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A New Method in Data Envelopment Analysis to Find Efficient Decision Making Units and Rank both Technical Efficient and Inefficient DMUs together

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Abstract

The inefficient DMUs are usually arranged after the technical efficient ones by DEA methods, however, it is possible that a technical efficient DMU neither be efficient nor be more efficient than some inefficient ones. This study distinguishes between the terms ‘technical efficiency’ and ‘efficiency’ and demonstrates that the technical efficiency is a necessary condition for being efficient and it is not an enough condition to call a DMU as efficient DMU. The study identifies the definitions of those terms and gives a new strong method to characterize efficient DMUs among the technical efficient ones. The new method, although, avoids the need for recourse to prices, weights or other assumptions between inputs and outputs of DMUs, it is also able to consider the prices and weights. A numerical example is also characterized the worth and benefits of the new proposed model in comparison with all current DEA models.

Mathematics Subject Classification: 90

Keywords: Data envelopment analysis, Efficiency, Technical efficiency, Super efficiency models

1. Introduction

Data envelopment analysis (DEA) is a nonparametric approach in operations research to estimate the performance evaluation, relative efficiency and productivity of homogenous decision making units (DMUs). This study distinguishes between the terms 'technical efficiency' and 'efficiency' in DEA after introduced them and proposes a new technique to characterize both technical efficient and inefficient DMUs among the observed DMUs. The paper is organized in six sections. Section 2 is the background and the problem statement is illustrated in Section 3. The proposed method is suggested in Section 4 which is examined in Section 5 with a numerical example. The paper is concluded in Section 6. The simulations are also performed with Microsoft Excel Solver due to have the simple linear programming problems.

2. Background

There are many measures of technical efficiency used in DEA models, though the most traditional ones is CCR, the radial measure which was proposed by Charnes et al. [3] on the basis of Farrell's work in 1957 [5]. Charnes et al. [4] also proposed a non-radial model, additive model (ADD), which is considered the possibility of simultaneous each input decreases and/or each output increases.

Since both CCR and ADD are not able to allow for a ranking of technical efficient DMUs, therefore, Andersen and Petersen [1] developed a modified version of DEA based upon comparison of technical efficient DMUs relative to a reference technology spanned by all other DMUs. The basic idea of Andersen and Petersen was to compare the DMU under evaluation with a linear combination of all other DMUs in the sample, i.e., the DMU itself is excluded. DEA has been constructed on the above techniques and increasingly developed in many various fields in last three decades.

Some similar common DEA models to CCR can be recognized as BCC [2], SBM [9] and ERM [8] and some analogous super-efficiency models to AP can be acknowledged as MAJ [7], SBM [10] and other models by Jahanshahloo et al. [6].

3. Efficiency, technical efficiency and problem statement

The ratio of output/input defines a measure of efficiency (or doing the jobs right) and a DMU $A(x,y)$ does the jobs righter than a DMU $B(x',y')$ (or A is more efficient than B), if the amount of y/x is greater than the amount of y'/x'. Moreover, Pareto-Koopmans definition in DEA declares that a DMU is to be rated as fully (100%) efficient (referred to as 'technical efficiency' in economics) on the basis of available evidence if and only if the performances of other DMUs do not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs. DEA is perfectly able to identify inefficient and technical efficient DMUs, however, where the weights are unknown it may not significantly be able to characterize the efficient DMUs i.e., the DMUs which do the jobs right. To illustrate the distinction between the terms 'technical efficiency' and 'efficiency', let us consider the DMUs in Table 1 which have two inputs and a single constant output without any other information.

Table 1: Three DMUs with two inputs and one output.

DMUs	Input1	Input2	Output	CCR Score	AP Rank
A	2	55	10	1.000	1.500
B	3	3	10	1.000	9.500
C	55	2	10	1.000	1.500

From Pareto-Koopmans definition the DMUs A, B and C are technical efficient because none of the inputs and output for each DMU can be improved without worsening some of other inputs or output. The AP ranks of these DMUs are illustrated in the last column of Table 1 which show the following ranking: $B > A = C$.

Now, let us add two inefficient DMUs to those DMUs in Table 1 according to Table 2. From AP method the ranking are as following: $A = C > B > D = E$.

Table 2: Five DMUs with two inputs and one output.

DMUs	Input1	Input2	Output	CCR Score	AP Rank
A	2	55	10	1.000	1.500
B	3	3	10	1.000	1.167
C	55	2	10	1.000	1.500
D	3	4	10	0.994	0.994
E	4	3	10	0.994	0.994

Although, both production possibility set (PPS) of those DMUs in Tables 1 and 2 are the same, the ranking of technical efficient DMUs are different with AP method. For instance, DMU B has the first ranking among the DMUs in Table 1 and the second ranking among the DMUs in Table 2, which shows that the ranking with AP method may not be significant. In fact, the inefficient DMUs D and E are very close to B and elimination of B in AP method has no significant effect in the corresponding PPS of DMUs in Table 2. Moreover, D and E in comparison with B are inefficient and other technical efficient DMUs do not dominate them. In other words, each technical efficient DMU may only dominate some inefficient DMUs and there may be some inefficient DMUs which are not dominated with some technical efficient ones. Therefore, it is possible that an inefficient DMU be more efficient than a technical efficient one which does not dominate it. In short, the Pareto-Koopmans definition is able to identify the DMUs which are on Farrell frontier, but the DMUs on Farrell frontier may neither do the jobs right nor be more efficient than some inefficient DMUs. Therefore, the Farrell frontier must be exactly examined and the definition of technical efficiency or Pareto-Koopmans definition should not be wrongly interpreted instead of doing the jobs right or efficiency in DEA, where the weights are unknown.

4. The Arash Method (AM)

In this section, a new method which is called Arash Method (**AM**) is proposed to examine the Farrell frontier, find the DMUs which do the jobs right and remove the previous shortcomings to arrange DMUs with a linear programming problem based on additive DEA model. In order to illustrate the method, assume that there are n DMUs ($\text{DMU}_i, i = 1,2, \dots, n$) with m nonnegative inputs ($x_{ij}, j = 1,2, \dots, m$) and p nonnegative outputs ($y_{ik}, k = 1,2, \dots, p$) for each DMU which at least one of its inputs and one of its outputs are not zero.

The **$\boldsymbol{\varepsilon}$-AM** in input-oriented case is as following where DMU_l ($l = 1,2, \dots, n$) is evaluated and $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m)$, $\varepsilon_j \geq 0$.

$$
\begin{aligned}
& max \ \textstyle\sum_{j=1}^{m} w_j^- s_j^- + \sum_{k=1}^{p} w_k^+ s_k^+, \\
& \text{Subject to} \\
& \textstyle\sum_{i=1}^{n} \lambda_i x_{ij} + s_j^- = x_{lj} + \varepsilon_j / w_j^-, \ j = 1,2, \dots, m, \\
& \textstyle\sum_{i=1}^{n} \lambda_i y_{ik} - s_k^+ = y_{lk}, \ k = 1,2, \dots, p, \\
& \lambda_i \geq 0, \ \ i = 1,2, \dots, n, \\
& s_j^- \geq 0, \ \ j = 1,2, \dots, m, \\
& s_k^+ \geq 0, \ \ k = 1,2, \dots, p.
\end{aligned}
$$

The **$\boldsymbol{\varepsilon}$-AM** target and score are as following

$$
\begin{aligned}
& x_{lj}^* = x_{lj} + \varepsilon_j / w_j^- - s_j^{-*}, \ j = 1,2, \dots, m, \\
& y_{lk}^* = y_{lk} + s_k^{+*}, \ k = 1,2, \dots, p.
\end{aligned}
$$

$$
A^* = \frac{\sum_{k=1}^{p} w_k^+ y_k \,/\, \sum_{j=1}^{m} w_j^- x_j}{\sum_{k=1}^{p} w_k^+ y_k^* \,/\, \sum_{j=1}^{m} w_j^- x_j^*}.
$$

The weights w_j^- and w_k^+ are defined as below for $j = 1,2, \dots, m$ and $k = 1,2, \dots, p$.

$$
w_j^- = \begin{cases} N_j & x_j = 0 \\ 1/x_j & x_j \neq 0 \end{cases} \quad \text{and} \quad w_k^+ = \begin{cases} M_k & y_k = 0 \\ 1/y_k & y_k \neq 0 \end{cases}.
$$

The N_j and M_k in the above equations can be selected in the natural numbers set or positive real numbers set which depend to the goals of each DMU for its resources and productions. Moreover, the score of **$\boldsymbol{\varepsilon}$-AM** is marked with A_ε^* where $\boldsymbol{\varepsilon} = (\varepsilon, \varepsilon, \dots, \varepsilon)$. It is compares each technical efficient DMU with a technical efficient target which is suggested from a little different amount in its data by the model and it characterizes whether that technical efficient DMU is efficient or not with the real definition of efficiency i.e., output/input. In other words, although, we would evaluate DMU_l, the input constraints in the model identifies that the corresponding virtual DMU of DMU_l is under evaluation due to examine how much an epsilon error in input values of DMU_l changes its technical efficient target. For instance, suppose that $x_j \neq 0$ and $y_k \neq 0$. The **0.01-AM** examines that only one hundredth error in each input of a DMU which is a DMU with these input values, $x_j + \varepsilon_j x_j$, for $j = 1,2, \dots, m$, how much affects on its efficiency score which is calculated with following equation

$$
A_\varepsilon^* = \frac{p/m}{\sum_{k=1}^{p} (y_k^* / y_k) \,/\, \sum_{j=1}^{m} (x_j^* / x_j)}.
$$

It is obvious that the above equation is independence of units and it assumes the input and output values of a DMU which is evaluated as measures to compare the DMU and its **AM** target. When $A_\varepsilon^* < 1$ for a DMU, the **$\boldsymbol{\varepsilon}$-AM** suggests it to change its input and output values to the **ε-AM** target and otherwise i.e., when $A_\varepsilon^* \geq 1$, **$\boldsymbol{\varepsilon}$-AM** warns that the DMU has a good combination of its input and output values in PPS and it should not change its data because it may decrease its efficiency score. Furthermore, **ε-AM** is always feasible for all $\varepsilon \geq 0$, because the virtual DMUs are always dominated with the real ones and ADD is always feasible. In addition, when A_ε^* is the same for two DMUs A and B, it means that both DMUs A and B are equivalent in the combination of their data where an ε error occurs in those data.

5. A numerical example and examine AM

Let us consider 23 DMUs with five inputs and two outputs in Table 3 which Jahanshahloo et al. [6] used them to compare 11 super-efficiency models which none of those models is able to arrange both technical efficient and inefficient DMUs together.

Table 3: An example of 23 DMUs with five inputs and two outputs

DMU	x_1	x_2	x_3	x_4	x_5	y_1	y_2	A_0^* --Rank		$A_{0.01}^*$--Rank		$A_{0.05}^*$--Rank	
1	583	8	2.75	16.731	17129300	285	0.8848	0.9421	11	0.9420	11	0.9412	7
2	741	8	2	18.999	8903705	95	0.8597	0.6810	22	0.6759	22	0.6562	22
3	**600**	**7**	**2.75**	**19.437**	**15864760**	**307**	**0.9226**	**1.0000**	**1**	**1.0000**	**1**	**1.0000**	**1**
4	593	8	2.75	19.326	14802089	260	0.8928	0.8940	12	0.8940	12	0.8940	10
5	**746**	**7**	**2**	**20.125**	**8398300**	**154**	**0.812**	**1.0000**	**1**	**0.9845**	**7**	**0.9227**	**9**
6	992	9	2.75	21.821	19330020	254	0.8624	0.7169	21	0.7169	21	0.7169	19
7	**775**	**8**	**2.75**	**13.333**	**17182320**	**292**	**0.9109**	**1.0000**	**1**	**0.9952**	**4**	**0.9758**	**4**
8	1852	14	3.25	21.696	30126900	473	0.8632	0.7424	20	0.7424	20	0.7424	18
9	**625**	**5**	**2**	**16.285**	**7638220**	**106**	**0.8898**	**1.0000**	**1**	**0.9938**	**6**	**0.9695**	**6**
10	**7673**	**6**	**2**	**16.789**	**8659940**	**148**	**0.8668**	**1.0000**	**1**	**0.9615**	**10**	**0.8281**	**16**
11	**423**	**6**	**2**	**13.304**	**10799980**	**151**	**0.9435**	**1.0000**	**1**	**1.0000**	**1**	**1.0000**	**1**
12	**1292**	**18**	**3.25**	**18.333**	**47102720**	**782**	**0.9571**	**1.0000**	**1**	**1.0000**	**1**	**1.0000**	**1**
13	1300	8	2.75	17.73	17451040	288	0.8996	0.8294	16	0.8293	16	0.8291	15
14	582	8	2.75	19.178	15850628	260	0.9054	0.8921	13	0.8921	13	0.8921	11
15	**620**	**8**	**2**	**16.056**	**7938560**	**124**	**0.8744**	**1.0000**	**1**	**0.9843**	**8**	**0.9280**	**8**
16	1256	10	2.75	21.516	23034560	378	0.8465	0.8123	17	0.8119	17	0.8112	17
17	765	10	2.75	19.145	15692740	303	0.8945	0.8830	14	0.8838	14	0.8838	13
18	**842**	**7**	**2.75**	**16.972**	**8029240**	**153**	**0.9074**	**1.0000**	**1**	**0.9947**	**5**	**0.9735**	**5**
19	1011	4	2.75	17.692	7702609	57	0.8764	0.7622	18	0.7521	18	0.7145	21
20	1128	9	2.75	21.927	22143650	357	0.9028	0.8445	15	0.8445	15	0.8445	14
21	3456	18	3.5	20.217	24892550	393	0.9195	0.6438	23	0.6438	23	0.6438	23
22	**1008**	**3**	**2.25**	**10.213**	**7405200**	**36**	**0.8611**	**1.0000**	**1**	**0.9757**	**9**	**0.8909**	**12**
23	910	4	2.25	12.941	8839280	72	0.7735	0.7509	19	0.7437	19	0.7162	20

There are 10 technical efficient DMUs such as DMUs 3, 5, 7, 9, 10, 11, 12, 15, 18 and 22 that **0-AM** can identify them similar to SBM input-oriented with the score $A_0^* = 1$ as depicted in ninth column of Table 3. Moreover, eleventh and thirteenth columns of Table 3 depict the efficiency scores of DMUs i.e., $A_{0.01}^*$ and $A_{0.05}^*$ by applying **0.01-AM** and **0.05-AM**, respectively. **0.01-AM** strongly suggests that DMUs 3, 11 and 12 are efficient and other DMUs are not. In other words, only one hundredth errors in each input of DMUs identifies that only three technical efficient DMUs have no significant different in their targets and other DMUs should improve its data to be more efficient. From the table, **0.05-AM** characterizes that the inefficient DMU 1 by 0.05 errors in data is more efficient than technical efficient DMUs 15, 5, 22 and 10. Moreover, **AM** method identifies that the technical efficient DMU 10 is not more efficient than inefficient DMUs 1, 4, 13, 14, 17, 20, where 0.05 errors are occurred in its data. This valuable information examines the Farrell frontier significantly and arranges both technical efficient and inefficient DMUs together as the twelfth and last columns of Table 3 illustrate it.

6. Conclusion

This study characterizes the distinctions between the terms ‘technical efficiency’ and ‘efficiency’ (i.e., doing the jobs right) and suggests a new method with a new measure which not only avoid the need for recourse to prices and weights, but also it is able to consider them. The measure significantly arranges both technical efficient and inefficient DMUs together and also removes the shortcomings of current super-efficiency models to arrange DMUs. The method was proposed in input-oriented case; however, it can be also extended in output-oriented case with little different conditions.

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