Technical Notes and Correspondence

A New Method for Stabilization of Networked Control Systems With Random Delays

Liqian Zhang, Yang Shi, Tongwen Chen, and Biao Huang

***Abstract*—We consider the stabilization problem for a class of networked control systems in the discrete-time domain with random delays. The sensor-to-controller and controller-to-actuator delays are modeled as two Markov chains, and the resulting closed-loop systems are jump linear systems with two modes. The necessary and sufficient conditions on the existence of stabilizing controllers are established. It is shown that state-feedback gains are mode-dependent. An iterative linear matrix inequality (LMI) approach is employed to calculate the state-feedback gains.**

***Index Terms*—Discrete-time systems, linear matrix inequality (LMI), Markov chains, networked control systems, network-induced delays.**

I. Introduction

Networked control systems are feedback control loops closed through a real time network. That is, in networked control systems, communication networks are employed to exchange information and control signals (reference input, plant output, control input, etc.) between control system components (sensors, controllers, actuators, etc.) [21]. The main advantages of networked control systems are low cost, reduced weight, simple installation and maintenance, and high reliability. As a result, networked control systems have great potential in applications in manufacturing plants, vehicles, aircrafts, and spacecrafts [15].

Despite the advantages and potentials, communication networks in control loops make the analysis and design of a networked control system complicate. One main issue is the network-induced delays (sensor-to-controller and controller-to-actuator), which occur when sensors, actuators, and controllers exchange data across the network. The delays may be constant, time-varying, and in most cases, random. It is known that the occurrence of delay degrades the stability and control performance of closed-loop control systems. Many researchers have studied stability, controller design for stabilization and performance of networked control systems in the presence of network-induced delays. In [14], the stability analysis and control design of networked control systems were studied when the network-induced delay at each sampling instant (sensor-to-controller delay $\tau_k +$ controller-to-actuator delay d_k) is random and less than one sampling time. The controller given there depends on sensor-to-controller delay τ_k. The results in [14] have recently been extended to the case with longer delays in [9]. In [21], the stability of networked control systems was analyzed by a hybrid system approach when the induced delay is deterministic (constant or time-varying) and the controller gain is constant; and in [13], a switched system approach was used to study the stability of networked control systems with constant controller gain. In [9], the maximum of the network-induced delay preserving the closed-loop stability for a given plant and controller was considered. In [17], the network-induced delay is assumed to be time-varying and less than one sampling time, and the stabilizing state-feedback gain is constant. It is noticed that in all of the aforementioned papers, the plant is in the continuous-time domain. For the discrete-time case, in [12] and [16], the network-induced random delays were modeled as Markov chains such that the closed-loop system is a jump linear system with one mode. It is noticed that in [12], the state-feedback gain is mode-independent, and in [16], the state-feedback gain only depends on the delay from sensor to controller. Recently, the stabilization of networked control systems with discrete-time plant was considered in [19] and [18] for constant and time-varying network-induced delays, respectively, and the state-feedback gains are constant.

In this note, we consider the stabilization problem of networked control systems with a discrete-time plant. The two random delays (sensor-to-controller and controller-to-sensor) are modeled as two different Markov chains, and the resulting closed-loop systems are jump linear systems with two modes characterized by two Markov chains. At each sampling time, the current states, the current sensor-to-controller delay (τ_k), and previous controller-to-actuator delay (d_{k-1}) are known by, e.g., the time-stamping technique. Thus, our goal is to design a state-feedback controller whose gain depends on both τ_k and d_{k-1}. In this way, the conservativeness of the stabilization conditions should be reduced. The necessary and sufficient conditions on the existence of stabilizing controllers are given, and an iterative linear matrix inequality (LMI) approach is used to calculate the controllers. An inverted pendulum example is considered to illustrate the proposed method.

II. Problem Statement

Consider the networked control setup in Fig. 1, where the plant is a linear time-invariant discrete-time system, $\tau_k \geq 0$ is the random time delay from the sensor to the controller, $d_k \geq 0$ is the random time delay from the controller to the actuator, and the controller is to be designed.

Here, it is assumed that both τ_k and d_k are bounded, that is

$$0 \leq \tau_k \leq \tau, 0 \leq d_k \leq d.$$

In real communication systems, current time delays are usually correlated with the previous time delays. It is reasonable to model two random delays τ_k and d_k as two homogeneous Markov chains that take values in $\mathcal{M} = \{0, 1, \ldots, \tau\}$ and $\mathcal{N} = \{0, 1, \ldots, d\}$, and their transition probability matrices are $\Lambda = [\lambda_{ij}]$ and $\Pi = [\pi_{rs}]$, respectively. That is, τ_k and d_k jump from mode i to j and from mode r to s, respectively, with probabilities λ_{ij} and π_{rs}, which are defined by

$$\begin{aligned} \lambda_{ij} &= \Pr(\tau_{k+1} = j | \tau_k = i) \\ \pi_{rs} &= \Pr(d_{k+1} = s | d_k = r) \end{aligned} \tag{1}$$

where $\lambda_{ij}, \pi_{rs} \geq 0$ and

$$\sum_{j=0,}^{\tau} \lambda_{ij} = 1 \quad \sum_{s=0}^{d} \pi_{rs} = 1 \tag{2}$$

for all $i, j \in \mathcal{M}$ and $r, s \in \mathcal{N}$.

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L. Zhang and B. Huang are with the Department of Chemical and Materials Engineering, University of Alberta, Edmonton, AB T6G 2G6, Canada.

Y. Shi and T. Chen are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada (e-mail: tchen@ece.ualberta.ca).

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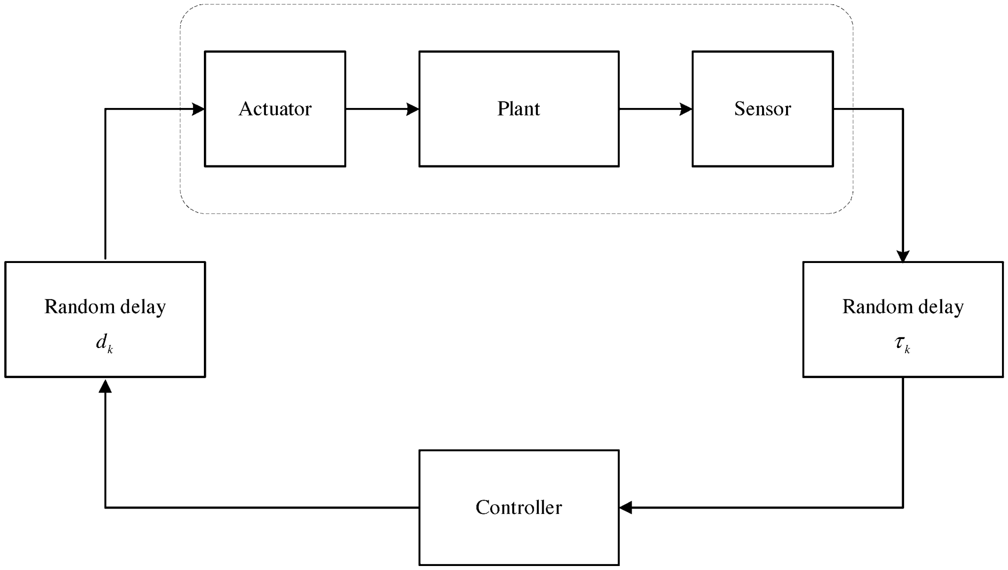

Fig. 1. Networked control system.

Remark 1: It is noted in [16] that modeling of τ_k and d_k as two Markov chains is quite general, and the package loss can be included naturally. It is usually assumed that the controller always uses the most recent data. Thus, if at sampling time $k, x(k-\tau_k)$ is available, then at sampling time $k+1$, if there are delays longer than 1 or package loss, we still have $x(k-\tau_k)$ to use. This means that the delay τ_k can increase at most 1 at each step, or

$$\Pr(\tau_{k+1} > \tau_k + 1) = 0.$$

From (1), we can see that this means that

$$\lambda_{ij} = 0, \qquad \text{if } j > i+1.$$

For d_k, we have similar comments. That is

$$\pi_{rs} = 0, \qquad \text{if } s > r+1.$$

Assume that the model of the plant is a linear time-invariant discrete-time model as follows:

$$x(k+1) = Ax(k) + Bu(k). \tag{3}$$

It is noticed that when the control action is calculated, we do not know the exact d_k, but τ_k and d_{k-1} are available. Consequently, the controller gain can be designed to be dependent on τ_k and d_{k-1}, that is

$$\begin{aligned} u(k) &= F(\tau_k, d_{k-1})x(k-\tau_k-d_k) \\ x(t) &= \phi(t), t \in \{-\tau-d, \ldots, 0\}. \end{aligned} \tag{4}$$

Hence, the closed-loop system from (3) and (4) can be expressed as

$$x(k+1) = Ax(k) + BF(\tau_k, d_{k-1})x(k-\tau_k-d_k). \tag{5}$$

It can be seen that the closed-loop system in (5) is a discrete-time jump linear system with two modes (τ_k and d_k) and mode-dependent delays. It is worth to be mentioned that the closed-loop system in (5) can not be reduced to a jump linear system with one mode by simply combining τ_k and d_k as one Markov chain. The reason can be seen from the fact that (5) depends not only on τ_k and d_k, but also on d_{k-1}.

In [1]–[3], stabilization and control design for discrete-time jump linear systems with one mode and mode-dependent delays were studied, and sufficient conditions for stochastic stability were given based on LMIs. The results can be extended to deal with the two-mode case. However, it is noticed that the sufficient conditions given in [1]–[3] require that the plant must be stable. This means that the sufficient conditions cannot be applied to stabilize the closed-loop systems when the plant is unstable. In the following, through the augmentation technique, necessary and sufficient conditions for the stochastic stability of the closed-loop system in (5) will be established.

At sampling time k, if we augment the state-variable as

$$X(k) = [x(k)^{\mathbb{T}} x(k-1)^{\mathbb{T}} \ldots x(k-\tau-d)^{\mathbb{T}}]^{\mathbb{T}} \tag{6}$$

then the closed-loop system in (5) can be written as

$$\begin{aligned} X(k+1) &= (\tilde{A} + \tilde{B}F(\tau_k, d_{k-1})\tilde{E}(\tau_k, d_k))X(k) \\ X(0) &= [\phi(0)^{\mathbb{T}} \phi(-1)^{\mathbb{T}} \cdots \phi(-\tau-d)^{\mathbb{T}}]^{\mathbb{T}} \end{aligned} \tag{7}$$

where

$$\tilde{A} = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \in \mathbb{R}^{n(1+\tau+d)\times n(1+\tau+d)}$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^{n(1+\tau+d)\times p}$$

$$\tilde{E}(\tau_k, d_k) = [0 \quad \cdots \quad 0 \quad I \quad \cdots 0] \in \mathbb{R}^{n\times n(1+\tau+d)}$$

and $\tilde{E}(\tau_k, d_k)$ has all elements being zeros except for the $(1+\tau_k+d_k)$th block being identity. It can be seen that the closed-loop system in (7) is a delay-free jump linear system with two modes modeled by different homogeneous Markov chains. Throughout this note, we use the following definition.

Definition 1: The system in (7) is stochastically stable if for every finite $X_0 = X(0)$ and initial mode $\tau_0 = \tau(0) \in \mathcal{M}$ and $d_{-1} = d(-1) \in \mathcal{N}$, there exists a finite $W > 0$ such that the following holds:

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} \|X(k)\|^2 | X_0, \tau_0, d_{-1}\right\} < X_0^{\mathbb{T}} W X_0. \tag{8}$$

We have noted that the closed-loop system in (7) is a delay-free discrete-time jump linear system with two modes. In the literature, the stochastic stability and stabilization for discrete-time jump linear systems with one mode has been well studied, e.g., [4] and [11]. Besides the difference in the number of modes, the other difference of the closed-loop system in (7) from the one-mode jump linear systems is that it depends not only on the current mode d_k, but also on the previous mode d_{k-1}. The objective in this note is to find the state-feedback gain $F(\tau_k, d_{k-1})$ such that the closed-loop system in (7) is stochastically stable. In the following, if we know that $\tau_k = i, d_{k-1} = r, F(\tau_k, d_{k-1})$ is denoted as $F(i, r)$.

III. Main Results

With Definition 1, the necessary and sufficient conditions on the stochastic stability of closed-loop system in (7) can be obtained.

Theorem 1: The closed-loop system in (7) is stochastically stable if and only if there exists $P(i, r) > 0$ such that the following matrix inequality:

$$L(i,r) = \sum_{s=0}^{d} \pi_{rs}(\tilde{A} + \tilde{B}F(i,r)\tilde{E}(i,s))^{\mathsf{T}}\bar{P}(i,s) \times(\tilde{A} + \tilde{B}F(i,r)\tilde{E}(i,s)) - P(i,r) < 0 \quad (9)$$

holds for all $i \in \mathcal{M}$ and $r \in \mathcal{N}$, where

$$\bar{P}(i,s) = \sum_{j=0}^{\tau} \lambda_{ij}P(j,s). \quad (10)$$

Proof: Sufficiency: For the closed-loop system in (5), consider the quadratic function which is given by

$$V(X(k),k) = X(k)^{\mathsf{T}}P(\tau_k, d_{k-1})X(k).$$

Noticing (10) and (7), we have

$$\begin{aligned}\mathcal{E}\{\Delta V(X(k),k)\} &= \mathcal{E}\{X(k+1)^{\mathsf{T}}P(\tau_{k+1}, d_k)X(k+1)|X_k \\ &\quad \tau_k = i, d_{k-1} = r\} - X(k)^{\mathsf{T}}P(i,r)X(k) \\ &= \sum_{s=0}^{d}\sum_{j=0}^{\tau} \pi_{rs}(X(k)^{\mathsf{T}}(\tilde{A} + \tilde{B}F(i,r)\tilde{E}(i,s))^{\mathsf{T}})\lambda_{ij} \\ &\quad \times P(j,s)((\tilde{A} + \tilde{B}F(i,r)\tilde{E}(i,s))X(k)) \\ &\quad - X(k)^{\mathsf{T}}P(i,r)X(k) \\ &= X(k)^{\mathsf{T}}\left[\sum_{s=0}^{d}\pi_{rs}(\tilde{A} + \tilde{B}F(i,r)\tilde{E}(i,s))^{\mathsf{T}}\bar{P}(i,s)\right. \\ &\quad \left.\times(\tilde{A} + \tilde{B}F(i,r)\tilde{E}(i,s)) - P(i,r)\right]X(k).\end{aligned}$$

Thus, if $L(i, r) < 0$, then

$$\begin{aligned}\mathcal{E}\{\Delta V(X(k))\} &= \mathcal{E}\{V(X(k+1), k+1)|X_k, \tau_k = i, d_{k-1} = r\} \\ &\quad - V(X(k),k) \\ &\le -\lambda_{\min}(-L(i,r))X(k)^{\mathsf{T}}X(k) \\ &\le -\beta X(k)^{\mathsf{T}}X(k) = -\beta\|X(k)\|^2\end{aligned}$$

where $\beta = \inf\{\lambda_{\min}(-L(i,r)), i \in \mathcal{M}, r \in \mathcal{N}\} > 0$. From the previous inequality, we can see that for any $T \ge 1$

$$\mathcal{E}\{V(X(T+1), T+1)\} - \mathcal{E}\{V(X_0, 0)\} \le -\beta\mathcal{E}\left\{\sum_{t=0}^{T}\|X(t)\|^2\right\}$$

or

$$\begin{aligned}\mathcal{E}\left\{\sum_{t=0}^{T}\|X(t)\|^2\right\} &\le \frac{1}{\beta}(\mathcal{E}\{V(X_0,0) - \mathcal{E}\{V(X(T+1), T+1)\}) \\ &\le \frac{1}{\beta}\mathcal{E}\{V(X_0, 0)\}\end{aligned}$$

which implies that

$$\mathcal{E}\left\{\sum_{t=0}^{\infty}\|X(t)\|^2\right\} \le \frac{1}{\beta}\mathcal{E}\{V(X_0,0)\} = \frac{1}{\beta}X(0)^{\mathsf{T}}P(\tau_0, d_{-1})X(0).$$

From Definition 1, the stochastic stability is obtained.

Necessity: Assume that the closed-loop system in (7) is stochastically stable. That is, we have

$$\mathcal{E}\left\{\sum_{k=0}^{\infty}\|X(k)\|^2|X_0, \tau_0, d_{-1}\right\} < X_0^{\mathsf{T}}WX_0. \quad (11)$$

Consider the following function:

$$X(t)^{\mathsf{T}}\tilde{P}(T-t, \tau_t, d_{t-1})X(t) \triangleq \mathcal{E}\left\{\sum_{k=t}^{T}X(k)^{\mathsf{T}}Q(\tau_k, d_{k-1})X(k)|X_t, \tau_t, d_{t-1}\right\} \quad (12)$$

with $Q(\tau_k, d_{k-1}) > 0$. Assume that $X(k) \ne 0$. Since $Q(\tau_k, d_{k-1}) > 0$, as T increases, either $X(t)^{\mathsf{T}}\tilde{P}(T-t, \tau_t, d_{t-1})X(t)$ is monotonically increasing or it increases monotonically until

$$\mathcal{E}\{X(k)^{\mathsf{T}}Q(\tau_k, d_{k-1})X(k)|X_t, \tau_t, d_{t-1}\} = 0$$

for all $k \ge k_1 \ge t$. From (11), it can be seen that $X(t)^{\mathsf{T}}\tilde{P}(T - t, \tau_t, d_{t-1})X(t)$ is bounded above and, thus, the limit shown in (13)

$$\begin{aligned}X(t)^{\mathsf{T}}P(i,r)X(t) &\triangleq \lim_{T\to\infty} X(t)^{\mathsf{T}}\tilde{P}(T-t, \tau_t = i, d_{t-1} = r)X(t) \\ &= \lim_{T\to\infty} \mathcal{E}\left\{\sum_{k=t}^{T}X(k)^{\mathsf{T}}Q(\tau_k, d_{k-1})X(k)|X_t, \tau_t = i, d_{t-1} = r\right\}.\end{aligned} \quad (13)$$

Since this is valid for any $X(t)$, we have

$$P(i,r) = \lim_{T\to\infty}\tilde{P}(T-t, \tau_t = i, d_{t-1} = r). \quad (14)$$

From (13), it can be seen that $P(i, r) > 0$ since $Q(\tau_k, d_{k-1}) > 0$. Let us consider

$$\begin{aligned}&\mathcal{E}\{X(t)^{\mathsf{T}}\tilde{P}(T-t, \tau_t, d_{t-1})X(t) - X(t+1)^{\mathsf{T}} \\ &\quad \tilde{P}(T-t-1, \tau_{t+1}, d_t)X(t+1)|X_t, \tau_t = i, d_{t-1} = r\} \\ &\qquad = X(t)^{\mathsf{T}}Q(i,r)X(t).\end{aligned} \quad (15)$$

Notice that

$$\begin{aligned}&\mathcal{E}\{X(t+1)^{\mathbb{T}}\tilde{P}(T-t-1,\tau_{t+1},d_t)\\&\quad X(t+1)|X_t,\tau_t=i,d_{t-1}=r\}\\&\quad=X(t)^{\mathbb{T}}\sum_{j=0}^{\tau}\sum_{s=0}^{d}\pi_{rs}\lambda_{ij}(\tilde{A}+\tilde{B}F(i,r)\tilde{E}(i,s))^{\mathbb{T}}\\&\qquad\times\tilde{P}(T-t-1,j,s)(\tilde{A}+\tilde{B}F(i,r)\tilde{E}(i,s))X(t).\end{aligned}$$

This, together with (15), implies that for any $X(t)$

$$\begin{aligned}&X(t)^{\mathbb{T}}[\tilde{P}(T-t,\tau_t,d_{t-1})-\sum_{s=0}^{d}\pi_{rs}(\tilde{A}+\tilde{B}F(i,r))\tilde{E}(i,s)^{\mathbb{T}}\\&\quad\times\sum_{j=0}^{\tau}\lambda_{ij}\tilde{P}(T-t-1,j,s)(\tilde{A}+\tilde{B}F(i,r)\tilde{E}(i,s))]X(t)\\&\quad=X(t)^{\mathbb{T}}Q(i,r)X(t).\end{aligned}$$

Letting $T\to\infty$ and noticing that (14) and $Q(i,r)>0$, we have

$$\begin{aligned}P(i,r)&-\sum_{s=0}^{d}\pi_{rs}(\tilde{A}+\tilde{B}F(i,r)\tilde{E}(i,s))^{\mathbb{T}}\\&\times\sum_{j=0}^{\tau}\lambda_{ij}P(j,s)(\tilde{A}+\tilde{B}F(i,r)\tilde{E}(i,s))>0.\end{aligned}$$

□

Theorem 1 gives necessary and sufficient conditions on the existence of the state-feedback stabilizing gain. However, since the given conditions in (9) are nonlinear in the controller gains, we need to find a method to solve them. To this end, in the following theorem, the equivalent conditions to (9) are given.

Theorem 2: There exists a controller in (4) such that the closed-loop system in (5) is stochastically stable if and only if there exist $\bar{X}(i,s)>0$, $P(i,r)>0$ and $F(i,r)$ such that the following LMI:

$$\begin{bmatrix}-P(i,r) & V(i,r)\\ V(i,r)^{\mathbb{T}} & -D(i)\end{bmatrix}<0 \tag{16}$$

with

$$\begin{aligned}V(i,r)&=\Big[(\pi_{r0})^{\frac{1}{2}}(\tilde{A}+\tilde{B}F(i,r)\tilde{E}(i,0)\cdots\\&\qquad(\pi_{rd})^{\frac{1}{2}}(\tilde{A}+\tilde{B}F(i,r)\tilde{E}(i,d)\Big]\\D(i)&=\mathrm{diag}\{\bar{X}(i,0),\ldots,\bar{X}(i,d)\}\end{aligned}$$

holds for all $i\in\mathcal{M}$ and $r,s\in\mathcal{N}$ with the constraint of

$$\bar{X}(i,s)=\bar{P}(i,s)^{-1} \tag{17}$$

where $\bar{P}(i,s)=\sum_{j=0}^{\tau}\lambda_{ij}P(j,s)$.

Proof: The proof is obtained by the Schur complement and letting $\bar{X}(i,s)=\bar{P}(i,s)^{-1}$. □

The conditions stated in Theorem 2 are in fact a set of LMIs with some matrix inversion constraints. Though they are nonconvex, there are some existing methods to solve them, such as, the alternating projections method [8], the min–max algorithm [7], the XY-centering algorithm [10], and the cone complementarity linearization (CCL) algorithm (or product reduction (PR) algorithm) [5], [6]. Notice that the XY-centering algorithm is very closely related to the min–max algorithm. In [5], the four algorithms just mentioned were compared, and numerical experiments showed that the CCL (PR) algorithm was the best since it is simple and very efficient in numerical implementation, and seldom fails to find a global optimum. Thus, in this note, it is suggested to use the CCL (PR) algorithm, which is an iterative LMI approach, to calculate $F(i,r)$ from Theorem 2 (see [6] and [20] for details).

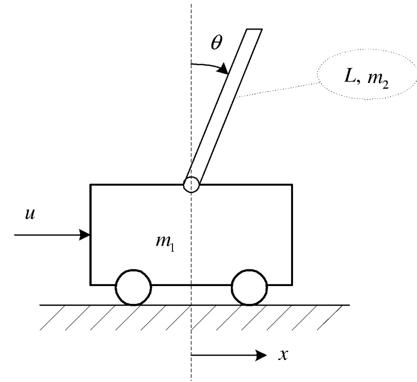

Fig. 2. Cart and inverted pendulum.

IV. Numerical Example

Consider the cart and inverted pendulum problem in Fig. 2, [16], where m_1 is the cart mass, m_2 is the pendulum mass, L is the length from the point of rotation to the center of gravity of the pendulum, x is the cart position, θ is the pendulum angular position, and u is the input force.

The state variables are $x_1=x, x_2=\dot{x}, x_3=\theta$, and $x_4=\dot{\theta}$. Assume that $m_1=1$ kg, $m_2=0.5$ kg, $L=1$ m, and the surface is friction free. The sampling time is $T_s=0.1$ second, and the random delays exist in $\tau_k\in\{0,1,2\}$ and $d_k\in\{0,1\}$, and their transition probability matrices are given by

$$\Lambda=\begin{bmatrix}0.5&0.5&0\\0.3&0.6&0.1\\0.3&0.6&0.1\end{bmatrix}\quad\Pi=\begin{bmatrix}0.2&0.8\\0.5&0.5\end{bmatrix}.$$

The controllers are designed using the discretized model, linearized when the pendulum is in the up-position ($\theta=0$), with a state–space model

$$x(k+1)=A_dx(k)+B_du(k)$$

where

$$A_d=\begin{bmatrix}1.0000&0.1000&-0.0166&-0.0005\\0&1.0000&-0.3374&-0.0166\\0&0&1.0996&0.1033\\0&0&2.0247&1.0996\end{bmatrix}$$
$$B_d=\begin{bmatrix}0.0045\\0.0896\\-0.0068\\-0.1377\end{bmatrix}.$$

Since A_d has eigenvalues at 1, 1, 1.5569, 0.6423, the discretized system is unstable. In [16], this example was considered with the assumption that $d_k=0$. With $d_k\in\{0,1\}$, to stabilize the system, by Theorem 2, we can obtain the controllers as follows:

$$\begin{aligned}F(0,0)&=[0.1690\quad 0.8824\quad 19.5824\quad 4.3966]\\F(0,1)&=[0.5625\quad 0.6259\quad 24.8814\quad 5.1886]\\F(1,0)&=[-0.3076\quad 0.9370\quad 12.0069\quad 5.9910]\\F(1,1)&=[-0.0097\quad 0.7109\quad 15.2518\quad 7.3154]\\F(2,0)&=[-0.3212\quad 1.0528\quad 11.9330\quad 6.3809]\\F(2,1)&=[0.0427\quad 0.8640\quad 16.0874\quad 7.8361].\end{aligned}$$

The state trajectories of the closed-loop system caused by the discretized model and the obtained controller are shown in Fig. 3 when $x(-3)=x(-2)=x(-1)=x(0)=[0\ 0\ 0.1\ 0]^{\mathbb{T}}$. It can be seen that the closed-loop system is stochastically stable.

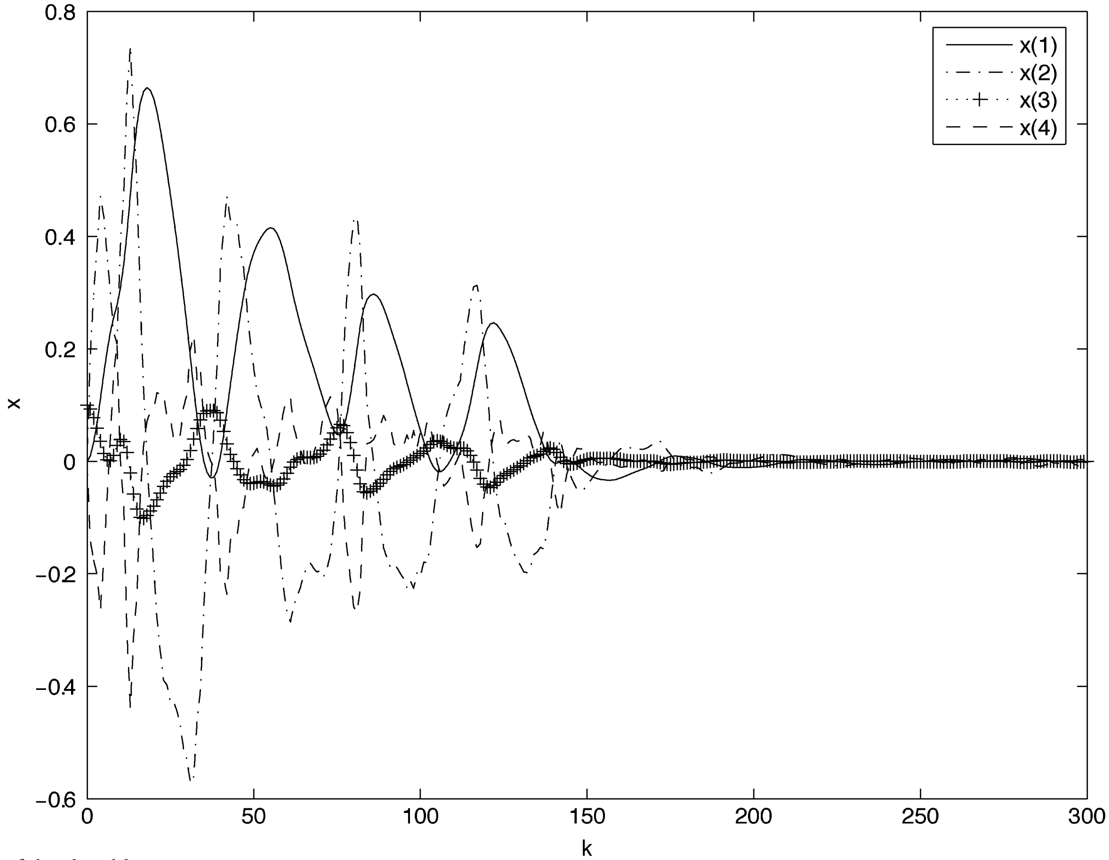

Fig. 3. States of the closed-loop system.

V. CONCLUSION

This note has presented a new method for the stabilization of a class of networked control systems with random communication delays. By modeling the delays as Markov chains, the closed-loop systems can be expressed as jump linear systems with two modes. Necessary and sufficient conditions of stochastic stability for the jump linear systems are obtained in terms of a set of LMIs with matrix inversion constraints, from which the state-feedback gain can be solved by an existing iterative LMI algorithm. It is shown that the state-feedback gain depends on the two modes. A numerical example has been considered to illustrate the main results.

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