A New Method for Power Signal Harmonic Analysis

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***Abstract*—With the increasing use of nonlinear loads in power systems, the harmonic pollution becomes more and more serious. It is well known that fast Fourier transform (FFT) is a powerful tool for power signal harmonic analysis, but leakage effect, picket fence effect, and aliasing effect make FFT suffer from specific restrictions. In this paper, we proposed a new method for power signal harmonic analysis. The major components of this method are a frequency and phasor estimating algorithm, a finite-impulse-response comb filter, and a correction factor. We also combine other methods to enhance our performance, such as discrete Fourier transform and least square error (LSE) method. To verify this method, we provided the comparisons of this method with FFT.**

***Index Terms*—Fast Fourier transform (FFT), finite-impulse-response (FIR) comb filter, harmonic analysis.**

I. Introduction

With the progress of industry, power-electronic equipment is widely used in power systems, but the nonlinear characteristics of this equipment have also produced serious harmonic pollution. In addition, many ill effects (i.e., worse power quality for end users, more loss in transmission lines, overheating of machines, and malfunction of relays and breakers) are due to harmonic pollution. It goes without saying that harmonic analysis is a very important subject in power systems.

About harmonic analysis, several algorithms have been proposed [1]–[5], and fast Fourier transform (FFT) is the most widely used computation algorithm for harmonic analysis [5]–[7]. However, leakage effect, picket-fence effect, and aliasing effect make FFT suffer from specific restrictions. Therefore, some methods [7]–[13] have also been provided to improve these drawbacks.

The harmonic analysis method we proposed in this paper is composed of three different parts: an accurate frequency and phasor estimation algorithm, an FIR comb filter to filter out specific signal components, and a correction factor to eliminate the side effect of the FIR comb filter.

The organization of this paper is as follows: we describe an algorithm for estimating the frequency and phasor in Section II. Next, in Section III, an FIR comb filter is derived from the results of Section II. Then, we successfully eliminate the side effect of the FIR comb filter in Section IV. The method we use to conduct harmonic analysis is proposed in Section V. In Section VI, simulation results are presented. Finally, we give conclusions in Section VII.

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II. Frequency and Phasor Estimation Algorithm

This section presents the algorithm for estimating frequency and phasor from a power signal. Suppose the signal $x(t)$ is assumed to be of the form

$$x(t) = X\cos(2\pi ft + \phi) \tag{1}$$

where X is the amplitude of the signal; f is the frequency of the signal; and ϕ is the phase angle of the signal. Suppose that $x(t)$ is sampled with a fixed time interval (Δt) to produce the sampled set $\{x(k)\}$

$$x(k) = X\cos(2\pi fk\Delta t + \phi). \tag{2}$$

Since we know that

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}. \tag{3}$$

Then, $x(k)$ can be expressed as

$$x(k) = \frac{X}{2}e^{j\phi}e^{j2\pi fk\Delta t} + \frac{X}{2}e^{-j\phi}e^{-j2\pi fk\Delta t}. \tag{4}$$

For further explorations, we use the following definitions:

$$a = e^{j2\pi f\Delta t} \tag{5}$$

$$A = \frac{X}{2}e^{j\phi} \tag{6}$$

$$z = \mathrm{Re}(a). \tag{7}$$

Then, (4) can be expressed as

$$x(k) = Aa^k + A^*a^{-k}. \tag{8}$$

Therefore, we can find the following relations between $x(k)$ and $x(k+1)$.

$$\begin{aligned} x(k) &= Aa^k + A^*a^{-k} \\ &\quad\ \ \downarrow{\times a}\qquad \downarrow{\times a^{-1}} \\ x(k+1) &= Aa^{k+1} + A^*a^{-k-1} \end{aligned}$$

After some algebraic manipulations, we find

$$z = \frac{x(k) + x(k+2)}{2x(k+1)}. \tag{9}$$

According to the definitions given above, we can compute the parameters of signal by the following equations:

$$f = \frac{\cos^{-1}(z)}{2\pi\Delta t} \tag{10}$$

$$A = \frac{x(k+1)a - x(k)}{(a^2 - 1)} \tag{11}$$

$$X = 2|A| \tag{12}$$
$$\phi = \text{angle}(A). \tag{13}$$

From the above equations, we find that this algorithm is easy to implement and not affected by frequency deviation. In addition, we find an important relation from (9)

$$x(k) - 2zx(k+1) + x(k+2) = 0. \tag{14}$$

Equation (14) reveals an important massage: if we can find the correct value of "z," then we can filter out this component of the frequency from signal. This idea can be expressed by the following.

$$x(k) - 2zx(k+1) + x(k+2) = y(k+2)$$

III. FIR Comb Filter

According to the results of Section II, we continue to develop advanced features. We want to transfer the algorithm in Section II into an FIR comb filter. Consider the waveform of a power signal as follows:

$$x(t) = \sum_{i=1}^{\mathrm{m}} X_i \cos(2\pi f_i t + \phi_i). \tag{15}$$

Suppose that $x(t)$ is sampled with a fixed time interval (Δt) to produce the sampled set $\{x(k)\}$

$$x(k) = \sum_{i=1}^{m} X_i \cos(2\pi f_i k\Delta t + \phi_i) \quad \mathrm{k} = 0, 1, 2, \ldots \tag{16}$$

Then, we define

$$a_i = e^{j2\pi f_i \Delta t} \tag{17}$$
$$A_i = \frac{X_i}{2} e^{j\phi_i} \tag{18}$$
$$z_i = \mathrm{Re}(a_i). \tag{19}$$

After some algebraic manipulations, we can find out the following results:

$$\sum_{n=0}^{2m} C(n)x(k+n) = 0 \tag{20}$$
$$C = \{\{1, -2z_1, 1\} * \{1, -2z_2, 1\} * \ldots * \{1, -2z_m, 1\}\} \tag{21}$$

where $*$ means convolution operator.

According to (20), we can arbitrarily filter the harmonic components in the signal if we obtain the correct values of $C(n)$. Since harmonics are multiple of the fundamental frequency, f_1 is the key to obtain the correct values of $C(n)$.

IV. Window Correction Factor

Although the proposed FIR comb filter can filter out the desired frequency components from the signal, it still has some side effects. It changes the amplitude and phase of the remaining components of the signal. To eliminate the side effect of the proposed filter, we provide window correction factor (WCF). Suppose a sampled set $\{x(k)\}$ becomes a filtered set $\{\widetilde{x}(k)\}$ by an FIR window filter W

$$\widetilde{x}(k) = \sum_{n=0}^{M-1} W(n)x(k+n). \tag{22}$$

We rearrange (22) to obtain

$$\widetilde{x}(k) = Aa^k \sum_{n=0}^{M-1} W(n)a^n + A^* a^{-k} \sum_{n=0}^{M-1} W(n)a^n. \tag{23}$$

Then, we define

$$\widetilde{A} = A \sum_{n=0}^{M-1} W(n)a^n. \tag{24}$$

Therefore, (23) can be expressed as follows:

$$\widetilde{x}(k) = \widetilde{A}a^k + \widetilde{A}^* a^{-k}. \tag{25}$$

From (25), we can find the relations between $\widetilde{x}(k)$ and $\widetilde{x}(k+1)$ are the same as the relations between $x(k)$ and $x(k+1)$.

$$\widetilde{x}(k) = \widetilde{A}a^k + \widetilde{A}^* a^{-k}$$
$$\times a \qquad \times a^{-1}$$
$$\widetilde{x}(k+1) = \widetilde{A}a^{k+1} + \widetilde{A}^* a^{-k-1}$$

After some algebraic manipulations, we obtain

$$z = \frac{\widetilde{x}(k) + \widetilde{x}(k+2)}{2\widetilde{x}(k+1)}. \tag{26}$$

According to the definitions given above, we can compute the parameters of the signal by the following equations:

$$f = \frac{\cos^{-1}(z)}{2\pi\Delta t} \tag{27}$$
$$A = \frac{\widetilde{x}(k+1)a - \widetilde{x}(k)}{(a^2 - 1) \times WCF}. \tag{28}$$

We define WCF as follows:

$$\mathrm{WCF} = \sum_{n=0}^{M-1} W(n)a^n. \tag{29}$$

Moreover, according to the definition of W, WCF is suitable for any finite sequence of the digital filter. Therefore, not only is the filter we proposed suitable, but so are window filters such as Hamming, Hanning, and Blackman window. Even discrete Fourier transform (DFT), a finite complex sequence, is also included.

V. Proposed Harmonic Analysis Method

The way we conduct harmonic analysis is different from FFT. Simply speaking, we compute each harmonic separately. Meanwhile, to prevent the influence of other harmonics, we have to filter them out. Moreover, if we obtain the fundamental frequency, then we can obtain the frequency of each harmonic.

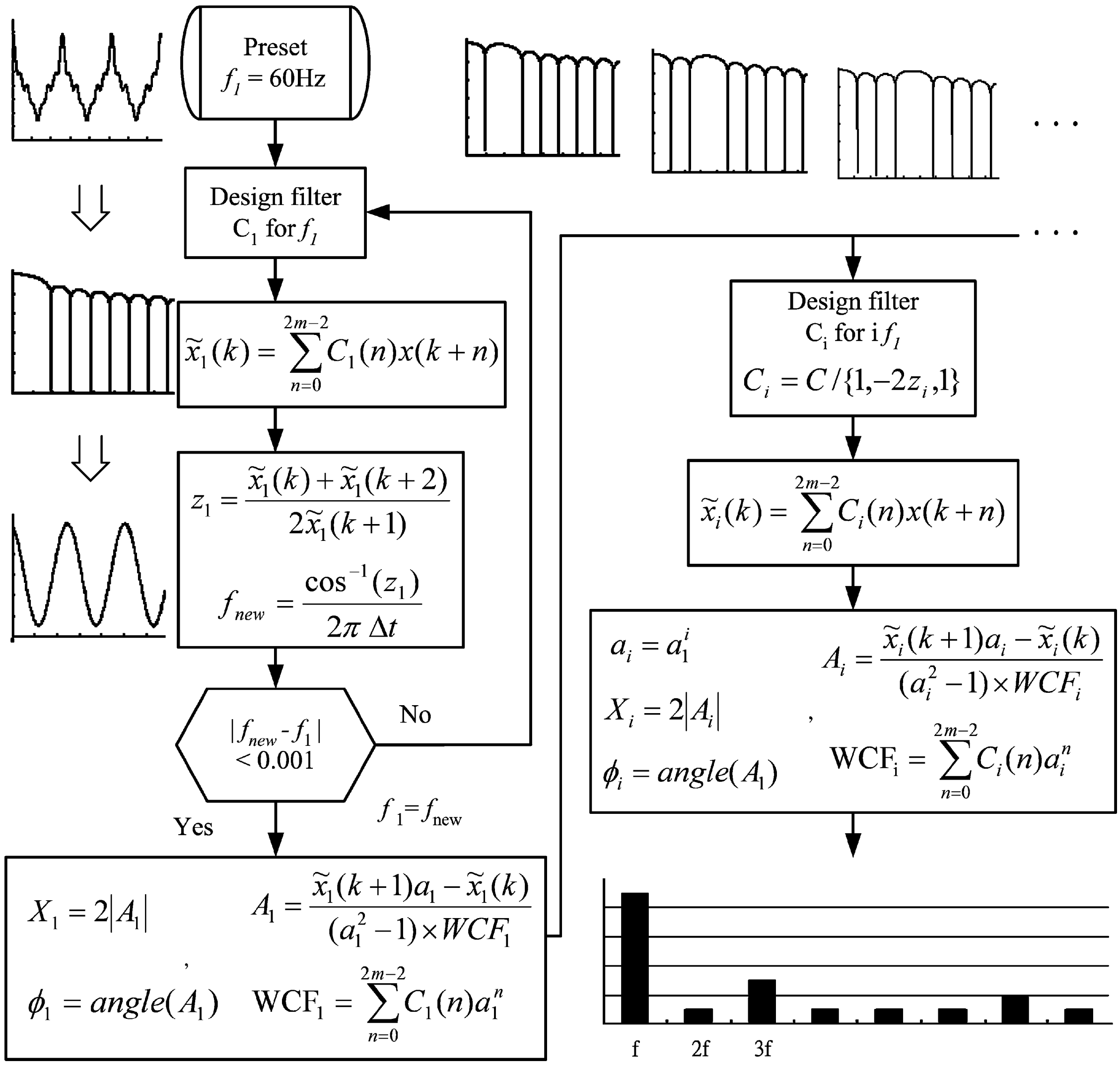

Fig. 1. Process of harmonic analysis.

Therefore, knowing how to estimate fundamental frequency is the key point. The procedure of our harmonic analysis is discussed: First, assume fundamental frequency is 60 Hz, then use (21) to create a comb filter, which filters out every harmonic. After filtering, we find the new fundamental frequency. Then, we redesign the comb filter for the new fundamental frequency and repeat these steps until the fundamental frequency converges. Next, produce the combs filter to filter out every harmonic except the ith harmonic and compute the phasor of the ith harmonic. The process is shown in Fig. 1.

In Fig. 1

$$C_1 = \{\{1, -2z_2, 1\} * \ldots * \{1, -2z_m, 1\}\}. \tag{30}$$

To reduce the computation time, we utilize deconvolution to get C_i. After getting f_1, we also get C_1 at the same time. Then, we can get C by convolution

$$C = C_1 * \{1, -2z_1, 1\}. \tag{31}$$

Next, we get C_i by deconvolution (/)

$$C_i = \frac{C}{\{1, -2z_i, 1\}}. \tag{32}$$

We can get the $\widetilde{x}_i(k)$ by the following equation:

$$\widetilde{x}_i(k) = \sum_{n=0}^{2m-2} C_i(n)x(k+n). \tag{33}$$

Next, we only compute the phasor of each harmonic by

$$A_i = \frac{\widetilde{x}_i(k+1)a_i - \widetilde{x}_i(k)}{(a_i^2 - 1) \times WCF_i}. \tag{34}$$

By using this method for power signal harmonic analysis, we will not seriously suffer the leakage effect, picket fence effect, and aliasing effect. Moreover, the sampling frequency is not necessary to be 2n multiples of the fundamental frequency. Although this method is not as fast as FFT, it is suitable for both online and offline applications. Additionally, it is obvious that this method is more efficient in parallel than in serial computation.

Since the accuracy of the fundamental frequency is the key point, we provide the LSE method to enhance the performance. We rewrite (26) as follows:

$$z_1 \begin{bmatrix} 2\tilde{x}(k+1) \\ \vdots \\ 2\tilde{x}(k+L+1) \end{bmatrix} = \begin{bmatrix} \tilde{x}(k)+\tilde{x}(k+2) \\ \vdots \\ \tilde{x}(k+L)+\tilde{x}(k+L+2) \end{bmatrix}. \tag{35}$$

Then, z_1 can be obtained by

$$z_1 = (A'A)^{-1}A'B \tag{36}$$

where L is the window size of the LSE method and

$$A = \begin{bmatrix} 2\tilde{x}(k+1) \\ \vdots \\ 2\tilde{x}(k+L+1) \end{bmatrix}, \quad B = \begin{bmatrix} \tilde{x}(k)+\tilde{x}(k+2) \\ \vdots \\ \tilde{x}(k+L)+\tilde{x}(k+L+2) \end{bmatrix}.$$

VI. Simulation Results

Here, we present two simulation results comparing the method we proposed with the FFT. For convenience, the method we proposed is called a combined method (CM). In the first case, the fundamental frequency is 59.5 Hz, and the harmonic contents are shown in Table I. We add 0.5% white noise into the signal. The sampling frequency is $60 \times 128 = 7680$ Hz because of the restriction of FFT. The computation results are also shown in Table I.

In this case, we use Matlab built-in function "FFT" to compute the results of FFT, and the window size of FFT is 128 samples. There are two different window sizes of CM: one is for computing the fundamental frequency and another is for computing harmonic contents. The window size of the computing fundamental frequency is 265 samples (127 for the comb filter, 128 for DFT, and ten for LSE). The window size of computing harmonic contents is changing with the order i of the harmonic (127 for comb filter, round (128/i) for DFT). From Table I, we can compare the performances of both methods. CM is better than FFT; this is because CM is not affected by frequency deviation. Moreover, if we want the performance of CM to be much better than FFT, we can add smoothing window like Hanning or Blackman window to filter out noise. In addition, the side effect of the smoothing window also can be eliminated by WCF.

In the second case, we discuss the aliasing effect. Assume the fundamental frequency is 60.5 Hz and the sampling frequency is $60 \times 16 = 960$ Hz. The harmonic contents and simulation results are shown in Table II. Since the fundamental frequency is 60.5 Hz, the frequency of the eighth harmonic is 484 Hz, which is above half of the sampling frequency. According to the Nyquist sampling theorem, this part of the signal cannot be recovered from samples. However, the influences of these harmonics still exist. The computation results of FFT are affected, which is called the aliasing effect. From Table II, we can find that CM gets the correct harmonic contents under no-noise condition, but it does not mean CM is out of the Nyquist sampling theorem. Actually from (14), we can find

$$z_8 = \cos(2\pi 484\Delta t) = \cos(2\pi 476\Delta t).$$

TABLE I

Simulation Signal and Results ($\mathrm{F1} = 59.5$ Hz With 0.5% Noise)

H	Amp.	FFT	CM	H	Amp.	FFT	CM
1	1	0.9931	0.9997	33	0	0.0031	0.0003
2	0.07	0.0737	0.0702	34	0	0.0034	0.0004
3	0.05	0.0526	0.0503	35	0	0.0033	0.0002
4	0	0.0016	0.0005	36	0	0.0032	0.0003
5	0.04	0.0411	0.0399	37	0	0.0033	0.0001
6	0	0.0018	0.0003	38	0	0.0041	0.0005
7	0.03	0.0320	0.0298	39	0	0.0040	0.0001
8	0	0.0036	0.0001	40	0	0.0044	0.0001
9	0	0.0028	0.0003	41	0	0.0048	0.0002
10	0	0.0023	0.0005	42	0	0.0051	0.0004
11	0	0.0023	0.0004	43	0	0.0060	0.0004
12	0	0.0018	0.0002	44	0	0.0077	0.0004
13	0	0.0017	0.0001	45	0	0.0108	0.0002
14	0	0.0017	0.0000	46	0	0.0217	0.0003
15	0	0.0018	0.0002	47	0.03	0.0130	0.0298
16	0	0.0023	0.0005	48	0	0.0209	0.0001
17	0	0.0019	0.0003	49	0.05	0.0396	0.0499
18	0	0.0020	0.0001	50	0	0.0123	0.0002
19	0	0.0020	0.0002	51	0	0.0075	0.0003
20	0	0.0028	0.0003	52	0	0.0056	0.0003
21	0	0.0036	0.0002	53	0	0.0042	0.0001
22	0	0.0066	0.0002	54	0	0.0038	0.0001
23	0.02	0.0154	0.0197	55	0	0.0039	0.0002
24	0	0.0065	0.0002	56	0	0.0044	0.0001
25	0.04	0.0394	0.0399	57	0	0.0060	0.0002
26	0	0.0087	0.0003	58	0	0.0150	0.0003
27	0	0.0056	0.0002	59	0.02	0.0106	0.0203
28	0	0.0044	0.0001	60	0	0.0055	0.0001
29	0	0.0040	0.0002	61	0.01	0.0067	0.0098
30	0	0.0036	0.0000	62	0	0.0034	0.0003
31	0	0.0035	0.0002	63	0	0.0089	0.0003
32	0	0.0033	0.0001	64	0.01	0.0017	0.0102

TABLE II

Simulation Signal and Results ($\mathrm{F1} = 60.5$ Hz)

f_1=60.5Hz		No Noise		0.5% Noise	
H	Amp.	FFT	CM	FFT	CM
1	1	1.0071	1.0000	1.0061	0.9998
2	0.07	0.0668	0.0700	0.0673	0.0704
3	0.05	0.0481	0.0500	0.0481	0.0500
4	0	0.0009	0.0000	0.0011	0.0006
5	0.04	0.0388	0.0400	0.0384	0.0402
6	0	0.0019	0.0000	0.0028	0.0003
7	0.03	0.0271	0.0300	0.0264	0.0293
8	0.01	0.0141	0.0100	0.0152	0.0102

The signal components over half of the sampling frequency are reflected back, and we cannot tell what these are. However, using the method we proposed will allow us to avoid an aliasing effect when the fundamental frequency is not exactly at 60 Hz. In this case, we add a Blackman window to enhance the preference while we add the noise into the signal.

VII. Conclusion

In this paper, we provide a new measurement method for harmonic analysis. This method is easy to implement and very flexible. Users can change the window (smoothing window, DFT, or LSE) and window size to get better performance. Meanwhile, it does not seriously suffer the drawbacks like leakage effect, picket-fence effect, and sampling frequency have to be 2^n multiple of the fundamental frequency. It is also more able to deal with the aliasing effect. This method really meets the need of offline applications. Furthermore, if we can implement this method in parallel computation, it should meet the need of online applications and be more practical.

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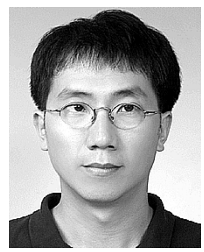

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