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A New Method of Predicting the Performance of Gas Turbine Engines

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ABSTRACT

This paper points out that the turbine performance computation method used widely at present in solving the performance of gas turbine engines is a numerically instable algorithm. So a new method, namely inverse algorithm, is proposed.

This paper then further proposes a new mathematical model of solving the stable performance of gas turbine engines. It has the features of not only being suitable for inverse algorithm for turbine performance, but also having less dimensions than existing models. So it has the advantages of high accuracy, rapid convergence, good stability, less computations, and so forth. It has been fully proven that the accuracy of the new model is much greater than that of the common model for gas turbine engines. Additionally, the time consumed for solving the new model is approximately 1 / 4~ 1 / 10 of that for the common model. Therefore, it is valuable in practice.

MOMENCLATURE

be	fuel rate
Cp	constant pressure specific heat
D	mean diameter at inlet of first stage rotor blade of turbine
$\mathbb{E}$	residual values, vector of order (nx1)
E	residual value
f	fuel–air ratio, or correction factor
G	mass flow quantity
$\overline{G}$	reduced flow quantity
k	adiabatic exponent
N	rotational speed
P	Power output, or pressure
R	gas constant
T	initial temperature
X	independent variables, vector of order (nx1)
x	independent variable
η	adiabatic efficiency
η_m	mechanical efficiency
η_{m1}	mechanical efficiency of low pressure rotor
η_{m2}	mechanical efficiency of high pressure rotor
λ_u	corrected rotational speed in dimensionless form
π	expansion ratio of turbine, or compression ratio of compressor
σ	recovery factor of total pressure
$\tau(\lambda)$	temperature ratio, $\tau(\lambda) < 1$
ω	relaxation factor

Subscripts

B	combustion chamber
C	compressor
cr	critical value
e	effective
ex	exit
g	gas
H	high pressure
in	inlet
L	low pressure, or load
M	intermediate pressure
T	turbine, or intermediate transition sector
o	design condition
1	inlet of low pressure compressor
2	inlet of high presure compressor
3	inlet of combustion chamber
4	inlet of high pressure turbine
5	inlet of intermediate pressure turbine
6	inlet of low pressure turbine
7	exit of low pressure turbine

Superscripts

*	stagnation
k	(k)th iteration
k+1	(k+1)th iteration

INTRODUCTION

In the calculation of the gas turbine engine performance, it is required to solve the following set of nonlinear equations

$$\mathbb{E}(X) = 0 \ , \quad X \in D \subset R^n \ , \quad \mathbb{E} \in R^n \tag{1}$$

Although the methods of solving Eqs. (1) have been published earlier in literature, calculation practice shows that one of the most effective methods is the iterative calculation of the following formula of Newton– Raphson

$$X^{k+1} = X^k - J(\mathbb{E},X)^{-1}_{X=X^k} \mathbb{E}^k \tag{2}$$

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where $J(\mathbb{E},X)$ is Jacobian matrix of the residual values $\mathbb{E}$, of order (n × n), i. e.,

$$J(\mathbb{E},X)=\begin{bmatrix} \frac{\partial E_1}{\partial x_1} & \frac{\partial E_1}{\partial x_2} & \cdots & \frac{\partial E_1}{\partial x_n} \\ \frac{\partial E_2}{\partial x_1} & \frac{\partial E_2}{\partial x_2} & \cdots & \frac{\partial E_2}{\partial x_n} \\ \frac{\partial E_n}{\partial x_1} & \frac{\partial E_n}{\partial x_2} & \cdots & \frac{\partial E_n}{\partial x_n} \end{bmatrix} \tag{3}$$

When the performance of gas turbine engines is solved with the help of N–R method, it has been found that there exist the following problems which have not been solved satisfactorily to date.

1. When a gas turbine engine works at part load, the surge of compressor and the over–temperature of turbine have great influences upon the work of the gas turbine engine. It can greatly decrease the work range of the engine and make the local convergence region narrower. It follows that the intial approximate solution has to be chosen close to the correct solution, but that is hard to do. Thus, the iteration in the computational process sometimes fails to converge.
2. Experience from computation tells us: When the dimensions of the Jocobian Matrix in Newton–Raphson iteration formula (2) is greater than or equal to 3, its inverted matrix is liable to become ill–conditioned.
3. The mathematical model (1) cannot be expressed by an analytic formula. Therefore, the Jacobian matrix in formula (2) can be determined only by finite difference method. If X is a vector of the order (n × 1), then using the forward or backward difference, the time required for calculating $J(\mathbb{E},X)$ is n times that for calculating $\mathbb{E}(x)$. Thus, the amount of work for computation is great.
4. In a variety of current algorithms, computation of performance of turbine in the gas turbine engine is conducted in the sequence of high pressure, intermediate pressure, and low pressure turbine. The π_T^* value of every turbine is obtained from the given $\overline{G}_{HT}$, according to the characteristic curves of every turbine. This algorithm will be called "sequential algorithm" in this paper. It has been found through research that the sequential algorithm is a numerically instable alghorithm. It will result in significant errors on turbine performance and overall performance under certain conditions (see the next section).

This paper will discuss why the sequential algorithm resulted in error and a new method, that is, the inverse algorithm will be proposed. Then, a new model for solving the performance of gas turbine engines will be recommended to obtain a more rapidly converging and more accurate solution.

THE ANALYSIS AND STUDY OF TURBINE PERFORMANCE

While solving the set of equations (1), the computation of turbine performance will be involved. The reason leading to error in the sequence algorithm, which is used widely at present, may be analysed as follows.

With respect to the flow characteristic curve at a certain value of λ_u in a conventional turbine characteristic diagram, we have the following differential equation

$$\frac{d\overline{G}}{d\pi_T^*}=f^2\frac{(\pi_{cr}^*-\pi_T^*)}{\overline{G}} \tag{4}$$

where f is the correction factor when the flow characteristic curve at a certain value of λu does not coincide with the ellipse rule completely. By re–arranging Eq. (4) we have

$$\delta\pi_T^*=k_1\delta\overline{G} \tag{5}$$

where operator δ is the relative error, for example, $\delta\overline{G}=\frac{d\overline{G}}{\overline{G}}$, $\delta\pi_T^*=\frac{d\pi_T^*}{\pi_T^*}$ and so on. K_1 is the propagation factor of error and it is given by

$$K_1=\frac{\overline{G}^2}{f^2\pi_T^*(\pi_{cr}^*-\pi_T^*)} \tag{6}$$

It is seen in Eq. (5) that when π_T^* is obtained from $\overline{G}$ by the sequential algorithm and the $\overline{G}$ at the inlet of a turbine has a deviation of $\delta\overline{G}$ because of round–off error and interpolation error, the value of $\delta\pi_T^*$ is surely K_1 times $\delta\overline{G}$. The relationship between K_1 and π_T^* for a conventional turbine, by the use of equation (6), is shown in Fig.1. It is shown in the figure that K_1 increases with the increase of π_T^* . When π_T^* approaches its critical value π_{cr}^* , $K_1 \to \infty$, theoretically. Therefore, when the turbine works at high load, a significant error may result.

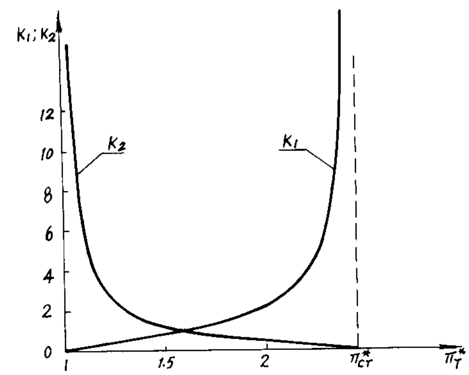

Fig. 1 Variations of K_1 and K_2 with π_T^* in a certain turbine

To reduce errors, a new computation method for predicting turbine performance, which is called inverse algorithm, is proposed here. By inverse algorithm it is meant that by the use of the π_T^* of the last turbine the $\overline{G}$ of every preceding turbine can be found according to the turbine characteristics curves. Re–arranging equation (5), we have

$$\delta\overline{G}=k_2\delta\pi_T^* \tag{7}$$

where k_2 is the propagation factor of error and it is given by

$$K_2=\frac{1}{K_1}=\frac{f^2\pi_T^*(\pi_{cr}^*-\pi_T^*)}{\overline{G}^2} \tag{8}$$

It is shown in Eq. (7) that when $\overline{G}$ is obtained from π_T^* by inverse algorithm, the $\delta\overline{G}$ is surely K_2 times the $\delta\pi_T^*$. Fig.1 also shows the relationship between K_2 and π_T^* for conventional turbine by the use of Eq. (8). It is shown in the figure that K_2 decreases with the increase of π_T^*, when π_T^* approaches its critical value π_{cr}^* , $K_2 \to 0$ theoretically. Thus, high load of turbine will result in a smaller propagated error. The situation of $\pi_T^*=1$ and $K_2 \to \infty$, is impossible to occur in a gas turbine. This is because the expansion ratio of all turbines is always larger than 1 even if in idle running.

In order to further analyse the differences between the two algorithms, we cite an example for explanation. The high pressure, intermediate pressure, and low pressure turbines of a certain marine gas turbine engine operate in series connection. The π_T^* and $\overline{G}$ of every turbine and the total expansion ratio $\pi_{T\Sigma}^*$ are calculated by the sequential algorithm and inverse algorithm respectively, assuming the λ_{UHT} values, the λ_{UMT} values, the λ_{ULT} values, and the $\overline{G}_{HT}$ values are the same. The relative error is also calculated to be

$$\delta\pi_T^*=\frac{(\pi_T^*)sequential-(\pi_T^*)inverse}{(\pi_T^*)sequential}$$

$$\delta\overline{G}=\frac{(\overline{G})sequential-(\overline{G})inverse}{(\overline{G})sequential}$$

The calculated results are shown in Fig. 2. It is known from the figure that:

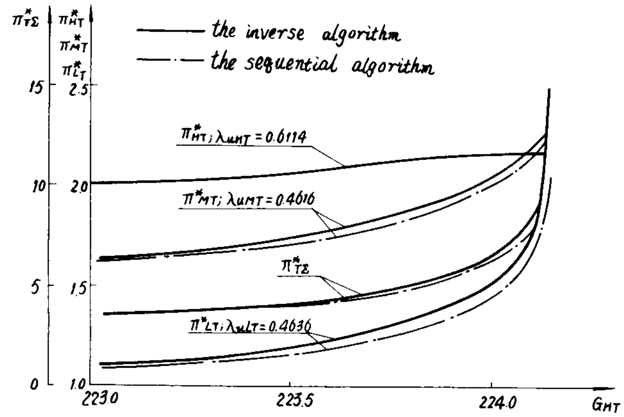

Fig. 2 Variations of π^*_{HT} , π^*_{MT} , π^*_{LT} , and $\pi^*_{T\Sigma}$ with G_{HT}

1. When $\bar{G}_{HT}$ increases, the difference between two π^*_T obtained respectively from two algorithms increases gradually, while the difference between two $\pi^*_{T\Sigma}$ increases more rapidly. On the other hand, the difference between two π^*_T decreases gradually with the decrease of load and so does the difference between two $\pi^*_{T\Sigma}$. The differences almost vanish at low load. It shows that the calculated results of these two algorithms are almost the same when the turbine works at low load.
2. The high pressure turbine has the smallest $\delta\pi^*_{HT}$, but low pressure turbine has the largest $\delta\pi^*_{LT}$ under the same $\bar{G}_{HT}$.
3. Increase of the number of turbines will result in a larger cumulative error. The turbines downstream on the expansion line have more notable calculating errors than those upstream. Take the design point as an example, $\delta\pi^*_{HT} = -0.02\%$; $\delta\pi^*_{MT} = 1.57\%$; $\delta\pi^*_{LT} = 14.79\%$; $\delta\pi^*_{T\Sigma} = 16.11\%$.
4. $\bar{G}_{HT}$ is assumed to have no error in our example. In common computation for gas turbines, the $\bar{G}_{HT}$ is obtained through interpolation of compressor characteristics and other calculations, Thus, it surely has an error $\delta\bar{G}_{HT}$. It is obvious that calculating the turbine performance on the $\delta\bar{G}_{HT}$ basis via sequantial algorithm will result in more significant errors.

As has been said before, the round–off errors and the interpolation errors have a great influence upon the computational results of the sequential algorithm under some conditions. It is called a numerically instable algorithm in mathematics. When the sequential algorithm is used to compute turbine performance, it will result in errors in turbine performance and overall performance.

To sum up, it is necessary to apply the inverse algorithm in solving the performance of gas turbine engines in order to raise the accuracy of the computation.

A NEW MATHEMATICAL MODEL FOR STEADY STATE PERFORMANCE OF GAS TURBINE ENGINES

Following is the derivation of a new mathematical model for a typical form of marine gas turbine engine which has three turbines and two compressors (HT drives HC, MT drives LC, and LT drives load).

Listed below are the flow continuity equations and power balance equations of high pressure rotor and low pressure rotor, and the coupling equations of the two rotors.

$$\frac{G_{HC}\sqrt{T_2^*}}{P_2^*} = \frac{G_{HT}\sqrt{T_4^*}}{P_4^*}\sqrt{\frac{T_2^*}{T_4^*}}\frac{\sigma_B\pi^*_{HC}}{(1+f)\zeta_2} \quad (9)$$

$$\frac{T_2^*}{T_1^*} = [(\pi^*_{LC})^{\frac{\kappa_{LC}-1}{\kappa_{LC}}} - 1]\frac{1}{\eta_{LC}} + 1 \quad (10)$$

$$[(\pi^*_{HC})^{\frac{\kappa_{HC}-1}{\kappa_{HC}}} - 1]\frac{1}{\eta_{HC}}$$

$$= (1+f)\zeta_2\frac{T_4^*}{T_2^*}\frac{C_{PHT}}{C_{PHC}}[1-\tau^*(\lambda)_{HT}]\eta_{m2} \quad (11)$$

$$[(\pi^*_{LC})^{\frac{\kappa_{LC}-1}{\kappa_{LC}}} - 1]\frac{1}{\eta_{LC}}$$

$$= \zeta_1\zeta_2\zeta_3(1+f)\frac{C_{PMT}}{C_{PLC}}\tau^*(\lambda)_{HT}\frac{T_4^*}{T_2^*}\frac{T_2^*}{T_1^*}[1-\tau^*(\lambda)_{MT}]\eta_{m1} \quad (12)$$

$$\frac{G_{LC}\sqrt{T_1^*}}{P_1^*} = \frac{G_{HC}\sqrt{T_2^*}}{P_2^*}\sqrt{\frac{T_1^*}{T_2^*}}\pi^*_{LC}\zeta_1 \quad (13)$$

$$\pi^*_{HT}\cdot\pi^*_{MT}\cdot\pi^*_{LT} = \pi^*_{HC}\cdot\pi^*_{LC}\cdot\sigma_{in}\cdot\sigma_{ex}\cdot\sigma_B\cdot\sigma_T \quad (14)$$

where $\zeta_1 = \frac{G_{HC}}{G_{LC}}$, $\zeta_2 = \frac{G_B}{G_{HC}}$, $\zeta_3 = \frac{G_{MT}}{G_{HT}}$, *and* $\zeta_4 = \frac{G_{LT}}{G_{MT}}$. In order to simplity the solving of the equations, the aerodynamic parameters of compressors can be expressed as a function of the gas parameters in turbines obtained by the inverse algorithm. Substitution of Eq. (9) in Eq. (11) gives

$$(\frac{G_{HC}\sqrt{T_2^*}}{P_2^*})^2[(\pi^*_{HC})^{\frac{\kappa_{HC}-1}{\kappa_{HC}}} - 1]\frac{1}{\eta_{HC}}\frac{1}{(\pi^*_{HC})^2}$$

$$= \frac{\sigma_B^2}{(1+f)\zeta_2}\frac{C_{PHT}}{C_{PHC}}[1-\tau^*(\lambda)_{HT}](\frac{G_{HT}\sqrt{T_4^*}}{P_4^*})^2\eta_{m2} \quad (15)$$

Similarly, from Eqs. (10), (12), (13) and (14)

$$[(\pi^*_{LC})^{\frac{\kappa_{LC}-1}{\kappa_{LC}}} - 1]\frac{1}{\eta_{LC}}(\frac{G_{LC}\sqrt{T_1^*}}{P_1^*})^2$$

$$= \frac{\zeta_3}{(1+f)\zeta_1\zeta_2}\frac{C_{PMT}}{C_{PLC}}\tau^*(\lambda)_{HT}$$

$$\cdot[1-\tau^*(\lambda)_{MT}]\eta_{m1}(\frac{G_{HT}\sqrt{T_4^*}}{P_4^*})^2(\frac{\pi^*_{HT}\pi^*_{MT}\pi^*_{LT}}{\sigma_{in}\sigma_{ex}\sigma_T})^2 \quad (16)$$

Eq. (16) and Eq. (15) can be written as

$$E_1 = [(\pi^*_{LC})^{\frac{\kappa_{LC}-1}{\kappa_{LC}}} - 1]\frac{1}{\eta_{LC}}(\frac{G_{LC}\sqrt{T_1^*}}{P_1^*})^2 - C_1 = 0 \quad (17)$$

$$E_2 = [(\pi^*_{HC})^{\frac{\kappa_{HC}-1}{\kappa_{HC}}} - 1]\frac{1}{\eta_{HC}}(\frac{G_{HC}\sqrt{T_2^*}}{P_2^*})^2\frac{1}{(\pi^*_{HC})^2} - C_2 = 0 \quad (18)$$

Where

$$C_1 = \frac{\zeta_3}{(1+f)\zeta_1\zeta_2}\frac{C_{PMT}}{C_{PLC}}\tau^*(\lambda)_{HT}[1-\tau^*(\lambda)_{MT}]$$

$$\cdot\eta_{m1}(\frac{G_{HT}\sqrt{T_4^*}}{P_4^*})^2(\frac{\pi^*_{HT}\pi^*_{MT}\pi^*_{LT}}{\sigma_{in}\sigma_{ex}\sigma_T})^2$$

$$C_2 = \frac{\sigma_B^2}{(1+f)\zeta_2}\frac{C_{PHT}}{C_{PHC}}[1-\tau^*(\lambda)_{HT}](\frac{G_{HT}\sqrt{T_4^*}}{P_4^*})^2\eta_{m2}$$

C_1 and C_2 include only aerodynamic parameters of turbines. They can be obtained when the performance computation of turbines in series is done by inverse algorithm, assuming a certain π^*_{LT} . In fact, the π^*_{LT} here is the parameter representing a certain part load of gas turbine engine. Some coefficients in the expressions can be assumed to be the same as design values when lacking data. C_{PHT} , C_{PMT} , C_{PLT} C_{PHC} , C_{PLC} , and f can also be corrected in successive iterations.

Eqs. (17) and (18) may be expressed in vector form (1), wherein $\mathbb{E} = (E_1 , E_2)^T$. The $\mathbb{E}$ here is a hybrid residual expressing power, flow rate, pressure ratio and so forth. Therefore, Eqs. (17) and (18) can also be called hybrid residual equations. The hybrid residual equations involve six unknown quantities: π^*_{LC} , η^*_{LC} , $G_{LC}\sqrt{T_1^*}/P_1^*$, π^*_{HC}, η^*_{HC} and $G_{HC}\sqrt{T_2^*}/P_2^*$. Therefore, we still need four supplementary equations to slove equations (17) and (18) , two of which are coupling equations of high and low pressure rotors. Incorporating and re–arranging Eqs. (17), (10) and (13), we have

$$(\frac{G_{LC}\sqrt{T_1^*}}{P_1^*})^2 = \zeta_1^2(\frac{G_{HC}\sqrt{T_2^*}}{P_2^*})^2(\pi^*_{LC})^2 - C_1 \quad (19)$$

Eq. (14) can be rewritten as

$$\pi^*_{HC}\cdot\pi^*_{LC} = C_3 \quad (20)$$

Where $C_3 = \pi^*_{HC}\pi^*_{MT}\pi^*_{LT}/(\sigma_{in}\sigma_{ex}\sigma_B\sigma_T)$

The other two equations are the characteristic equations of high and low pressure compressors.

$$\eta^*_{LC} = f(\frac{G_{LC}\sqrt{T^*_1}}{P^*_1}\ ,\ \ \pi^*_{LC}) \tag{21}$$

$$\eta^*_{HC} = f(\frac{G_{HC}\sqrt{T^*_2}}{P^*_2}\ ,\ \ \pi^*_{HC}) \tag{22}$$

By simultaneously solving the six equations, that is Eqs. (17), (18), (19), (20), (21) and (22), we can determine the values of the six unknowns and obtain a set of unique solutions. Of the six unknowns, $\eta^*_{HC}, \eta^*_{LC}, \pi^*_{HC}$ and $\frac{G_{HC}\sqrt{T^*_2}}{P^*_2}$ are all functions of π^*_{LC} and $\frac{G_{LC}\sqrt{T^*_1}}{P^*_1}$. If we put $x_1 = \pi^*_{LC}$ and $x_2 = \frac{G_{LC}\sqrt{T^*_1}}{P^*_1}$, Eqs (17) and (18) can be written as

$$E_1 = f_1(x_1,\ X_2) = 0$$
$$E_2 = f_2(x_1,\ X_2) = 0 \tag{23}$$

By Newton–Raphson's iteration method, we obtain the solution of Eqs. (23) to be

$$X^{k+1} = X^k - \omega_k J^{-1}(E,X)_{x=x^k} E^k \qquad k = 0,1,\cdots \tag{24}$$

where ω_k represents the relaxing factor of (k)th iteraion, and J(E, X) represents Jacobian matrix (2×2 dimensions) with respect to residuals. The hybrid residual equations (23) do not satisfy the rotational speed equality equation. Therefore, it is necessary to use a rotational speed iterative equation to do the correction of rotational speed. The general expression of rotational speed iterative equation can be expressed as follows

$$x^{k+1} = \psi(x^k) \qquad\qquad (k = 0,1,2,\cdots) \tag{25}$$

Where x corresponds to N_{HT} , or N_{MT} , or N_{LT} ; $\psi(x)$ represents the process which is used to calculate rotational speeds of compressors or a propeller from the given rotational speed of the turbine. $\psi(x)$ may be called an iterative function, while $\{x^k\}$ is called an iterative sequence or iterative format. In the first iteration, x may be selected as the design value of $N^{(0)}_{HT}$, or $N^{(0)}_{MT}$, or $N^{(0)}_{LT}$. Then the rotational speed is corrected in terms of two different situations.

The determination of corredted rotational speed of the high and intermediate pressure turbines

$N^{(1)}_{HC}$ and $N^{(1)}_{LC}$ can be determined from characteristic curves of high or low pressure compressors respectively after solving Eqs. (23). If $N^{(1)}_{HC} \neq N^{(0)}_{HT}$ or $N^{(1)}_{LC} \neq N^{(0)}_{MT}$, we put $N^{(1)}_{HC} = N^{(1)}_{HT}$ or $N^{(1)}_{LC} = N^{(1)}_{MT}$. The correction formulas of reduced rotational speeds of high or low pressure turbines can be expressed as follows:

$$\lambda^{(1)}_{UHT} = \frac{\pi D_{HT} N^{(1)}_{HT}}{60\sqrt{\frac{2k}{k+1} R T^*_4}} \tag{26}$$

or

$$\lambda^{(1)}_{UMT} = \frac{\pi D_{MT} N^{(1)}_{MT}}{60\sqrt{\frac{2k}{k+1} R T^*_5}} \tag{27}$$

The determination of corrected rotational speed of the low pressure turbine (or power turbine)

The power output of a gas turbine engine $P^{(1)}_e$ can be obtained after solving Eqs. (23). Assume the design power output of a gas turbine engine is $P^{(0)}_e$ and design rotational speed of the load is $N^{(0)}_L$ Since the propeller is driven by the low pressure turbine, we take approximatively

$$\frac{P^{(1)}_e}{P^{(0)}_e} = (\frac{N^{(1)}_L}{N^{(0)}_L})^3$$

$$i.e. \qquad N^{(1)}_L = N^{(0)}_L \sqrt[3]{\frac{P^{(1)}_e}{P^{(0)}_e}}$$

If $N^{(1)}_L \neq N^{(0)}_{LT}$, we put $N^{(1)}_L = N^{(1)}_{LT}$. Then, the reduced rotational speed of low pressure turbine may be corrected with following equation

$$\lambda^{(1)}_{ULT} = \frac{\pi D_{LT} N^{(1)}_{LT}}{60\sqrt{\frac{2k}{k+1} R T^*_6}} \tag{28}$$

To sum up, by successively iterating Eqs. (26), (27), and (28) we can obtain three sequences of approximate solutions: $\lambda^{(0)}_{UHT}$, $\cdots$, $\lambda^{(k)}_{UHT}$,$\cdots$; $\lambda^{(0)}_{UMT}$, $\cdots$, $\lambda^{(k)}_{UMT}$, $\cdots$; $\lambda^{(0)}_{ULT}$, $\cdots$, $\lambda^{(k)}_{ULT}$, $\cdots$, until $|N^{(k+1)}_{HC} - N^{(k)}_{HT}| < \varepsilon_1$; $|N^{(k+1)}_{LC} - N^{(k)}_{MT}| < \varepsilon_2$; $|N^{(k+1)}_{L} - N^{(k)}_{LT}| < \varepsilon_3$. In the next section we will explain roughly that this iterative equation has a good convergence.

DISCUSSION

The features of the new mathematical model

In the last section we proposed a new mathematical model in which the problem of solving three–dimensional residual equations is reduced to a mixed problem of solving two–dimensional residual equations together with extracting a root of a one–dimensional non–linear eauation. Such compound model has the following features.

1. This model is suitable for the use of these algorithms which has good accuracy and numerical stability to obtain the performance of turbines and compressors.
2. The dimensions of hybrid residual equations (23) is less than that of the model used widely at present. Therefore, it is easy to converge in iteration and needs less computation.
3. When the inverse algorithm is used in the computation of turbine performance, it is very effective to correct rotational speed with the iterative method. Besides, it also has rapid convergence and good stability.

As to the last feature mentioned above, whether the iteration of rotational speed will surely converge is a problem of common interest. Here we explain it briefly as follows:

In higher mathematics, the following convergence principle about iteration is given: If the equation $x = \psi(x)$ satisfies the conditions: $\psi(x)$ remains continuous on [a, b] and if to any x [a, b], there exists $|\psi'(x)| \leqslant L < 1$, then the iteration process $x^{k+1} = \psi(x^k)$ (k = 0, 1, $\cdots$) will converge with any initial approximate x_0 [a, b], and $\lim\limits_{k \to \infty} x^k = x^*$.

The $\psi(x)$ in this paper is only an algorithm and therefore has no concrete analytical expression in the course of solving of gas turbine engine performance. Thus, it is difficult to derive ψ' (x). But it is shown in the example calculation that the iterations of rotational speed will converge rapidly when the inverse algorithm is used to calculate turbine performance.

It is mainly because the inverse algorithm is numerically more stable and the round–off error only slightly affects the calculated accuracy. Thus, $\psi(x)$ is a continuous function (see Fig. 3). It is also shown in the example calculation that when the load of the gas turbine engine is high, ψ' (x) is much less than 1. Thus, the iterative sequence $x^{k+1} = \psi(x^k)$ (k = 0, 1, $\cdots$) will rapidly converge to the solution of the original equation according to the principle of convergence. But since the sequential algorithm is numerically instable, it results in the accumulation and enlargement of the error. Besides, it also leads to a fluctuation and jump of the $\psi(x)$, so it can not guarantee $|\psi'(x)| < 1$ everywhere. Thus, the iteration of rotational speed is not only slow but also frequently can not converge (see Fig.4).

The accuracy and computation time of the new type of mathematical model

Because this model is on the basis of simultaneously solving continuity equations, power balance equations, pressure ratio equality equation and rotational speed equality equation, there is no doubt that it is theoretically correct. In practice this method has been used for the computation of the performance of a marine gas turbine engine and has obtained satisfactory results.

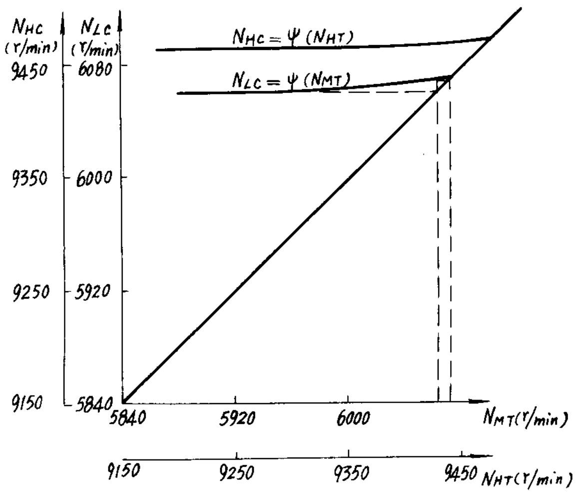

Fig. 3 Relation between n_{HC} (or n_{LC}) and n_{HT} (or n_{MT}) under the condition of the inverse algorithm, with $Ne/Ne_0 = 0.64$ and $|n_{HC}^{k+1} - n_{HT}^{k}| < 1(r/min)$; or $|n_{LC}^{k+1} - n_{MT}^{k}| < 1(r/min)$.

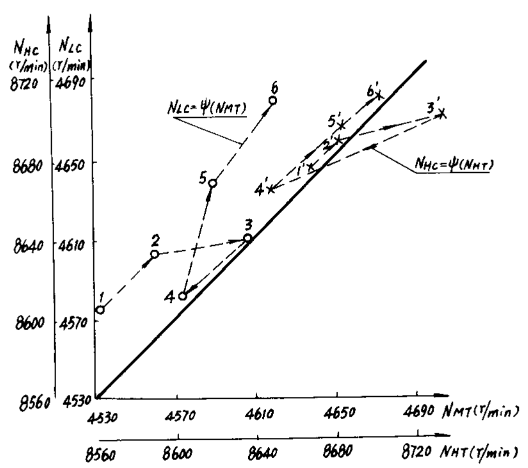

Fig. 4 Relation between n_{LC} (or n_{HC}) and n_{HT} (or n_{MT}) under the condition of the sequential algorithm, with $Ne/Ne_0 = 0.23$ and $|n_{HC}^{k+1} - n_{HT}^{k}| < 50(r/min)$ or $|n_{LC}^{k+1} - n_{MT}^{k}| < 50(r/min)$

In order to explain clearly the advantages of this method, the sequential algorithm and the inverse algorithm are respectively used to compute turbine performance under the conditions of the same type of gas turbine engine and the same form of mathematical model (23). The computational results are shown in Fig.5. It can be seen from Fig.5 that the computational results of the two algorithms almost coincide at low load, but the calculating error from these two algorithms increases gradually with the increase of load. When the inverse algorithm is used in new model (23), the difference of absolute rotation between turbine and compressor, that is, the accuracy ε of iterated rotational speed can be taken as 1 (r / min) at different part load. When the sequential algorithm is used in the new model (23), the value of ε has to increase with the increase of load, otherwise the computation fails to converge. This is because that the numerical instability of the sequential algorithm will be larger with the increase of load. For the example above, the values of ε under two different algorithms are shown in table 1.

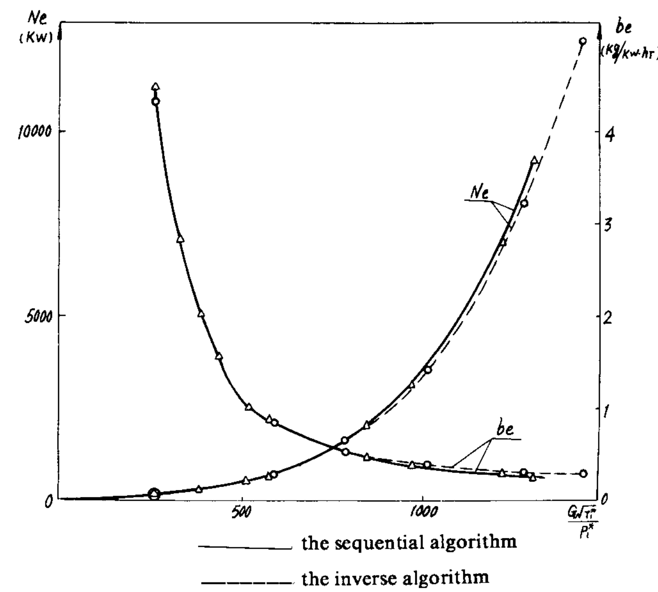

Fig. 5 Variations of Pe and be with $\frac{G\sqrt{T_1^*}}{P_1^*}$ in a certain gas turbine engine under two different algorithms

Table 1. The values of ε under two different algorithms

$\frac{G\sqrt{T_1^*}}{P_1^*}$		442.71	583.60	980.79	1319.8
ε (r / min)	sequential algorithm	1	5	50	175
	inverse algorithm	1	1	1	1

It may be seen from this table that, with the increase of the values of ε, the sequential algorithm will doubtlessly result in remarkable errors under the conditions of the higher load of the gas turbine engine.

The new model (23) involved in the inverse algorithm will rapidly converge with high accuracy under the condition of any part load of gas turbine engine, because the values of accuracy ε may be taken to be a small and constant quantity with the variation of load. Besides, it also involves less computation. Listed below are the CPU times consumed for the computation under the conditions of different part loads with DPS 8 computer.

Table 2. The CPU times comsumed for the computation under the different part loads

Ne / Neo	0.057	0.130	0.284	0.642	1.0
CPUtime	1min 19sec	1min 18sec	46sec	40sec	36sec

It can be seen in the table that larger loads result in less time consumed for the same accuracy. The time comsumed for this method is approximately $\frac{1}{4} \sim \frac{1}{10}$ less than that for common methods.

CONCLUSIONS

1. It is proposed in this paper to use the compound model composed of two–dimensional hybrid residual equations and a one–dimensional non–linear equation as the new model for calculation of steady state performance of marine gas turbine engines. It has the advantages of high accuracy, rapid convergence, good stability, and less computation. The model is also apropriate for land gas turbine engines and aero–gas turbine engines.
2. The inverse algorithm, a new method proposed in this paper for the computation of the performance of turbines in series connection, has good numerical stability. It also has a good effect on reducing the accumulation and propagation of round–off errors and interpolation errors. Using this algorithm to solve the performance of gas turbine engines, the accuracy and stability of computation can be greatly improved.
3. The use of two–dimensional hybrid residual equations proposed in this paper will avoid the equations to become ill–conditioned. It has a rapid convergence due to the application of the inverse algorithm. It has not been found that the inversion of the Jacobian Matrix has ever failed.
4. This paper proposes to use one–dimensional non–linear equation for solving the rotational speed by use of iteration method. Because, in this method, the performance of series turbines is obtained by inverse algorithm, it can well satisfy the convergence principle of iteration. The example computation shows that it is not difficult to limit the difference between rotational speeds of turbines and that of compressors in less than 1 (r / min).

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