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2 A New Method for Estimating of Evapotranspiration and Surface Soil  
3 Moisture from Optical and Thermal Infrared Measurements: The  
4 Simplified Triangle

5  
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15  
16 **ABSTRACT**

17 Earth Observation (EO) provides a promising approach towards deriving accurate  
18 spatiotemporal estimates of key parameters characterizing land surface interactions, such as  
19 latent (LE) and sensible (H) heat fluxes as well as soil moisture content. This paper proposes a  
20 very simple method to implement, yet reliable to calculate evapotranspiration fraction (EF)  
21 and surface moisture availability ( $M_o$ ) from remotely sensed imagery of Normalized  
22 Difference Vegetation Index (NDVI) and surface radiometric temperature ( $T_{ir}$ ). The method is  
23 unique in that it derives all of its information solely from these two images. As such, it does  
24 not depend on knowing ancillary surface or atmospheric parameters, nor does it require the  
25 use of a land surface model. The procedure for computing spatiotemporal estimates of these  
26 important land surface parameters is outlined herein stepwise for practical application by the  
27 user. Moreover, as the newly developed scheme is not tied to any particular sensor, it can also  
28 be implemented with technologically advanced EO sensors launched recently or planned to be  
29 launched such as Landsat 8 and Sentinel 3. The latter offers a number of key advantages in  
30 terms of future implementation of the method and wider use for research and practical  
31 applications alike.

32  
33 **Keywords:** *surface soil moisture, evapotranspiration, triangle method, thermal remote  
34 sensing*

## 38 1. INTRODUCTION

39 Currently there is an urgent need for a better understanding of Earth's natural processes and  
40 interactions, which is underlined even more in the face of increased pressures from climate  
41 change and global food and water security issues (Ireland et al., 2015). In this regard, exact  
42 information on the spatiotemporal variation of parameters such as surface soil moisture (SSM)  
43 and evapotranspiration (so-called as latent heat flux, LE) is of key significance (Piles et al.,  
44 2016). This is due to the influence of these parameters on various physical processes of the  
45 Earth system, where they exert a strong control on the Earth's water cycle and ecosystem  
46 functioning in general (Shen et al., 2013; Srivastava et al., 2017). Their accurate estimation is  
47 also of prime interest for a number of environmental and commercial applications, from  
48 sustainable water resource management to evaluating parameterization schemes for weather  
49 and climatic models (Liu and Xie, 2013; Bao et al., 2018).

50 The advent of Earth Observation (EO) technology has provided economically feasible means  
51 to derive temporally consistent coverage of those parameters at different spatial scales (Tian et  
52 al., 2014). Several EO-based approaches have thus been developed over the past few decades  
53 varying from statistical semi-empirical to analytical ones with physical-based algorithms (see  
54 reviews by Petropoulos et al., 2015; Petropoulos et al., 2018). These modelling schemes are  
55 characterized by varying mechanisms and degrees of complexity, data requirements, basic  
56 assumptions, and accuracy. Evidently, there is a specific group of EO-based techniques which  
57 aim at deducing surface fluxes of LE, H and/or SSM at a variety of spatial and temporal scales  
58 based on the synergy of satellite data from optical (visible and infrared - VNIR) and thermal  
59 infrared (TIR) radiometers. Those methods, commonly termed in the literature as  $T_s/VI$   
60 methods, are based on the physical relationships that exist when a satellite-derived land  
61 surface temperature ( $T_s$ ) is plotted against a spectral vegetation index (VI).

62 It has been demonstrated that the derivation of spatially distributed estimates of energy fluxes  
63 and SSM using the  $T_s/VI$  'triangular' scatterplot is feasible without the use of a boundary  
64 layer model. Yet, more sophisticated approaches tend to involve the use of a land biosphere  
65 model, specifically of a Soil Vegetation Atmosphere Transfer (SVAT) model, via a technique  
66 commonly termed as the "triangle". Various validation studies have demonstrated its ability to  
67 provide estimates of both surface heat fluxes and SSM with accuracies in the order of 40 to 70  
68  $Wm^{-2}$  and within 5 % vol vol<sup>-1</sup> for SSM over homogenous areas (Gilles et al., 1997; Owen et  
69 al. 1998; Jiang et al., 2001; Carlson, 2007; Tang et al., 2010; 2013). The significant prospect  
70 of the  $T_s/VI$  scatterplot methods and of the "triangle" in particular, is documented by the fact  
71 that variants of this technique are considered at present in operational products development  
72 of energy fluxes and/or SMC on a global scale (Chauhan et al., 2003; ESA STSE, 2012). Also,  
73 a variant of the "triangle" it already deployed today over Spain to operationally deliver SSM  
74 maps at 1 km spatial resolution from ESA's own SMOS satellite (Piles et al., 2011; 2014).  
75 Thus, from the above it becomes evident that research focusing on the "triangle"  
76 implementation is undoubtedly of key interest, particularly so given the fact that variants of  
77 this method are being explored today for operational implementation.

78 In this context, the present study aims at introducing the “simplified triangle” method, which  
79 represents an extension of the so-called ‘triangle’ method. This new method allows one to  
80 estimate surface evapotranspiration fraction (EF) and surface soil moisture availability ( $M_o$ )  
81 over an area using just a few simple calculations in conjunction with satellite or aircraft  
82 images made at optical wavelengths and in the thermal infrared. Two parameters are derived  
83 from the simplified triangle method, a surface wetness, represented by the parameter  $M_o$ , and  
84 the evapotranspiration fraction EF. The former is defined as the ratio of soil surface  
85 evaporation  $ET_s$  to the potential evapotranspiration ( $ET_s/ET_{pot}$ ), but is also loosely equated  
86 with the ratio of soil water content to that at field capacity. EF is defined as the ratio of  
87 evapotranspiration to net radiation ( $R_n$ ). The simplified triangle method has a great advantage  
88 over other methods belonging to this same group of models in that it does not require a land  
89 surface model or ancillary surface or atmospheric data for its execution and, as such, it is  
90 practical and easy to apply. It should be noted that recently, a pair of similar models (referred  
91 to as trapezoid models has been published (Sadeghi et al., 2017; Babaeian et al., 2018). As in  
92 the simplified triangle model, these two models also require no ancillary data, but use the  
93 short wave solar radiation both in addition to and as an alternate to surface radiometric  
94 temperature, the former aimed for use with satellites for which no thermal sensors exist.

95

## 96 **2. INPUT PARAMETERS**

97 To implement the “simplified triangle”, two image fields required, as obtained from satellite  
98 (or aircraft) measurements: the surface radiometric temperature ( $T_{ir}$ ) and the normalized  
99 difference vegetation index (NDVI). The latter is derived from a pair of radiances measured at  
100 two wavelengths in the solar spectrum, one in the visible and one in the near infrared. NDVI  
101 is defined as:

$$102 \quad NDVI = (R_{NIR} - R_{RED}) / (R_{NIR} + R_{RED}) \quad (1)$$

103 Here,  $R_{NIR}$  and  $R_{RED}$  are the reflectance values measured, respectively, in the near infrared  
104 (e.g. a wavelength just above 0.7 microns, as from channel 4 of Landsat 8 sensor) and in the  
105 visible (e.g. a wavelength near 0.65 microns, as from channel 3 of Landsat 8 sensor).  
106 Calculation of the fractional vegetation cover from NDVI is described below.

107 As defined,  $M_o$  applies only to the top few millimeters of the bare soil surface. Similarly, the  
108 bare soil surface radiometric temperature ( $T_s$ ) applies to the bare soil surface.  $T_{ir}$ , of course,  
109 pertains to a mixture of the bare soil surface ( $T_s$ ) and the vegetation canopy temperature  $T_{veg}$ .

110

## 111 **3. IMAGE SELECTION**

112 In order to minimize measurement errors, the two images, those of  $T_{ir}$  and NDVI, must  
113 represent a reasonably uniform terrain height (not varying by more than about 10%) and  
114 should not contain a large fraction of standing water or cloud. Although different vegetation  
115 types may be present in the image without great loss of accuracy, highly inhomogeneous  
116 vegetation such as a forest situated aside a field of corn or grass might introduce some error

117 (including edge effects) in the derived surface parameters (Carlson & Sanchez-Azofeifa,  
118 1999). If the image contains some standing water or cloud, these effects first must be  
119 removed, a process that can be done by a judicial analysis of the reflectance pattern.

120 Estimate of the salient features discussed in Section 6 depends on having at least some bare  
121 soil and fully vegetated pixels in the image. It seems quite likely that bare soil and vegetation  
122 coexist in at least a few pixels in images taken over a larger enough area. Fractional vegetation  
123 cover ( $F_r$ ; defined below) for each pixel is calculated and the end point parameters discussed  
124 in the next section are then able to be determined.

125

#### 126 **4. METHOD DEMONSTRATION**

127 Now, consider the two images made over an area, one for  $T_{ir}$  and the other for NDVI. The first  
128 step is to identify some pixels representative of dense vegetation (full vegetation cover) and  
129 some of dry, bare soil, as illustrated in the example shown in Figure 1. In these two images,  
130 the pixels with highest and lowest values of radiometric surface temperature are indicated in  
131 the upper left image and those pixels with the highest and lowest values of NDVI are  
132 indicated in the lower right image.

133 Corresponding surface pixels selected in these two images represent the hottest and least  
134 vegetated pixels (over dry, bare soil) such as found over a paved urban area (parking lot, city  
135 center, etc.). Two other pairs of arrows denote densely vegetated terrain, such as found in the  
136 countryside. Highest values of surface temperature (red areas in the temperature image) likely  
137 represent pixels with zero surface soil moisture, while those over dense vegetation correspond  
138 to a full vegetation cover and a source of potential transpiration (the dense green areas in  
139 Figure 1).

140 If pixels can be found that lie over patches of dense vegetation that is not wilted, they define  
141 the full vegetation cover condition where the fractional vegetation cover ( $F_r$ ) equals 1.0 There,  
142  $T_{ir}$  is equal to  $T_{min}$ , and NDVI is defined as  $NDVI_s$  (the arrow pointing to a green patch in the  
143 NDVI image of Figure 1). Similarly, the maximum temperatures (the arrow pointing to a red  
144 patch in the temperature image and a white patch in the NDVI image of Figure 1) define the  
145 bare soil condition ( $F_r=0$ ) and the maximum temperature  $T_{max}$ , where the bare soil NDVI is  
146 defined as  $NDVI_o$ .

147

148 **[FIGURE 1 GOES HERE]**

149

150 Having defined these points on the image, the next step is to calculate fractional vegetation  
151 cover and Surface Temperature from NDVI. A useful relationship (Gilles et al., 1997) is:

152

$$153 \quad F_r = ((NDVI - NDVI_o) / (NDVI_s - NDVI_o))^2 \quad (2)$$

154 Similarly, we introduce the very important concept of a *scaled* infrared surface temperature  
155  $T^*$  defined as

$$156 \quad T^* = (T_{ir} - T_{min}) / (T_{max} - T_{min}) \quad (3)$$

157  
158

159 Both the scaled  $T_{ir}$  (now called  $T^*$ ) and the fractional vegetation cover  $F_r$  are thus constrained  
160 to vary between 0 and 1.0.  $T^*$  is sometimes referred to as the ‘temperature-dryness index’  
161 (Sandholt et al., 2002).

162  
163 As shown by Carlson and Ripley (1997),  $F_r$  is highly insensitive to atmospheric attenuation  
164 effects on NDVI, so that the method requires no correction for NDVI. Although it has not  
165 been conclusively demonstrated, it is quite likely that the scaled temperature  $T^*$  is less  
166 sensitive to atmospheric attenuation than  $T_{ir}$  itself because scaling should tend to at least  
167 partially cancel the atmospheric correction. This is not a settled issue and the user is free to  
168 introduce corrections to  $T_{ir}$  for atmospheric attenuation. Our assumption for not correcting  $T_{ir}$   
169 for atmospheric attenuation rests partly on analogy with NDVI, but also on the fact that the  
170 values of  $M_o$  and EF are constrained to vary within the triangle between the values of zero and  
171 one. Neglect of atmospheric attenuation greatly reduces image processing time without  
172 necessarily engendering serious error. Scaling will also remove errors due to sensor  
173 calibration.

174 As an example of calculating  $T^*$ , consider Figure 1. Here, maximum and minimum NDVI and  
175  $T_{ir}$  for the case represented, chosen subjectively by eye, are NDVIs = 0.82, NDVIo = 0.18,  
176  $T_{max} = 296.3K$  (23.0 ° C),  $T_{min} = 287.5K$  (14.2 ° C), so that.  $T^*$  varies over a range of 8.8 ° C.  
177 A value of  $T^*=0.5$  corresponds to  $T_{ir} = 18.6$ . Similarly, the range of NDVI is 0.64, so that a  
178 value of NDVI = 0.50 yields a value of  $F_r$  (Equation 2) of 0.25.

179

## 180 **5. TRIANGLE CONSTRUCTION**

181 The triangle is best viewed by plotting pixels for  $T^*$  and  $F_r$  on a two dimensional space as in  
182 Figure 2. The latter represents a raw image uncorrected for standing water and cloud. It  
183 exhibits considerable scatter due to standing water and cloud near the bottom and especially  
184 on the left.

185 One can filter these effects by realizing that clouds tend to be cold and highly reflective but  
186 yield low values of NDVI while standing water tends to be cold but with very low reflectance  
187 and very low values of NDVI. Once, cloud and standing water pixels have been removed the  
188 resulting configuration is a better defined triangular or trapezoidal feature (Figure 3).  
189 Although the shape of the triangle tends to degrade as the resolution of the radiometer  
190 decreases it appears even at 1 km resolution, such as for AVHRR images (Figure 2).

191

[FIGURE 2 GOES HERE]

192  
193  
194

## 195 6. ESSENTIAL FEATURES OF THE TRIANGLE

196 A striking feature of this kind of pattern shown in Figures 2 and 3 is the very sharp edge on  
197 the warm side of the pixel envelope, which we define as the *warm edge*. It could just as well  
198 be called the dry edge as it represents the limit of *surface* soil dryness which we assume also  
199 corresponds to a line along which  $M_o=0$ . The base of the triangle, which is sometimes referred  
200 to as the *soil line*, corresponds to  $F_r=0$ . A corresponding *cold edge*, defining the isopleth  
201  $M_o=1.0$ , is also shown, though this line is sometimes blurred by scatter. Finally, the top  
202 (*vertex*) of the triangle corresponds to full vegetation cover,  $F_r=1.0$ , although at that point the  
203 soil, being largely obscured by vegetation,  $M_o$  values are not resolvable.

204 Figure 3 shows the warm and cold edges, the soil line and the triangle's vertex. Now, let us  
205 examine these salient features of the triangle in more detail.

206 **warm edge:** The characteristic sharply defined warm side of the triangle can,  
207 provided that the feature is a right triangle, be ruled by eye from the lower right-hand vertex  
208 ( $F_r=0$  ( $NDVI_o$ ),  $T_{max}$ ) to the upper vertex ( $F_r=1.0$ ;  $NDVI_s$ ). If the vertex is well-defined, this  
209 point is found at  $NDVI_s$ ,  $T_{min}$ . Some researchers (Tang et al. 2010) divide up slices of  $F_r$   
210 between the cold and warm edges into segments of  $T^*$ , and define the warm edge as the point  
211 where the pixel density in these segments, in moving from cold to warm, decreases to some  
212 small number or where, say, 99% of the pixels have been sampled in that slice, at which point  
213 the value of  $T^*$  is recorded. Once these points have been determined for a series of points at  
214 various values of  $F_r$ , a straight-line represents the best fit of the warm edge to these end points  
215 at different values of  $F_r$ .

216 For a triangle with a well-defined upper vertex, the slope of the line between  $T^*$  of zero and  
217 one is always.

$$218 \quad T^*(\text{warm edge}) = 1 - F_r \quad (4a)$$

219

220 In the more general case, a regression line (Equation 4b) is fit to the warm edge using the  
221 unscaled  $T_{ir}$  (e.g., Sandholt et al., 2002; Tang et al., 2010).

222

$$223 \quad T_{ir}(\text{warm edge}) = \alpha + \beta \times (NDVI) \quad (4b)$$

224 where constants  $\alpha$  and  $\beta$  define the best fit linear regression of NDVI versus  $T_{ir}$  along the  
225 warm edge. Note that  $T_{ir}$  must always be equal to or less than  $T_{ir}$  along the warm edge.



226 The triangular scatterplots in shown in Figure 4 are among many similar ones from this same  
227 data set (Silva-Fuzzo & Rocha, 2016). Here, the slanting red lines (determined visually) make  
228 reasonable (if not precise) fits to the warm edge of the pixel envelope.

229 Some researchers contend that the scatter of points beyond the warm edge represents water  
230 stressed vegetation, but this has not been proved and can also be due to sloping terrain.

231

232

[FIGURE 3 GOES HERE]

233 **cold edge:** This feature, which represents the limit of wetness, tends to be less well  
234 defined than the warm edge. Pixels usually form a less sharp border than for the slanting warm  
235 edge, but the border constituting the cold edge tends to be vertically orientated along a straight  
236 line drawn between the point ( $F_r=1.0, T_{min}$ ) and ( $F_r=0; T_{min}$ ), as discussed by Jiang et al.  
237 (2001, Sandholt et al. (2002) and Kasim (2015). Figure 4 (and many others not shown here)  
238 attests to the verticality of the cold edge. While some triangles sometimes tilt toward the left,  
239 as can be seen in Figure 3, we think that the absence of pixels near the lower left part of the  
240 triangle in these cases is due to the rarity of truly wet, bare soil surfaces.

241

242 **Soil line:** The bare soil line, ( $NDVI_o$ ) can also be determined by eye or by a statistical  
243 test as just mentioned with regard to the warm edge. Figure 4 shows triangles with fairly well-  
244 defined bases, although one might contend with the exact locations of the soil line in Figure 3  
245 and the warm edges in Figure 4.

246

247

[FIGURE 4 GOES HERE]

248

249 **Triangle vertex:** Some triangles appear with rounded or flattened tops, more closely  
250 resembling trapezoids. This could be due to various factors. Some researchers believe  
251 the variation of  $T^*$  at the point where  $F_r=1.0$  is due to a real variation in leaf  
252 temperature due to water stress (Petropoulos et al., 2009). An alternate possibility is  
253 that the highest values of NDVI do not represent a truly 100% vegetation cover.

254 Simulations with a Soil-Vegetation-Atmosphere-Transfer (SVAT) model, the model  
255 being described in Petropoulos et al. (2009), suggest that a flattened top can appear in  
256 a full vegetation cover not under stress if the leaf area index (LAI) is not very large,  
257 say close to 3, in which case some holes exist in the vegetation canopy. If so, sharp  
258 vertices in a triangle may signify that the LAI is very much larger than 3.

259

260 **7 SOLUTIONS FOR  $M_o$  AND  $E_f$**

261 The geometry presented here presumes that  $M_o$  and EF vary linearly across the pixel domain.  
262 Thus, from Figure 5 we see that  $M_o$  is just the ratio of the segments (a/d) (Petropoulos et al.,  
263 2009). Mathematically, geometry requires that

264

265

266

$$M_o = (1 - T_{\text{pixel}}^*) / T^* \text{ (warm edge)} \quad (5)$$

267 where  $T_{\text{pixel}}^*$  must always be less than or equal to that along the warm edge. Thus, if the pixel  
268 envelope is a triangle:

269

270

$$M_o = (1 - T_{\text{pixel}}^*) / (1 - F_r) \quad (6)$$

271

$$EF = EF_s \times (1 - F_r) + EF_{\text{veg}} \times F_r \quad (7)$$

272 Equation 7,  $F_r$  is the fractional vegetation cover, EF is weighted by the fractional vegetation  
273 cover, where the soil component  $EF_s$  (the ratio of soil evaporation to net radiation) is equal to  
274  $M_o$  and  $EF_{\text{veg}} = 1.0$  for the vegetative component. This equation simplifies to the very simple  
275 expression

276

277

278

$$EF = M_o \times (1 - F_r) + F_r \quad (8)$$

279 Note that one can substitute Equation 4b in Equation 5 if the triangle has a flat top and the  
280 warm edge is determined by a regression line.

281

282

**[FIGURE 5 GOES HERE]**

283

284 Figure 6 is a graphical representation for the solution to Equations 5-8, illustrating that the  
285 isopleths of  $M_o$  and EF are sloping straight lines within the triangle. As such, Figure 6  
286 constitutes a universal solution for all triangles. If triangles are plotted on the same scale in  
287 which vertices vary from zero to one, they are all congruent.

288

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295

Imagine a series of such triangles created over a period of days and stacked vertically in  
chronological order, the triangle representing the earliest image on the bottom and later ones  
above. The vertical axis is now that of time. Such a representation allows the user to chart the  
temporal movement of  $M_o$  and EF at specific land surface points, so the progress of surface  
drying can be monitored (Carlson & Arthur, 2000; Owen et al., 1998; Carlson & Sanchez-  
Azofeifa, 1999). An example of this type of time variation of a pixel is illustrated  
schematically by the arrow in Figure 6, showing a progressive drying of the surface point as  
the point moves downward and to the right with time.

296 Cases where the scatterplot more closely resembles a trapezoid than a triangle with a sharp  
297 upper vertex may occur because pixels chosen to represent a dense vegetation canopy may  
298 actually not be representative of a full canopy in which no direct solar radiation is reaching the  
299 soil surface. A better choice of  $NDVI_s$  might then be at the apex of a virtual triangle created  
300 using the regression line (Equation 4b) to extrapolate the warm edge to the point where  $T^*=0$   
301 (along the cold edge). Similarly, where the line representing the warm edge does not fit snugly  
302 against the pixel envelope, a judicious revision of the initial choices of  $T_{max}$  or  $NDVI_s$  can be  
303 achieved using the regression Equation 4b to extrapolate  $T_{ir}$  to better choices of these end  
304 points.

305 **[FIGURE 6 GOES HERE]**

306

## 307 **8. VALIDATION OF METHOD**

308 Figure 7 shows isopleths of  $M_o$  and EF derived from a full soil /vegetation/  
309 atmosphere/transfer (SVAT) model for a triangular pixel distribution over crops in Costa Rica  
310 (Carlson & Sanchez-Azofeifa, 1999). Note that isopleths of  $M_o$  are nearly straight lines that  
311 slope upward to the left, very similar to those for  $M_o$  in the universal triangle, shown in Figure  
312 6.

313 Isopleths of EF, however, though deviating from straight lines in the SVAT model  
314 representation (Figure 7) still agree reasonably closely with those from the universal triangle  
315 (Figure 6), the largest differences occurring near the lower left side of the triangles. In  
316 comparing these two figures, differences in  $M_o$  and EF are generally less than 0.15. Recent  
317 simulations (Kasim, 2019; private communication) show no significant differences for  $M_o$   
318 estimated from the simplified triangle method and that derived from a full SVAT model  
319 (described in Petropoulos et al., 2009). Yet, the latter remains to be investigated in detail in  
320 the future. The methodology appears to have worked well when applied to a study of soybean  
321 productivity in Brazil in which EF was used to predict crop yield (Silva-Fuzzo & Rocha,  
322 2016). Certainly, more validation studies are needed to lend further credence to the STM.

323

324 **[FIGURE 7 GOES HERE]**

325

## 326 **9. SUMMARY**

327 The simplified triangle method allows one to estimate the surface soil moisture availability  
328 and the evapotranspiration fraction without the aid of a land surface (e.g., SVAT) model and  
329 without the need for ancillary surface and atmospheric information. As such, the method is not  
330 only fast and easy to apply, but it is especially useful in regions with little ancillary  
331 atmospheric or surface data. Those who are unfamiliar with mathematical representations of  
332 land surface and atmospheric parameters or who wish only a quick and easy method to  
333 estimate the surface soil wetness and evapotranspiration might find this method more

334 appealing than one that requires more complexity. Moreover, the ‘universal triangle’ affords a  
335 graphic representation of land surface changes with time.

336 Although more investigation is necessary in order to fully assess the accuracy of the  
337 simplified triangle method, in comparison with estimates made with the aid of more complex  
338 land surface models, preliminary indications based on papers in print or under review (cited  
339 above) suggest that the results from both methods would be very similar. Two papers by  
340 Silava-Fuzzo and Silva-Fuzzo et al, cited in this paper, show that the values of EF generated  
341 from the simplified triangle method produced good estimates of soybean yields over Brazil.

342 Two aspects of this model appear to trouble some of its users. One is the idea of scaling and  
343 the other pertains to the extraction of the relevant end values for NDVI and  $T_{ir}$ . We hope that  
344 this paper will help clarify the scaling process. To date, however, no satisfactory method has  
345 been demonstrated that allows the user to extract these parameters objectively. Instead, one is  
346 forced to do this manually, with the interplay of hand and eye, while noting the values  
347 designated by the cursor at the appropriate bare soil and dense vegetation locations. A highly  
348 practical advance in this method, therefore, would be to develop an operational system in  
349 which the triangles and their relevant parameters are determined operationally with intelligent  
350 software using only NDVI and  $T_{ir}$  images.

351 We suggest that, once the method is satisfactorily validated, the next step in its development  
352 would be to implement it operationally. The use of the “triangle” exploiting EO data from  
353 with these new satellite sensors remains to be seen.

354

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362

## 363 **REFERENCES**

364 ARTHUR-HARTRANFT, S.; CARLSON, T. N.; CLARKE, K., 2003. Satellite and  
365 ground-based microclimate and hydrologic analyses coupled with a regional urban  
366 growth model. *Remote Sensing of Environment*, **86**, 385-400.

367 BABAERIAN, E., SADEGHI, M., FRANZ , T. E., JONES, S., TULLEER, M., 2018.  
368 Mapping soil moisture with the Optical TRAppezoid Model (OPTRAM) based on long-  
369 term ODIS observations Ebrahim. *Remote Sensing of Environment*, **211**, 425-440.

370 BAO, Y., L. LIN, S. WU, K.A.K. DENG & G.P. PETROPOULOS, 2018. Surface Soil  
371 Moisture Retrievals Over Partially Vegetated Areas From the Synergy of Sentinel-1 &  
372 Landsat 8 Data Using a Modified Water-Cloud Model. *International Journal of*

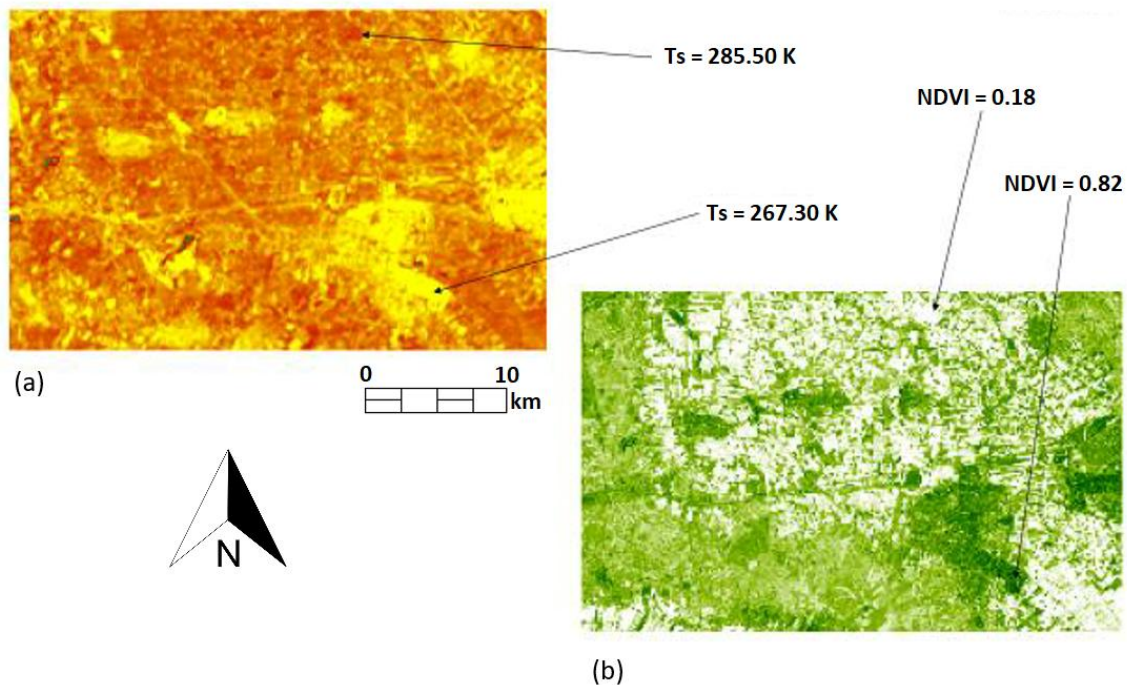
- 373 *Applied Earth Observation & Geoinformation*, **72**, 76-85,  
374 /doi.org/10.1016/j.jag.2018.05.026.
- 375 CARLSON, T. N. 2007. An overview of the triangle method for estimating surface  
376 evapotranspiration and soil moisture from satellite imagery. *Sensors MDPI*, **7**, 1612-  
377 1629.
- 378 CARLSON, T. N., 2013. Triangle models and misconceptions. *Int. J. of Remote Sensing*  
379 *Applications*. **3**, 155-158.
- 380 CARLSON, T. N.; ARTHUR, S. T., 2000. The impact of land use-land cover changes due  
381 to urbanization on surface microclimate and hydrology: a satellite perspective. *Global*  
382 *and Planetary Change*, **25**, 40-65.
- 383 CARLSON, T. N.; RIPLEY, D. A., 1997. On the relation relationship between NDVI,  
384 fractional vegetation cover and leaf area index. *Remote Sensing of Environment*, **62**,  
385 241-252.
- 386 CARLSON, T. N.; SANCHEZ-AZOFEIFA, G. A., 1999. Satellite remote Sensing of land  
387 use changes in and around San Jose', Costa Rica. *Remote Sensing of Environment*, **70**,  
388 247-256.
- 389 CHAUHAN, N. S., MILLER, S., ARDANUY, P., 2003. Spaceborne soil moisture  
390 estimation at high resolution: a microwave-optical/IR synergistic approach.  
391 *International Journal of Remote Sensing*, **22**, 4599-46.
- 392 EUROPEAN SPACE AGENCY: SUPPORT TO SCIENCE ELEMENT, 2012. A  
393 pathfinder for innovation in Earth Observation, 41 pp., available at:  
394 [http://due.esrin.esa.int/stse/files/document/STSE\\_report\\_121016.pdf](http://due.esrin.esa.int/stse/files/document/STSE_report_121016.pdf) (last access: 10  
395 July 2013), ESA, 2012.
- 396 GILLIES, R. R.; CARLSON, T. N; CUI, J.; KUSTAS, W. P.; HUMES, K. S., 1997. A  
397 verification of the 'triangle' method for obtaining surface soil water content and  
398 energy fluxes from remote measurements of the Normalized Difference Vegetation  
399 Index (NDVI) and surface radiometric temperature. *International Journal of Remote*  
400 *Sensing*, **18**, 3145-3166.
- 401 IRELAND, G., PETROPOULOS, G. P., CARLSON, T. N., PURDY, S., 2015. Addressing  
402 the ability of a land biosphere model to predict key biophysical vegetation  
403 characterisation parameters with Global Sensitivity Analysis. *Environmental*  
404 *Modelling & Software*, **65**, 94-107.
- 405 JIANG, L.; ISLAM, S., 2001. Estimation of surface evaporation map over southern Great  
406 Plains using remote sensing data. *Water Res. Research*. **37**, 329-340.
- 407 KASIM, A. A., 2015. Derivation of surface soil water content using a simplified geometric  
408 method in Allahabad District, Uttar Pradesh, India. *International Journal of Scientific*  
409 *Research*. **6**, 1631-1637.

- 410 LIU, J. G., XIE, Z. H., 2013. Improving simulation of soil moisture in China using a  
411 multiple meteorological forcing ensemble approach. *Hydrology and Earth System*  
412 *Sciences Discussions*, **10**(3): 3467-3500.
- 413 OWEN, T. W.; CARLSON, T. N.; GILLIES, R. R., 1998. An assessment of satellite  
414 remotely-sensed land cover parameters in quantitatively describing the climatic effect  
415 of urbanization. *International Journal of Remote Sensing*, **9**, 1663-1681.
- 416 PETROPOULOS, G.; CARLSON, T. N.; WOOSTER, M.; S. ISLAM, S., 2009. A review  
417 of Ts/VI remote sensing based methods for the retrieval of land surface energy fluxes  
418 and soil surface moisture. *Progress in Physical Geography*. **33**, 224-250.
- 419 PETROPOULOS, G.P., IRELAND, G. & B. BARRETT, 2015. Surface Soil Moisture  
420 Retrievals from Remote Sensing: Evolution, Current Status, Products & Future  
421 Trends. *Physics and Chemistry of the Earth*. DOI: 10.1016/j.pce.2015.02.009.
- 422 PETROPOULOS, G.P., P.K. SRIVASTAVA, K.P. FEREDINOS & D. HRISTOPOULOS,  
423 2018. Evaluating the capabilities of optical/TIR imagine sensing systems for  
424 quantifying soil water content. *Geocarto International*, DOI  
425 10.1080/10106049.2018.1520926
- 426 PILES, M, A. CAMPS, M. VALL-LLOSSERA, A. MONERRIS, M. TALONE, 2011.  
427 Downscaling SMOS-derived soil moisture using MODIS visible/infrared data. *IEEE*  
428 *Transactions on Geoscience and Remote Sensing*. **49**, pp. 3156 – 3166, doi:  
429 10.1109/TGRS.2011.2120615.
- 430 PILES, M, M VALL-LLOSSERA, A. CAMPS, N. SÁNCHEZ, J. MARTÍNEZ-  
431 FERNÁNDEZ, J. MARTÍNEZ, V. GONZÁLEZ-GAMBAU, 2013. On the synergy of  
432 SMOS and Terra/Aqua MODIS: high resolution soil moisture maps in near real-time,  
433 *Proc. IEEE International Geoscience and Remote Sensing Symposium*, pp. 3423-26.  
434 doi: 10.1109/IGARSS.2013.6723564.
- 435 PILES, M., G.P. PETROPOULOS, N. SANCHEZ, A. GONZÁLEZ-ZAMORA & G.  
436 IRELAND, 2016. Towards Improved Spatio-Temporal Resolution Soil Moisture  
437 Retrievals From the Synergy of SMOS & MSG SEVIRI Spaceborne Observations.  
438 *Remote Sensing of Environment*, **180**, pp:403-471, DOI 10.1016/j.rse.2016.02.048
- 439 PILES, M., SÁNCHEZ, N., VALL-LLOSSERA, M., CAMPS, A., MARTÍNEZ-  
440 FERNÁNDEZ, J., MARTÍNEZ, J., & GONZÁLEZ-GAMBAU, V., 2014. A  
441 downscaling approach for SMOS land observations: long-term evaluation of high  
442 resolution soil moisture maps over the Iberian Peninsula. *IEEE Journal of Selected*  
443 *Topics in Applied Earth Observations and Remote Sensing*, **7**, 3845-3857.
- 444 PRICE, J. C., 1990. Using spatial context in satellite data to infer regional scale  
445 evapotranspiration. *IEEE. Transactions of Geosciences and Remote Sensing* **28**, 940-  
446 958.

- 447 SADEGHI, M., BABAEIAN, E., TULLER, JONES, S. B., 2017: The optical trapezoid  
448 model: A novel approach to remote sensing of soil moisture applied to Sentinel-2 and  
449 Landsat-8 observations. *Remote Sensing of Environment*, **198**, 52-68.
- 450 SANDHOLT, I.; RASMUSSEN, K.; ANDERSEN, J., 2002. A simple interpretation of the  
451 surface temperature/vegetation index space for assessment of surface moisture status.  
452 *Remote Sensing of Environment*. **79**, 213-224.
- 453 SHEN, C., NIU, J., PHANIKUMAR, M. S., 2013. Evaluating controls on coupled  
454 hydrologic and vegetation dynamics in a humid continental climate watershed using a  
455 subsurface-land surface processes model. *Water Resources Research*, **49**(5), 2552-  
456 2572.
- 457 SILVA-FUZZO, CARLSON, T.N., KOURGIALAS, N., PETROPOULOS, G.P., 2019. A  
458 new technique for estimating soybean yield through remote sensing of soil  
459 moisture and evapotranspiration – A case study in Brazil. *Water MDPI*, submitted.
- 460 SILVA-FUZZO, D.F.; ROCHA, J.V., 2016. Simplified triangle method for estimating  
461 evaporative fraction over soybean crops. *Applied Remote Sensing*. **10**, 15pp.
- 462 SRIVASTAVA, P.K., D. HAN, A. YADUVANSHI, G.P. PETROPOULOS, S. K. SINGH,  
463 R. K. MALL & R. PRASAD, 2017. Reference Evapotranspiration Retrievals From a  
464 Mesoscale Model Based Weather Variables for Soil Moisture Deficit Estimation.  
465 *Sustainability*, **9**, 1971-88, doi:10.3390/su9111971
- 466 TANG, R.; LI, Z.; TANG, L., 2010. An application of the Ts-VI triangle method with  
467 enhanced edges determination for evapotranspiration estimation from MODIS data in  
468 arid and semi-arid regions: implementation and validation. *Remote Sensing of  
469 Environment*, **114**, 540-551.
- 470 TIAN, F., QIU, G., LÜ, Y., YANG, Y., XIONG, Y., 2014. Use of high-resolution thermal  
471 infrared remote sensing and “three-temperature model” for transpiration monitoring in  
472 arid inland river catchment. *Journal of Hydrology*, **515**, 307-315.
- 473

474 **List of Figures:**

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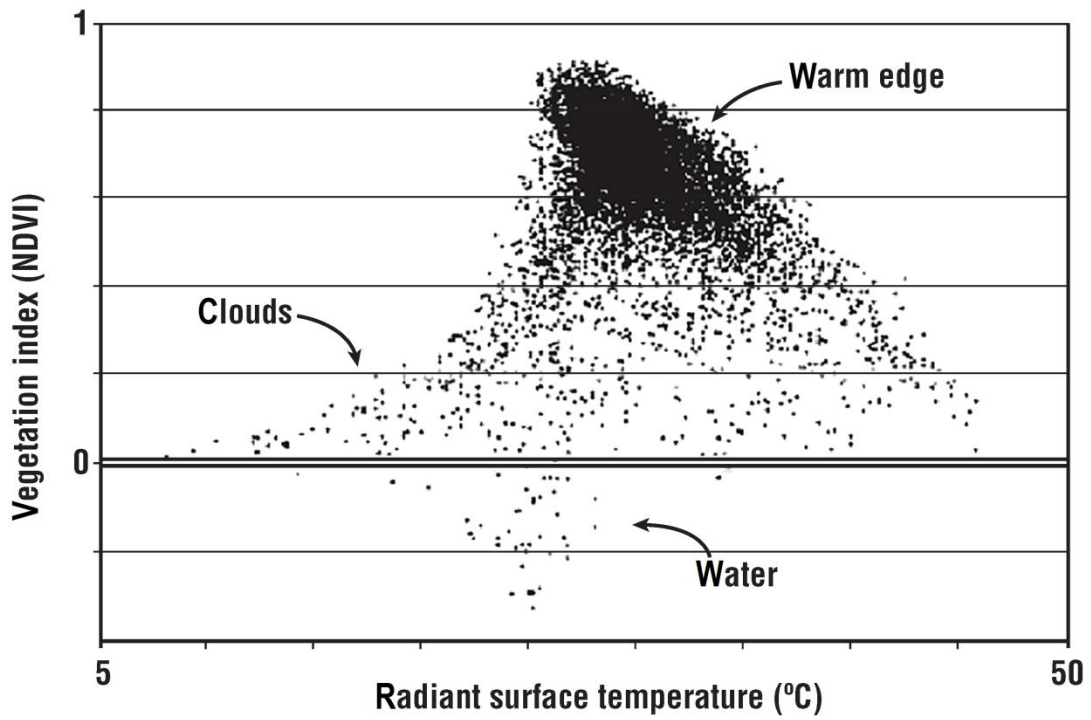


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477 **Figure 1:** An example of a Surface Radiometric Temperature image (a) and  
478 NDVI image (b) derived from ASTER satellite Surface Kinetic Temperature  
479 Surface Reflectance products acquired for a region in The Netherlands on 30  
480 March 2004. In (a), reds are the hottest and yellows the coolest pixels. Hottest and  
481 coolest pixels are labeled with arrows in degrees K. In the NDVI image (b) whites  
482 are the least vegetated and greens the most vegetated pixels with highest and  
483 lowest NDVI values labeled with arrows.

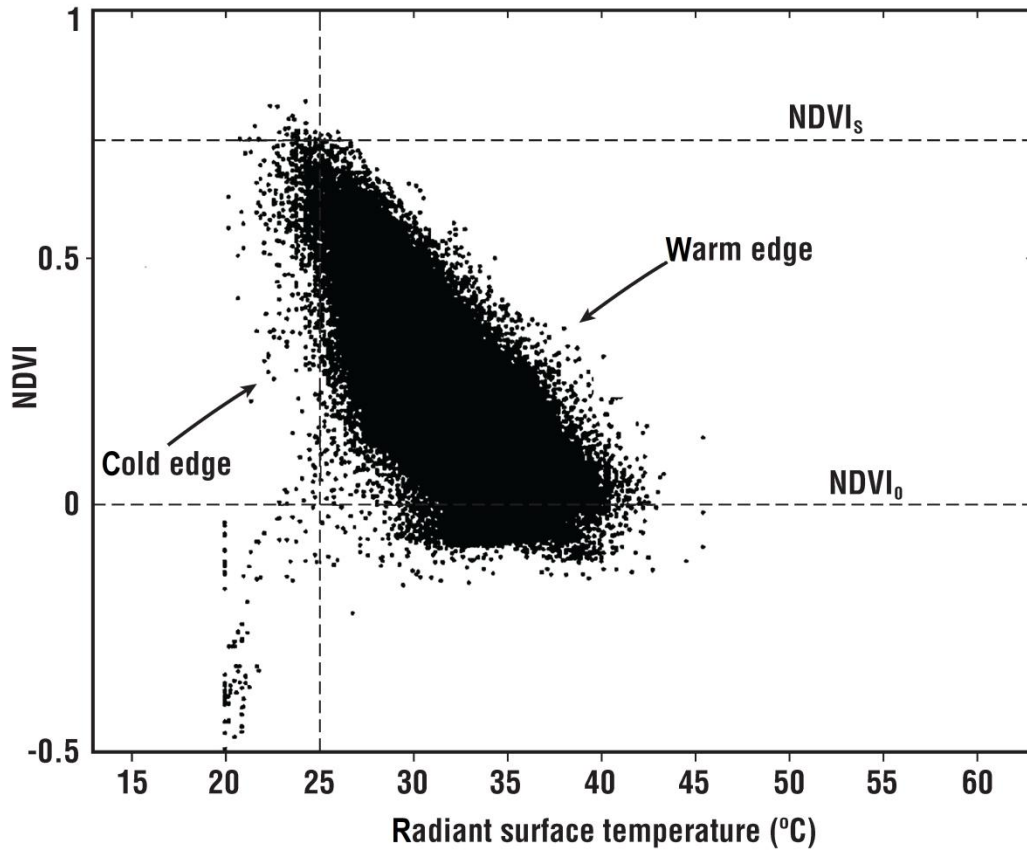
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486 **Figure 2:** An example of a scatterplot created from the  $T_{ir}$  versus NDVI for an  
 487 AVHRR image (referred to also in Arthur-Hartranft et al., 2003), originally from  
 488 Owen et al., (1998). The horizontal axis is  $T_{ir}$  and the vertical axis is NDVI, showing  
 489  $NDVI_0$  and  $NDVI_s$  marked by horizontal lines.



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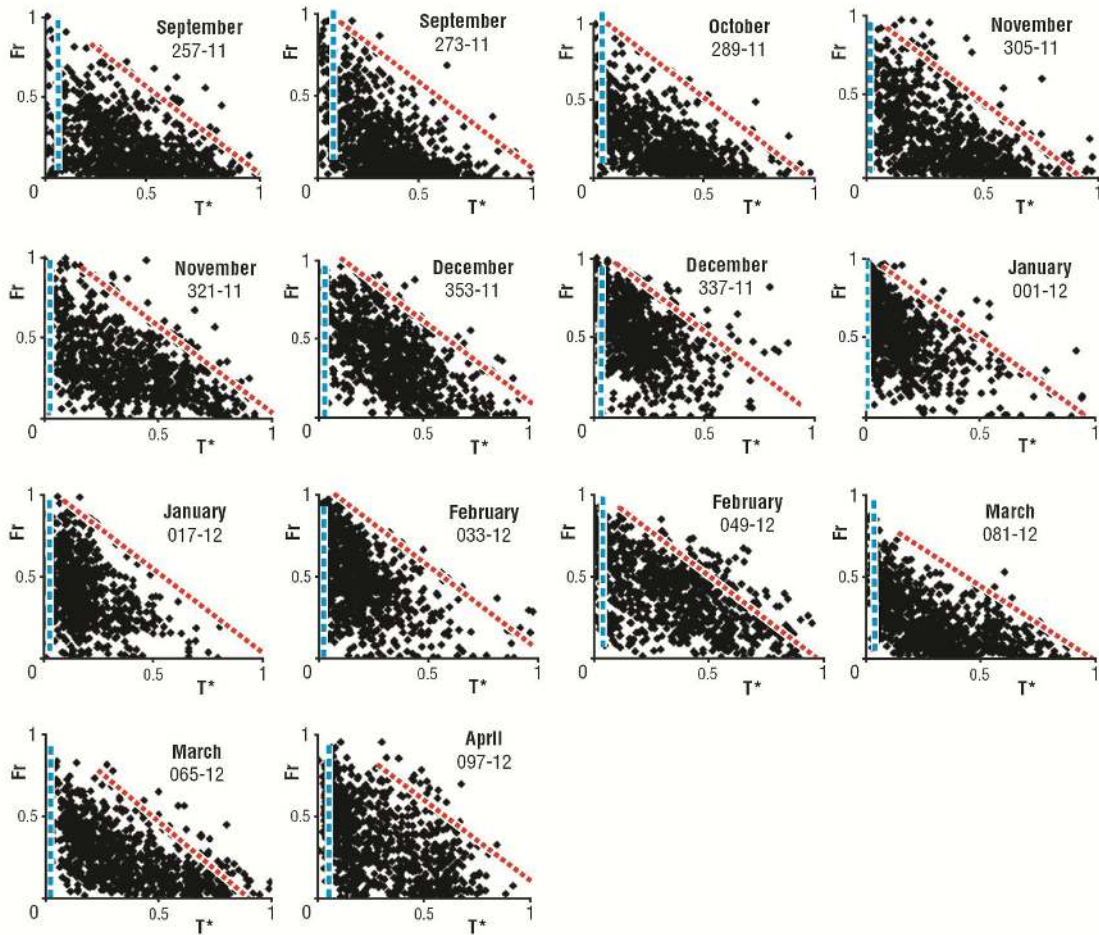
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**Figure 3:** Triangle plotted as NDVI versus  $T_{ir}$  created from pixels measured by an aircraft radiometer (adopted from Gilles et al., 1997), showing the warm and cold edges.



495

496 **Figure 4:** Examples of triangular shapes derived from images over soybean fields for  
 497 Toledo County in Brazil, plotted as in the previous figures, with fractional vegetation  
 498 (here called  $F_r$ ) along the vertical and  $T^*$  along the horizontal. The sloping red line is  
 499 the defined warm edge and the vertical blue line is the cold edge. The soil line  
 500 corresponds to the lower (horizontal) axis (adapted from Silva-Fuzzo & Rocha, 2016)

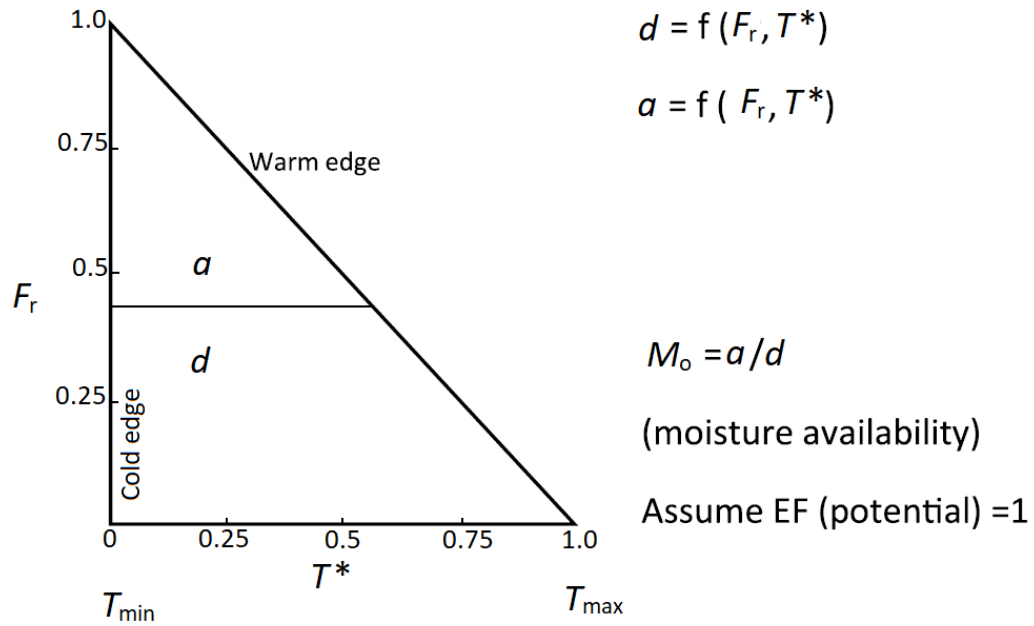
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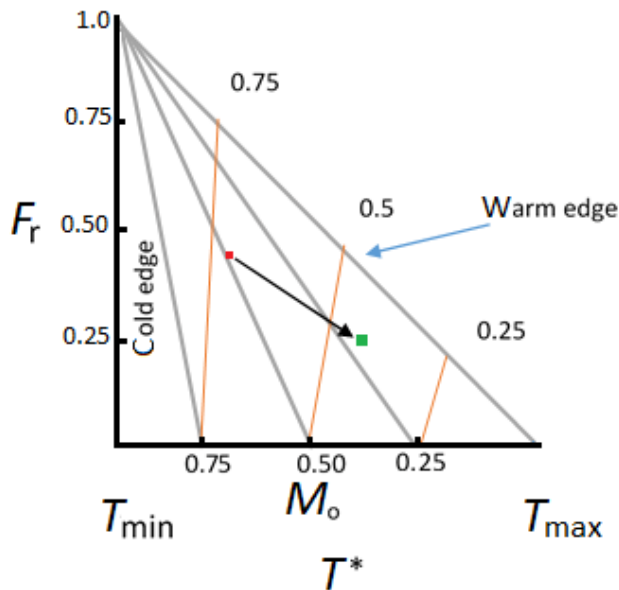
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507 **Figure 5:** Conceptualization of the structure and solution of the triangle domain  
 508 using the “simplified triangle” method based on Equations 5-8. The horizontal  
 509 segment represents any slice across the triangle at constant  $F_r$ , where the letters  $a$   
 510 and  $d$  represent, respectively, a part of the segment and its entire length.

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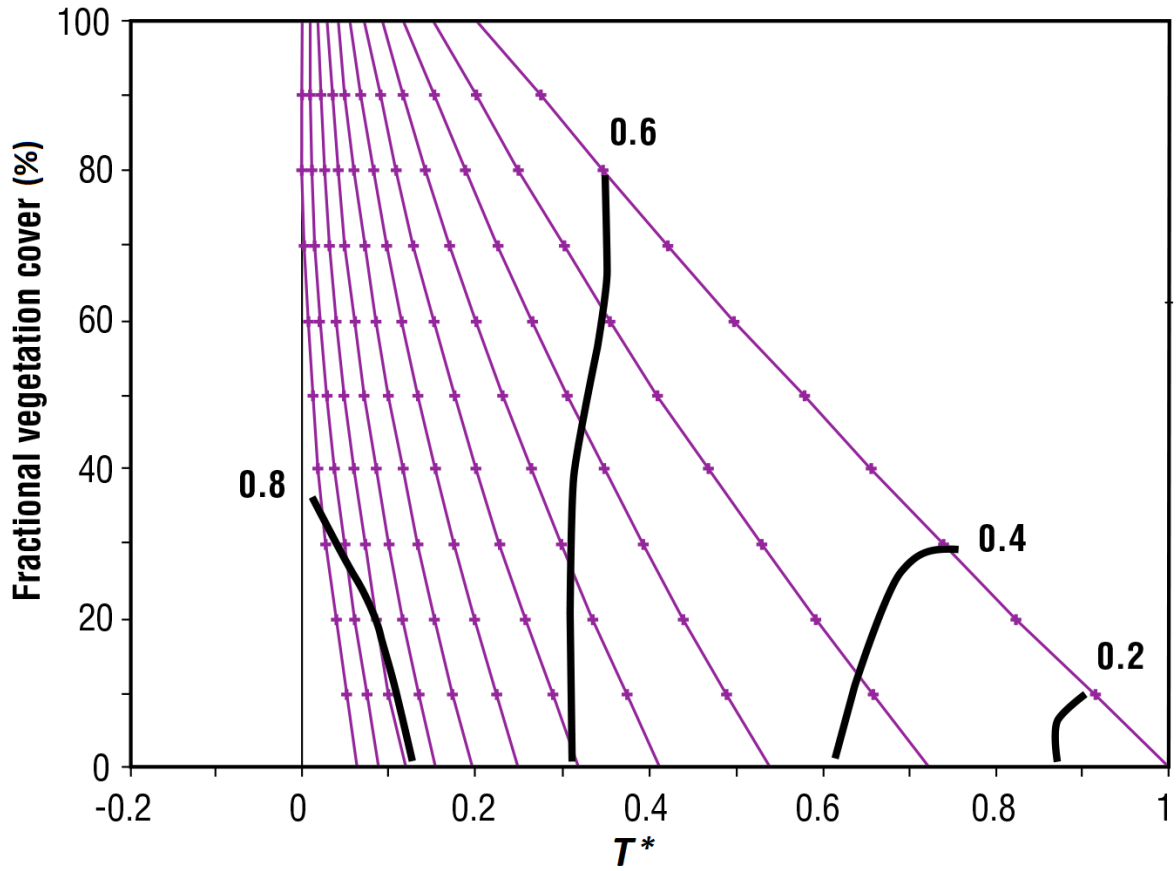


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513 **Figure 6:** The ‘universal triangle’ ( $F_r$  versus  $T^*$ ). Isopleths of EF (slanting lines  
 514 toward the right, labelled accordingly along the warm edge) and  $M_o$  (slanting  
 515 lines to the left, labelled along the horizontal for the solution of Equations (5)-  
 516 (8). The arrow segment represents a surface pixel drying with time (from red

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518

square to green square).  $T^*$  varies from zero at the lower left vertex to 1.0 at the lower right vertex.



519

520 **Figure 7:** Numerical solution for isopleths of  $M_o$  (solid lines sloping upward to the left and  
521 labeled at 0.1 increments) and EF (solid black lines) versus  $F_r$  (percent) and  $T^*$  (horizontal  
522 axis) (adopted from Carlson & Sanchez-Azofeifa, 1998).

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