation can be included in the mathematical model.

The main idea of the mathematical model can be described as follows:

- If it is possible to restore the EDS without violating the operational constraints, all variables y_i must be equal to zero, and the demands at the load nodes must be satisfied by the substations. In this case, the model optimizes the number of switching operations. Furthermore, all artificial variables are zero, and the artificial substation is isolated.
- If it is not possible to restore the EDS without violating the operational constraints, then some variables y_i are equal to 1, which means that the complete demand of the corresponding nodes is disconnected. The mathematical model does not permit the partial disconnection of a demand (i.e., there is full attendance or complete disconnection). In this case, the model must prioritize the disconnection of the lower demands if the same value for α_i is chosen. Moreover, the different selection of values for α_i allows the optimization model to disconnect preferential non-priority loads. Under these conditions, the model optimizes the number of operations of the switches as a secondary function. The load nodes that are not reconnected to the EDS must be linked to the artificial substation.

It must be noted that the proposed mathematical model is general and comprehensive, which permits it to solve particular cases. So, in its general form, by controlling the parameter α_i, the model prioritizes the satisfaction of the demand. Within this process, a node of the system, which was not disconnected (in-service customers), can be disconnected in order to reconnect nodes in the disconnected part of the system (out-of-service areas) with larger demand, as proposed in [5].

This mathematical formulation is a mixed integer second-order cone programming model that can be solved using commercial solvers. The main advantage of this kind of formulation is that classical optimization techniques can guarantee the optimal solution to the problem. This formulation can require a long processing time to find the optimal solution because any node or circuits can be disconnected. For particular cases, however, the model can be solved very quickly, as is show in the Tests and Results Section.

III. Tests and Results

The mathematical model was tested using a 53-node test system with 61 circuits, which was based on the system in [28] adapted from [29]. In the latter work, a specialized tabu search algorithm was used to solve the restoration problem in EDS. The 53-node test system had 3 substations and 50 load nodes and under normal conditions presented an active power demand of 45 668.7 kW and a reactive power demand of 22 118.24 kVAr. The nominal voltage of the system was 13.8 kV, while the lower and upper voltage limits were 0.95 p.u. and 1.00 p.u., respectively. The data for this test system is shown in the Appendix. The value of the parameters used were: $\alpha_i = 0.1$, $\beta_{ij} = 1$, $\rho_{ij} = 0.1$, $\mu_{ij} = 1$, and $\lambda_{ij} = 0.1$ (reflecting the preference for using automatic switches over open manual switches). Fig. 2 shows the normal operation of the system with a configuration of open and closed switches that form a radial topology. The nominal capacity of the substations at nodes 101, 102, and 104 is 33 400, 30 000, and 22 000 kVA, respectively. The model was implemented in AMPL [30] and was solved with CPLEX [31] using a computer with an Intel $i7$ 4770 processor. The stopping criterion was a maximum gap of 0, i.e., the optimization process carried out by CPLEX ended when the optimal solution was guaranteed.

A. General Tests

For the general tests, any switch could be open or closed, and the demand of any node could be disconnected. All of the switches were of the same type. There were no priority loads, and each circuit of the system had a switch (allowing the circuit to be open or closed). These kind of tests are complex and, therefore can require longer processing times. However, they permit a more complete conceptual analysis. In the Section III-B, specialized tests involving short processing times are described.

1) Fault at Node 3: In this case, a fault at node 3 was analyzed. As such, node 3 and circuits 101-3 and 4-3 were disconnected from the EDS, and the number of related switch operations was not taken into account. Also, the load at node 3 (485.10 kW and 234.93 kVAr) was not restorable. After isolating the fault, the EDS had 52 nodes (3 substations and 49 load nodes) and 59 circuits. The following results were obtained: (a) it was not possible to restore all of the demands of the remaining EDS. Nodes 5, 6, and 26 were disconnected because it was not possible to satisfy the complete demand at these nodes. Therefore, the curtailment in the restored EDS was 3 118.50 kW and 1 510.35 kVAr, totaling, 3 465 kVA of apparent power (i.e., the total demand at nodes 5, 6, and 26); the switches in circuits 5-4, 28-6, 27-8, 26-27, and 34-33 needed to be open, totalizing 5 open switches, and the switches in circuits 28-27, 8-33, 35-40, and 28-50 needed to be closed, totalizing 4 closed switches; (b) the restored EDS operated with 49 nodes (3 substations and 46 load nodes) and 46 circuits. The restored EDS is shown in Fig. 3; it can be seen that the system operates with a radial configuration; (c) the node with the lower voltage was the node 4,

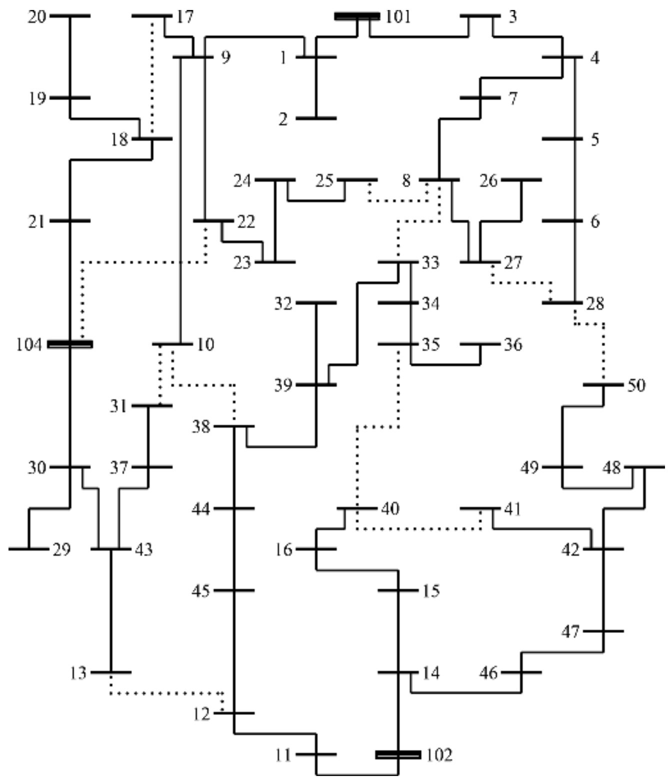

Fig. 2. Topology of the test system before the fault.

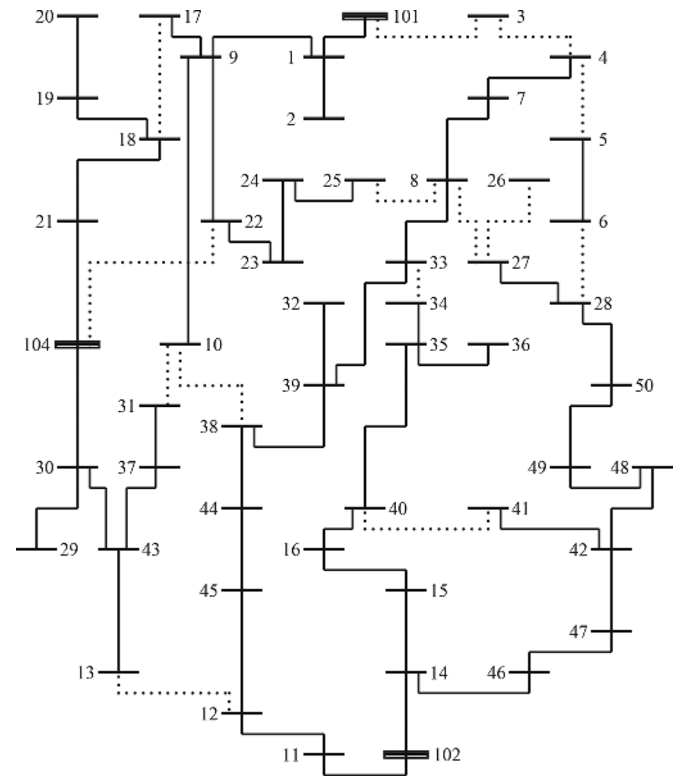

Fig. 3. Topology of the test system restorated after the fault at node 3.

which had a voltage of 0.9635 p.u. The optimal solution was found in 38.6 s.

2) Fault at Node 11: In this case, a fault at node 11 was analyzed. Node 11 and circuits 102-11 and 12-11 were disconnected from the EDS, and the number of related switch operations was not taken into account. Also, the load at node 11 (207.90 kW and 100.72 kVAr) was not restorable. After isolating the fault, the EDS had 52 nodes (3 substations and 49 load nodes) and 59 circuits. The following results were obtained: (a) it was possible to restore all of the demands of the remaining EDS. Just node 11, at fault, was isolated in the EDS; the switches in circuits 45-12, 39-38, and 34-33 had to be open, totalizing 3 open switches, and the switches in circuits 13-12, 8-33, 35-40, and 10-38 had to be closed, totalizing 4 closed switches. Therefore, 7 operations of the switches were executed; (b) the node with the lower voltage was node 32, which had a voltage of 0.9685 p.u. The optimal solution was found in 8.1 s.

3) Fault at Node 14: In order to restore the system after a fault at node 14, node 14 and circuits 102-14, 15-14, and 14-46 were disconnected. Also, the load at node 14 (693.00 kW and 335.64 kVAr) was not restorable. After the isolation of the fault, the remaining EDS had 52 nodes (3 substations and 49 load nodes) and 58 circuits. After solving the problem, it was verified it was not possible to restore all of the demand. The solution had the following characteristics: (a) nodes 15, 16, 36, 46, and 47 were disconnected from the system because it was not possible to completely satisfy the demands at these nodes. Therefore, the curtailment in the restored EDS was 4 435.20 kW and 2 148.06 kVAr, totalizing 4 928 kVA of apparent power (i.e., the sum of the loads at nodes 15, 16, 36, and 47); (b) the switches in circuits 36-35, 16-40, 42-41, and 47-42 had to be open, totalizing 4 open switches. The switches in circuits 40-41, 35-40, and 28-50 had to be closed, totalizing 3 closed switches. Therefore, the state of 7 switches was modified. The EDS operated with 47 nodes (3 substations and 44 load nodes) and 44 circuits; and (c) the node with the lower voltage was node 41, which had a voltage of 0.9600 p.u. The optimal solution was found in 27.8 s.

4) Comments on the General Tests: From the aforementioned tests, the following observations can be made about the performance of the mathematical model:

- For the fault at node 3, a better quality solution was obtained than the one found in [28] using the tabu search algorithm. The solution found had load disconnection.
- For the fault at node 11, the solution obtained was of the same quality as the one found in [28] by the tabu search algorithm. In this case, there was not need for load disconnection. As a result, it can be concluded that there is no curtailment, and there are alternative optimal solutions.
- For the fault at node 14, a better quality solution was obtained than the one found in [28] by the tabu search algorithm. The solution obtained needed load disconnection.
- For all cases, the voltage was above the lower limit, and the substations operated within their limits.

Another important fact is that there is usually voltage regulation at the substation. For these cases, it would be appropriate to fix the voltage at its upper limit (1.05 p.u.). For the 53-node test system, additional tests showed that this strategy did not provide significant improvements because the problem lay in the current capacity of some circuits. However, for other systems, this strategy could be relevant. By increasing the voltage at the substation, there should be a slight reduction in the current of the circuits, and therefore there is a small possibility that some additional loads will be reconnected. In order to verify this premise, the tests for the three faults were carried out again, but this time the voltage at the substations was fixed at 1.05 p.u. The results showed that for a fault at node 3, the same solution was found and for a fault at node 11 there was an alternative optimal solution. But for a fault at node 14, however, there was a better quality solution. In this solution, the switches in circuits 40-41, 35-40, and 28-50 had to be closed, while the switches in circuits 16-40, 42-41, and 47-42 had to be open, totalizing 6 switch operations. It is important to note that the solution found restored the demand at node 36 (which was disconnected in the solution found in Section III-A3) and the demand at node 41 (which was disconnected in the solution found by the tabu search algorithm). The theoretical explanation for this behavior may be related to the fact that an increase in the voltage produces a small reduction in the circuit currents, which may be enough to incorporate an additional load into the restoration of the EDS studied. So, the solution found disconnects a load of 4227.3 kW and 2047.3 kVAr, that is, 4697 kVA.

Additionally, tests were performed on faults in each bus of the electric system. The tests presented here were the most important and significant ones. For faults in each of the other buses of the system, the processing times were much smaller and less critical than for the faults in the buses connected to the substations. Therefore, those tests have not been included, as they do not add any relevant information.

Since the parameters ρ_{ij} and β_{ij} are concurrent, choosing $\beta_{ij} > \rho_{ij}$ could be enough. However, a more conservative choice for an electric system with k_{ap} open automatic switches would be the values $\rho_{ij} = 1$ and $\beta_{ij} = (k_{ap} + 1)$. For the choice of parameter α_i, one should take into account that the coefficients of the variables y_i should be significantly higher than the coefficients of the other variables in the objective function according to the same logic guiding the choice of the M parameter in the big-M method in linear programming. Thus, these parameters must be several times larger than the other parameters. Therefore, the lowest α_i must be sufficiently higher than the other parameters. In the performed tests, the lowest value that α_i could assume was calculated using the data of the test system. So, $S_{ij}^{\min} = \min\{P_n^D + Q_n^D \mid n \in \Omega_b\}$ and A was the highest value of the other parameters (β_{ij}, ρ_{ij}, and λ_{ij}). Then, the minimum value of α_i was $(K \cdot A)/S_{ij}^{\min}$ with K being relatively large, for example between 20 and 50 units. In the system tested, $S_{ij}^{\min} = 308.62$ (see Table II) and $A = 1$. In addition, we chose $K = 30$, which resulted in $\alpha_i = 0.097$. Thus, we used a value of $\alpha_i = 0.1$. This means that the lowest coefficient value for the y_i variables was the coefficient y_{11} with a value of $\alpha_{11} = 30.86$. All other coefficients for α_i were higher. Obviously, the same logic can be applied if power is represented in the p.u. system.

B. *Specialized Tests*

In this subsection, specialized tests that consider specific technical characteristics of the EDS are shown. Such characteristics are priority loads, the requirement that the part of the EDS not isolated by the fault must not be disconnected by the restoration strategy, the discrimination between manual and automatic switches, and the fact that not every circuit can be open/closed, that is, just a portion of the circuits have interconnection switches. The main result of these tests was that the processing time was significantly reduced, making the proposed optimization method applicable to real-time situations. Only faults at node 3 were considered, and, for these specific tests, the solutions found were the same as those presented in Section III-A1. (Only the test E1, in which the disconnection of loads not isolated by the faults was not permitted, produced a different solution.) So, the only difference was in the processing times.

1) Test E1: For this test, it was assumed that in the restoration process it would not be possible to disconnect nodes that were not isolated after the fault. This requirement meant that $y_i = 0$ for all nodes that were not isolated and $x_{ij} = 1$ for all circuits that were in operation after the fault. So, the switches that could be used in the restoration were the ones that were in the region isolated by the fault, including the switches at the open circuits that could connect that region to the remaining part of the EDS in operation. Consequently, the number of binary variables of the model was reduced drastically. Under these exigencies, the optimal solution disconnected nodes 4, 5, 6, 7, and 26, totalizing 4573.80 kW and 2215.21 kVAr (5621 kVA). In addition, the switches in circuits 8-7, 27-8, 26-27, and 28-6 were open, while the switches in circuits 8-25, 28-27, and 28-50 were closed, totalizing 7 switch operations. This solution was the same as the one found using the tabu search algorithm presented in [28]. The processing time was 0.7 seconds. For the other tests, the optimal solution was the same as was shown in Section III-A1.

2) Test E2: In this test, priority loads were considered. A priority load not disconnected by the fault must not change the feeder where it is connected, and if the priority load was disconnected it should be reconnected with priority. The loads at nodes 25, 27, 31, and 33 were priority. (Note that load at node 27 was disconnected by the fault). For this test, the variables $x_{30-104}, x_{30-43}, x_{37-43}$, and x_{31-37} were fixed to 1 for the priority load at node 31; $x_{11-102}, x_{11-12}, x_{12-45}, x_{44-45}, x_{38-44}, x_{38-39}$, and x_{33-39} are fixed to 1 for the priority load at node 33; $x_{1-101}, x_{1-9}, x_{9-22}, x_{22-23}, x_{23-24}$, and x_{24-25} were fixed to 1 for the priority load at node 25. Additionally, the variables $y_{30}, y_{43}, y_{37}, y_{31}, y_{11}, y_{12}, y_{45}, y_{44}, y_{38}, y_{39}, y_{33}, y_1, y_9, y_{22}, y_{23}, y_{24}$, and y_{25} had to be fixed to 0. Furthermore, the value for α_{27} (related to the priority load at node 27) was chosen to be larger than the other α_i, aiming at restoring that load with priority. The optimal solution was found in 8.4 s.

3) Test E3: In this test, besides the priority load considered in Test E2, some automatic switches were considered. Open automatic switches were in circuits 8-33, 35-40, 28-50, 10-38, and 17-18, therefore, the values for ρ_{ij} related to those devices had to be lower that the values for β_{ij}. Closed automatic switches were in circuits 33-34, 26-27, 8-27, 13-43, and 44-45, therefore, the values for λ_{ij} related to those devices had to be lower than the values for μ_{ij}. Also, the variables fixed in the Test E2 remained fixed. The optimal solution for this test was obtained in 5.5 s.

4) Test E4: For this test, besides the priority loads and automatic switches considered in Test E3, it was assumed 15 switches did not exist in the system. So, the variables x_{ij} related to the circuits 19-20, 18-19, 18-21, 13-43, 32-39, 4-7,

TABLE I
SUMMARY OF THE RESULTS

Case	Load curtailment (kW)	Power Losses (kW)	# of open switches	# of closed switches
Fault at node 3	3118.50	563.65	5	4
Fault at node 11	0	552.44	3	4
Fault at node 14	4435.20	506.39	4	3
Test E1	4573.80	511.79	4	3
Test E2	3118.50	563.65	5	4
Test E3	3118.50	563.65	5	4
Test E4	3118.50	563.65	5	4

7-8, 16-40, 15-16, 14-15, 48-49, 49-50, 41-42, 42-47, and 46-47 had to take a value of 1 (i.e., they had to be closed). The optimal solution for this test was found in 1.2 s.

It should be noted that the results for these specialized tests showed that when particular cases (existing in the EDS) are considered, the processing time is reduced considerably.

A summary of the results is presented in Table I.

C. Additional Comments on the Tests and the Mathematical Model

The results showed that when the general model is used, the processing time is relatively large for solving the restoration problem. Therefore, this model can be used off-line to carry out preventive tests, to optimize maintenance programs, and to generate optimal solutions that can be used to verify the quality of faster heuristic or metaheuristic methods. Furthermore, it is known that branch and bound algorithms can demand a large amount of time to verify the optimal solution, but they usually found the optimal or near optimal solutions very quickly. In this context, the EDS operator does not need to wait until the process is finished; the first feasible solution without load curtailment obtained by the optimization method can be used; furthermore, a stopping criterion, such as a preestablished optimality gap different from 0, can be defined. Both new research about the generation of feasible constraints for optimization problems and improvements in commercial solvers have improved the performance of the optimizations methods and reduced the computational effort demanded when solving the models. For the specialized tests, the processing times are compatible with the requirements for the restoration problem in EDSs.

Concepts from operational research were applied to the formulation of the mathematical model. Thus, the following observations are appropriate: (a) the objective function is a generalization of the two-phase method widely used in the simplex algorithm in linear programming. So, in phase 1 of the simplex method, artificial variables are used, and they are systematically eliminated in the optimization process (see big-M method in linear programming [32]), i.e., the normal variables and the artificial ones are treated hierarchically; (b) in the mathematical model, concepts of graph theory are also used to ensure the radial topology of the final solution, while the minimum number of switching operations needed to isolate the disconnected part of the system and connect it to the fictitious substation is guaranteed; and (c) the transformation of a mixed integer nonlinear programming model into a mixed integer second-order conic programming formulation is essential for making the proposed method viable. Preliminary work using mixed integer nonlinear models has been abandoned due to the lack of efficient solvers for these kind of formulations.

TABLE II
NODE DATA

Node	Active power (kW)	Reactive power (kVAr)	Node	Active power (kW)	Reactive power (kVAr)
101	0.00	0.00	25	623.70	302.07
102	0.00	0.00	26	831.60	402.78
104	0.00	0.00	27	1039.50	503.42
1	2910.60	1409.64	28	485.10	234.93
2	1039.50	503.43	29	970.20	469.85
3	485.10	234.93	30	1801.80	872.64
4	762.30	369.22	31	485.10	234.93
5	1801.80	872.64	32	1178.10	570.57
6	485.10	234.93	33	2009.70	973.36
7	693.00	335.64	34	831.60	402.79
8	1316.70	637.71	35	623.70	302.07
9	831.60	402.79	36	207.90	100.72
10	2009.70	973.36	37	1455.30	704.86
11	207.90	100.72	38	762.30	369.21
12	1247.40	604.14	39	693.00	335.64
13	762.30	369.22	40	970.20	469.85
14	693.00	335.64	41	623.70	302.07
15	970.20	469.85	42	831.60	402.79
16	1316.70	637.71	43	900.90	436.36
17	485.10	234.93	44	970.20	469.85
18	831.60	402.79	45	554.40	268.50
19	970.20	469.85	46	1247.40	604.14
20	554.40	268.50	47	693.00	335.64
21	1247.40	604.14	48	554.40	268.50
22	762.30	369.22	49	346.50	167.78
23	693.00	335.64	50	554.40	268.50
24	346.50	167.78			

The proposed method determines the final configuration, i.e., providing the information about the switches that must be open or closed. An algorithm to solve the problems separately and sequentially of how to identify the switches that must be operated and generate the sequence in which they must be activated is proposed in [8]. However, in the specialized literature, the problem of finding the sequence of switching operations is still under analysis. The model proposed in this work can be expanded to consider that problem and we consider that this important issue should be addressed in other research work.

IV. CONCLUSION

In this paper, a comprehensive mathematical model for the restoration problem in balanced radial distribution systems was presented. The mixed integer second-order conic model can be solved efficiently using commercial solvers. Thus, the proposed model and the proposed solution presented can be used offline or online, depending on the time needed to solve the problem.

Test results show excellent performance with regard to the quality of the solution (i.e., the model guarantees that the solutions are optimal). The drawback of the model is the relatively long processing time. However, further research can be conducted to improve this feature. Finally, we consider this work to be an initial proposal that can lead to further research in related topics such as (a) improving the mathematical model with the generation of valid constraints in order to reduce the processing time, (b) developing slightly relaxed models that can be solved more quickly using commercial solvers, while does not significantly degrading the quality of solutions, (c) considering the

TABLE III
CIRCUIT DATA

Initial node	Final node	Resistence (Ω)	Reactance (Ω)	Current limit (A)
101	1	0.0543	0.0675	600
101	3	0.0421	0.0524	600
4	3	0.0603	0.0749	600
7	4	0.0483	0.0600	600
5	4	0.1472	0.1499	250
8	7	0.0603	0.0749	600
6	5	0.1179	0.1201	250
9	1	0.0663	0.0824	600
2	1	0.1472	0.1499	250
10	9	0.3388	0.3449	250
102	14	0.0725	0.0901	600
15	14	0.1769	0.1802	250
16	15	0.1326	0.1350	250
102	11	0.0543	0.0675	600
12	11	0.0603	0.0749	600
13	12	0.3194	0.2202	150
20	19	0.2281	0.1572	150
19	18	0.1828	0.1260	150
18	17	0.2968	0.2046	150
17	9	0.1256	0.1060	400
21	18	0.1472	0.1499	250
104	21	0.0730	0.0617	400
104	22	0.1769	0.1802	250
22	9	0.2208	0.2248	250
23	22	0.2507	0.1729	150
24	23	0.2054	0.1416	150
25	24	0.1594	0.1099	150
8	25	0.2054	0.1416	150
27	8	0.1769	0.1802	250
26	27	0.2507	0.1729	150
28	27	0.2281	0.1572	150
28	6	0.3655	0.2520	150
104	30	0.0543	0.0675	600
29	30	0.2281	0.1572	150
43	30	0.1916	0.1950	250
37	43	0.1828	0.1260	150
31	37	0.1367	0.0942	150
10	31	0.2281	0.1572	150
43	13	0.1095	0.0925	400
45	12	0.0483	0.0600	600
44	45	0.0421	0.0524	600
38	44	0.0603	0.0749	600
39	38	0.0809	0.0824	500
32	39	0.2968	0.2046	150
33	39	0.1326	0.1350	250
8	33	0.2208	0.2248	250
33	34	0.1367	0.0942	150
34	35	0.1594	0.1099	150
35	36	0.1594	0.1099	150
40	41	0.2741	0.1890	150
16	40	0.1828	0.1260	150
42	41	0.2741	0.1890	150
48	42	0.1828	0.1260	150
49	48	0.2741	0.1890	150
50	49	0.1594	0.1099	150
47	42	0.0911	0.0769	400
46	47	0.1472	0.1499	250
14	46	0.1002	0.0846	400
35	40	0.1301	0.0897	150
10	38	0.1828	0.1260	150
28	50	0.1126	0.0776	150

effect of distributed generation, and (d) discussing the problem of how to implement the switching operations after identifying the switches that must be operated in order to energize the portion of the electrical system de-energized.

APPENDIX A
53-NODE TEST SYSTEM DATA

The data for this test system is shown in Tables II and III.

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Rubén Romero (M’93–SM’08) received the B.Sc. and P.E. degrees in electrical engineering from the National University of Engineering, Lima, Perú, in 1978 and 1984, respectively, and the M.Sc. and Ph.D. degrees from the University of Campinas, Brazil, in 1990 and 1993, respectively.

Currently he is a Professor in the Electrical Engineering Department at the São Paulo State University (FEIS-UNESP), Ilha Solteira, Brazil. His general research interests are in the area of electrical power systems planning.

John F. Franco (S’11–M’13) received the B.Sc. and M.Sc. degrees in electrical engineering from the Universidad Tecnológica de Pereira, Colombia, in 2004 and 2006, respectively, and the Ph.D. degree in electrical engineering from the São Paulo State University (UNESP), Ilha Solteira, Brazil, in 2012.

His areas of research are the development of methodologies for the optimization, planning and control of electrical power systems.

Fabio B. Leão (S’09–M’12) received the B.Sc. degree with honors in electrical engineering and the Ph.D. degree from UNESP, São Paulo State University, Ilha Solteira, Brazil, in 2006 and 2011, respectively.

He is currently Assistant Professor in the Department of Electrical Engineering, UNESP, Ilha Solteira, Brazil. His research focuses on protection, automation, fault diagnosis and reliability in electric power systems, artificial intelligence applied to electric power systems, distribution power systems, distributed generation and smart grid.

Marcos J. Rider (S’97–M’06) received the B.Sc. (Hons.) and P.E. degrees in electrical engineering from the National University of Engineering, Lima, Perú, in 1999 and 2000, respectively; the M.Sc. degree from the Federal University of Maranhão, Maranhão, Brazil, in 2002; and the Ph.D. degree from the University of Campinas, Brazil, in 2006, all in electrical engineering.

Currently he is a Professor in the Electrical Engineering Department at the São Paulo State University (FEIS-UNESP), Ilha Solteira, Brazil. His areas of research are the development of methodologies for the optimization, planning and control of electrical power systems, and applications of artificial intelligence in power systems.

Eliane S. de Souza received the B.Sc. degree in computing science from the Matto Grosso University, Colider, Brazil, in 2008 and the M.Sc. degree in electrical engineering from the São Paulo State University (UNESP), Ilha Solteira, Brazil, in 2014. Currently, she is pursuing the Ph.D. degree in electrical engineering in the UNESP, Ilha Solteira, Brazil.

Her areas of research are the development of methodologies for the optimization, planning and control of electrical power systems.