A New Method of Tracing Alternating Current Curves — Source link

Fitzhugh Townsend

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TRANSACTIONS

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Vol. XVII. JANUARY TO DECEMBER. 1900.

New York, January 24th, 1900.

The 139th meeting of the INSTITUTE was held this date at 12 West 31st Street, and was called to order by President Kennelly at 8.25 P. M.

THE SECRETARY:—At the meeting of the Council this afternoon, the following associate members were elected:

Name.	Address.	Endorsed by
BEHREND, BERNHARD A.,	Erie, Pa.	C. P. Steinmetz. T. C. Martin. W. D. Weaver.
BEVERIDGE, EDMUND WALTER,	Assistant Engineer, The G. I. P. Railway Co., India, Sirud Khandish District, Bombay, India.	Ralph W. Pope. N. S. Keith. F. E. Kinsman.
DENHAM, JOHN	Electrician, Cape Government, Cape Town, South Africa.	W. M. Mordey. J. E. Lloyd. A. E. Worswick.
EVANS, PAUL H.	Chief Engineer, Mexican General Electric Co., Apartado 403 Mexico City.	C. F. Beames. A. E Worswick. JohnW.Thompson
GREENWOOD, GEORGE,	Electrical Engineer and Superintendent, Jalapa Railway and Power Co., Jalapa, V. C. Mexico.	JohnW.Thompson C. F. Beames. A. E. Worswick.
HOLMES, GWYLLYM R.	Holmes-Rose Electric Co., 2842 Parkwood Ave., Baltimore, Md.	H. K. McCay. Gano S. Dunn. M. C. Schwab.
HYDE, J. E. HINDON,	120 Broadway; residence, 48 West 11th St., New York City.	C. F. Chandler. C. A. Doremus. Henry Morton.
MAGNUS, BENJAMIN	Electrical Engineering Student, Columbia University; residence, 22 E. 111th Street, New York City.	Geo F. Sever. Fitzh. Townsend. F. B. Crocker.
[illegible]LES, HARRY B.	Electrical Engineer, Ferrocarrillos del Distrito, American Club, Mexico City.	JohnW.Thompson C. F. Beames. A. E. Worswick.

PFEIFFER, ALOIS J. J.	Engineer, Thomson - Houston Co., 5 Piazza Castello Milano, Italy.	H. A. Foster. W. J. Davis, Jr. A. H. Armstrong.
POOLE, CHARLES OSCAR	General Superintendent Electrical Dept. San Francisco Gas and Electric Co,, 229 Stevenson St.; residence, 452 Bryant St., San Francisco, Cal.	J. A. Lighthipe. Geo. P. Low. F. A. C. Perrine.
POWELSON, WILFRED VAN NEST,	Government Inspector, Electrical Appliances. General Electric Co., Schenectady, N.Y.; Lieutenant U. S. Navy.	Eskil Berg. Ernst Berg. Chas. P. Steinmetz
SAYLOR, FREDERICK ALEXANDER,	Chief Electrician, U. S. S. Chicago ; residence, Reading, Pa.	W. E. Chappell. Arthur L. Rice. T. P. Thompson.
SELDEN, R. L. JR.,	Deep River, Conn.	F. G. Strong. R. W. Pope T. C. Martin.
SHEARER, J. HARRY	Electrical Engineer, National Electric Light Co., Apartado, 639 Mexico City, Mexico.	John W. Thompson C. F. Beames. A. E. Worswick.
SHEPHARD, ROBERT R.	Erecting Engineer, Mexican General Electric Co., Apartado 403, Mexico City, Mexico.	C. F. Beames. A. E. Worswick. John W. Thompson
THOMPSON, MILTON T.	Constructing Engineer, Mexican General Electric Co., Apartado 403, Mexico City, Mexico.	C. F. Beames. A. E. Worswick. John W. Thompson
ZABEL, MAX W.	Draughtsman and Student of Patent Law with John A. Brown & Cragg, 1450 Manadnock Bg.; residence, 454 North Ave., Chicago, Ill.	D. C. Jackson C. F. Burgess M. C. Beebe.

Total 18.

The following associate members were transferred to membership:

Approved by Board of Examiners, September 7th, 1899.

WALTER DOUGLAS YOUNG, Electrical Engineer, B. & O. R. R., Baltimore, Md.

Approved by Board of Examiners, October 20th, 1899.

LOZIER, ROBERT T. E. Electrical Engineer, Member of Firm of Bullock Electric Co., St. Paul Bldg., New York City.

Approved by Board of Examiners, Dec. 8th, 1899.

H. A. STORRS, Assistant Engineer, U. S. Engineer's office, New London, Conn.

THE PRESIDENT:—Since our last meeting the INSTITUTE has suffered by the death of two of our well-known members, Mr. James Hamblet has served the INSTITUTE for some years, both as vice-president and manager. His contribution to the electrical interests, work and fraternity extend over half a century.

Mr. Dana Greene had rendered valuable and active service to the INSTITUTE, not only as Manager on the Council, but also in contributing papers. By his extensive general knowledge and genial influence he greatly promoted electrical engineering progress in those branches which he made peculiarly his own. His untimely death will be greatly deplored not only by all our members who knew of his work, but by the favored few who were acquainted with his charming personality. The Council has at its meeting to-day passed resolutions to be communicated to the families of these gentlemen, and has ordered suitable obituary notices to be printed in the TRANSACTIONS.

The following paper was then read by Mr. Fitzhugh Townsend on "A New Method of Tracing Alternating Curves."

A paper presented at the 139th Meeting of the American Institute of Electrical Engineers, New York, January 24th, 1900, President Kennelly in the Chair.

A NEW METHOD OF TRACING ALTERNATING CURRENT CURVES.

BY FITZHUGH TOWNSEND.

The following method of determining the instantaneous values of alternating current waves recommends itself by its simplicity, accuracy, and rapidity. In addition to this, the method is capable of determining directly a curve of induction.

Fig. 1 shows the connections for taking the curves of electromotive force and magnetic induction simultaneously. E is the E. M. F., the curve of which is to be determined, R_1, R_2 and R_3 are ohmic resistances, e is a small transformer with open magnetic circuit, and C is the contact maker. This consists of a disk, the perimeter of which, in a bipolar alternator, is divided into two sections. One section only is conducting, and since the disk is mounted on the shaft of the machine, or on that of a synchronous motor, the current can flow in the circuit of the contact maker only during one-half of a period. d is a Weston portable voltmeter, connections being made directly to the movable coil.

Determination of a Curve of Induction.

The double throw switch b shown in Fig. 1 should be closed so as to connect points 3 and 4 to 5 and 6, switch a being open. The required curve can then be obtained by noting the deflections of the instrument, d, corresponding to successive angular positions of the contact brush f. The theory on which this is based is as follows: Owing to the construction of the contact maker, the current i, which passes through the instrument d, flows during one-half of a period only. During the active half period, however, i is proportional to E, the electromotive force: i will therefore be an interrupted current having the form of the electromotive force as shown in Fig. 2. The position of the

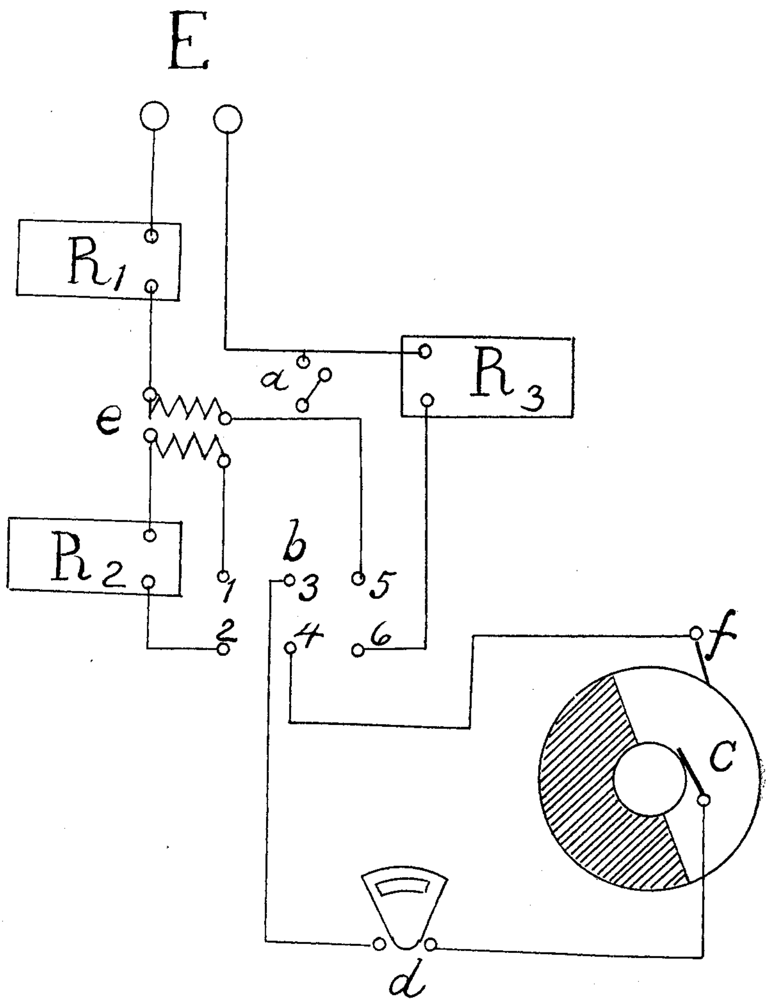

Fig. 1.

time interval $t_2 - t_1$ will depend on the angular position of the contact brush f.

The deflection of the voltmeter will be proportional to the number of impulses per second. Each impulse being proportional to the time integral of the current during the interval $t_2 - t_1$.

Therefore,

$$\theta = K\int_{t_1}^{t_2} i\, dt = K_1 \int_{t_1}^{t_2} E\, dt = K_2 \int_{t_1}^{t_2} \frac{d\, B}{d\, t}\, d\, t = K_2 (B)_{t_1}^{t_2} = K_3\, B$$

If the brush f, and consequently the interval $t_2 - t_1$, are moved through a period, and the values of θ at different points noted, the curve of the induction B will be obtained.

DETERMINATION OF A CURVE OF ELECTROMOTIVE FORCE.

The switch b should now be closed so as to connect points 3 and 4 to 1 and 2. The contact maker and the voltmeter will

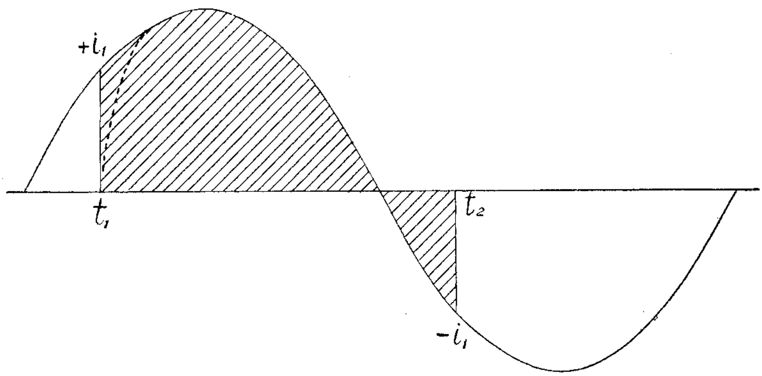

FIG. 2.

then be in the secondary circuit of the small transformer e. Close switch a. The curve of the electromotive force may then be obtained, as in the case of the curve of induction, by noting the deflections of the voltmeter for different angular positions of the brush f. It can be shown readily that this is the case. If the constants of the transformer e are suitably related to each other, the primary current will be the same in form as the electromotive force E, and practically independent of the value of the secondary current i. This will be brought about when the electromotive force of mutual induction in the primary, due to the secondary current is only a small percentage of the E. M. F., E. We will then have

$$i = K \frac{d\,E}{d\,t} \text{ and } \theta = K_1 \int_{t_1}^{t_2} \frac{d\,E}{d\,t}\, d\,t = K_2\,(E)_{t_1}^{t_2} = K_3\,E$$

Fig. 3 shows curves of B and E in a transformer obtained as described above. The form of these curves requires a few words of comment.

Any complex harmonic electromotive force generated by a properly designed alternator, may be represented by the equation $E = A_1 \sin p\,t + A_3 \sin 3\,p\,t + A_5 \sin 5\,p\,t +$ etc. $+ B_1 \cos p\,t + B_3 \cos 3\,p\,t + B_5 \cos 5\,p\,t +$ etc. Therefore,

$$B = \int E\,dt = -\left(\frac{A_1}{p} \cos p\,t + \frac{A_3}{3\,p} \cos 3\,p\,t + \frac{A_5}{5\,p} \cos 5\,p\,t + \text{etc.}\right)$$

$$+ \frac{B_1}{p} \sin p\,t + \frac{B_3}{3\,p} \sin 3\,p\,t + \frac{B_5}{5\,p} \sin 5\,p\,t + \text{etc.}$$

It appears from this that the curve of induction should differ less from the sinusoidal form than the curve of electromotive force, since the relative amplitudes of the upper harmonics are made less by the process of integration in proportion as their frequency exceeds that of the fundamental. Thus the third harmonic in the curve of induction has one-third the relative amplitude which it has in the curve of E. M. F. Similarly, the fifth harmonic has its relative amplitude divided by five, and so on.

Also the complexities in the induction should be the reverse of those in the electromotive force. In other words, a peak in curve B should correspond to a depression in curve E.

In order to prove the accuracy of the method, and to show that the curve B in Fig. 3 is correct, the true curve of induction was constructed from the electromotive force curve by a process of integration. The constructed curve was found to coincide with the experimental curve as shown.

The method of obtaining the curve of induction from the curve of E. M. F. by integration, is extremely laborious. The method described in this paper is the only one to-day which is capable of determining the curve of induction directly, thereby saving much time and trouble.

In determining the curve of induction, the phase depends on the relative values of the resistance and inductance of the circuit through the interrupter. A variation of the amplitude of

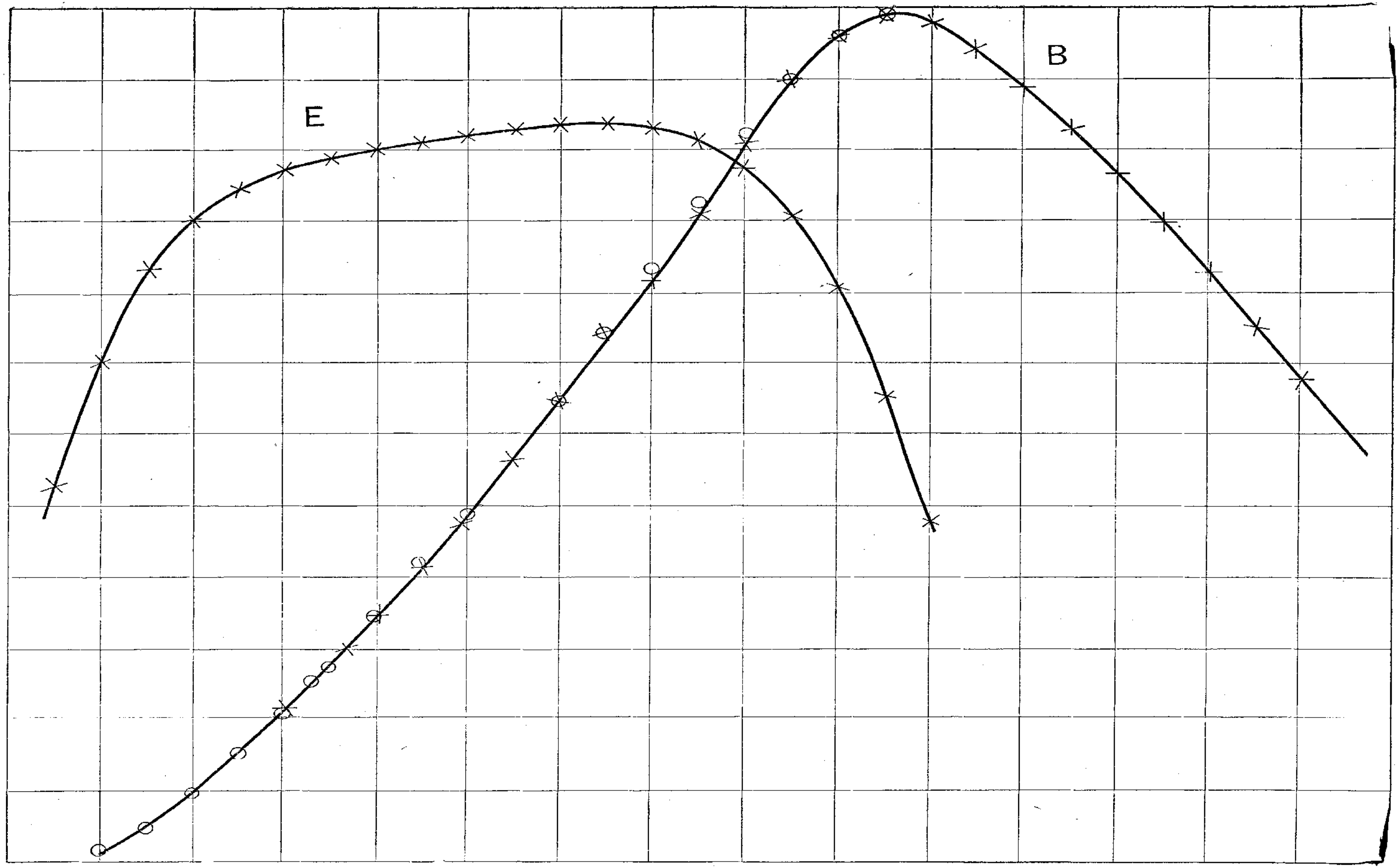

FIG. 3. Points Determined Experimentally, ×. Points Determined by Calculation, O.

E will not change the phase of the curve to a perceptible extent.

In the case of the determination of E, the phase depends on the values of resistance and inductance in both the primary and secondary circuits of the transformer e. As in the case of the B curve, varying E with the same resistance and inductance in both primary and secondary circuits, will not change the phase appreciably.

The shape of the wave E will remain unchanged, so long as three conditions which are essential to the accuracy of the method are maintained.

The first of these is that the electromotive force of mutual induction in the primary circuit due to the secondary current shall be small compared with the total E. M. F. impressed on the primary circuit.

If this were not the case, the secondary E. M. F., and therefore the secondary current would not be proportional to the rate of change of the primary current.

In the transformer used in taking the curves shown, the coefficient of mutual induction M was found to be .00076. The secondary current never could be more than 0.03 amperes without producing too large a deflection of the voltmeter needle, therefore the electromotive force of mutual induction due to the secondary current could not be more than

$$E_m = p\, M\, C_2 = 760 \times .00076 \times 0.03 = .017,$$

which shows that the first condition of accuracy mentioned above would be fulfilled to within 1% for an impressed electromotive force as low as 1.7 volts.

The second condition is that the resistance R_1 and R_2 in Fig. 1 shall be great compared with the inductances of the primary and secondary circuits of the transformer e.

In the case of the curve shown in Fig. 4, the values of R_1 and R_2 were 30 and 50 ohms respectively. If E is a complex harmonic, R_2 must be greater than R_1.

Since

$$i = K \frac{d\,E}{d\,t},$$

the upper harmonics in the secondary current i will be much more pronounced than they are in the curve E.

If R_2 were not large compared with the secondary inductance, the form of the current i might be quite different from that which it should have in order to give accurate results.

The third condition which must be fulfilled in order that the curve obtained may be correct in shape, is that the time constant of the secondary circuit shall not exceed a certain value.

When a circuit containing inductance is closed, the current

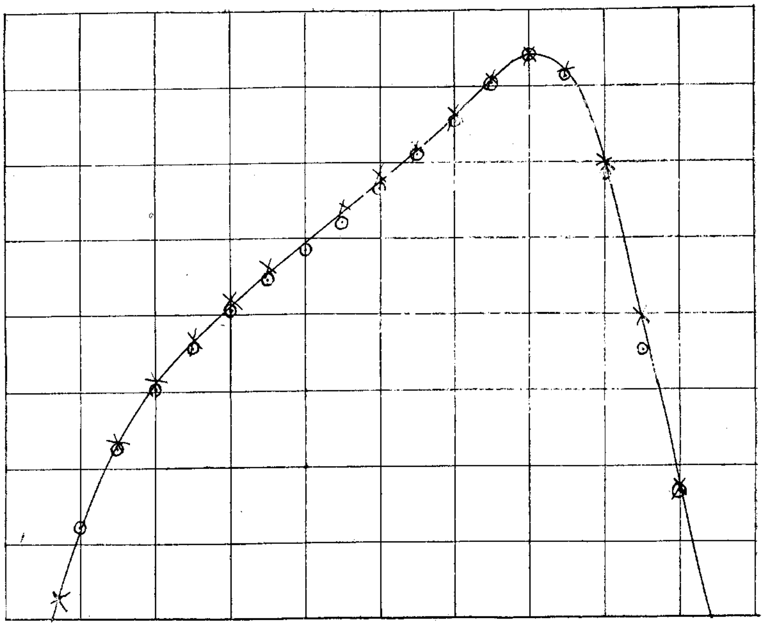

FIG. 4. Points Obtained by Zero Method, ⊙. Points Obtained by Present Method, ×.

does not reach its value E/R until a certain time has elapsed, and during this interval the current is

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Evidently if the time constant is too large, or if R/L is too small, the integral of the secondary current will not be the entire shaded area of Fig. 2, but will be bounded at the make by some curved line instead of a perpendicular, as shown by the dotted line in the figure. This would be a source of error, which would come in more and more as the secondary current integral drew near to its zero value. When this integral is a maximum, the make takes place at $i = 0$, hence at this point of the curve the effect would not occur. The result of the time constant of the circuit would therefore be a decrease in the relative size of the ordinates near the zero points, and also a slight shifting of the zero points with respect to the maximum, the maximum ordinate remaining unchanged both in intensity and phase. It is probable, then, that the effect of too great a time constant in the secondary circuit would result in making the curve come out more pointed than it really is, and slightly bent over sideways.

The curve of a complex harmonic electromotive force shown in Fig. 4 was determined as a test of accuracy: It furnishes a comparison between the method described here, and the well-known zero method in which a telephone receiver is used as the indicating instrument. As may be seen, there is a satisfactory agreement in the readings obtained by the two methods.

The curve of Fig. 4 was determined at a frequency of 120 periods per second. By increasing R_1 and R_2, it would evidently be possible to obtain curves at much higher frequencies. Mechanically, the method lends itself readily to this, inasmuch as the action does not depend on an instantaneous contact. This is a very advantageous feature, especially at high frequencies.

The transformer *e* used in these experiments was wound on a wire core $1\frac{1}{2}''$ in diameter. There were 100 turns both in the primary and secondary windings.

The following are the principal constants of this transformer:

L_1 (on open secondary)	.00207	henrys.
R_1 " " "	.179	ohms.
L_2 " " primary	.00152	henrys.
R_2 " " "	.096	ohms.
M...	.00076	henrys.
Resistance of voltmeter	.87	ohms.
Inductance " "	.00034	henrys.

Of course, only relative values are given by the readings of the voltmeter. In order to obtain absolute values, some means of calibration must be adopted. The easiest way to accomplish this is to switch the apparatus in series with some electromotive force, either direct or alternating, the maximum value of which is known. The deflection obtained divided by the true value of the electromotive force it represents will be the constant of the instrument for the particular amounts of resistance and self-induction which may be connected in the primary and secondary circuits of the transformer *e*.

The fact that the brush *f* makes contact for a half period and not for a very small fraction of a period, is a great advantage, inasmuch as the difficulties of a method of instantaneous contact are usually due to the short time of contact, making it hard to keep the apparatus in good working order. With the interrupter described here, however, there is no more difficulty than with an ordinary direct current commutator, and in fact rather less, as sparking, which is always the main evil of the commutator is practically absent in the interrupter.

In addition to the above, the present method seems to be the only one by which the curves of magnetic induction or quantity of electricity can be determined simultaneously with those of electromotive force and current.

Alternating Current Laboratory,
Electrical Engineering Dept.,
Columbia University,
New York City, January, 1900.

Discussion.

Dr. Samuel Sheldon:—I have been very much interested in reading this paper and hearing the description of this ingenious piece of apparatus. But it seems to me that in order that the instrument may give directly the curve of induction, that the two halves of the cycle of alternation must be perfectly symmetrical; that is, the positive flux of current must have the same time relation that the negative flux has. I cannot see from an examination of the mathematical equation that there is any difficulty with the equation. But a general equation should, if correct, be applicable to every case. It does not seem to apply to the case of the exaggerated curve shown in Fig 5, or at least, the contact-maker would not yield the correct curve of induction. Let the rectangular curve represent the time relation of the E. M. F. With proper arrangement of apparatus this will also be the curve of

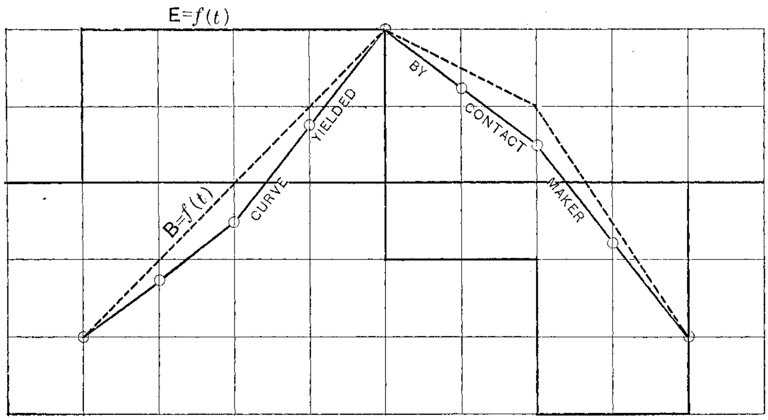

Fig. 5.

current. The relation which must exist between the induction and the time, in order to produce the curve of E.M.F., is represented by the dotted curve. The dotted curve is the integral curve of the E. M. F. curve. Now the curve which would be yielded by the contact-maker as described in the paper, the contact being maintained for half a cycle, would not coincide with this dotted curve of induction. The voltmeter readings will be proportional to the algebraic sum of the areas lying between the E. M. F. curve, the time axis, and the time intercepts of making and breaking contacts. A slight inspection will show that the curve passing through the points enclosed in circles would be the one which would be yielded by the contact-maker.

The difficulty in using a telephone and contact-maker with, say, the Mershon method is two-fold. In the first place the ordinary frequencies of alternating currents give a musical tone by means of the contact-maker, to which the ear is not particularly sensitive, and it is difficult to distinguish the intensity of a minimum, providing one cannot get absolute silence. In the second place, with a contact-maker having any breadth of contact at all, it is impossible to balance a variable E. M. F. against a constant E. M. F. Therefore, unless the contact's breadth be infinitesimal silence cannot be obtained. If, however, one uses a contact of sufficient breadth to be sure that the contact is made with every revolution of the contact-maker (of breadth, not necessarily to half a cycle), and makes use of the voltmeter as described in the paper *i. e.*, really as a ballistic galvanometer, then one can get a curve such as is yielded by an ordinary contact-maker and a very satisfactory one, without the liability of the contacts getting out of order.

In this use of the Weston voltmeter we have still another important adaptation of that valuable instrument. I have had occasion to use a Weston voltmeter for ballistic work, and it perhaps may be of interest to some who may not know it, that the Weston voltmeter will give deflections or throws in response to instantaneous quantities of electricity passing through it which are directly proportional to these quantities. Now a ballistic galvanometer has been defined by a good many different people as a galvanometer which has a needle which has a large moment of inertia and which has a long period of swing.

DR. PUPIN:—And small damping.

DR. SHELDON:—That is sometimes put in, yes.

DR. PUPIN:—Always.

DR. SHELDON:—Well, all right. It does not make any difference. It is not necessary that the ballistic galvanometer should have these qualities. If we calibrate a Weston voltmeter in a proper manner it can serve the purposes of a ballistic galvanometer just as well as the old fashioned kind. Those who have ever had occasion to take hysteresis curves by the step-by-step method, using a galvanometer which has a long swing of from 12 to 15 seconds, with two or three minutes' wait before the needle comes to rest after each swing, know what a tedious process it is to get a hysteresis curve. Yet the hysteresis curve can be taken with a Weston voltmeter of the ordinary round pattern switch-board style, having its series resistance removed, and it can be taken in less than two minutes, with a sufficient number of points to give accurate results, if the curve is properly integrated.

MR. FITZHUGH TOWNSEND:—With regard to that objection about determining the curve of induction, I have never attempted to determine the curve of induction from an unsymmetrical electromotive force. In all practical machines, good machines, and almost all alternating current machines, the positive half of the wave is the same as the negative half of the wave, and in that case I think the method holds good. A slight modification of the method which I have not yet tried, which would involve the use of two brushes, would I think obtain a symmetrical curve of induction, but that is more or less a *tour de force*. I have, therefore, only devoted my attention to the treatment of curves which were symmetrical with respect to the zero line.

MR. CHARLES P. STEINMETZ:—During late years no end of different methods have been brought out to take instantaneous readings of electrical quantities, mostly differing from each other by a different kind of contact-maker or a different material used, or other such minor qualities, or an improved method of using it. Now the method brought before us to-day essentially differs from previous methods, as far as I know them, by, I believe very favorable features. In its general principle this method described here consists of the following: To get instantaneous readings of a quantity, we derive from this quantity another quantity, which is a differential of the former, and take integral readings, that is average values of the latter quantity. We thereby get rid of most of the difficulties incident to instantaneous readings. We are independent of the width of the contact. We are independent of the impossibility of getting an absolute point-contact of no duration, and of the difficulties due to the arc following the contact-brush for some appreciable time. There are some limitations to this method, which after they are pointed out will probably be overcome. The source of error shown in Fig. 2 as due to the current not assuming instantly the full value, I do not consider so essential, because there is another error compensating for it. In Fig 2, the current in the moment when the circuit is broken, does not instantly drop to zero, but gradually dies out by a small arc continuing to carry the current. This gradual dying-out of the current compensates for the error mentioned in the paper to a certain extent. If you think of it still more, the error introduced by the current not stopping instantly, depends on the voltage at which you operate, and gets larger with increase of voltage, while the error due to the current not instantly reaching full value gets less with increase of voltage, thus theoretically you can always find a voltage, where the errors just neutralize each other. Practically, therefore, the method should give very accurate results provided that the errors are small enough *per se*, so as not to require perfect compensation.

The principle of this method as brought out here, however, can be applied in many different ways. It is shown only for determining the induction curve of the machine. This I do not consider as so very important, because you can get this curve of induction, although a little more laboriously, from the electromotive force curve; but you can apply the same method wherever you want instantaneous values of a quantity, and are able to get differential values of this quantity. To illustrate, for instance, one of the problems of great importance at present is to determine the variation of the rate of rotation of flywheels of slow-speed engines which are intended for use with synchronous apparatus as alternators, synchronous motors, etc. What you want to get there is the instantaneous displacement of the flywheel from the position it would have at uniform rate of rotation. Now the differential of this displacement is the velocity of the flywheel. Assume that on the shaft of the flywheel you have a small dynamo with constantly excited field, the field, for instance, excited by a storage battery. Take average or integral values of the electromotive force given by this dynamo, then you see these integral values give you direct the position of the flywheel. You would in such a case probably counterbalance the largest part of the electromotive force of the small dynamo by a storage battery with a constant electromotive force, to get a larger variation relatively than the very small variation of speed, and you would have to develop this method like you would have to develop any method.

But I believe that integrating or averaging the differential values of the quantity of which you want instantaneous values can be applied to many cases, and in some cases which are probably more important than the determination of instantaneous values of electromotive force, since there are no suitable methods in existence for such other cases.

Dr. M. I. Pupin:—I do not think that Dr. Sheldon's objection is a very serious one, for the reason that Mr. Townsend already pointed out. The method which Mr. Townsend describes is intended to analyze curves that are obtained in ordinary machines, well designed, not badly designed machines. If a machine is badly designed it is unworthy of investigation; the less you fuss with it the better. But even with a badly designed machine such as that would be which Dr. Sheldon presented on the blackboard, by a slight modification of the method as Mr. Townsend has already indicated, using two brushes, the difficulty could be gotten around. In all our commercial machines the electromotive forces and the currents are perfectly symmetrical on the two sides of the axis of the abscissæ. In that case the method will work all right. Now so far as any errors are concerned, I think that Mr. Townsend has rather enlarged them or made them appear more serious than they really are,—the source of error, namely the time constant. The time constant comes in the curve not rising instantaneously to its full value when you close the circuit and not falling off to zero when you break the circuit. That does not amount to much, because for other reasons you have to make the resistance large in comparison to the inductance of the circuit. In that case of course you have the time constant, a very large quantity; the current will in a very small fraction of the half period come to its full value. So that error really does not amount to anything appreciable. It certainly is very much smaller than other errors of observation that we meet with in investigations of this kind. The errors of observation in investigations of this kind are nearly one per cent. If any body does as well as that he is fortunate, and this error introduced by having the time constant a little bit too small will not be anything more than a very small fraction of one per cent.

Mr Steinmetz has said that we do not care so very much for the curve of quantity. Well, I think we do. You see in our present measurements we have three different kinds of instruments, fundamentally different. We have the voltmeter and the ammeter and the ballistic galvanometer—not in the way that Dr. Sheldon defined it, but in the way that it ought to be defined. The one measures the rate of variation of the quantity of electricity; that is the ammeter. The other measures in dynamo-electric machines the rate of variation of a magnetic quantity, this is the voltmeter; and the third one, the ballistic galvanometer, measures the quantity, either the quantity of electricity or the quantity of magnetism. Now in this method that Mr. Townsend brought before us to-night, we have another method which resembles the ballistic, to be sure, but still it is a different story from the ballistic, altogether a different story —measuring quantities, measuring the quantity of electricity and the quantity of magnetism directly, not in a roundabout way by integrating a curve. Whoever has made the calculations of integrating a periodic curve knows that there are a great many sources of error, very much greater sources of error than you meet with in experimental work of this kind. Some of us might say; what is the use of measuring the quantity of magnetism or the quantity of electricity? There are investigations and that too not only of purely scientific interest but also of very great commercial and engineering value in which we do care for quantity of magnetism, quantity of induction and quantity of electricity.

I mention this for the purpose of bringing out something which Mr. Townsend has not brought out. This method was evolved in our laboratory in connection with an investigation which should be mentioned here. He has worked now for over a year very faithfully over a difficult problem—the problem of determining, what I choose to call, the dynamical hysteresis loop.

If you will allow me, I will point out the importance of this loop. If you determine an alternating electromotive force curve, and the corresponding current curve by a sliding contact, or any other way, and from the secondary E. M. F. curve, determine the curve of induction in a transformer, and then you plot a curve of Fig. 6, taking from the abscissæ the ampere turns in the primary, for the ordinates you take the total induction B of the transformer. You get the curve of Fig. 6 which I call the dynamical hysteresis loop. Its area represents the hysteresis and Foucault current loss per cycle. That is the total loss in the transformer, provided, of course, the secondary is not closed. But if the secondary is closed, then that is the total work done in the transformer, taking in hysteresis, Foucault current primary $I^2 R$ and secondary load.

Well, you see, the determination of curve in Fig. 6 is a very important thing. It enlarges our idea of the hysteresis loop. If you have no secondary current then you can study exactly what is going on in the iron while it is being magnetized and demagnetized. If you have a very high degree of lamination there are no appreciable Foucault current losses at ordinary commercial frequencies and in accordance with investigations which have been done abroad (and we have done some work of this kind in Columbia University owing principally to the perseverance and exertions of Mr. Townsend) then the hysteresis loop of Fig. 6 becomes more nearly like the statical hysteresis loop, determined with direct current and very slow cycles. If you magnetize the iron very slowly, the curve develops sharp corners as you all know. But you probably never get corners with alternating currents. Now, of course, if the impressed primary electromotive force is a simple harmonic, the induction will be a simple harmonic, and we can derive the induction curve very easily from the curve of the secondary electromotive force. There is no difficulty at all in doing that. But if the primary impressed electromotive force is a complex harmonic, the induction will be

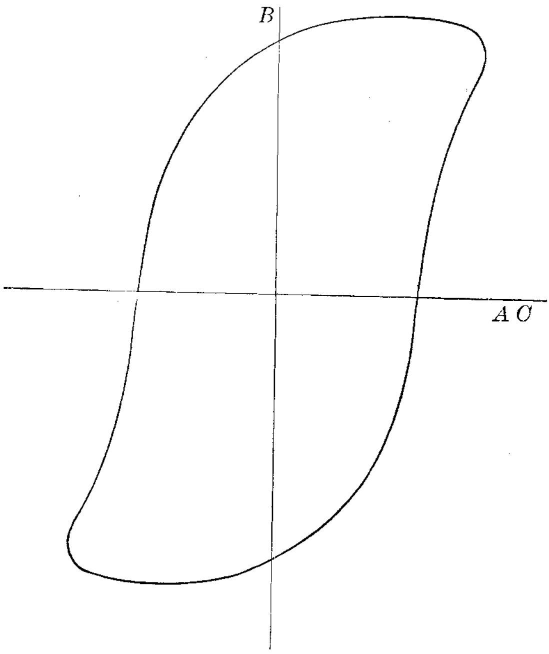

FIG. 6.

a complex harmonic, and if you try to deduce the induction from the secondary electromotive force you have to go through a lot of most tedious calculations. Mr. Townsend's method avoids this completely. It saves, therefore, a tremendous amount of labor and liability of error.

There is another thing that we have been trying to do for years and could not get it. How could this extended idea of the hysteresis loop be applied just as well to the losses in condensers? The reason that we get a hysteresis loop which is not an ellipse is because the permeability of the iron is a variable quantity. Now if you take a condenser you will find that the condenser capacity varies with the voltage just as the permeability varies with the magneto-motive force applied to the transformer. The static permeability, that is the specific inductive capacity, varies with the applied electromotive force; and just as in the case of the transformer, you get a hysteresis loop, so in the case of a condenser you also get a hysteresis loop. Now one way in which this variability or permeability manifests itself in the transformer is this: If you have a simple harmonic electromotive force the current is distorted; it has complex harmonics; now if you take a condenser, and impress a simple harmonic electromotive force upon it the current will be complex harmonic. But mind you, the difference of potential in the condenser will be a simple harmonic just the same as the secondary electromotive force in the transformer is a simple harmonic of the impressed E. M. F. You can plot the curve of hysteresis and other losses, whatever they may be in exactly the same way as you do in the case of iron. You take, not the magnetomotive force but the electromotive force for abscissæ (that is the difference of potential of condenser). For ordinates you plot, not the quantity of magnetic induction, but the quantity of electric induction Q, that is the quantity of electricity, and then you get the points of the hysteresis loop which looks something like curve of Fig. 6. You get the hysteresis loop for condensers just the same as for iron and you can estimate your losses and study how the whole thing acts. You do not only get the losses but you actually get the curve and see how those losses are brought about. Now this is a very interesting thing from a purely scientific standpoint and very important from the engineering standpoint and ought to be studied. Now in the ordinary sliding contact method of plotting curves we cannot determine the quantity of electricity directly; but if we use Mr. Townsend's method we can get that just as accurately as we can determine the voltage by a voltmeter. That seems to me to be a very important point.

MR. TOWNSEND:—With regard to the error of the time constant, it depends, of course, on the ratio of self-induction and resistance in the circuit containing the contact-maker and the instrument. If the resistance is sufficiently high for the current to have the true form of the rate of change of the E. M. F., it is found in practice that no error due to the time constant of the circuit can be detected.

In order to prove this, I determined a curve with no external resistance in the circuit. In other words, R_2 in Fig. 1 was made equal to zero, the instrument in series with the contact-maker being connected directly across the secondary of the transformer *e*. Under these circumstances, at a frequency of 120, the error due to the time constant amounted to about two per cent. In this case, however, the method was not correctly used. To comply with the theory, R_2 must be sufficiently great to make the current through the instrument proportional to the time rate of change of the electromotive force to be measured. When this condition is fulfilled, it will be found that the error due to the time constant cannot be detected.

MR. STEINMETZ:—This method of getting the dielectric hysteresis loop of the condenser was first applied, as far as I remember, some years ago by Dr. Bedell at Cornell University. Unfortunately Dr. Bedell has not been so successful as Dr. Pupin in getting a condenser with considerable internal losses but had condensers of very high efficiency. The hysteresis loop he got was so extremely flat and narrow as to be within the errors of observation, amounting to something less than three or four per cent. of the total volt-ampere capacity. But there are condensers which have an excessive internal loss so as to show a large loop and make this phenomenon more pronounced. However, no change of capacity with the voltage was observed by Dr. Bedell nor by me in condensers.

I do not want to be misunderstood by Dr. Pupin. I did not say that I do not care for the magnetic induction, but I did not consider it so very important to get a new method for it, because it can be derived from the current and electromotive force by integration. As a rule, in the usual method of getting instantaneous values of electromotive force you get them say from five to five degrees apart, and then you merely add the total value of 180 degrees and from this value you always add one value at one side,—say for 185°, 190°, etc., and subtract one value at the other side, say 5°, 10° etc., and you have the curve of induction. Now, that can de done for the whole curve in a very few minutes. However, obviously it is an advantage to derive it directly, only it is not so absolutely important.

I fully realize the importance of getting the curve of magnetic induction, the more as for many years I have been satisfied that the general custom of claiming some great and unknown advantages for a sine wave of electromotive force, is not warranted, but that in many cases the sine wave of electromotive force is about the worst you can get. Certain forms of distorted waves give a considerably lower magnetic flux in alternators, transformers and other apparatus, at the same induced E. M. F., and consequently a higher efficiency. Now you see that it is quite interesting to get the easy way of illustrating why it is, that for instance with the distorted wave of the ironclad unitooth alternator at the same impressed E. M. F. and same frequency, the hysteresis loss in transformers is from eight to twelve per cent. lower than with the sine wave, while certain forms of distributed armature windings give hysteresis losses in excess to those given by a sine wave.

Dr. Pupin:—I am aware, of course, of Dr. Bedell's article, but at that time when the article was published we had already done our work at Columbia University on that very thing, so we still claim the priority although we did not publish anything on the subject.

Another thing, Dr. Bedell's curve of hysteresis was, as Mr. Steinmetz remarked, quite within the errors of observation, so we feel rather uncertain as to whether what he got was really a hysteresis curve or simply a record of his errors. These errors can be obviated by employing very large condenser capacity. We have condensers at Columbia University which mount up to over 240 microfarads, so that we can draw on that as much as we please, and in that way of course we can increase the hysteresis loop to anything we desire. Another point was touched by Mr. Steinmetz, namely, the disputed question whether a peaked electromotive force curve, since it produced a flat induction curve, is not preferable in certain cases, can be decided by experiment in following the use of this contact-maker. It is a disputed question. Mr. Steinmetz maintains (and he does it of course partly for selfish reasons, because he must support his rule of hysteresis loss), that if a curve is a flat curve, although, of course, the area enclosed would be a larger one, the total loss would be smaller, because the loss depends only on the maximum of induction. Now, if that is really so it is a very interesting fact. I am inclined to believe it to be so. But still we would like to have some direct experimental evidence, and if this new method of plotting the curves of induction directly can help us to that, why that of itself is worth all the labors that Mr. Townsend spent on the designing and construction of this sliding contact.

Mr. Steinmetz:—I may say in this respect that not everything is published immediately after it is found out, but sometimes very many years after, even more than seven years. The first proof that the distorted wave gave the lesser hysteresis in the transformer was found by the wattmeter about ten years ago, before the laws of hysteresis were discovered. At the time it was not understood. Afterwards I hunted up the old tests and found the first experiment regarding the effect of the wave distortion or rather the effect of different types of alternators made by wattmeter reading, dating back before the time of my studies of the hysteresis law.

Dr. Pupin:—Yes, but I am afraid if we go into this discussion there will not be any time left for this paper, because I do not believe implicitly in the wattmeter readings. I would not accept them without an investigation as to the construction of the wattmeter and the way that it was used; so that I would not pay much attention to that. We all know that wattmeter readings are not reliable when we have harmonics in the circuit; that special precautions must be taken to ensure the accuracy of the readings. Now whether in these wattmeter readings to which Mr. Steinmetz referred, those precautions were taken or not I do not know. I do not think they were.

MR. STEINMETZ:—I do not quite agree that wattmeter readings are not reliable. It is true that theoretically an error is introduced by the inductivity of the potential circuit, and in scientific investigations of extreme accuracy this error would undoubtedly be serious. In practice, however, where as a rule no attempt is made to get a higher accuracy than one-fourth per cent., this wattmeter error is generally negligible.

Sometime ago I made an investigation in this respect and found that with certain commercial voltmeters the inductance error is still below one per cent. at 1500 cycles. The wattmeter error is of the same magnitude for non-inductive load, and increases with the increasing phase displacement of the main current, but becomes noticeable only at very low power factors, below 20%. The exciting current or open circuit current of a transformer, however, has quite a high power factor, from 50 to 65%, and in this case the wattmeter error due to the fundamental wave, is still negligible, that due to the higher harmonic small, and since the power of the higher harmonics is a small part of the total power, the maximum possible error due to the wattmeter is of far lower magnitude than the difference of hysteresis loss observed with different wave shapes.

There being no further discussion, the following paper entitled "Notes on Single-Phase Induction Motors and the Self-Starting Condensing Motor," was read by Mr. Steinmetz.