

A New Method for Solving Bottleneck-Cost Transportation Problems

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Abstract

A new method namely, blocking method for finding an optimal solution to bottleneck transportation problems which is very different from other existing methods. Then, for finding all efficient solutions of a bottleneck-cost transportation problem, another method namely, blocking zero point method is proposed which is based on zero point method [10]. Both the proposed methods will provide the necessary decision support to decision makers while they are handling time oriented logistic problems.

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1. Introduction

The time-minimizing or bottleneck transportation problem (BTP) is a special case of a transportation problem in which a time is associated with each shipping route. Rather than minimizing cost, the objective is to minimize the maximum time to transport all supply to the destinations. In a BTP, the time of the transporting items from origins to destinations is minimized, satisfying certain conditions in respect of availabilities at sources and requirements at the destinations. Many researchers [7,9,12,13,4,3,2] developed various algorithms for solving time minimizing transportation problems. The transportation time is relevant in a variety of real

transportation problems, too. In [8], Ilija Nikolić has developed an algorithm to find the minimum of the total transportation time to time transportation problems.

Bottleneck-cost transportation (BCTP) is a kind of a bicriteria transportation problem. The bicriteria transportation problem is a particular case of multiobjective transportation problem which had been proposed and also, solved by Aneja and Nair [1] and until, now many researchers [11, 6, 5] also, have great interest in this problem, and some method used their special techniques in finding the solutions for two objective functions approximately approaching to the ideal solution.

In this paper, we propose a new method namely, blocking method for finding an optimal solution to BTPs which is very different from other existing methods. Then, we propose another method namely, blocking zero point method for finding all efficient solutions to a bottleneck-cost transportation problem (BCTP) which is based on zero point method [10]. The proposed methods will provide the necessary decision support to the user while there are handling time oriented logistic problems. The blocking zero point method enables the decision maker to evaluate the economical activities and make the correct managerial decisions.

2. Bottleneck transportation problem

Consider the following BTP:

$$(P) \text{ Minimize } z = [\text{Maximize } t_{ij} / x_{ij} > 0]_{(i,j)}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \dots, m \quad (1)$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n \quad (2)$$

$$x_{ij} \geq 0, \text{ for all } i \text{ and } j \text{ and integers,} \quad (3)$$

where m is the number of supply points; n is the number of demand points; x_{ij} is the number of units shipped from supply point i to demand point j ; t_{ij} is the time of transporting goods from supply point i to demand point j ; a_i is the supply at supply point i and b_j is the demand at demand point j .

In a BTP, time matrix $[t_{ij}]$ is given where t_{ij} is the time of transporting goods from the i th origin to the j th destination. For any given feasible solution

$X = \{x_{ij} : i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$ of the problem (P), the time transportation is the maximum of t_{ij} 's among the cells in which there are positive allocations. This time of the transportation remains independent of the amount of commodity sent so long as $x_{ij} > 0$.

2.1. Blocking method

We, now introduce a new method namely, the blocking method for finding an optimal solution to bottleneck transportation problems.

The blocking method proceeds as follows.

Algorithm:

Step 1: Find the maximum of the minimum of each row and column of the given transportation table. Say, T .

Step 2: Construct a reduced transportation table from the given table by blocking all cells having time more than T .

Step 3. Check if each column demand is less than to the sum of the supplies in the reduced transportation problem obtained from the Step 2.. Also, check if each row supply is less than to sum of the column demands in the reduced transportation problem obtained from the Step 2.. If so, go to Step 6.(Such reduced transportation table is called the active transportation table).
If not, go to Step 4.

Step 4: Find a time which is immediately next to the time T . Say U .

Step 5: Construct a reduced transportation table from the given transportation table by blocking all cells having time more than U and then, go to the Step 3..

Step 6: Do allocation according to the following rules:

- (a) allot the maximum possible to a cell which is only one cell in the row / column . Then, modify the active transportation table and then, repeat the process till it is possible or all allocations are completed.
- (b) If (a) is not possible, select a row / a column having minimum number of unblocked cell and allot maximum possible to a cell which helps to reduce the large supply and/ or large demand of the cell.

Step 7: This allotment yields a solution to the given bottleneck transportation problem.

Now, we prove the solution to a BTP obtained by the blocking method is an optimal solution to the BTP.

Theorem 1. A solution obtained by the blocking method to the BTP, (P) is optimal.

Proof : Let T_0 be the time transportation of a feasible solution of the given problem.

Let W be the time transportation of the feasible solution to the given BTP by the blocking method. It means that all transportation can be made in the time of W . In the active table, maximum time is W . As per the Step 3 to 5, W is the minimum time to transport all items from the origins to destinations.

If $T_0 \geq W$, the solution obtained by the blocking method is optimal.

If $T_0 < W$, it means that all transporting work can be made in the time of T_0 and $T_0 < W$. Therefore, T_0 is a time for transportation which is less than W . This is not possible.

Thus, the time for transportation obtained using the blocking method to the bottleneck transportation problem is optimal.
Hence the theorem.

The blocking method for solving a BTP is illustrated by the following example.

Example 1: Consider the following bottleneck transportation problem:

								Supply
	12	13	34	7	8	29	19	15
	7	18	36	40	38	6	10	7
	11	20	30	21	21	29	31	45
	27	12	39	31	5	36	12	30
	15	17	32	36	22	16	14	12
	17	38	16	33	23	30	29	16
Demand	20	13	11	27	9	5	40	

Now, the maximum of minimum of each row and the minimum of each column =16.

Now, using the Step 1. to the Step 5., we have the following complete allocation table :

								Supply
	12	13		7	8		19	15
	7	18				6	10	7
	11	20		21	21			45
		12			5		12	30
	15	17				16	14	12
	17		16					16
Demand	20	13	11	27	9	5	40	

Now, using the Step 6., the optimal solution to the bottleneck problem is given below:

								Supply
					8(9)		19(6)	15
						6(5)	10(2)	7
	11(15)	20(3)		21(27)				45
							12(30)	30
		17(10)					14(2)	12
	17(5)		16(11)					16
Demand	20	13	11	27	9	5	40	

and the minimum time transportation is 21.

Remark 1: The time budgetary time transportation problem by the blocking method can also be solved.

3. Bottleneck-Cost Transportation Problem

Consider the following BCTP:

$$(MP) \text{ Minimize } z_1 = \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij}$$

$$\text{Minimize } z_2 = [\text{Maximize } t_{ij} / x_{ij} > 0]_{(i,j)}$$

subject to (1), (2) and (3) are satisfied.

In BCTP, faster/ slower mode of the transportation results in lesser/ higher time of transportation but involves a high/low cost of transportation. This implies that the commissioning of a project is influenced by the bottleneck. Thus, for a sequence of various times in a specified time interval, one can have a sequence of less-cost transportation schedules. This type of analysis enables the decision makers to select an appropriate transportation schedule, depending on his financial position and the extent of bottleneck that they can afford

Definition 1: A point (X, T) where $X = \{x_{ij}; i = 1, 2, \dots, m \text{ and } j = 1, 2, 3, \dots, n\}$ and T is a time, is said to be a feasible solution of (MP) if X satisfies the conditions (1) to (3).

Definition 2: A feasible point (X_0, T_0) is said to be efficient for (MP) if there exists no other feasible point (X, T) in (MP) such that $z_1(X) \leq z_1(X_0)$ and $z_2(T) < z_2(T_0)$ or $z_1(X) < z_1(X_0)$ and $z_2(T) \leq z_2(T_0)$.

Definition 3: A cost transportation problem of a BCTP is said to be active for any time M if the minimum time transportation corresponding to the cost transportation problem is M .

3.1. Blocking Zero point method:

We, now introduce a new method, called the blocking zero point method for finding all efficient solutions to the BCTP.

The blocking zero point method proceeds as follows.

Algorithm:

Step 1: Construct the time transportation problem from the given BCTP.

Step 2: Solve the time transportation problem by the blocking method. Let the optimal solution be T_0 .

Step 3. Construct the cost transportation problem from the given BCTP.

Step 4: Solve the cost transportation problem by the zero point method and also, find the corresponding time transportation. Let it be T_m .

Step 5: For each time M in $[T_0, T_m]$, compute $\alpha = \frac{T_m - M}{T_m - T_0}$ which is the level of time satisfaction for the time M .

Step 6 : Construct the active cost transportation problem for each time M in $[T_0, T_m]$ and solve it by zero point method.

Step 7: For each time M , an optimal solution to the cost transportation problem, X is obtained from the Step 6 with the level of time satisfaction α . Then, the vector (X, M) is an efficient solution to BCP.

Now, the blocking zero point method is illustrated by the following example.

Example 2: Consider the following 3X4 bottleneck-cost transportation problem. The upper left corner in each cell gives the time of transportation on the corresponding route and the lower right corner in each cell gives the unit transportation cost per unit on that route.

					Supply
	10	68	73	52	8
	5	6	10	11	
	66	95	30	21	19
	6	7	12	14	
	97	63	19	23	17
	14	11	9	7	
Demand	11	3	14	16	

Now, the time transportation problem of BCTP is given below:

					Supply
	10	68	73	52	8
	66	95	30	21	19
	97	63	19	23	17
Demand	11	3	14	16	

Using the blocking method, we have that the optimal solution of the time transportation problem of BTP is 66.

Now, the cost transportation table of BCTP is given below

					Supply
	5	6	10	11	8
	6	7	12	14	19
	14	11	9	7	17
Demand	11	3	14	16	

By zero point method, the optimal solution is $x_{13} = 8$; $x_{21} = 11$; $x_{22} = 3$; $x_{23} = 5$; $x_{33} = 1$ and $x_{34} = 16$ with the minimum transportation cost is 348 and the minimum time transportation is 95.

Now, we have $T_0 = 66$; $T_m = 95$ and the time $M = \{66,68,73,95\}$

Now, the active cost transportation problem of BCTP for $M = 66$ is given below:

					Supply
	5	-	-	11	8
	6	-	12	14	19
	-	11	9	7	17
Demand	11	3	14	16	

Using zero point method, the optimal solution is $x_{11} = 6$; $x_{14} = 2$; $x_{21} = 5$; $x_{23} = 14$; $x_{32} = 3$ and $x_{34} = 14$ with total minimum transportation cost = 381.

Now, the active cost transportation problem of BCTP for $M = 68$ is given below

					Supply
	5	6	-	11	8
	6	-	12	14	19
	-	11	9	7	17
Demand	11	3	14	16	

Using zero point method, the optimal solution is $x_{11} = 5$; $x_{12} = 3$; $x_{21} = 6$; $x_{23} = 13$; $x_{33} = 1$ and $x_{34} = 16$ with the minimum transportation cost 356.

Now, the active cost transportation table of BCTP for $M = 73$ is given below

					Supply
	5	6	10	11	8
	6	-	12	14	19
	-	11	9	7	17
Demand	11	3	14	16	

Using zero point method, the optimal solution is $x_{12} = 3$; $x_{13} = 5$; $x_{21} = 11$; $x_{23} = 8$; $x_{33} = 1$ and $x_{34} = 16$ with the minimum total transportation cost = 351.

Now, the efficient solutions to the BCTP is given below:

S.No.	Efficient solution of BCTP	Objective value of BCTP	Satisfaction Level α
1	$x_{11} = 6$; $x_{14} = 2$; $x_{21} = 5$; $x_{23} = 14$; $x_{32} = 3$ and $x_{34} = 14$ with time 66	(381, 66)	1
2	$x_{11} = 5$; $x_{12} = 3$; $x_{21} = 6$; $x_{23} = 13$; $x_{33} = 1$ and $x_{34} = 16$ with time 68	(356, 68)	$\frac{27}{29} \approx 0.93$
3	$x_{12} = 3$; $x_{13} = 5$; $x_{21} = 11$; $x_{23} = 8$; $x_{33} = 1$ and $x_{34} = 16$ with time 73	(351, 73)	$\frac{22}{29} \approx 0.76$
4	$x_{13} = 8$; $x_{21} = 11$; $x_{22} = 3$; $x_{23} = 5$; $x_{33} = 1$ and $x_{34} = 16$ with time 95	(348, 95)	0

Remark 2: The time or / and cost budgetary transportation problem by the blocking zero point method can also be solved.

4. Conclusion

The time of transport might be significant factor in several transportation problems. The proposed methods are quite simple from the computational point of view and also, easy to understand and apply. By blocking zero point method, we obtain a sequence of optimal solutions to a bottleneck-cost transportation problem for a sequence of various time in a time interval. This method provides a set of transportation schedules to bottleneck-cost transportation problems which helps the decision makers to select an appropriate transportation schedule, depending on his financial position and the extent of bottleneck that he can afford. The blocking zero point method enables the decision maker to evaluate the economical activities and make the correct managerial decisions.

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